Color Transparency and Fermi Motion

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Abstract

It is argued that color transparency in quasi elastic scattering of electrons and hadrons on nuclei is possible only due to Fermi-motion. We found a strong dependence of nuclear transparency on Bjorken x in (e,e’p), it is close to the Glauber model expectations at $x > 1$, but increases and even exceeds one at $x < 1$. It is argued that color transparency is accompanied by large longitudinal momentum transfer to nuclear matter during the passage of the small size wave packet.

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The phenomenon of color transparency (CT) provides an unique test of the dynamical role of color in QCD. The screening of the intrinsic color of colorless hadrons leads to a weak nonexponential attenuation of high energy hadrons in nuclear matter \[1\], filtering of transverse sizes through nuclei \[2\] and to the vanishing of initial and final state interactions of hadrons participating in a hard reaction on nuclei \[3, 4\].

The strength of attenuation of hadrons in nuclei can be judged by measuring the A dependence of cross-sections for nuclear reactions. Data are usually presented in the form of a ratio \(T_r = \sigma^A / (A\sigma^N)\) called nuclear transparency. Here \(\sigma^{A,N}\) are cross sections of a reaction on a nucleus and on a free nucleon respectively, with the same kinematics in each case.

Quantum-mechanical interference effects violate many of the expectations of the classical approach. It is demonstrated in ref. \[5, 6\] that nuclear transparency, \(T_r\), can exceed one. So it doesn’t have a meaning of transparency of the nucleus. It also leads to an enhancement of polarization effects\[7\], an obvious indication of interference. Contrary to naive expectations attenuation of photoproduction of \(\psi'\) charmonium is much weaker then of \(J/\psi\) which has a smaller radius\[3\].

The purpose of the present paper is to develop the quantum mechanical treatment of CT on, for example, the \(A(e,e'p)A'\) reaction. We find dramatic effects. Nuclear transparency is strongly dependent on the Bjorken variable \(x_B\) or equivalently the lost momentum of the struck nucleon. At \(x_B > 1\) CT effects are negligibly small. The main effects are localized at \(x_B < 1\), and should be easily observed at \(Q^2 > 10 GeV^2\). At lower \(Q^2\) CT effects are predicted to be so weak that their experimental detection is doubtful.

According to wide spread folklore interaction of small size objects disappears, due to the effect of color screening. However literally speaking this is not true. We argue here that such objects strongly interact and transfer considerable longitudinal momentum to the nuclear matter, only the net attenuation of these objects vanishes.

CT, as first stated in the paper ref. \[1\], is a particular case of Gribov’s inelastic shadowing \[8\]. In hadronic state representation diagonal and off diagonal diffractive am-
plitudes cancel and attenuation vanishes. The possibility of such fine tuning of diffractive amplitudes was considered earlier [9, 10]. This was also realized later in a ref. [11, 12]. Evolution can also be calculated in the quark basis and a path integral method was developed in [5, 6].

In this note we investigate the role of Fermi motion and longitudinal momentum transfer in nuclear transparency. Let us consider elastic ep scattering with high $Q^2$. We will consider the imaginary part of the amplitude corresponding to all intermediate particles being on mass shell. According to expectations of PQCD average size of the ejectile is small and it has to demonstrate CT. To have CT different intermediate states have to cancel each other. However at fixed $x_B = 1$ only the proton can be produced on shell if the target proton is at rest. Thus no cancellation is possible, i.e. no CT. However in quantum mechanics how the size is measured must be defined. To observe CT the size detector (second scattering center) must be put at a short distance from the target proton. Then the target proton can not be treated as at rest, it has uncertainty in momentum. As a result, some heavier states can be produced. The closer the target is to the size detector the more states are produced, the complete is the cancellation.

Thus the Fermi motion of the bound nucleon is a source of CT in quasielastic scattering. On the other hand it restricts the amount of CT. At $x_B = Q^2/(2m_p\nu) = 1$, to produce an excited state of mass $m^*$ in the final state, the target nucleon must preferentially have an initial momentum in the direction opposite to the photon, $k_p < -(m^{*2} + m_p^2)/2\nu$. So the mass spectrum of produced states is restricted by:

$$m^{*2} < m_p^2 + 2\nu k_F. \quad (1)$$

Consequently even if the ejectile has a very small size, $\rho^2 \approx 1/Q^2$, the nucleus, as a quantum size detector, is insensitive to such small sizes. It can resolve only sizes larger than:

$$\rho^2 \geq \frac{m_p}{k_F} \frac{1}{Q^2}. \quad (2)$$

We see that the $(e,e'p)$ and $(p,2p)$ reactions have poor kinematics for CT studies. One
has to increase $Q^2$ which decreases the cross-section while one just wants to increase the laboratory frame momentum of the final particle.

Nevertheless the sensitivity of the nucleus to the size of the ejectile can be enhanced, effectively broadening the mass spectrum of produced particles. This is done by decreasing $x_B$ to about $1 - k_F/m_p$. Then all Fermi momenta are used, the range of $m^2 - m_p^2$ is doubled and the nucleus can analyze $\rho^2$ a factor of two smaller.

For numerical estimates we need some reasonable approximations. We replace the full expansion of the hadron wave function over states with a fixed transverse size $\rho$ by the two-component approximation,

$$ |h\rangle \approx \alpha |0\rangle + \sqrt{1-\alpha^2} |1\rangle \tag{3} $$

This model, as well as the eigen state method was first proposed for calculations of inelastic correction in [9, 10]. The two component method has more recently been used by in ref. [11, 12].

The state $|0\rangle$ in eq. 3 effectively includes all states $|\rho\rangle$ with small attenuation in the nucleus, ie. $\text{Im} f(\rho) \ll 1/\rho_A R_A$ where $f(\rho)$ is the eigenvalue of the scattering amplitude, $\rho_A$ and $R_A$ are the nuclear density and radius respectively. We consider the state $|0\rangle$ as non-interactive, ie $f_0 = 0$. The corresponding effective amplitude $\text{Im} f_1 = \sigma_{\text{tot}}^{hN}/2(1-\alpha^2)$.

The only unknown parameter, $\alpha$, can be fixed, using the forward diffractive dissociation cross-section, $\sigma_{\text{dd}}^{hN}$ and elastic cross-section $\sigma_{\text{el}}^{hN}$ [3]:

$$ \frac{\sigma_{\text{dd}}}{\sigma_{\text{el}}} = \frac{\alpha^2}{1-\alpha^2}. \tag{4} $$

Here the eigenvalues of the scattering amplitude are averaged over the eigenstates of the interaction. The value of the ratio in eq. 4 is known [13] to be about 0.1 for pp scattering, so we use $\alpha^2 = 0.1$. The value of 0.5 was used in ref. [12]. The results are quite insensitive to the exact value of $\alpha^2$.

The next step is to assume that only the small size state is produced in the $(e,e'p)$ reaction. The Fermi motion provides different weight factors for different hadronic states,
depending on their mass. Instead of the pure initial, produced on a a free proton, 
\[ |i⟩ = |0⟩ = α|p⟩ + \sqrt{1-α^2}|p^*⟩ \] we get:
\[ |\tilde{i}⟩ = α|p⟩ + \sqrt{1-α^2}|p^*⟩ \sqrt{\frac{W_A(x_B - Δx_B)}{W_A(x_B)}}. \] (5)

Here \(|p^*⟩\) is a state of mass \(m^*\), which takes effectively into account all the diffractive excitation of the proton. The hadronic basis \(|p⟩, |p^*⟩\) are eigenstates of the free Hamiltonian, \(W_A(x_B)\) is the probability for a struck proton to carry a momentum fraction, \(x_B/A\), of the nucleus momentum in the infinite momentum frame. The heavier state \(|p^*⟩\) is produced with a shifted value of \(x_B\):
\[ Δx_B = \frac{m^{*2} - m_p^2}{2m_pν}. \] (6)

Thus the \(|p⟩\) and \(|p^*⟩\) get different weight factors in eq. (5). The dominant resonances produced in the diffractive dissociation are the \(N^*(1440)\), the \(N^*(1520)\) and the \(N^*(1680)\). However the \(N^*(1440)\) is strongly suppressed in electroproduction with high \(Q^2\). So we fixed \(m^* = 1.6GeV\).

The next step is to study the evolution of the wave packet with initial wave function, eq. (3) as it propagates through the nucleus. This can be done with the equation obtained in [10]:
\[ i \frac{d}{dz} |P⟩ = \hat{U} |P⟩ \] (7)
where \(|P⟩\) denotes the set of states \(|p⟩\) and \(|p^*⟩\). The evolution operator \(\hat{U}\) has the form [10]:
\[ \hat{U} = \begin{pmatrix} p - iσ_{tot}/2ρ_A(z) & \frac{α}{\sqrt{1-α^2}} iσ_{tot}^pρ_A(z) \\ \frac{α}{\sqrt{1-α^2}} iσ_{tot}^pρ_A(z) & p - Δp - iσ_{tot}^p/2ρ_A(z) \end{pmatrix} \] (8)

Here \(z\) is the longitudinal coordinate along the proton trajectory, \(p\) is the proton momentum and \(Δp = Δx_Bm_p\).

We performed numerical calculations for the \(^{56}\text{Fe}\) nucleus with the Woods Saxon density distribution [15]. The Bjorken variable \(x_B\) is related to the Fermi momentum of the proton by:
\[ x_B ≈ 1 - \frac{k_z}{m_p} \] (9)
where the positive direction of $k_z$ corresponds with the direction of the photon. This expression is valid at small $|k| \leq k_F$, where $k_F$ is the average Fermi momentum. We have used a Gaussian parameterization of the Fermi momentum distribution:

$$W_A(k) = \frac{3}{2\pi k_F^2} \exp\left(-\frac{3k^2}{2k_F^2}\right). \quad (10)$$

We have calculated the nuclear transparency as a function of $x_B$ at fixed $Q^2$. It is defined as

$$Tr = \sum_\alpha \int d^2b \int_{-\infty}^{\infty} dz_1 e^{ik_z z_1} \psi_\alpha^\dagger(b, z_1) \langle p|\hat{V}(z_1, \infty)|i\rangle \int_{-\infty}^{\infty} dz_2 e^{-ik_z z_2} \psi_\alpha^\dagger(b, z_2)^* \langle p|\hat{V}(z_2, \infty)|i\rangle^* \quad (11)$$

where $k$ is the missing momentum in the (e,e$'p$) reaction and $\psi_\alpha^\dagger$ is the nuclear wave function in the shell $\alpha$. $\hat{V}(z, z')|i\rangle \exp(ik_z(z - z'))$ is a solution of eq. (7) at the point $z'$ with the initial state $|i\rangle$ at the point $z$. $W_A(k_z) = \int d^2k_T W_A(\vec{k})$. The nuclear density matrix $\rho(\vec{r_1}, \vec{r_2}) = \sum_\alpha \psi_\alpha(\vec{r_1})\psi^*(\vec{r_2})$ is connected with the Fermi momentum distribution by a Fourier transformation: $W_A(k) = \int d^3r_1 d^3r_2 \exp(i\vec{k} \cdot (\vec{r_1} - \vec{r_2})) \rho(\vec{r_1}, \vec{r_2})$, so the correlation length of $\rho(r_1, r_2)$, $(\langle r_1 - r_2 \rangle^2)^{1/2} \approx \frac{6}{k_F} \approx 2fm$. This correlation length satisfies the conditions, $m_p \Delta z \Delta x_B/2 \ll 1$ at energies above a few GeV, and $\sigma_{tot}\rho_A(\vec{r})\Delta z/4 \ll 1$. Therefore eq. (11) can be modified in the first order on $\Delta z$ in the form,

$$Tr(x_B) = \int d^2b \int_{-\infty}^{\infty} dz \rho_A(b, z) \langle p|\hat{V}(z, \infty)|i\rangle^2 \quad (12)$$

The evolution is calculated along the $z$-axis starting from the point $(b, z)$. The results are shown in fig.1 as a function of $x_B$ for $Q^2$ equal to 7 GeV$^2$, 15 GeV$^2$ and 30 GeV$^2$. At $x_B > 1$ the nuclear transparency is small and close to the expectation of the Glauber model. The reason is obvious: at $x_B > 1$ the quasi elastic scattering takes place on protons having large negative Fermi momenta. To make the nucleus transparent excited states must be produced as well. The latter however prefer higher Fermi momenta which are suppressed. So at large $x_B$ predominately protons are produced.

At $x_B < 1$, on the contrary excited states are preferentially produced; to the extent that the transparency even becomes larger than 1 for small $x_B$. This is understood as
follows: at small \( x_B < 1 \), or equivalently large positive \( k_z > k_F \), direct proton production is strongly suppressed by the nuclear wave function. At the same time the production of higher states needs \( k_z \) smaller; they are suppressed much less or even enhanced. The excited state can convert to a proton during its propagation through the nucleus. As a result the value of \( \text{Tr}(x_B) \) can be larger than one.

A measurement of the \( x_B \) dependence of nuclear transparency would be the best test of CT, however that might be experimentally difficult. Instead we can simply divide all events into two samples; one with \( x_B > 1 \) and the other with \( x_B < 1 \). The CT effects are expected to be quite different in these two samples with a much larger effect seen for \( x_B < 1 \). Results of such a calculation are shown in fig. 2 as a function of \( Q^2 \). The curve corresponding to \( x_B > 1 \) does not appreciably deviate from the Glauber approximation. The curve corresponding to \( x_B < 1 \), on the other hand increases steeply as a function of \( Q^2 \).

Let us briefly list the main observations of this paper:

i) CT is a result of strong cancellations of the amplitudes for the production of the proton and the production of excited states. This cancellation leads to a high nuclear transparency, but does not mean the absence of interaction. Production of different intermediate states use different Fermi momenta of the target proton and different momenta returned to the nucleus during evolution of these states through the nucleus. So naive expectations that CT, as a suppression of final(initial) interactions, provides a good method to measure Fermi momentum distribution is wrong — the amount of transparency depends strongly on the Fermi momentum of the struck particle. There is an, in principle, uncertainty in the initial momentum of struck proton of the order of \( (m^* - m_p^2)/2\nu \), which is essential even at high energies, as the effective value of \( m^* \) increases with energy.

ii) The Fermi momentum distribution is essential for the phenomenon of CT, it gives the possibility of simultaneous production of states of different masses. The Fermi momentum distribution modifies the relative weights of different hadronic states in the produced wave packet. As a result the fine tuning needed for CT is destroyed. It can be
iii) The distortion of the tuning of the relative contributions of different states, imposed by the Fermi momentum distribution, depends strongly on $x_B$. It is most important at large $x_B > 1$, where CT disappears. The weakest distortion of the tuning takes place at $x_B \approx 1 - k_F/m_p$. At smaller $x_B$ the tuning is also violated, but the result is opposite: the nuclear transparency increases and exceeds one.

iv) It is very difficult to observe CT effects below $Q^2 < 7 \, \text{Gev}^2$. This includes the region explored by the recent experiment at SLAC[16] and the region that can be explored by CEBAF. On the other hand considerable effects are predicted at $Q^2 > 10 \, \text{GeV}^2$.

v) The calculations here have mostly a demonstrative character. The mass distributions are different in deep inelastic scattering and diffractive dissociation, with different centers of gravity. The two channel models unable to incorporate this effect. In addition $m^{*2}$ should depend on the energy of the interactions. More realistic Fermi momentum distributions should be used taking into account the nuclear shell distributions and non-diagonal effects. Never-the-less the present calculation are indicative of the results that would be obtained with a better calculation.

vi) All the above statements are qualitatively valid for the wide angle quasi elastic $(p, 2p)$ scattering. Some of the results of the experiment performed at BNL[17], that seemed puzzling, now become clearer.

Measurements[17] were done at three incident energies. The kinematics of the experiment was then used to reconstruct the initial Fermi momentum of the target proton (neglecting binding energy and final/initial state interactions). At each beam energy all events were spread over bins corresponding to different Fermi momenta, i.e. different energies in the cm. frame. Usually theoretical predictions are compared with the energy dependence of these points. However transparency depends on the energy of the particles in the lab frame, rather than on the cm. energy. For fixed beam energy higher $s$ corresponds to higher Fermi momenta in the direction opposed to the incoming proton. Taking into account the results of the present paper, we expect decreasing CT with increasing
s for fixed incident energy. This effect is weak at 6 GeV/c due to the strong mixing of eigenstates, and the points do not demonstrate strong energy dependence. At beam momentum of 12 GeV/c the expected decrease of nuclear transparency is considerable, of the order of that observed\cite{17}. Thus the points, that were considered as contradicting the expected energy behavior now confirm the CT phenomenon more then other points. There are some problems with the explanation of the data corresponding to the beam momentum of 10 GeV/c. However they originate from only one point which shows too high a transparency.

vii) As we stressed before the quasi elastic electron and proton scattering have poor kinematics — too low energy, $\nu$, of the recoil proton in comparison with the square of the momentum transfer, $Q^2$. What processes are better? First, the quasielastic $A(p, 2p)A'$ reaction is better with asymmetrical geometry, with $Q^2$ and consequently the energy of the recoil proton fixed. The size of the recoil particle then does not depend on the incident energy as well so all the incident energy dependence comes from the time evolution of the proton wave function. Thus we can increase the incident momentum without decreasing the cross section dramatically.

Acknowledgment: We would like to thank S.J. Brodsky, O. Haüsser, W.B. Lorenzon, G.A. Miller and N.N. Nikolaev for interest in the work and useful discussions. B.Z.K thanks TRIUMF for its hospitality during his visit and B.K.J thanks the Natural Sciences and Engineering Research Council of Canada for financial support.

References


Fig. 1. The nuclear transparency as a function of $x_B$. The curves correspond to $Q^2$ of 7 GeV$^2$ (long dashed curve), 15 GeV$^2$ (solid curve) and 30 GeV$^2$ (short dashed curve). The dash-dotted curve is the Glauber model.

Fig. 2. The nuclear transparency as a function of $Q^2$. The solid curve is for $x_B > 1$ while the dashed curve is for $x_B < 1$. The dash-dotted curve is the Glauber model.
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