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Recent results on the strong coupling constant from the DELPHI experiment at LEP

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Abstract

New results on the determination of the strong coupling constant from the DELPHI experiment are presented. They include measurements from global event shapes, scaling violations, the Z^0 line shape and the ratio of hadronic to leptonic widths of τ decays. The strong coupling constant has also been measured separately for b quarks enabling the predicted flavour independence of α_s to be examined.

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RECENT RESULTS ON THE STRONG COUPLING CONSTANT FROM THE DELPHI EXPERIMENT AT LEP

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ABSTRACT

New results on the determination of the strong coupling constant from the DELPHI experiment are presented. They include measurements from global event shapes, scaling violations, the Z^0 line shape and the ratio of hadronic to leptonic widths of τ decays. The strong coupling constant has also been measured separately for b quarks enabling the predicted flavour independence of α_s to be examined.

1. Introduction

The strong coupling constant is *the* fundamental parameter of QCD, and its determination is consequently of tremendous experimental endeavour, encompassing several renewed efforts as the theoretical understanding advances. Here, the most recent developments from the DELPHI experiment at LEP are presented and discussed. They include α_s measurements from:

- topological studies of hadronic events based on QCD calculations including leading and next-to-leading logarithmic contributions,
- scaling violations in the charged particle momentum fraction distributions,
- the Z^0 line shape,
- the ratio of the hadronic and electronic branching fractions of the τ lepton,
- an analysis of inclusive lepton events initiated by b quarks.

2. α_s from Hadron Distributions

One of the most precise measurements of α_s to date has been obtained from analyses of event shape distributions of hadronic final states of Z^0 decays at LEP ($Q \approx M_Z$) [1]. Originally, the method involved fitting the experimental data to the theoretical expectations of the different observables calculated to $O(\alpha_s^2)$ [2], in regions where the contribution from three-jet events is significant. Within this framework,

however, there exists a large variation of the value of $\alpha_s(M_Z)$ as a function of the renormalization scale parameter, μ . This unphysical dependence on the scale is due to terms of higher than second order contribution to the observable, which are missing in the perturbative calculation. Since the QCD theory does not specify the choice of scale, several different criteria have been applied in an attempt to justify the *best* scale value. These suggest that a selection of the scale other than that of the *natural* value of $\mu^2 = M_Z^2$ may account for the effect of the unknown higher order contributions. Predictions of the optimum scale value are of the order of $\mu \approx M_Z/4$, and such low values are indeed seen to give a better description of the data. Nevertheless, the central value of $\alpha_s(M_Z)$ and its associated ‘scale’ error quoted by experiments typically encompasses all $\alpha_s(M_Z)$ determinations between scale values corresponding to the b quark mass and the Z^0 mass.

Clearly, the problem would be largely overcome with the availability of the $O(\alpha_s^3)$ calculations, but unfortunately these are not yet within immediate reach. Rather, recent theoretical activity has concentrated on the resummung of the leading logarithmic (LL) and next to leading logarithmic (NLL) contributions in the perturbative series to all orders of α_s , for certain specific event shape variables [3]. These are thrust ($1 - T$), heavy jet mass ($\rho^{(M,T)}$), back to back energy energy correlations ($EEC(\chi)$) and, more recently, jet broadening (B). The resulting expressions are thus expected to give a good description of the data in the two-jet back-to-back region where these LL and NLL terms dominate. Furthermore, it is expected that in a scheme where the exact $O(\alpha_s^2)$ calculation is combined with the resummed LL and NLL expressions, the fit range considered could be extended and the scale dependence further reduced [4, 5].

The resummed calculation is performed in terms of the variable R which is defined as the number of events not exceeding a determined cut-off, y , which symbolizes any of the above mentioned observables [3]:

$$R(y, \alpha_s(Q^2)) = \frac{1}{\sigma_{tot}} \sigma(\eta < y) . \quad (1)$$

The expression is then parametrized as follows:

$$R(y, \alpha_s(Q^2)) = C(\alpha_s(Q^2)) \Sigma(y, \alpha_s(Q^2)) + F(y, \alpha_s(Q^2)) \quad (2)$$

where,

$$\begin{aligned} C(\alpha_s(Q^2)) &= 1 + \sum_{n=1}^{\infty} \alpha_s^n(Q^2) C_n \\ \ln(c \Sigma(y, \alpha_s(Q^2))) &= \sum_{n=1}^{\infty} \alpha_s^n(Q^2) \sum_{m=1}^{n+1} G_{nm} \ln^m y \\ F(y, \alpha_s(Q^2)) &= \sum_{n=1}^{\infty} \alpha_s^n(Q^2) F_n(y) \end{aligned} \quad (3)$$

Here C_n and G_{nm} are constants while $F_n(y)$ are functions which vanish at small y . The parameter c is unity for all variables apart from $EEC(\chi)$ where it equals 2. As can

easily be derived from Eq. (3), the function $\ln R(y, \alpha_s(Q^2))$ involves terms satisfying the condition $m \leq n + 1$ while the series expansion of $R(y, \alpha_s(Q^2))$ contains all terms fulfilling $m \leq 2n$.

The evaluation of $R(y, \alpha_s(Q^2))$ for the different variables contains some important features which are well worth pointing out explicitly in order to gain a proper understanding of the calculation:

- The exponential component of Eq. (3), i.e. Σ , is calculated for small y where the equality $R(y, \alpha_s) = \Sigma(y, \alpha_s)$ holds. The results obtained are thus valid for the back-to-back jets region ($y \ll 1$).
- The calculation is not exact to all orders and for this reason there is a remaining dependence on the renormalization scale, μ , which materializes through the exchange $\alpha_s(Q^2) \rightarrow \alpha_s(\mu^2)$ and through the introduction of terms of the form $\ln(Q^2/\mu^2)$. It is noted, however, that for small scale values, these logarithmic terms lead to large contributions which may not have been properly accounted for in the QCD calculation of $R(y, \alpha_s(Q^2))$. The procedure of scale optimization, unlike in the case of the $O(\alpha_s^2)$ expression, is therefore not recommended.
- All leading (*LL*) and next-to-leading (*NLL*) terms, i.e. those with $m \geq n$, are resummed to all orders. Since the results for R are fully known to $O(\alpha_s^2)$, the sub-dominant terms ($m < n$) as well as all the constants C_n and functions $F_n(y)$ can then be computed, up to second order, by comparing the fixed order calculation with the resummed expressions.

While the above points offer a new approach to the analysis of the data, they also lead to new ambiguities which need to be dealt with:

- The resummed expression, Σ , which appears in R as an exponential, cannot be directly identified with the full $O(\alpha_s^2)$ formula. Different results are then obtained when combining the two expressions, depending on whether the $\ln R$ of the exact $O(\alpha_s^2)$ terms, or Σ , are expanded to enable the matching. Furthermore, there are also ambiguities when expanding the series in Σ . Here, either the sub-leading coefficient G_{21} is included in the exponential and then later subtracted from the full $O(\alpha_s^2)$ expression, or alternatively, it is simply neglected. The different recombinations schemes are referred to in the trade as $\ln cR$, $cR - G_{21}$ and cR respectively, and lead to the so called *matching ambiguities* as they introduce differences proportional to the sub-leading terms, $O(\alpha_s^3 \ln^2 y)$.
- The validity of the calculation largely depends on the *relative* weight of the sub-leading terms with respect to the remainder. Indeed only for a limited region of $y \leq 0.03$ are the sub-leading logarithms, $O(\alpha_s^n \ln^m y)$, $m < n$, below 1%. Thereafter they increase dramatically with increasing y and can already be of importance for not too large y values. For instance, in the region $0.03 \leq y \leq 0.3$ the following ratios are found when comparing the sub-leading terms of third

order:

$$\frac{\sum_{m=1}^{m=2} O(\alpha_s^3 \ln^m y)}{\sum_{m=3}^{m=6} O(\alpha_s^3 \ln^m y)} \equiv 1\% - 20\% \quad \frac{\sum_{m=1}^{m=2} O(\alpha_s^3 \ln^m y)}{\sum_{m=4}^{m=8} O(\alpha_s^4 \ln^m y)} \equiv 1\% - 200\% \quad (4)$$

The explicit values of the constants G_{nm} accompanying the logarithms therefore play a prominent role in defining the y interval in which the data can be reliably fitted, or whether a specific distribution can even be considered at all.

The DELPHI analysis [4] uses two different methods whereby the data is fitted in two well separated variable regions, thereby testing two different theoretical approaches [6].

1. Since for small y the LL and NLL terms dominate over those corresponding to the second order calculation and the sub-leading logarithms, the latter contribution can be safely neglected. Within this region of small y the resummed calculation constitutes an expression which is valid to all orders and can therefore be employed in its own right to compute α_s . Unfortunately this method does not work for all distributions; the $EEC(\chi)$ distribution, for example, tends to infinity for small y , and within the accessible y -range the sensitivity to the LL and NLL terms is not sufficiently large.
2. In the intermediate y region, the combined second order and LL and NLL result (where the theory essentially becomes an approximation at third order) can be applied. Here the *matching ambiguities* need to be taken into account.

The DELPHI data corresponds to 180,000 multihadron events (after all selection cuts) collected during 1991. Both charged and neutral particles have been used to construct the observables with the exception of $EEC(\chi)$ where neutrals have been omitted to avoid large acceptance corrections. The measured distributions were corrected for detector and hadronization effects and initial state photon radiation using Monte Carlo simulated data. The corrected data were then compared to the theoretical expectation. For the purpose of extracting a measurement of α_s , a correlation matrix for all observables was constructed and a simultaneous fit to all the distributions performed. The ranges considered were, in method 1:

$$0.01 \leq 1 - T \leq 0.03, \quad 0.01 \leq \rho^{(M,T)} \leq 0.02$$

and in method 2:

$$0.06 \leq 1 - T \leq 0.30, \quad 0.03 \leq \rho^{(T)} \leq 0.24, \quad 0.02 \leq \rho^{(M)} \leq 0.15, \quad 90^\circ \leq \chi \leq 162^\circ$$

The error on the α_s measurement from the uncertainties in acceptance and selection cuts were studied and quantified (*exp*), as was that due to changes in the hadronization model and parameters tunings (*had*), and the fit range considered. To estimate the theoretical uncertainty in method 2, fits were made over several

values of the renormalization scale, $\mu^2/M_Z^2 = 0.5, \dots, 2.0$, and with the different matching schemes. The various results for $\alpha_s(M_Z)$ obtained were distributed with a certain variance whose value was regarded as an estimate of the error due to the theoretical uncertainties.

The results for $\alpha_s(M_Z)$ and its associated error, from method 1, as determined at the scale $\mu^2 = M_Z^2$, is:

$$\alpha_s(M_Z) = 0.121 \pm 0.003(\text{exp}) \pm 0.003(\text{had}) .$$

The variation of $\alpha_s(M_Z)$ as a function of the scale for the different observables, as determined from method 2, is shown in Fig. 1. For thrust like quantities, i.e. $1 - T$ and $\rho^{(M,T)}$, the results of method 1 agree with those of method 2 when using the $\text{ln}cR$ or $cR - G_{21}$ combination schemes. Only the results from the $\text{ln}cR$ scheme for these distributions are therefore shown in Fig. 1. In the case of the $EEC(\chi)$ distribution however, as method 1 is not applicable, both $\text{ln}cR$ and cR matchings have been applied to determine the central value. Taking into account the correlation between these variables, and adopting the measurement specifically obtained at the scale of $\mu^2 = M_Z^2$, the average result quoted is:

$$\alpha_s(M_Z) = 0.123 \pm 0.002(\text{exp}) \pm 0.002(\text{had}) \pm 0.002(\text{fit range})^{+0.004}_{-0.003}(\text{scale} + \text{matching ambiguity})$$

$$\alpha_s(M_Z) = 0.123^{+0.006}_{-0.005}$$

(preliminary) .

In method 2 the variation of $\alpha_s(M_Z)$ with the scale exhibits a greater stability when compared with the equivalent determination using the fixed second order calculation [7]. The error due to the scale is thus reduced, though once again there is no concrete formalism to constrain the extent of its uncertainty. The data in any case prefer scale values close to the natural choice. Another point of note is that the value of $\alpha_s(M_Z)$ quoted is about 9% larger than that obtained from the second order analysis, although this former determination corresponds to a value obtained at a much smaller scale.

The advantage of Method 1, as already discussed, is that it compares certain distributions with the QCD expectation as constructed from all orders in $\alpha_s(M_Z)$. This certainly looks an attractive proposition as it induces small theoretical errors. The disadvantage however is the large hadronization correction factors associated with small y values, which might lessen the credibility of the result. Method 2 on the other hand, despite having a larger intrinsic theoretical uncertainty, uses a fit region which is better understood and where hadronic corrections are under control. The two methods thus complement one another, and it is indeed gratifying that they lead to similar results.

Figure 1: $\alpha_s(M_Z)$ as a function of the scale μ^2/M_Z^2 as derived from method 2. For $1 - T$ and $\rho^{(M,T)}$ only the results from the *IncR* matching scheme are shown, while for $EEC(\chi)$, results from both the *IncR* and *cR* schemes are given.

3. α_s from Scaling Violations

The determination of α_s from scaling violations in fragmentation functions is a well established procedure in deep-inelastic scattering experiments [8]. Only recently however, with the vast statistics and bigger lever arm in energy provided by LEP, has it become fruitful to apply this analysis within e^+e^- annihilation [9]. Data collected at PETRA ($Q^2 = 35^2 \text{ GeV}^2$) and LEP ($Q^2 = 91^2 \text{ GeV}^2$) clearly display scaling violation in the corrected charged particle fractional momentum distributions, $x = p_{hadron}/E_{beam}$ (Fig. 2). The source of this scaling violation is attributed mainly to the radiation of gluons from quarks, the phase space for which increases with Q^2 , thereby becoming increasingly inciting for LEP physicists to obtain a measurement for α_s .

The fragmentation functions are directly related to the scaled hadron momentum distributions according to the equation:

$$D(x, Q^2) = \sum_i F_i(Q^2) D_i(x, Q^2) \equiv \frac{1}{\sigma_{tot}} \frac{d\sigma}{dx}(e^+e^- \rightarrow had + X), \quad (5)$$

where $x = 2p_{had}/Q$, $D_i(x, Q^2)$ is the fragmentation function of quark flavour, i , and, F_i , its corresponding weight given by the standard model. The idea is thus to input the fragmentation function at some reference energy scale, Q_0 , and then to evolve to another scale, Q_1 , by varying the QCD parameter, Λ , such that the best match to the data at the Q_1 scale is achieved.

Figure 2: (a) the corrected inclusive momentum distribution at two Q^2 values, and (b) their ratio. The solid curves represent the results of the fit; the dashed line in (b) shows the predicted behaviour assuming no variation in the b fraction with Q^2 .

The practicalities are as follows. The x distribution at the reference energy of $Q_0^2 = 35^2 \text{ GeV}^2$, using data from TASSO, is first parametrized using the successful string Monte Carlo model [10] incorporating the Lund symmetric fragmentation function [11] (with parameters a (free) and b (fixed)) for the light uds quarks, and the functional form of Peterson et al. [12] (with parameters ϵ_b (free) and $\epsilon_c = 9.4\epsilon_b$) for the heavy bc quarks. Other parameters in the Monte Carlo model are left at their default values as these are already known to give a good description at the reference energy. Having parametrized the x dependence at the given reference energy, the Q^2 dependence of the fragmentation function is determined from the integration of the exact second order QCD matrix elements [13] incorporated in the Lund string model [10]. Whilst the absence of the next-to-leading QCD corrections in the evolution equations may remain a point of concern here [14, 15], the use of matrix elements offers the distinct advantage of being able to account for the significant variation in flavour composition over the energy range considered, i.e. $F_i(Q^2)$ in Eq. (5). A first measurement for α_s can then be obtained by fitting the inclusive spectra at $Q_1^2 = 91^2 \text{ GeV}^2$ keeping the parametrization for the x dependence obtained at $Q_0^2 = 35^2 \text{ GeV}^2$. That the same fragmentation parameter values are being used to describe the x dependence over such a large energy range is a direct consequence of deliberately maintaining the same minimum invariant mass between quarks and gluons in the Monte Carlo model at all energy scales. The value of 9.1 GeV was chosen, corresponding to a $y_{min} = m_{ij}^2/Q^2$ of 0.01 and 0.068 at Q^2 of 91^2 GeV^2 and 35^2 GeV^2 respectively. Further progress can, however, be made by leaving the three variables, a , ϵ_b and $\Lambda_{\overline{MS}}$, as free parameters, thereby taking into account the correlations between them, and performing a simultaneous fit to all the

available data from PEP, PETRA, TRISTAN and LEP experiments. The energy range covered is thus $29^2 < Q^2 < 91.2^2 \text{ GeV}^2$, which is somewhat wider than that studied in deep-inelastic scattering. Only the x range between 0.18, below which soft gluons dominate and higher order corrections and infrared divergences need be considered, and 0.8, above which the experimental correction factors become unacceptably large owing to the poorer momentum resolution of the detectors, is considered. Several fits were performed and the result quoted is:

$$\alpha_s(M_Z) = 0.119 \pm 0.006 \quad (\text{preliminary}) ,$$

where the biggest source of error is the uncertainty in the choice of renormalization scale and the dependence on the value of the minimum invariant mass cut-off. These errors were added linearly. Other systematic errors from experimental uncertainties, differences between experiments (unlike deep-inelastic scattering experiments, data used here have been amassed by several different experiments), variation of the x range considered and fragmentation effects were added in quadrature, but proved to be small by comparison. The fit results are displayed as solid lines in Fig. 2.

A couple of important observations are also noted. Firstly, it is evident that the change in shape in the x distribution over the wide range of energies cannot be mimicked merely by the different quark composition. This is typified by the relatively small correlation coefficient between $\Lambda_{\overline{MS}}$ and the fragmentation parameters a ($\rho = 38\%$) and ϵ_b ($\rho = 10\%$). The fraction of beam energy taken by the B mesons at the Z^0 pole was also found to be 0.70 ± 0.01 , in agreement with results obtained from inclusive lepton analyses [16]. Secondly, the uncertainty due to unknown higher order contributions (estimated by varying the renormalization scale) which so plagues α_s determinations from event shapes, is of less significance in this analysis since it is the *difference* between the spectra at difference energies which is being studied, leading to a partial cancellation of the higher order contributions (and fragmentation effects).

This result for α_s is the first such determination from the scaling violation in the fragmentation functions in e^+e^- annihilation. The experimental systematics appear to be understood and under control leading to an overall uncertainty in α_s of 5%, although the effect of not incorporating the next-to-leading QCD corrections to the QCD evolution equations remains to be quantified [14, 15].

4. α_s from the Z^0 Line Shape

The single most attractive point concerning the determination of α_s from the ratio, R_Z , of the hadronic to leptonic Z^0 decay widths is embedded in the fact that the result does not depend on the non-perturbative hadronization effects. By simply counting the number of hadronic and leptonic events, α_s can be determined

from the relation:

$$R_Z \equiv \left(\frac{\Gamma_{had}}{\Gamma_{lep}} \right) = \left(\frac{\Gamma_{had}}{\Gamma_{lep}} \right)_0 \times (1 + \delta_{QCD}) , \quad (6)$$

where the QCD correction δ_{QCD} has been calculated to complete third order [17] and with the inclusion of mass corrections takes on the form [18]:

$$\delta_{QCD} = 1.05 \left(\frac{\alpha_s}{\pi} \right) + 0.9 \left(\frac{\alpha_s}{\pi} \right)^2 - 13 \left(\frac{\alpha_s}{\pi} \right)^3 , \quad (7)$$

and where the standard model prediction for $(\Gamma_{had}/\Gamma_{lep})_0$ of 19.97 ± 0.03 has a slight dependence on the unknown top and Higgs masses.

However, rather than apply the straightforward method of event counting, a more significant α_s measurement can be obtained from a simultaneous fit to both the cross section and electroweak asymmetry data, leaving R_Z as an additional free parameter. The DELPHI result [19] of $R_Z = 20.82 \pm 0.16$, derived in this way from an analysis of 450,000 Z^0 decays collected in 1990 and 1991, corresponds to a value of:

$\alpha_s(M_Z) = 0.129 \pm 0.025$

(DELPHI, preliminary) ,

where the error is dominated by statistics. Of course, better precision is obtained by combining results from all four LEP experiments. This gives [20] $R_Z = 20.85 \pm 0.07$, which yields:

$\alpha_s(M_Z) = 0.130 \pm 0.012$

(OPAL, DELPHI, ALEPH, L3, prelim.) ,

where the error further includes small systematic contributions from the unknown top and Higgs masses and the uncertainty in the QCD mass correction.

The statistical significance of the result can be further improved upon by performing a ‘Minimal Standard Model’ constrained fit to the line shape data leaving only α_s , M_Z and m_t as free parameters, as discussed in [21].

5. α_s from $\Gamma_{\tau \rightarrow had}/\Gamma_{\tau \rightarrow lep}$

As in the previous method, the determination of α_s from the ratio of hadronic to leptonic tau-decay rates, $R_\tau = \Gamma_{\tau \rightarrow had}/\Gamma_{\tau \rightarrow lep}$, does not rely on knowledge of the hadronization process, particularly when expressed in the form $R_\tau = (1 - B_e - B_\mu)/B_e$, where B_e and B_μ refer to the electronic and muonic branching fractions of the tau. Theoretically, R_τ can be parametrized [22] as:

$$R_\tau = 3 (|V_{ud}|^2 + |V_{us}|^2) (1 + \delta_{EW} + \delta_{pert} + \delta_{non-pert}) , \quad (8)$$

where the electroweak and non-perturbative corrections are estimated to be $\delta_{EW} \simeq 0.019$ and $\delta_{non-pert} = -0.007 \pm 0.004$, respectively. The perturbative correction has been

calculated to third order and, at the tau mass scale, has the form:

$$\delta_{pert} = \frac{\alpha_s}{\pi} + 5.202 \left(\frac{\alpha_s}{\pi} \right)^2 + 26.37 \left(\frac{\alpha_s}{\pi} \right)^3 . \quad (9)$$

From an analysis of the leptonic tau decays [23], DELPHI finds [24] $R_\tau = 3.59 \pm 0.15$, leading to a value of $\alpha_s(M_\tau) = 0.338 \pm \begin{smallmatrix} 0.053 \\ 0.074 \end{smallmatrix}$. Extrapolating to the Z^0 mass scale gives:

$\alpha_s(M_Z) = 0.116 \pm \begin{smallmatrix} 0.007 \\ 0.009 \end{smallmatrix}$

 (DELPHI, preliminary) .

Once again, a better precision is attained by combining all the experimental information available from the four LEP experiments. A recent average [24] yields a value for $\alpha_s(M_Z)$ of:

$\alpha_s(M_Z) = 0.121 \pm 0.005$

 (OPAL, DELPHI, ALEPH, L3, prelim.) .

The above result promises to be one of the most accurate determinations of α_s to date, although there is some suggestion [14] that the size of the non-perturbative correction could be more significant than realized. With regards to the perturbative correction, the main theoretical uncertainty is due to the dependence on the renormalization scale, μ . Similarly to the $O(\alpha_s^2)$ analysis of event shapes, this renormalization-scheme dependence can be significantly reduced with the inclusion of recent resummed calculations [25].

6. α_s for b Quarks

The different methods employed in the determination of the strong coupling constant test both the consistency and universality of QCD. Nevertheless, in order to extend the validity of the theory as far as possible, tests probing the predicted flavour independence of the theory also suggest themselves. In this respect, DELPHI has addressed the question of the relative strength of the strong coupling constant for b quarks, from an analysis of lepton candidates in approximately 430,000 hadronic Z^0 decays [26].

The main approach of the analysis is to divide the inclusive lepton data into two- and three (or more)-jet event samples, and to fit the predicted spectra of the different processes yielding leptons to the observed distributions in momentum and transverse momentum space in both samples simultaneously. In this way, the number of m -jet events at *parton* level contributing to the number of n -jet events at *detector* level is unfolded separately for b , c and background events, taking account of detector and hadronization corrections, and the relative difference between the detection efficiencies for leptons in two- and three-jet events. Fig. 3 shows the results of a fit to the 1991 inclusive muon sample. A measurement for the strong

Figure 3: the p and p_T distributions of muon candidates in the 1991 two- and three-jet hadronic data, together with predictions of the fit. In the computation of the p_T , the lepton was excluded in the calculation of the jet direction.

coupling constant for b quarks is then extracted using the complete second order QCD prediction [2] and taking into account the effect of the b quark mass. This latter correction was estimated at $\approx +3\%$ using the calculation of [27] where, albeit, only the photon propagator is considered. Recent calculations [28] which specifically include the contribution from the Z^0 propagator, computed using exact matrix elements with massive quarks, suggest however, that the correction is somewhat larger ($\approx +5\%$). It is interesting to note that such a large depletion in the number of three-jet events for b quarks, could be experimentally measurable, even if, a flavour independent coupling constant is assumed. Using for the moment the prediction of [27] for the mass corrections, the result quoted, at the M_{Z^0} scale is:

$$\alpha_s^b(M_Z) = 0.121 \pm 0.004(stat) \pm 0.003(syst) \pm 0.007(scale) \quad (\text{prelim.}).$$

A comparison with $\alpha_s(M_Z)$ for all flavours as measured from the three-jet fraction of all the hadronic events further allowed the b coupling strength to be expressed in terms relative to that for the $udsc$ quarks. Such a comparison has the advantage of cancelling certain common systematic errors, in particular that due to the scale. The relative coupling strengths are thus computed to be:

$$\frac{\alpha_b(M_Z)}{\alpha_{udsc}^3(M_Z)} = 1.02 \pm 0.04(stat) \pm 0.03(syst) \quad (\text{preliminary}) .$$

7. Summary

The measurements of α_s presented here are summarized in table 1, and are seen to be in good agreement with one another.

The analyses of hadronic event shapes and scaling violations both measure α_s to an accuracy of 5% where the main contribution to the error arises from the uncertainties in higher order corrections. However, it is also acknowledged that the method by which the effect of the unknown higher order corrections is evaluated, may be regarded as ‘arbitrary’ owing to the absence of any concrete formalism that may otherwise allow experimentalists to place constraints acceptable to the community at large. With this in mind, a more conservative approach to the treatment of theoretical errors suggests that the theoretical uncertainty is more of the order of 8% [14]. Further major improvements thus await the availability of complete third order calculations.

The accuracy of $\alpha_s(M_Z)$ from the Z^0 line shape, as determined from DELPHI data alone, currently stands at 20% and is limited by statistics. However, with the combination of measurements from the four LEP experiments, an accuracy comparable to that presently obtained from hadron distributions, will be achieved from a total of $\approx 3 \times 10^6$ Z^0 decays per experiment. More data is eagerly awaited.

The value of α_s from τ decays, as extrapolated to the Z^0 mass scale, is in nice agreement with values obtained from the other analyses presented here, and gives credible support for the running of the strong coupling constant. The result obtained also provides one of the most accurate determinations of α_s , although the size of the non-perturbative corrections remains a subject of debate.

The strong coupling constant has been measured separately for b quarks, and the flavour independence of α_s , as predicted by QCD, verified to within 5%.

Analysis	$\alpha_s(M_Z)$
(1) hadron distributions	$0.123 \pm^{+0.006}_{-0.005}$
(2) scaling violations	0.119 ± 0.006
(3) Z^0 line shape	0.129 ± 0.025
(4) R_τ	$0.116 \pm^{+0.007}_{-0.009}$
(5) α_s for b quarks	0.121 ± 0.009

Table 1: summary of α_s measurements from DELPHI.

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