CM-P00057471

Archives

CERN-TH.6639/92
MPG-VT-UR 05/92

Quark exchange contribution to the effective meson-meson interaction potential

D. Blaschke *

Theory Division, CERN, CH-1211 Geneva 23, Switzerland.

G. Röpke

MPG AG "Vielteilchensysteme"

Universität Rostock, O-2500 Rostock, Germany.

Abstract

A many-quark system with confining interactions (strings) within colourless $q\bar{q}$ pairs is considered in the low-density limit. From a diagrammatic representation of the four quark ($q^2\bar{q}^2$) propagator, an effective meson-meson interaction is derived. This repulsive hadronic interaction is a consequence of the Pauli blocking on the quark level and can be interpreted as a flip of pairing. The example of elastic scattering of s-wave mesons ($\pi\pi$) is presented. Choosing the meson wave functions and the phenomenological $q\bar{q}$ interaction as Gaussian functions, a separable meson-meson interaction potential is obtained.

CERN-TH.6639/92
MPG-VT-UR 05/92
October 1992

*Permanent address: Universität Rostock, O-2500 Rostock, Germany.

The description of hadrons and their interactions at finite temperatures and densities is a lively and debated field of current research. Predictions for detectable consequences of the modification of hadron properties in dense matter may be tested in experiments with ultrarelativistic heavy ion beams at CERN and Brookhaven and are therefore subject to many recent studies [1].

Besides studies within QCD, which are based on lattice gauge simulations [2], there have been effective approaches developed to account for bound-state formation in quark matter such as the Nambu–Jona-Lasinio model [3]. However, the problem of hadron-hadron scattering in hadronic matter could not yet be handled within these models.

Another, rather successful type of approaches to the quark substructure of hadrons is based on the non-relativistic quark potential models [4]. These approaches account for the confinement of quarks by attributing a phenomenological confining force to the string (flux tube) that joins a quark and antiquark in a meson or three quarks in a baryon. Within these approaches, not only single hadron properties obtained, but there are also some attempts to treat the two-hadron scattering process [5]-[10]. Of particular interest is the generalization of these models to the case of a many-quark system. In [11, 12], a confinement potential approach to a many-quark system has been formulated, where the string interaction is saturated within colour neutral clusters of nearest neighbours quarks¹.

Within this model, we derive the meson-meson interaction using a diagrammatic approach to the quark exchange process in mesonic matter. The first Born approximation coincides with a result recently obtained in Ref. [14]. In contrast to these authors, we give a formulation, within the thermodynamic Green function approach, which has the advantage of being applicable to quark matter at finite temperatures and densities, see also [12]. The formation of mesonic bound states of quarks is obtained from the two-particle Green function describing the propagation of a quark-antiquark pair.

We start with the single-quark Green function

$$G(1, z) = (z - E_1)^{-1}, \quad (1)$$

where $1 = \nu_1, p_1$ describes the spin-flavour-colour (SFC) degrees of freedom ν_1 and the momentum p_1 of the quark 1, $E_1 = m + p_1^2/(2m)$. The two-particle Green function is given in the low-density limit by the ladder approximation:

$$G_2^L(12, 1'2', \Omega_2) = G_2^0(12, \Omega_2) \left[\delta_{11'} \delta_{22'} + \sum_{1''2''} V(12, 1''2'') G_2^L(1''2'', 1'2', \Omega_2) \right], \quad (2)$$

¹The most likely string configuration which is given by the minimum potential energy can also be found numerically by solving an optimization problem [13].

with the free two-quark Green function

$$\frac{G_2^0(1, 2, \Omega_2)}{G_2^0(1, z_r, \Omega_2)} = \frac{\sum_{z_r} G(1, z_r) G(2, \Omega_2 - z_r)}{1 - f(E_1) - f(E_2)} = \frac{(3)}{\Omega_2 - E_1 - E_2}.$$

In the following, Fermi functions are neglected in the low-density limit, $1 - f_1 - f_2 \approx 1$.

With the help of the solution of the two-particle Schrödinger equation E_α and the wave function n , we find the bilinear representation for G_L^L , which reads in operator notation

$$G_2^L = \sum_{\alpha} |\alpha\rangle \mathcal{G}_{meson}(\alpha, \Omega_2) \langle \alpha| \quad (4)$$

with

$$\mathcal{G}_{meson}(\alpha, \Omega_2) = (\Omega_2 - E_\alpha)^{-1} \quad (5)$$

being the meson propagator. In general, the summation over α in (4) includes bound states and scattering states. In the low-density/temperature limit considered here, we will restrict ourselves to bound states (hadrons) only, since in that region the contribution of scattering states (quasifree quarks) is negligible, owing to the confining property of the quark interaction. In order to derive the effective hadron-hadron interaction potential $U(\alpha_1\alpha_2, \alpha'_1\alpha'_2)$ arising from the quark substructure of the hadrons, we consider the 4-quark Green function G_4^q , which is equivalent to the 4-quark T-matrix T_4^q by a usual amputation procedure [12, 15]. The upper label L indicates that diagrams of the ladder type have been included. We consider two species quark (q) and antiquark ($\bar{q}$) and an instantaneous interaction $V_{q\bar{q}}$ that is only operative within colour-neutral pairs. The diagrams representing G_4^q have the general structure of the antisymmetrized free 4-quark propagator $G_0^{q,4x}$ and blocks of interactions. For example,

can be represented by the sequence

$G_{2+2}^I G_4^{0-1} G_{2+2}^{II} G_4^{0-1} G_{2+2}^I G_4^{0-1} G_{2+2}^{II} G_4^{0-1} G_{2+2}^I G_4^{ex}$

Here, the amputation of the four-quark propagators (crosses) is described by the inverse of G_4^0 ,
 $(G_4^0)^{-1}(1234, 1'2'3'4', \Omega_4) = (\Omega_4 - E_1 - E_2 - E_3 - E_4)\delta_{11'}\delta_{22'}\delta_{33'}\delta_{44'}$ (6)
in the low-density limit, and

$$G_{2+2}^I(1234, 1'2'3'4', \Omega_4) = \sum_{\alpha_1\alpha_2} \frac{<12|\alpha_1 > <34|\alpha_2 > <\alpha_2|3'4' > <\alpha_1|1'2' >}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} - \frac{\delta_{11'}\delta_{22'}\delta_{33'}\delta_{44'}}{\Omega_4 - E_1 - E_2 - E_3 - E_4}. \quad (7)$$

The alternative 2 + 2 pairing for the interaction corresponds to G_{2+2}^{II} and follows from Eq. (7) by interchanging the variables ($1 \leftrightarrow 3, 1' \leftrightarrow 3'$) on the r.h.s. The subtraction procedure in Eq. (7) guarantees that each "block" contains at least one interaction defining the pairing. The antisymmetrized $G_{4,ss}^0$ is given by

$$G_{4,ss}^0(1234, 1'2'3'4', \Omega_4) = (\Omega_4 - E_1 - E_2 - E_3 - E_4)^{-1} (\delta_{11'}\delta_{22'}\delta_{33'}\delta_{44'} - \delta_{13'}\delta_{22'}\delta_{31'}\delta_{44'} - \delta_{11'}\delta_{23'}\delta_{32'}\delta_{44'} + \delta_{13'}\delta_{24'}\delta_{31'}\delta_{42'}). \quad (8)$$

Each possible diagram of G_4 can be decomposed in a single way into the blocks $G_{4,ss}^0$, G_{2+2}^I and G_{2+2}^{II} , such that

$$G_4^I = G_4^I + G_{2+2}^{II} - G_{4,ss}^0, \quad (9)$$

$$G_4^I = G_{4,ss}^0 + G_{2+2}^I U_{2+2} G_4^I, \quad (10)$$

$$G_4^{II} = G_{4,ss}^0 + G_{2+2}^{II} U_{2+2} G_4^{II}, \quad (11)$$

where

$U_{2+2}(1234, 1'2'3'4', \Omega_4) = -(\Omega_4 - E_1 - E_2 - E_3 - E_4)\delta_{13'}\delta_{22'}\delta_{31'}\delta_{44'}$.
This decomposition of G_4 into blocks of two-particle propagators gives the possibility to identify the effective potential U_{2+2} with the flip of pairing in the four-quark ladder, e.g.,

$$G_{2+2}^I G_{2+2}^{0-1} G_{2+2}^I G_{2+2}^I = G_{2+2}^I U_{2+2} G_{2+2}^I U_{2+2} G_{2+2}^I. \quad (13)$$

By introducing the pair-flip potential (12) in Eqs. (10), (11), these two equations of motion for the propagation of the four-quark state in either of the two possible quark pairings get formally decoupled. Once the asymptotic pairing channel is fixed by, say G_{2+2}^I , the homogeneous wave equation for G_4^I is a Schrödinger equation for a two-meson state with the interaction given by the pair-flip potential (12).

Considering the combination $G_{2+2}^I U_{2+2}$, which is the repeated element of Eq. (10), we have²

$$\begin{aligned} G_{2+2}^I(1234, 1'2'3'4', \Omega_4) U_{2+2}(1''2''3''4'', 1'2'3'4', \Omega_4) = \\ = - \left(\sum_{\alpha_1\alpha_2} \frac{<12|\alpha_1 > <34|\alpha_2 > <\alpha_2|1'4' > <\alpha_1|3'2' >}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} - \frac{\delta_{13'}\delta_{22'}\delta_{31'}\delta_{44'}}{\Omega_4 - E_1 - E_2 - E_3 - E_4} \right) (\Omega_4 - E_1 - E_2 - E_3 - E_4) \\ = - \sum_{\alpha_1\alpha_2} \frac{<12|\alpha_1 > <34|\alpha_2 > <\alpha_2|1'4' > <\alpha_1|3'2' >}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} \times \\ \times \left(V_{44}(1''4'', 1'4')\delta_{2''2'}\delta_{3''3'} + V_{44}(3''2'', 3'2')\delta_{1''1'}\delta_{4''4'} \right). \end{aligned} \quad (14)$$

For the last step, we have used the identity $\Omega_4 - E_1 - E_2 - E_3 - E_4 = (\Omega_4 - E_{\alpha_1} - E_{\alpha_2}) + (E_{\alpha_1} - E_2 - E_3) + (E_{\alpha_2} - E_1 - E_4)$ and the Schrödinger equation $(E_{\alpha_1} - E_1 - E_2)|12 > = \sum_{1'2'} V(12, 1'2')|1'2' >$. The main result of our investigation of the 4-quark ladder propagator G_4^I is the definition of the effective meson-meson interaction potential $U_{meson-meson}$ which follows if one interprets the sequence $G_{2+2}^I U_{2+2} G_{2+2}^I U_{2+2}$ occurring in the iteration of Eqs. (9)-(11) in the meson picture,

$$\begin{aligned} \dots & G_{2+2}^I(1\dots4, 1''\dots4'', \Omega_4) U_{2+2}(1''\dots4'', 1'''\dots4''', \Omega_4) \\ & \times G_{2+2}^I(1'''\dots4''', 1''''\dots4'''', \Omega_4) U_{2+2}(1''''\dots4'''', 1'''\dots4''', \Omega_4) \dots = \\ & = \dots \sum_{\alpha_1, \dots, \alpha_2} \frac{<12|\alpha_1 > <34|\alpha_2 > <\alpha_2|1'4' > <\alpha_1|3'2' >}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}} \times \\ & \times U_{meson-meson}(\alpha_1, \alpha_2; \alpha_1', \alpha_2') G_{2,meson}(\alpha_1', \alpha_2'; \alpha_1', \alpha_2', \Omega_4) \\ & \times <\alpha_1'|1''2'' > <\alpha_2'|3''4'' > \\ & \times \left(V_{44}(1''4'', 1'4')\delta_{2''2'}\delta_{3''3'} + V_{44}(3''2'', 3'2')\delta_{1''1'}\delta_{4''4'} \right) \dots \end{aligned} \quad (15)$$

²Here and in the following we use the convention that doubly occurring variables have to be summed.

where

$$U_{\text{meson-meson}}(\alpha_1, \alpha_2, \alpha'_1, \alpha'_2) = - \sum_{1...4'} < \alpha_1 | 12 > < \alpha_2 | 34 > [V_{\pi\pi}(14, 1'4') \delta_{\pi\pi} \delta_{33'} + V_{\pi\pi}(32, 3'2') \delta_{11} \delta_{44'}] < 1'4' | \alpha'_1 > < 3'2' | \alpha'_2 > \quad (16)$$

and

$$G_{2,\text{meson}}(\alpha_1, \alpha_2, \Omega_4) = \sum_{\Omega_2} G_{\text{meson}}(\alpha_1, \Omega_2) G_{\text{meson}}(\alpha_2, \Omega_4 - \Omega_2) = \frac{1 + g(E_{\alpha_1}) + g(E_{\alpha_2})}{\Omega_4 - E_{\alpha_1} - E_{\alpha_2}}. \quad (17)$$

As before in the case of the two-quark propagator (3), the phase-space occupation effects given by the Bose functions $g(E)$ in the two-meson propagator are neglected in the low-density limit, $1 + g_1 + g_2 \approx 1$. Applying the above procedure to G_{2+2}^H gives the same result for the interaction $\mathcal{U}$.

Since the potential $V_{\pi\pi}$ does only depend on the quark momenta³ the hadron wave function $< 12 | \alpha >$ separates into an orbital and an SFC part according to

$$< 12 | \alpha > = \phi_\alpha(p_1, p_2) \chi_\alpha(\nu_1, \nu_2). \quad (18)$$

Specific expressions for these wave functions are given in textbooks, e.g. [16, 17]. Now, the effective potential $\mathcal{U}$ factorizes into a SFC and an orbital contribution

$$\mathcal{U}(\alpha_1 \alpha_2, \alpha'_1 \alpha'_2) = -C_{\text{SFC}}(n_1 n_2, n'_1 n'_2) \sum_{p_1 \dots p'_4} \left\{ \phi_{\alpha_1}^*(p_1, p_2) \phi_{\alpha_2}^*(p_3, p_4) \left[V_{\pi\pi}(p_1, p_4, p'_1, p'_4) \delta_{p_2, p'_2} \delta_{p_3, p'_3} + V_{\pi\pi}(p_2, p_3, p'_2, p'_3) \delta_{p_1, p'_1} \delta_{p_4, p'_4} \right] \phi_{\alpha'_1}(p'_2, p'_3) \phi_{\alpha'_2}(p'_1, p'_4) \right\}. \quad (19)$$

The matrix element $C_{\text{SFC}}(n_1 n_2, n'_1 n'_2)$ counts the "overlap" of hadrons in the SFC space

$$C_{\text{SFC}}(n_1 n_2, n'_1 n'_2) = \langle \chi_{\alpha_1}(\nu_1 \nu_2) \chi_{\alpha_2}(\nu_3 \nu_4) | \chi_{\alpha'_1}(\nu'_3 \nu'_2) \chi_{\alpha'_2}(\nu'_1 \nu'_4) \rangle. \quad (20)$$

Now, we can take only the ground states $|\alpha > = |0, k >$ and define this interaction as $\mathcal{U}(12, 1'2')$. This neglect of virtual excited states corresponds to the close coupling scheme and is an approximation that can be improved later on by including those states.

An explicit calculation of the "Pauli-potential" $\mathcal{U}$ for the quark-exchange process necessitates the potential $V_{\pi\pi}$ and the hadronic wave function ϕ_α . The general form of a non-relativistic $q\bar{q}$ potential for the description of s-wave mesonic bound states consists of an orbital part (o) and a spin-spin interaction part (ss)

$$V_{\pi\pi} = V_{\pi\pi}^o + V_{\pi\pi}^{ss}. \quad (21)$$

For the orbital part $V_{\pi\pi}^o$ of the potential, we do not take the usual Cornell form [4] but instead we use, for an exploratory calculation, a Gaussian form which has advantages for the analytical treatment. It reads in momentum space

$$V_{\pi\pi}^o(p_1, p_2, p'_1, p'_2) = - \frac{(2\pi)^3}{\Omega_0} V_0(\pi \sigma)^{-3/2} e^{-(p_1 - p'_1)^2 / \sigma} \delta_{p_1 + p_2, p'_1 + p'_2}. \quad (22)$$

³Relativistic corrections according to the Fermi-Breit-Hamiltonian contribute a parametric dependence on the quark spins (see [4]). For the s-wave mesons, the mass difference between spin singlet and triplet states ($\pi - \rho$) is described as hyperfine splitting, see below.

The spin-spin potential is given in momentum space by the Fourier transform of the hyperfine interaction in the Fermi-Breit Hamiltonian, see e.g. [4],

$$V_{\pi\pi}^{ss}(p_1, p_2, p'_1, p'_2) = - \frac{(2\pi)^3}{\Omega_0} \frac{\alpha_s}{3\pi^2 m_q^2} \left(1 - \frac{4}{3} S \right) \delta_{p_1 + p_2, p'_1 + p'_2}, \quad (23)$$

where $S = 0, 1$ is the spin of the $q\bar{q}$ state. It may be treated as a special case of the potential (22) for $\sigma \rightarrow \infty$. We employ here a Gaussian ansatz for the wave function of $q\bar{q}$ s-wave bound states

$$\phi_{\pi}(p_1, p_2) = \frac{(2\pi)^{3/2}}{\sqrt{\Omega_0}} (\pi b^2)^{-3/4} e^{-(p_1 - p_2)^2 / 4b^2} \delta_{p_1 + p_2} \quad (24)$$

with a root mean squared radius $\langle r^2 \rangle^{1/2} = \sqrt{3/2} b^{-1}$. Thanks to the convenient choice of the potential (22) and the wave function (24), the Pauli potential (19) can be evaluated analytically ($x = \sigma/(2b^2)$):

$$\begin{aligned} \mathcal{U}^o(12, 1'2') &= 16 C_{\text{SFC}}(12, 1'2') \frac{(2\pi)^3}{\Omega_0} V_0 (2\pi b^2 (4 + 3x))^{-3/2} \\ &\quad \times \exp \left[-\frac{1}{4b^2} \left(k'^2 + \frac{4+x}{4+3x} k^2 \right) \right] \delta_{K, K'} \\ &= \mathcal{U}^o(k, k') \delta_{K, K'}. \end{aligned} \quad (25)$$

Here, the centre of mass and relative momenta in the two-meson system have been introduced as $K = k_1 + k_2$ and $k = (k_1 - k_2)/2$, resp. (primed momenta accordingly). A similar calculation for $\mathcal{U}^{ss}$ yields in the spin singlet case ($S = 0$)

$$\begin{aligned} \mathcal{U}^{ss}(12, 1'2') &= \frac{16}{3\sqrt{3}} C_{\text{SFC}}(12, 1'2') \frac{(2\pi)^3}{\Omega_0} \frac{\alpha_s}{3\pi^2 m_q^2} \exp \left[-\frac{1}{4b^2} (k'^2 + k^2/3) \right] \delta_{K, K'}, \\ &= \mathcal{U}^{ss}(k, k') \delta_{K, K'}. \end{aligned} \quad (26)$$

This simple result is a consequence of the model for the wave function and the confinement potential. It can be improved by a more adequate form of the quark potential as well as the meson wave function. The coefficient C_{SFC} is obtained from the SFC parts of the mesonic wave functions in a straightforward way. For the example of $\pi\pi$ scattering, these coefficients are in shorthand notation $C_{\pi\pi\pi\pi} = C_{\pi\pi\pi\pi} = 1/6$, $C_{\pi\pi\pi\pi} = C_{\pi\pi\pi\pi} = 1/12$, $C_{\pi\pi\pi\pi} = 0$. Thus, quark-exchange effects in $\pi\pi$ scattering are most prominent in the isospin $= 2$ channel (repulsive δ_2^0 phase shift), whereas they are absent in the annihilation channel (ρ -resonance). Indeed, it has been shown within a Born approximation [14] to the isolated 4-quark propagator, that the $\pi^+\pi^+$ phase shift can be well reproduced assuming only the hyperfine part of the quark exchange potential with a standard r.m.s. radius of the pion wave function $\sqrt{\langle r^2 \rangle} = 0.67$ fm [18].

The derivation of the quark exchange potential for meson-meson scattering in a many-quark system given above allows for a straightforward generalization to the case of finite temperatures and densities. Going beyond the low-density limit one has to keep the occupation numbers occurring in the propagators (3,17) of the one-meson as well as the two-meson states. Furthermore, the inclusion of excited mesonic states as, for instance, the ρ -meson is an interesting future application of the method. Finally, we would like to mention that, in a self-consistent treatment, in addition to the Pauli-blocking terms in the meson propagator, the exchange potential $\mathcal{U}$ gives rise to density-dependent self-energies in the quark propagator. Both these effects may act such that at a critical density (temperature) the mesonic bound-state poles in the two-quark propagator vanish. This phenomenon is known as the Mott effect, and is discussed as one mechanism for quark deconfinement in dense hadronic matter [12, 19]. A more detailed investigation of that issue goes beyond the present work and will be given separately [20].

References

- [1] T. Hatsuda, Nucl. Phys. A544 (1992) 27c.
- [2] D. Toussaint, Nucl. Phys. B (Proc. Suppl.) 26 (1992) 3.
- [3] For recent reviews on that subject see, for instance, S.P. Klevansky, Rev. Mod. Phys. 64 (1992) 649; U. Vogl and W. Weise, Prog. Part. and Nucl. Phys. 27 (1991) 195; T. Hatsuda, Ref. [1].
- [4] W. Lucha, F. Schöberl and D. Gromes, Phys. Rep. 200 (1991) 127.
- [5] M. Oka, Phys. Rev. D31 (1985) 2274; M. Oka and C.J. Horowitz, Phys. Rev. D31 (1985) 2773.
- [6] F. Lenz et al., Ann. Phys. (NY) 170 (1986) 65.
- [7] S. Gardner and E.J. Moniz, Phys. Rev. C36 (1987) 2504; S. Gardner, Phys. Rev. C42 (1990) 2103.
- [8] A.M. Green and G.Q. Liu, Nucl. Phys. A500 (1989) 529.
- [9] C. Alexandrou, T. Karapiperis and O. Morimatsu, Nucl. Phys. A518 (1990) 723.
- [10] R.R. Schröder and H.G. Dosch, Nucl. Phys. A451 (1986) 666.
- [11] D. Blaschke, F. Reinholz, G. Röpke and D. Kremp, Dubna Report No. E2-84-572 (1984), unpublished; Phys. Lett. 151 B (1985) 439.
- [12] G. Röpke, D. Blaschke and H. Schulz, Phys. Rev. D34 (1986) 3499.
- [13] C.J. Horowitz and J. Piekarewicz, Phys. Rev. C44 (1992) 2753; C.J. Horowitz, E. Moniz and J.W. Negele, Phys. Rev. D31 (1985) 1689.
- [14] T. Barnes and E.S. Swanson, Phys. Rev. D46 (1992) 131.
- [15] W.-D. Kraeft, D. Kremp, W. Ebeling and G. Röpke, Quantum statistics of charged particle systems (Plenum, New York, 1986).
- [16] F.E. Close, An Introduction to Quarks and Partons (Academic Press, London, 1979).
- [17] A. LeYaouanc, L. Oliver, O. Pène and J.-C. Raynal, Hadron Transitions in the Quark Model (Gordon and Breach, Amsterdam, 1988).
- [18] B. Povh and J. Hüfner, Phys. Lett. B245 (1990) 653.
- [19] Ch. Barter, D. Blaschke and H. Voss, Preprint CERN-TH. 3499/92 (1992), Phys. Lett. B (to appear).
- [20] D. Blaschke, G. Röpke and H. Voss, in preparation.