EE

CERN-SL 92-55 AP
W9307

EUROPEAN LABORATORY FOR PARTICLE PHYSICS

C1

CERN SL/92-55 (AP)

CM-P00062712

LATTICE AND INTERACTION REGION DESIGN FOR Z-FACTORIES

Eberhard Keil

Abstract

Methods for increasing the luminosity of LEP by about an order of magnitude or more are discussed when LEP is operated at the energy of the Z. The notation and scaling laws for luminosity, beam-beam tune shifts, and synchrotron radiation are presented first. Limitations of the bunch current and of the total beam current are discussed. Methods for the design of the lattice and the interaction regions are presented. Two schemes for raising the total circulating current and hence the luminosity – pretzels and bunch trains – are discussed in detail. The paper ends with a discussion of combining pretzels with bunch trains, and with lessons for future Z factories.

Lecture given at the Joint US-CERN School on Particle Accelerators,
Benalmadena, Spain, 29 October to 4 November 1992

Geneva, Switzerland

15 December 1992

1 INTRODUCTION

This paper discusses methods for increasing the luminosity L of LEP by about an order of magnitude or more when it is operated at the energy of the Z. The standard formulae and notation are introduced in Section 1.1. Effects of synchrotron radiation are summarized in Section 1.3. Bunch and beam current limitations are discussed in Sections 1.4 and 1.5, respectively. Since these topics are also covered by other lectures at this school, the results are just quoted, and no derivation is shown. The lattice and interaction region design is discussed in Section 2. Two methods of increasing the total circulating current and hence the luminosity – pretzels and bunch trains – are presented in Sections 3 and 4, respectively. Section 5 contains a discussion of combining pretzels with bunch trains, and lessons for the future.

1.1 Standard Formulae and Notation

When the vertical amplitude function at the interaction points β_y is much larger than the bunch length σ_s , i.e. for $\beta_y \gg \sigma_s$, the luminosity L and the beam-beam tune shift parameter ξ_z in the z -plane (z is x =horizontal or y =vertical) are given by:

$$L = \frac{N^2 f}{4\pi\sigma_x\sigma_y} = \frac{N^2 f_{\text{rev}} k}{4\pi\sigma_x\sigma_y} \quad \xi_z = \frac{Nr_e\beta_z}{2\pi\gamma(\sigma_x + \sigma_y)\sigma_z} \quad (1)$$

The meaning of frequently used symbols and their values are explained in Table 1. I assume throughout that the e^+ and e^- parameters are the same. Correction factors for L and ξ_y with finite β_y/σ_s are known [1, 2].

1.2 Manipulations of the Standard Luminosity Formula

To gain further insight we use ξ_y to eliminate one power of N from the L equation:

$$L \approx \frac{Nf\gamma\xi_y}{2r_e\beta_y} = \frac{Ik\gamma\xi_y}{2e r_e\beta_y} \quad (2)$$

These equations tell us that the products Nf or Ik have to be increased by the same factor as the desired increase in L , because γ is fixed, $\xi_x \approx 0.03$, $\xi_y \approx 0.03$ are fairly fixed, and β_y cannot vary much as will be discussed in Section 2.2. These equations also imply that ϵ_x is adjusted such that the design value of ξ_x is reached at the design current I :

$$\epsilon_x = \frac{Nr_e}{2\pi\gamma\xi_x} \quad (3)$$

The lattice design must ensure that this emittance is reached at the design energy and at the design current, and that the lattice is flexible enough to reach the beam-beam limit for a range of energies and bunch currents around the design values. The emittance ϵ_x increases in proportion to N or I . To have $\xi_x = \xi_y$, we need $\epsilon_y/\epsilon_x = \beta_y/\beta_x$. In a typical low- β insertion $\beta_y \ll \beta_x$, hence beams are flat with $\epsilon_y \ll \epsilon_x$ and $\sigma_y \ll \sigma_x$. There are two extreme ways for increasing Nf : One may either increase N and keep f fixed, or one may keep N fixed and increase f . Intermediate solutions don't contribute anything fundamentally new. The ultimate limit on L is determined by the total beam current Ik .

$C_q = 3.84 \times 10^{-13} \text{ m}$	$55\hbar/32\sqrt{3}mc \approx$ Compton wavelength
$f = f_{\text{rev}}k$	Bunch collision frequency
$f_{\text{rev}} = 11.245 \text{ kHz}$	Revolution frequency
$I = Nef = 0.5 \text{ mA}$	Bunch current
$J_s = 2$	Damping partition number for synchrotron oscillations
$J_x = 1$	Damping partition number for horizontal betatron oscillations
$J_y = 1$	Damping partition number for vertical betatron oscillations
$dJ_s/d\Delta p/p = 394$	Slope of damping partition number J_s
k	Number of bunches in one beam
$\ell_B = 70 \text{ m}$	Total length of dipoles in an arc cell
$\ell_Q = 3.2 \text{ m}$	Total length of quadrupoles in an arc cell
$N = 2.775 \times 10^{11}$	Bunch population
$Q = \rho\mu/\ell_B = 69$	Contribution of arcs to tune
$Q_s = 0.08$	Synchrotron tune
$R = 3494 \text{ m}$	Average radius of arcs
$r_e = 2.818 \times 10^{-15} \text{ m}$	Classical electron radius
$U_s = 137.3 \text{ MeV}$	Synchrotron radiation loss per turn
$\alpha = 1 \text{ mrad}$	Full horizontal crossing angle
$\beta_x = 1.25 \text{ m}$	Horizontal amplitude function at IP
$\beta_y = 0.05 \text{ m}$	Vertical amplitude function at IP
$\gamma = 89237$	Particle energy E in units of its rest energy E_e
$\epsilon_x = \sigma_x^2/\beta_x = 30 \text{ nm}$	Horizontal emittance
$\epsilon_y = \sigma_y^2/\beta_y = 1.2 \text{ nm}$	Vertical emittance
$\mu = \pi/2$	Phase advance in an arc cell
$\rho = 3076 \text{ m}$	Bending radius
$\sigma_e = 1.0654 \times 10^{-3}$	Rms momentum spread
$\sigma_s = 10.7 \text{ mm}$	Bunchlength
$\sigma_x = 0.194 \text{ mm}$	Horizontal rms beam radius at the collision points IP
$\sigma_y = 7.75 \text{ }\mu\text{m}$	Vertical rms beam radius at the collision points IP
$\tau_s = 29.5 \text{ ms}$	Damping time for synchrotron oscillations
$\tau_x = 59.0 \text{ ms}$	Damping time for horizontal betatron oscillations
$\tau_y = 59.0 \text{ ms}$	Damping time for vertical betatron oscillations

Table 1: Explanation and Standard Values of Frequently Used Parameters

1.3 Effects of Synchrotron Radiation

Synchrotron radiation causes several effects in e^+e^- storage rings. The radiation loss per turn U_s in terms of voltage which must be compensated by the acceleration in the RF system, and the radiation damping rates τ_z^{-1} for z -th mode are given by:

$$U_s = \frac{4\pi r_e E_e \beta^3 \gamma^4}{3\rho} \quad \tau_z^{-1} = \frac{U_s f_{\text{rev}} J_z}{2E} \quad (4)$$

Note that U_s and τ_z are both proportional to ρ^{-1} . Hence the smaller the storage ring the higher the synchrotron radiation losses and synchrotron radiation power, and the more expensive the RF system. This dependence on ρ differs from that of the length of the arcs which varies in proportion to ρ . The cost of the arc tunnel and of the equipment inside is also likely to be about proportional to ρ . One way of arriving at a choice for ρ is balancing the cost of the tunnel against that of the RF system [3]. The damping partition numbers J_z always satisfy the relation $J_x + J_y + J_s = 4$. If the machine has a median plane like LEP, the vertical damping partition number satisfies $J_y = 1$. If in addition the bending magnets have no gradient, then also $J_x \approx 1$, and hence $J_s \approx 2$. Errors in the alignment and excitation of the magnets in LEP cause errors in the closed orbit and in the J 's. The damping partition numbers also depend on the relative momentum error $\Delta p/p$ since the closed orbit for an off-momentum particle passes through the quadrupoles off-axis. Hence, they act like combined-function magnets. The slopes of J_s and J_x are given by:

$$\frac{dJ_s}{d\Delta p/p} = \frac{2\ell_B}{\ell_Q} \frac{4 + \sin^2 \mu/2}{\sin^2 \mu/2} = -\frac{dJ_x}{d\Delta p/p} \quad (5)$$

Since both J_s and J_x must be positive, this slope determines the damping aperture, bounded by: $-2(dJ_s/d\Delta p/p)^{-1} \leq \Delta p/p \leq (dJ_s/d\Delta p/p)^{-1}$. The balance between quantum excitation and radiation damping determines the relative energy spread σ_e and the horizontal emittance ϵ_x :

$$\sigma_e = \frac{C_q \gamma^2}{\rho J_s} \quad \epsilon_x \approx \frac{C_q \gamma^2 R}{J_x Q^3 \rho} \quad (6)$$

Both equations apply to a machine in which all dipoles have the same bending radius ρ . The equation for ϵ_x assumes that the bending angle and the phase advance μ in a cell are both small. The correction factor for finite bending angle and finite μ is known [4]. The desired value of ϵ_x can be achieved by adjusting the contribution Q of the arcs to the tune. Small variations in ϵ_x can be obtained by varying J_x in principle, by small variations of the momentum error $\Delta p/p$ on the closed orbit that are achieved by changes of the RF frequency. However, in practice such changes are not desirable in LEP because they interfere with the energy calibration.

In LEP with a rather low field in the dipoles in the arcs it is easy to increase the emittance and the energy spread with wiggler magnets, while it is difficult to reduce the emittance, and impossible to reduce the energy spread. Hence, it is advantageous to make the focusing in the arcs stronger than needed, i.e. to choose Q higher than needed from the equation above, and to increase the emittance with wiggler magnets. The wigglers in LEP have just three poles, a central pole with a field up to 1 T in the same direction as the main dipoles,

and outer poles with a field up to 0.4 T in the opposite direction to limit adverse effects on the polarization [5]. The total deflection in a wiggler vanishes. The damping wigglers are installed in the straight sections with $D_x = 0$. The emittance wigglers are positioned in the dispersion suppressors with $D_x \neq 0$. They increase the emittance from 12.26 nm to 30.25 nm.

1.4 Bunch Current Limitations

The transverse mode coupling instability limits the bunch current I at injection where the beam energy E is lowest. In the broad-band resonator model its threshold is approximately given by:

$$I \approx \frac{2\pi(E/e)Q_s f_{\perp}(\sigma_s)}{\sum_i \bar{\beta}_i (hR/Q_{\perp})_i} \quad (7)$$

Here, $Q_s \approx 0.08$ is the synchrotron tune, $h = f_r/f_{\text{rev}}$ is the harmonic number of the resonator resonating at frequency f_r , $R/Q_{\perp i}$ is the transverse impedance of the i -th group of equipment, $\bar{\beta}_i$ is the average value of the β -function there, and $f_{\perp}(\sigma_s)$ is a form factor that has a minimum of about unity when the bunchlength σ_s is about half the vacuum chamber radius, and increases roughly proportionally to σ_s for higher values. Therefore, the bunches are made longer with wigglers in routine operation. In LEP, about 2/3 of the sum in the denominator is due to the Cu RF cavities. Hence, it is attractive in the long run to replace them by s.c. cavities which by design have a larger beam hole and a lower transverse impedance. The remaining 1/3 of the sum is due to bellows and difficult to change. There is excellent quantitative agreement between observations of the threshold in LEP and the theory underlying the scaling law above [6]. All other single-beam collective effects do not limit the bunch current in LEP.

The beam size is proportional to the bunch current $I^{1/2 \dots 4/3}$ and will eventually reach the edge of the – physical or dynamic – aperture when I is increased. The value of the power of β_y depends on the scaling procedure. This limitation arises first in the low- β insertions, and will be discussed in Section 2.2.

Collimators protect the experiments from synchrotron radiation background. Synchrotron radiation background from the arcs is reduced by distance, i.e. by the length of the straight section, by weak dipoles, experiments not being in direct line of sight, and by the synchrotron radiation being at least scattered twice on collimators. Synchrotron radiation background from the quadrupoles in the straight section increases in proportion to the beam size, and must be shielded by collimators with an aperture proportional to the beam size. Observations in LEP show that there is a practical upper limit for the horizontal emittance at $\epsilon_x = 50$ nm.

All three reasons prevent increasing the bunch current by an order of magnitude.

1.5 Limitation of Total Beam Current

All schemes for increasing the LEP luminosity by an order of magnitude or more require an increasing of the total circulating current by about the same factor. Several phenomena may limit the total beam current. The voltage delivered by the RF system must be larger

Figure 1: Horizontal Orbit Function $\beta_x^{1/2}$ in a LEP Octant

than synchrotron radiation losses $U_s \approx 125$ MV. This figure is much smaller than voltage required at 90 GeV. The power P from the RF system

$$P = 2U_s I k \approx 2 \cdot 125 \text{ MV} \cdot 1 \text{ mA} \cdot 36 \approx 9 \text{ MW} \quad (8)$$

must be at least equal to the synchrotron radiation power. In this equation the factor of two takes care of the two beams, and the factor 36 is the maximum number of bunches which we will arrive at in Section 3. The power above is much smaller than the power installed for LEP-200. The power absorbed by the vacuum chamber per running metre $P/2\pi\rho \approx 0.5$ kW/m is within the capabilities of the present cooling system. The power P_{hom} of the higher-mode losses in the superconducting cavities is given by:

$$P_{\text{hom}} = 2k_{\parallel} k I^2 f_{\text{rev}}^{-1} \quad (9)$$

With a loss factor $k_{\parallel} \approx 0.25$ V/pC [7] and a revolution frequency $f_{\text{rev}} = 11245.5$ Hz, the specifications of the present higher-mode couplers [8] $P_{\text{hom}} = 200$ W impose a limit $k I^2 \approx 4.5$ mA². The maximum number of bunches k decreases quadratically with the bunch current from $k = 18$ at $I = 0.5$ mA, through $k = 8$ at $I = 0.75$ mA to $k = 4$ at $I = 1$ mA.

2 LATTICE DESIGN

The LEP lattice consists of eight – almost – identical octants. The octant boundaries coincide with the eight interaction points. The octants in turn consist of three modules each, half a

Figure 2: Horizontal Dispersion D_x in a LEP Octant

straight section, an arc, and half a straight section. Figures 1 and 2 show the orbit function β_x , and the dispersion D_x , respectively. The behaviour of β_y is similar to that of β_x . All figures in this lecture were made with MAD [9]. When the distance s along the orbit is used as abscissa, a schematic layout diagram often appears on top of the graph. Blocks above and below the central line represent dipoles, blocks above (below) the central line represent horizontally focusing (vertically focusing) quadrupoles, etc. The three orbit functions are periodic in the arcs which consist of FODO cells with a phase advance $\mu = \pi/2$. The dispersion D_x vanishes in the straight sections. This is achieved by dispersion suppressors at both ends of the arcs. A weak dipole at the entrance of the dispersion suppressor prevents synchrotron radiation from the arcs falling directly into the detectors at the interaction points.

2.1 Straight Section Design

Figure 3 shows β_x , β_y and D_x in a smaller region of LEP, starting at the interaction point in Pit 2, including the low- β insertion, the RF section with a periodic cell lattice for minimum $\bar{\beta}_y$, and the dispersion suppressor. Each of these three lattice modules has a specific purpose: The low- β insertion matches the optical functions α and β between the values in the RF section and the low values for the β -functions wanted at the interaction point in Pit 2. The RF section consists of $5/2$ cells of FODO lattice with the space between the quadrupoles chosen such that there is space for eight Cu RF cavities and associated equipment. The phase advance there is adjusted such that the average value $\bar{\beta}$ is at a minimum in order to raise the

threshold of the transverse mode coupling instability. The dispersion suppressor matches the horizontal dispersion D_x from zero in the straight section to the non-zero value in the arc, and the optical functions α and β between the values in the RF section and those in the arc, with prescribed values of the phase advances. This is achieved by adjusting the strengths of eight individually powered quadrupoles in each dispersion suppressor. In this manner, the dispersion suppressor can be matched to arc cells with a range of phase advances. Much simpler dispersion suppressors are possible if the phase advance in the arc cells is fixed [10]. I will show below that a high gradient in the first superconducting quadrupole is desirable, and that the minimum value of β_y which can be achieved in practice increases with ϵ_x .

Figure 3: Orbit Functions $\beta_x^{1/2}$, $\beta_y^{1/2}$ and D_x from Pit 2 into Arc

2.2 Interaction Region Design

The low- β insertion in LEP is a classical low- β insertion with $\beta_y \ll \beta_x$. It is matched by using a computer program. Approximate parameters and scaling laws can be obtained analytically. There are relations between the focal length d and the distance l_x of the centre of the first quadrupole from the interaction point, between d , the aperture a , the “pole-tip field” B_Q , and the length of quadrupole l_Q , and between l_Q and l_x :

$$d \approx l_x/2 \quad d \approx \frac{B \rho a}{l_Q B_Q} \quad l_Q \approx l_x/2 \quad (10)$$

The aperture a must exceed the beam size by a factor which is typically 10. With the optimum emittance ratio $\epsilon_y/\epsilon_x = \beta_y/\beta_x$, the beam is approximately round on the front face

of the first quadrupole. Allowing for “full coupling” with $\epsilon_y/\epsilon_x = 1/2$ the vertical beam size becomes larger than the horizontal one, and determines the aperture a . We therefore write for a , where $\beta_Q \approx \ell_x^2/\beta_y$ is taken at the quadrupole:

$$a \approx 10\sqrt{\epsilon_x\beta_Q/2} \approx 10\sqrt{\epsilon_x\ell_x^2/2\beta_y} \quad (11)$$

The two vertically focusing quadrupoles contribute Q'_y to the vertical chromaticity:

$$Q'_y \approx \beta_Q/2\pi d \approx \ell_x/\pi\beta_y \quad (12)$$

If one imposes the condition that Q'_y does not exceed a given value one obtains for the minimum value of β_y :

$$\beta_y^{3/2} \approx \frac{20B\rho\sqrt{2\epsilon_x}}{\pi B_Q Q'_y} \quad (13)$$

This scaling law reveals how β_y is related to energy, pole-tip field, and the contribution Q'_y to the chromaticity. There is no handy formula which yields the maximum tolerable contribution Q'_y for a given emittance ϵ_x , but a suspicion, supported by tracking results, that the dynamic aperture decreases when Q'_y increases. Thus the limit on the contribution Q'_y is reached when the dynamic aperture is just large enough for the emittance ϵ_x .

A second limit on β_y is obtained by inspecting the chromatic variation of β_y [11], and comparing it to the rms momentum spread σ_e :

$$\frac{1}{\beta_y} \frac{d\beta_y}{d\Delta p/p} \approx \frac{\ell_x}{\beta_y} \approx \frac{1}{\Delta p/p} \approx \frac{1}{10\sigma_e} \quad (14)$$

This leads to the following minimum value of β_y from the momentum aperture $\Delta p/p$:

$$\beta_y^{3/2} \approx \frac{20B\rho(\Delta p/p)\sqrt{2\epsilon_x}}{B_Q} \quad (15)$$

With the standard parameters and $B_Q = 2$ T, one finds $\beta_y = 24$ mm. Hence, there is room for reducing β_y . The other parameters follow from β_y by back substitution.

3 PRETZELS

In a pretzel scheme the e^+ and e^- beams are separated over most of the circumference by exciting forced, closed-orbit oscillations with $n\pi$ phase difference by electrostatic separators. It follows that a pretzel scheme increases the aperture needed by the amplitude of the orbit oscillation, and that it can only be installed in machines which have an aperture which is larger than that needed without pretzels. This is typically the case when machines are operated below their design energy. Such a scheme was first installed and used in the CESR storage ring at Cornell University, Ithaca NY, USA, and the name pretzel was invented there [12, 13]. The bunch spacing is arranged such that beam-beam collisions in the arcs occur only where the orbits are well separated. A pretzel scheme for LEP was proposed by Rubbia [14]. It led to a feasibility study [15], and eventually to a report of the Working Group on High Luminosity at LEP [16]. Figure 4 shows a typical e^+ orbit and the horizontal dispersion D_x in an octant of LEP. The e^- orbit and dispersion are almost mirror images.

Figure 4: Horizontal Orbit Offset x and Dispersion D_x for a Pretzel from Pit 2 to Pit 3

3.1 Pretzel Design Criteria

A pretzel scheme is designed such that the collisions at the interaction points still happen. However, the e^+ and e^- orbits are separated over most of the circumference. The separation is done in the horizontal plane for the following reasons:

- In a typical e^+e^- storage ring the horizontal aperture is larger than the vertical aperture.
- A vertical orbit offset in the sextupoles, caused by a vertical pretzel, would excite a skew gradient which in turn would cause coupling between the horizontal and vertical betatron oscillations.
- A vertical orbit offset in the sextupoles, caused by a vertical pretzel, would excite the vertical dispersion D_y which in turn would cause an increase of the vertical emittance ϵ_y by quantum excitation beyond the desirable value $\epsilon_y = \epsilon_x \beta_y / \beta_x$.

The pretzel scheme is also designed such that there is no offset in the RF cavities to avoid exciting synchro-betatron resonances. Symmetries in the ideal machine are used to compensate the consequences of the differences between the e^+ and e^- orbits. In particular, the pretzels are made anti-symmetric about the interaction points to avoid accumulating effects from the orbit differences. In a real machine, these symmetries are broken by alignment and RF and magnet excitation errors. Their consequences must be corrected. The choice of the pretzel amplitude is determined by three criteria that will be discussed in Section 3.3:

- beam-beam tune shifts

- beam-beam kicks
- separation between opposite beams in terms of σ_x

The quantities $Q_x, Q_y, Q'_x, Q'_y, \beta_x, \beta_y$ should be independent of the pretzel amplitude [12].

Figure 5: Scaled Horizontal Offset $x/\beta_x^{1/2}$ and Dispersion $D_x/\beta_x^{1/2}$ in $\mu\text{m}^{1/2}$ for a Pretzel in Octant from Pit 2 to Arc

3.2 Implementation of Pretzels in LEP

Clearer pictures are obtained by plotting the normalized pretzel $x/\beta_x^{1/2}$ as a function of the horizontal phase advance μ_x . Figure 5 shows the normalized pretzel starting at Pit 2. The orbit separation starts at the end of the straight section beyond RF system. Therefore, the two beams pass through the RF cavities without offset. The two beams collide head-on in Pit 2, where the dispersion D_x vanishes, i.e. $D_x = 0$ by symmetry, but its slope D'_x does not, i.e. $D'_x \neq 0$. Hence, D_x does not vanish in the straight sections, as shown in Figure 4. In the straight sections, D_x behaves like a betatron oscillation. Therefore, the normalized dispersion $D_x/\beta_x^{1/2}$ behaves like a sine wave when plotted against μ_x , as shown in Figure 5. A $D_x \neq 0$ in the RF system near Pit 2 drives synchro-betatron resonances [18]. The pretzel on the other side of Pit 2 is symmetrical. The pretzels in the other even pits are very similar.

Figure 6 shows the normalized pretzel ending at Pit 3. By symmetry, the orbit offset x and dispersion D_x vanish at Pit 3, i.e. $x = 0$ and $D_x = 0$, while the orbit and dispersion slopes, x' and D'_x do not vanish, i.e. $x' \neq 0$ and $D'_x \neq 0$. The pretzel on the other side of Pit 3 is symmetrical. The pretzels in the other odd pits are very similar. In LEP, we adjust the

horizontal phase advance μ_x in the straight sections next to the odd pits to close the pretzel. It should be noted that the partition numbers change on pretzel orbits for the same reason as on off-momentum orbits. The change of the J 's is quadratic in the pretzel amplitude. For the example shown in Figure 4 the changes are $\Delta J_x \approx -\Delta J_y \approx 0.17$.

Figure 6: Scaled Horizontal Offset $x/\beta_x^{1/2}$ and Dispersion $D_x/\beta_x^{1/2}$ in $\mu\text{m}^{1/2}$ for a Pretzel in Octant from Arc to Pit 3

3.3 Accelerator Physics of Pretzels

The beam-beam tune shifts ξ_x and ξ_y of beams with full horizontal separation $x \gg \sigma_x$ may not be negligible. They are given by:

$$\xi_x = -\frac{Nr_e\beta_x}{2\pi\gamma x^2} \quad \xi_y = \frac{Nr_e\beta_y}{2\pi\gamma x^2} \quad (16)$$

The beam-beam tune shifts for separated beams have opposite sign, contrary to those for head-on collisions. Expressions for ξ_x and ξ_y for arbitrary horizontal separation are known [16]. For vanishing separation x , ξ_x and ξ_y remain finite, and Eq. (1) applies. Inspection of these equations shows that it is advantageous to arrange for the separated beam-beam collisions with horizontal separation to occur close to a peak of the sine wave because ξ_x depends only on the horizontal phase, since the orbit offset is given by $x = \beta_x^{1/2} \sin(\mu_x + \phi_x)$, where ϕ_x is an arbitrary phase. Hence, the expression β_x/x^2 appearing on the right hand side of the equation for ξ_x is an invariant. The separated beam-beam collisions should also

be arranged close to a horizontally focusing quadrupole where β_y is small and x is large and ξ_y is small.

The following arguments lead to a choice of the bunch spacing s and hence of the number of evenly spaced bunches k :

- In a machine designed for pretzels, with a horizontally focusing quadrupole at all bunch collision points in the arcs, the minimum bunch spacing s should be approximately equal to the horizontal betatron wavelength in the arcs, i.e. $s \approx \lambda_\beta$. If this condition were satisfied in LEP with a horizontal phase advance $\mu_x = \pi/2$ the number of bunches would be $k \approx 84$. More daring designers might take $s \approx \lambda_\beta/2$, and collide at all pretzel phases which are an odd multiple of $\pi/4$.
- The bunch spacing s should be larger than the length of the straight sections to avoid bunch collisions there. This condition implies $k < 54$.
- The bunch spacing s should be an even multiple of the RF wavelength λ_{RF} . The following numbers of bunches k are compatible with the harmonic number in LEP $h = 31320 = 2^3 \cdot 3^3 \cdot 5 \cdot 29$: $k = 6, 8, 10, 12, 18, 20, 24, 30, 36, 40$.
- Checking the horizontal separation x at all collision points in the arcs for the tune shifts ξ_x and ξ_y shows that there are poorly separated encounters for some values of k in the above list. Table 2 shows the sums of the unwanted ξ_x and ξ_y [16]. It may be seen that $k = 20, 30, 40$ had to be dropped, and that $\sum \xi_y \ll \sum \xi_x$, as one would expect from the arguments above.

k	6	8	10	12	18	24	36
$\sum \xi_x$	.0187	.0018	.0079	.0386	.0253	.0437	.0571
$\sum \xi_y$	.0024	.0001	.0044	.0046	.0057	.0049	.0113

Table 2: Sums of the unwanted beam-beam tune shifts ξ_x and ξ_y for good values of the number of bunches k in the LEP pretzel scheme with a bunch current $I = 0.75$ mA

The separated beam-beam collisions also cause a horizontal beam-beam kick x'_{bb} which with full horizontal separation $x \gg \sigma_x$ and $N = 2.775 \times 10^{11}$, at 45.6 GeV, $x = 12$ mm is given by:

$$x'_{\text{bb}} = -\frac{2Nr_e}{\gamma x} \approx 1.5 \mu\text{rad} \quad (17)$$

These beam-beam kicks have the following consequences:

- Beam-beam kicks cause a horizontal orbit distortion that is proportional to the population of the encountered bunch N .
- All bunches travel along their private closed orbit, given by the sequence and populations of the bunches in the other beam that they meet.
- To get an idea about the magnitude of these kicks they may be compared to the horizontal divergence which with $\epsilon_x = 30$ nm, $\beta_x = 132$ m is given by $\sigma'_x = (\epsilon_x \gamma_x)^{1/2} \approx 15 \mu\text{rad}$ where $\gamma_x = (1 + \alpha_x^2)/\beta_x$.

- If the beam-beam kick is not small compared to the divergence the two bunches are likely to miss each other at the desired interaction points. Hence, we obtain a tolerance on the inequality of the bunch populations N which is easily satisfied:

$$\frac{\Delta N}{N} \ll \frac{\gamma x}{2Nr_e} \sqrt{\epsilon_x \gamma_x} \quad (18)$$

Figure 7: Full Horizontal Separation x and $5\sigma_x$ in Units of metres for a Bunch Train Near Pit 2

A further criterion on the pretzel amplitude follows from a comparison between the separation $2x$ and the horizontal beam radius σ_x . A separation of $5 \dots 6\sigma_x$ is needed to ensure a good lifetime of the two beams [17]. This observation is often discussed in a way analogous to the quantum lifetime in an RF system [19]. The probability that a particle reaches, by quantum excitation, a horizontal betatron amplitude large enough to make it pass through the core of the opposite beam is a steeply decreasing function of that amplitude, just as the probability for a particle to get its energy changed enough to jump out of the RF bucket. The separation will be discussed further in Section 5.1.

3.4 Status of Pretzels in LEP

Pretzels were added to LEP as an afterthought [14]. Half the electrostatic separators required were installed by spring 1991. This was enough to study the behaviour of two regular bunches with a pretzel bunch between them. The remaining separators were installed by May 1992,

and machine development with up to eight bunches in each beam was started. Physics runs with pretzels took place from 19 October 1992. The number of bunches is $k = 8$ for two reasons:

1. In the LEP RF system [20] the Cu RF cavities are coupled to storage cavities, and the coupled system operates with two frequencies. The beat frequency between them corresponds to $k = 8$.
2. The LEP experiments are able to cope with half the design bunch spacing of $11 \mu\text{s}$, but not with much smaller bunch spacings.

At the time of the school, the luminosity had reached the break-even point with 4.4 mA total current, and $L \approx 10^{31} \text{cm}^{-2}\text{s}^{-1}$ in all four experiments. A few problems remained:

- The maximum bunch current at injection is smaller than that reached with 4 on 4 bunches, 250 instead of $450 \mu\text{A}$. It is known from experiments that this reduction is due to the mid-arc collisions, and it can be increased to $320 \mu\text{A}$ by a higher pretzel amplitude.
- The lifetime is very sensitive to the inequality of the bunch currents.
- Deliberate orbit distortions near the injection kickers are different for normal and pretzel bunches.
- The luminosity is about 25 % below the values expected from currents and beam sizes.

Figure 8: Beam-Beam Tune Shifts ξ_x and ξ_y in Units of 0.001 for a Bunch Train Near Pit 2

4 BUNCH TRAINS

In LEP, four trains of closely spaced bunches might be an alternative to pretzels. In this case, the two beams have to be separated only over a distance equal to the length of the bunch train, near the interaction points. Since the two beams should still collide at the interaction point, an asymmetric separation at a crossing angle is appropriate. A crossing with a horizontal angle α makes it possible to satisfy one of the criteria for the crossing angle which is related to synchro-betatron resonances and requires that $\alpha \ll \sigma_x/\sigma_s$ [21]. The design of a horizontal separation scheme with a full crossing angle $\alpha = 1$ mrad is shown in Figure 7. Also shown there is $5\sigma_x$, assuming a horizontal emittance $\epsilon_x = 30$ nm to demonstrate that the criterion $x > 5\sigma_x$ is satisfied.

Figure 9: Beam-Beam Kicks x' in μrad and Ratio nx' between x' and Divergence σ'_x for a Bunch Train Near Pit 2

4.1 Accelerator Physics of Bunch Trains

As was the case for pretzels, beam-beam tune shifts and beam-beam kicks must be checked. Figure 8 shows the beam-beam tune shifts ξ_x and ξ_y for separated collisions near Pit 2, assuming that the bunch current is $I = 0.5$ mA. It may be seen that both tune shifts are small enough when the collisions occur at distances from 10 to 50 m from Pit 2. The horizontal tune shift ξ_x is larger in absolute value. This is not surprising when one looks at the β -functions in Figure 3. The collisions farthest away from the interaction point occur at a distance equal to half the length of the bunch train. It follows from the range given that

the length of the bunch train is at most 100 m and that the bunch spacing must be at least 20 m. Hence, a bunch train may consist of six bunches at most.

The beam-beam kicks x' and the ratio between x' and the horizontal divergence σ'_x are shown in Figure 9, assuming $\alpha = 1$ mrad, $I = 0.5$ mA, and $\epsilon_x = 30$ nm. The beam-beam kicks x' are of the order of μrad , and the ratio between x' and divergence is at the per cent level.

The bunch spacing s must be a multiple of both the LEP and SPS RF wavelengths to inject a perfectly synchronized bunch train. With the present RF harmonic numbers in LEP and SPS – $h_{\text{LEP}} = 31320$ and $h_{\text{SPS}} = 4620$ – the RF wavelengths are related by $58\lambda_{\text{LEP}} = 33\lambda_{\text{SPS}}$. Hence, the minimum bunch spacing is $s \approx 50$ m, thus allowing just three bunches in a train of 100 m length. If the harmonic number in the SPS is increased by 20 units to $h_{\text{SPS}} = 4640$, the RF wavelengths are related by $7\lambda_{\text{LEP}} = 4\lambda_{\text{SPS}}$, and the bunch spacing $s \approx 20$ m arrived at above becomes possible.

Figure 10: Beam-Beam Tune Shifts ξ_x and ξ_y in Units of 0.001 for a Betatron Wavelength in the Arc from Pit 2 to Pit 3

4.2 Remaining Problems

Two trains of n bunches, circulating in opposite direction, collide in $(2n - 1)$ places half the bunch spacing s apart. Each of the bunches in one train meets bunches in the other train in n neighbouring places. Where these places are depends on the position of the bunch along the bunch train. All bunches have one head-on collision at the interaction point, and $(n - 1)$

separated collisions. The leading bunch has the head-on collision first, and the separated ones afterwards. The opposite is true for the trailing bunch. In general, the m -th bunch in the train has $(m - 1)$ separated collisions, a head-on collision, and then $(n - m)$ separated collisions. Figures 8 and 9 show that the separated collisions do not have the same effect, i.e. the beam-beam kicks x'_{bb} and beam-beam tune shifts ξ_z are all different, even if all bunch populations are identical. Therefore one must expect effects in bunch trains similar to those discussed in Section 3.3 which are absent with single bunches, e.g. spreads in the orbits and spreads in the tunes.

Figure 11: Beam-Beam Kick $|x'|$ in μrad and Ratio $|nx'|$ between $|x'|$ and Divergence σ'_x for a Betatron Wavelength in the Arc from Pit 2 to Pit 3

This reminds us of the effects expected for the bunch trains in large hadron colliders like the LHC and SSC [22]. However, there is a difference between these bunch trains and those considered here. In the LHC or SSC, the bunch trains are much longer than the distance over which they circulate in the same ring and affect each other by their electromagnetic fields. Therefore, the number of collisions is different for bunches at or near the head or tail of the train from that for bunches near the centre. In LEP, the bunch trains circulate in the same ring over the whole circumference, and the number of collisions is the same for all bunches.

- Can the differences between bunches be avoided by symmetry if the bunch trains consist of just two bunches? Unfortunately, doubling the number of bunches will not yield more than a factor of two in luminosity.

- There may be interference between the solenoid and solenoid compensation and the distorted orbit due to the horizontal crossing angle. Can this be fixed by symmetry, or by perfect matching [23] on the actual orbit?
- The synchrotron radiation from the first few quadrupoles is enhanced by the orbit offset there. Can this be fixed by aligning these quadrupoles on the incoming beam, as in B factories [24]?
- Can the injectors deliver the bunch trains required?
- Closely spaced bunches leave little time to process events in experiments between bunch crossings. Must the experiments process events between crossings? Or are the bunch trains short enough that they can wait with the processing until the gap between bunch trains arrives? It should be remembered that most collisions do not have an event. Therefore, it appears that by accumulating data over a bunch train the experiment accumulates background, but events do not pile up. This is different from the situation in the LHC where the bunch spacing is similar, but every collision contains one or more events.

Figure 12: Separation $|x|$ in Units of σ_x for a Betatron Wavelength in the Arc from Pit 2 to Pit 3

5 CONCLUSIONS

5.1 Combination of Bunch Trains with Pretzels

The question may be raised whether it is possible and advantageous to combine bunch trains with pretzels, i.e. to consider more than four trains of bunches in LEP, up to 36, the maximum possible, as discussed in Section 3.3. Combining pretzels with bunch trains is also being studied for the CESR storage ring at Cornell University.

The effects close to the interaction points were already presented. We will now study the collisions between bunch trains in the arcs. Figure 10 shows the separated beam-beam tune shifts ξ_x and ξ_y in four cells of the LEP arcs or one betatron wavelength λ_β . The full line for ξ_x has a period equal to half the betatron wavelength λ_β . As discussed in Section 3.3, ξ_x is simply proportional to $1/\sin^2 \mu_x$. The flat minimum of ξ_x is about as long as a cell, implying that the bunch trains can be about 80 m long. The dashed line for ξ_y in Figure 10 shows additional maxima at the horizontally defocusing quadrupoles where β_y is large. The length over which ξ_y is small enough is only about 50 m. Hence, the bunch trains can at most have about that length.

Figure 11 shows the separated beam-beam kicks x' and the ratio between x' and the horizontal divergence σ'_x in four cells of the LEP arcs. The kicks x' are a few μrad and the ratio x'/σ'_x is a few per cent.

Figure 12 shows the ratio between the absolute value of the separation $|x|$ and the horizontal rms beam radius σ_x for a betatron wavelength in an octant of LEP, and demonstrates that the criterion $|x| > 5\sigma_x$ is well satisfied.

Figures 10, 11, and 12 show that there are regions in the arcs where bunch trains of up to 50 m length can collide without ξ_x , ξ_y , x' , x'/σ'_x and $|x|/5\sigma_x$ becoming too large. It remains to be demonstrated that this applies to all collision points foreseen in a specific implementation of a pretzel and bunch train scheme in LEP.

5.2 Lessons for the Future

What can be learned from this lecture for a hypothetical Z factory which includes pretzels and/or bunch trains as a design feature, not as an afterthought? In such a machine, collisions between bunches and/or bunch trains occur at regular intervals around the circumference. Therefore, its lattice should be designed such that they systematically occur in the vicinity of maxima of the pretzel separation. This is achieved if the lattice design includes the following features:

- Horizontally focusing quadrupoles are installed at the bunch collision points, and therefore are more regularly spaced through the whole factory, i.e. arcs, dispersion suppressors, insertions, than they are in LEP now.
- The horizontal phase advance is a simple rational fraction like $1/6$ or $1/4$ through the whole range of the pretzels.
- The phase of the pretzel is adjusted such that maxima of the separation occur in the horizontally focusing quadrupoles.

- The number of betatron wavelengths inside the pretzels allows an easy variation of the bunch spacing, by having many small factors.
- The betatron wavelength in the arcs is a multiple of the RF wavelength.

By observing these rules, collisions of bunches and/or bunch trains at unfavourable places with small separation are excluded, and maximum flexibility in the adjustment of their number is achieved.

References

- [1] G.E. Fischer, SPEAR-154 (1972).
- [2] SPEAR Group, IEEE Trans. Nucl. Sci. **NS-20** (1973) 838.
- [3] B. Richter, Nucl.Instr.Meth. **136** (1976) 47.
- [4] R.H. Helm and H. Wiedemann, PEP Note 303 (1979).
- [5] A. Hutton, Particle Accelerators **7** (1976) 177.
- [6] G. Besnier, D. Brandt, B. Zotter, Particle Accelerators **17** (1985) 51.
- [7] B. Zotter, CERN SL/92-29 (DI) (1992) 457.
- [8] E. Haebel, this school.
- [9] H. Grote and F.C. Iselin, CERN SL/90-13 (AP). Rev. 3 (1992)
- [10] E. Keil, CERN 77-13 (1977) 22.
- [11] B.W. Montague, LEP note 165 (1979).
- [12] R. Littauer, IEEE Trans. Nucl. Sci. **NS-32** (1985) 1610.
- [13] D.L. Rubin, Particle Accelerators **26** (1990) 63.
- [14] C. Rubbia, Proc. European Particle Accelerator Conf. (Rome 1988) 290.
- [15] J.M. Jowett, Proc. IEEE Particle Accelerator Conf. (Chicago 1989) 1806.
- [16] E. Blucher et al. (eds.), CERN 91-02 (1991).
- [17] D.H. Rice, CBN 92-9 (1992).
- [18] A. Piwinski and A. Wrulich, DESY 76-07 (1976).
- [19] M. Sands, SLAC-121 (1970).
- [20] LEP Design Report, Vol. II, CERN-LEP/84-01 (1984).
- [21] A. Piwinski, IEEE Trans. Nucl. Sci. **NS-24** (1977) 1408.
- [22] J.L. Tennyson, SSC-155 (1988).
- [23] S. Kamada, Particles & Fields '91 (Vancouver 1991) 1055.
- [24] An Asymmetric B Factory, LBL PUB-5303 (1991).