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EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CM-P00062720

CERN SL/93-04 (AP)

Summary of the Working Group on Short-Term Stability (Workshop on Stability of Particle Motion in Storage Rings)

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Abstract

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Submitted for the Proceedings of the
Workshop on Stability of Particle Motion in Storage Rings
Brookhaven National Laboratory, October 1992.

Geneva, Switzerland

19 January 1993

SUMMARY FOR WORKING GROUP A ON SHORT-TERM STABILITY

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ABSTRACT

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INTRODUCTION

Seventeen papers were presented during the ten sessions of the working group A on short-term tracking.¹⁻¹⁷ The reader is referred to their printed versions in these proceedings. A few remarks will also be made in the following sections.

The next section is the author's view on how opinions on particle tracking in circular accelerators and storage rings evolved over the last ten years. It is followed by the working group's recommendations on correction algorithms. Next come some clarifications on coupled motion, followed by an attempt to demonstrate some advantages of the object-oriented approach. The final section contains the authors conclusions and recommendations.

HISTORICAL REVIEW

Symplecticity

Mainly due to the lack of computing power, tracking programs available in the early 1980's would not track more than a few hundred turns. It is striking that one tracking program even contained DIMENSION statements which limited tracking to 400 turns. Most storage rings being built at that time used classical separate function magnets and exhibited a rather linear behaviour. In these machines short-term tracking could not show any adverse effects of non-symplectic tracking. It was thus not generally accepted that only symplectic tracking could produce meaningful results for long-term stability studies. Many accelerator physicists believed that symplectic tracking would cause an unnecessary slow-down in tracking.

The 1982 Brookhaven workshop finally stressed the importance of symplectic tracking. The available computing power had drastically increased, allowing many more turns to be tracked, and results were shown which clearly indicated beam blow-up for non-symplectic tracking programs. Nevertheless it took several years until this was generally accepted, but fortunately today there is no doubt left about the necessity of symplectic tracking.

Lie Algebra

Lie algebraic tools were first presented¹⁸⁻²⁰ as a tool for symplectic tracking. The authors hoped that composition of transfer maps for individual elements could produce a one-turn map, and that by repetitive squaring the one-turn map they could obtain a many-turn map, which would yield a very fast simulation method for long-term tracking studies.

Soon it was recognized that unavoidable truncations of high-order effects during composition could hide important non-linear effects, and that composition could only be used for short sequences of elements, typically for a single cell or even a half cell.

However, Lie algebraic tools have proven to be an excellent tool for analyzing transfer maps. Normal form and resonance analysis of the one-turn map provides insight on the behaviour of the machine, and quantities like chromaticity, tune shift with amplitude, distortion functions of moderate order, resonance terms, etc. can easily be obtained. These techniques have also been used with success to analyze maps of high orders generated by composition of truncated Taylor maps.

Thin Lens Tracking

Another method for symplectic tracking, using thin lenses was offered in specialized programs like PATRICIA,^{21,22} RACETRACK,^{23,24} PATRAC,^{25,26} PATRIS,²⁷ TEAPOT,²⁸ SIXTRACK,²⁹ and SSCTRK.³⁰ Thin lens maps are exactly symplectic, but they do not represent the accelerator lattice accurately. The thin lens method has been called the exact solution to a wrong problem.

Thin lens approximations provide a very high tracking speed, and they have been applied with success to many large storage rings.

Differential Algebra

Differential algebra^{11,31} packages have been proposed to permit map evaluation and composition to orders typically as high as 10 to 15. These packages provide truncated Taylor expansions ("jets") of the desired transfer maps. The truncated maps are not symplectic, but the error can be made small by using sufficiently high orders. However, the convergence problem has not been completely solved.

Truncated Taylor maps can be analyzed directly or using Lie algebraic techniques to find quantities like chromaticities and tune shifts with energy or with amplitude.

Differential algebra can easily represent the dependence of a map on various machine parameters. The effect of parameter variations can then be determined, studied, and corrected.

Differential algebra methods come in two flavours. The hard way evaluates the transfer map for each element up to a given order. The resulting maps are then concatenated to form a one-turn map. Evaluation of element maps to high orders is an error-prone process, and composition of maps requires a lot of computing work. For these two reasons this approach is not recommended.

It is easier to start with input phase space coordinates for the first element defined as a vector of differential algebra variables. After tracking this vector once around the ring, the phase space coordinates at the exit of the last element represent a differential

algebra map for one turn. This method requires no evaluation of maps beyond what is available in a tracking program; all one has to do is replace every arithmetic operation in tracking by the corresponding differential algebraic operation.

When used for long-term tracking, a differential algebra map must be replaced by an equivalent symplectic map. To this aim one has to convert the truncated Taylor series to a representation which agrees with the former in terms up to its order.

Differential algebra also has its merit for analyzing transfer maps. It has been used with success for extracting all kinds of global quantities.

Generating Function Methods

In many cases transfer maps are available as Taylor series. For practical reasons the series must be truncated, associated with a loss of symplecticity.

A canonical transformation generated by a generating function always produces a symplectic map. This fact can be used for the evaluation of truncated Taylor maps. Convenient generating functions are the mixed-variables ones:

$$F = F(q_1, p_2) \quad \Rightarrow \quad q_2 = \frac{\partial F(q_1, p_2)}{\partial p_2}, \quad p_1 = \frac{\partial F(q_1, p_2)}{\partial q_1}. \quad (1)$$

Or

$$F = F(q_2, p_1) \quad \Rightarrow \quad q_1 = \frac{\partial G(q_2, p_1)}{\partial p_1}, \quad p_2 = \frac{\partial G(q_2, p_1)}{\partial q_2}. \quad (2)$$

Programs like MARYLIE^{20,32} and MAD^{33,34} convert the Lie transformation to a generating function which expanded to a Taylor series matches the Lie series up to the truncation order. A similar approach is also suited for evaluating differential algebra maps.

The generating function is not unique, as the conversion adds terms above the truncation order, which may become rather large. This can result in large computational errors. Since the equations are implicit for either the positions or the canonical momenta, an iterative method is required to solve for these quantities.

Cremona Map Methods

A Cremona map is a symplectic map the Taylor expansion of which terminates at a finite order. Simple examples are those maps which can be described by generating functions of the form

$$F(q_1, p_1) = q_1 p_2 + G(p_2) \quad \Rightarrow \quad q_2 = q_1 + \frac{\partial G(p_2)}{\partial p_2}, \quad p_2 = p_1. \quad (3)$$

Or

$$F(q_2, p_1) = q_2 p_1 + H(q_2) \quad \Rightarrow \quad q_2 = q_1, \quad p_2 = p_1 + \frac{\partial G(q_2)}{\partial q_2}. \quad (4)$$

where G and H are polynomials. The first type resembles a drift map, e. g.

$$x_2 = x_1 + p_{x1}(1 - p_{t1}/\beta)L \quad (5)$$

and the second type looks like a thin lens kick map. Compositions of such maps also produce Cremona maps, if no terms are truncated.

This method (“Kick factorization”) has been proposed^{35,36} as a tool to replace truncated Taylor maps by a symplectic map composed of simple Cremona maps. The procedure builds a Cremona map which agrees up to the truncation order with the given map. Once the Cremona map is known, it can be evaluated by subsequently applying its “factors”. This is usually much faster than the iterative procedure required to apply a generating function, and introduction of spurious higher-order terms can be controlled more easily.

CORRECTION ALGORITHMS

The working group recommends the following correction procedure:

- Define quantities to be corrected, e. g.:
 - Tune shift with momentum,
 - Variation of lattice functions with momentum,
 - Orbit displacement,
 - Tune shift with amplitude,
 - etc.
- Translate to a mathematical criterion.
- Identify the parameters to change, e. g.:
 - Ordering of magnets,
 - Position of corrector magnets,
 - Excitation of corrector magnets,
 - etc.
- Set up an algorithm and execute it.

COUPLING

It is understood that the most complete information about linear coupling can be obtained from the normal form analysis. However, for certain purposes the formulations below may be useful.

Transverse Linear Coupling

Transverse coupling has been analyzed by several authors.^{6,15,37} Consider the linear transfer map M in two degrees of freedom partitioned into four 2×2 blocks:

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \quad (6)$$

The 4-dimensional phase space vector shall also be partitioned according to the horizontal and vertical planes. Edwards and Teng introduced a “symplectic rotation”

$$R = \begin{pmatrix} I \cos \phi & \bar{R} \sin \phi \\ -R \sin \phi & I \cos \phi \end{pmatrix}, \quad (7)$$

where R is a 2×2 matrix with unit determinant, and $\bar{R}$ denotes its symplectic conjugate:

$$R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad |R| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 1, \quad \bar{R} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}. \quad (8)$$

This leaves three free parameters for the elements of R , and a fourth free parameter ϕ . Edwards and Teng determine R such that M conjugated with R becomes block diagonal:

$$RMR^{-1} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix} \Rightarrow R = \frac{1}{\sqrt{|B + \bar{C}|}}(B + \bar{C}). \quad (9)$$

If $|B + \bar{C}| < 0$ both ϕ and all elements of R become imaginary. This can be avoided by redefining

$$R = \frac{1}{\sqrt{1 + |S|}} \begin{pmatrix} I & \bar{S} \\ -S & I \end{pmatrix}, \quad (10)$$

where

$$S = R \tan \phi, \quad \text{and} \quad \cos \phi = \frac{1}{\sqrt{1 + |S|}}. \quad (11)$$

All four elements of S are free parameters. The solutions is:

$$\begin{aligned} S &= - \left(\frac{1}{2}(\text{Tr } A - \text{Tr } D) + \text{sign}(|B + \bar{C}|) \sqrt{|B + \bar{C}| + \frac{1}{4}(\text{Tr } A - \text{Tr } D)^2} \right)^{-1} \\ &\quad \times (B + \bar{C}), \\ E &= A - BS, \\ F &= D + \bar{S}C. \end{aligned} \quad (12)$$

The block diagonalized matrix can be parametrized as usual. From the eigenvectors of the conjugated system

$$V_1 = \begin{pmatrix} \sqrt{\beta_1} & 0 \\ \frac{\alpha_1}{\sqrt{\beta_1}} & \frac{1}{\sqrt{\beta_1}} \end{pmatrix}, \quad V_2 = \begin{pmatrix} \sqrt{\beta_2} & 0 \\ \frac{\alpha_2}{\sqrt{\beta_2}} & \frac{2}{\sqrt{\beta_2}} \end{pmatrix} \quad (13)$$

one finds the eigenvectors of the coupled system:

$$W_1 = \frac{1}{\sqrt{1 + |S|}} \begin{pmatrix} V_1 \\ \bar{S}V_1 \end{pmatrix}, \quad W_2 = \frac{1}{\sqrt{1 + |S|}} \begin{pmatrix} -SV_2 \\ V_2 \end{pmatrix}. \quad (14)$$

Tracking the Edwards-Teng Functions

For tracking the coupled lattice functions we assume that the transfer matrix for one element is partitioned as above:

$$R_e = \begin{pmatrix} A_e & B_e \\ C_e & D_e \end{pmatrix}, \quad (15)$$

The symplectic rotation at element entrance changes the diagonal blocks to

$$E_e = (A_e - B_e S_1) / \sqrt{|A_e - B_e S_1|}, \quad F_e = (D_e + \bar{S}_1 C_e) / \sqrt{|A_e - B_e S_1|}, \quad (16)$$

and the new coupling matrix at exit becomes

$$S_2 = -(C_e - D_e S_1) \overline{(A_e - B_e S_1)} / |A_e - B_e S_1|. \quad (17)$$

We may track the decoupled lattice functions using the matrices E_e for mode 1 and F_e for mode 2. The above formalism has been extended to three dimensions by Teng.³⁸

Mais-Ripken Lattice Functions

An alternative way to handle coupling has been given by Mais and Ripken.^{39,40} Below an alternate derivation is given. A straightforward description of the three eigenmodes is contained in the three eigenvectors of the one-turn transfer matrix $R(0, C)$:

$$V_k(0) = \begin{pmatrix} v_{1,k} \\ v_{2,k} \\ v_{3,k} \\ v_{4,k} \\ v_{5,k} \\ v_{6,k} \end{pmatrix}, \quad k = 1, 2, 3. \quad (18)$$

They are ordered such that the first one is mainly horizontal, the second one mainly vertical, and the third one mainly longitudinal, and normalized such that

$$\sum_{m=1}^3 (\Re v_{2m-1,k} \Im v_{2m,k} - \Im v_{2m-1,k} \Re v_{2m,k}) = 1, \quad k = 1, 2, 3 \quad (19)$$

The real and imaginary parts of the eigenvectors, placed as columns in a matrix, then form a symplectic matrix. The general initial condition for the eigenmode k is

$$\begin{aligned} Z_k(0) &= \Re(V_k(0) \exp(-2\pi i \mu_k(0))) \\ &= \Re V_k(0) \cos(2\pi \mu_k(0)) + \Im V_k(0) \sin(2\pi \mu_k(0)), \quad k = 1, 2, 3, \end{aligned} \quad (20)$$

where $\mu_k(0)$ is the initial principal phase for this mode. The eigenvectors behave like ordinary trajectories, thus they can be tracked by

$$V_k(s) = R(0, s) V_k(0). \quad (21)$$

The phase for the projection of mode i on plane k is

$$\mu_{i,k}(s) = \arctan \frac{\Im v_{2k-1,i}}{\Re v_{2k-1,i}}. \quad (22)$$

All three projections of an eigenmode have the same phase advance Q_k for one turn around the machine. However, the projections of the eigenmode on the three planes need not all start at the same phase. Mais and Ripken use the projections of the Courant-Snyder lattice functions on the three planes:

$$\begin{aligned} \beta_{i,k}(s) &= v_{2k-1,i} v_{2k-1,i}^* \\ \gamma_{i,k}(s) &= v_{2k,i} v_{2k,i}^*, \\ \alpha_{i,k}(s) &= v_{2k-1,i} v_{2k,i}^* \end{aligned} \quad k, i = 1, 2, 3. \quad (23)$$

Here the index i refers to the eigenmode, and the index k to the plane. Note that the equation

$$\beta_{i,k} \gamma_{i,k} - \alpha_{i,k}^2 = \left(\beta_{i,k} \frac{d\mu_{i,k}}{ds} \right)^2, \quad \text{but} \quad \beta_{i,k} \frac{d\mu_{i,k}}{ds} \neq 1 \quad (24)$$

holds, where $\mu_{i,k}$ is the phase advance for the projection on plane k of the eigenmode i .

Relations between Different Formalisms

Expressing the beam widths in Mais-Ripken form and in Edwards-Teng form one obtains the following relations:

$$\begin{aligned} \beta_{1,x} &= \beta_1 \cos^2 \phi, & \beta_{1,y} &= \beta_1 \sin^2 \phi, \\ \beta_{2,x} &= \beta_2 \sin^2 \phi, & \beta_{2,y} &= \beta_2 \cos^2 \phi, \end{aligned} \quad (25)$$

where $\beta_{i,k}$ is the projection of β_i on the plane k . The apparent emittances for each mode are:

$$\begin{aligned} E_{1,x} &= J_1 \beta_1 \cos^2 \phi, & E_{1,y} &= J_1 \beta_1 \sin^2 \phi, \\ E_{2,x} &= J_2 \beta_2 \sin^2 \phi, & E_{2,y} &= J_2 \beta_2 \cos^2 \phi, \end{aligned} \quad (26)$$

As expected the total transverse energy remains constant for each mode.

OBJECT-ORIENTED PROGRAMMING

Object-oriented methods have been around for at least ten years.^{33,41} The concept was emerging from the evolution of new computer languages. Then the object-oriented languages were not yet generally available, so the concept could only be implemented with inadequate means. Dynamic memory management had to be implemented in FORTRAN, and PASCAL or C structure had to be simulated using PARAMETER and EQUIVALENCE statements.

In the meantime compilers for object-oriented languages like C++,⁴² Smalltalk, Eiffel, and others are generally accessible. This has greatly simplified the situation, and sets of classes for beam optics calculations could be implemented.¹¹

The object-oriented approach allows to build “plug compatible” software modules which may be interchanged without affecting any existing code. Assume we have defined three C++ classes to be used with a tracking program:


```

// virtual base class for beam elements
class element {
public: ...
    // track phase space vector:
    virtual void propagate(vector);
    ...
};

// class for drift spaces:
class drift: public element {
public: ...
    // track phase space vector:
    virtual void propagate(vector);
    ...
private:
    double length;
};

// class for sequences of elements:
class line: public element {
public: ...
    // track phase space vector:
    virtual void propagate(vector);
    ...
private:
    // description of sequence:
    ...
};

```

We have also defined a function to implement propagation through a sequence:

```

void line::propagate(vector v)
{
    // for all members of the sequence do
    ...
    member->propagate(v);
}

```

Having written, compiled and tested this code we may easily add a new element class:

```

// class for Siberian snakes:
class snake: public line {
public: ...
    virtual void propagate(vector);
    ...
private:
    // additional data:
    ...
}

```

```
};
```

```
void snake::propagate(vector v)  
{ ... }
```

No recompilation of existing code is required. Nevertheless, whenever a snake occurs in a line, the function propagate for the line knows that it should call snakes propagate.

CONCLUSIONS AND RECOMMENDATIONS

Map Methods versus Integrals

In the past global quantities like tune shifts or resonance widths were invariably evaluated using single or multiple integrations around the ring. Important terms could easily get lost in this process. An instructive example has been given by Jäger and Möhl,⁴³ when the chromaticity of a small ring (LEAR) was calculated with several programs and compared. Most programs which use integral methods ignore some terms which are important for this type of ring, while all programs using map methods give correct results.

Integral methods are considered to be dangerous and, if available, map methods should be preferred.

Survival Plots

Sometimes the dynamic aperture determined by a survival plot shows a sharp limit, and sometimes it depends logarithmically on the number of turns tracked. The general consensus of the working group is that with non-symplectic tracking it almost certainly exhibits a slope, while with symplectic tracking both cases are possible. In the latter case the Lyapunov exponent may help decide on stability of particles.

Comparison of Programs

One often hears statements like “We have run this problem with the three programs ..., and found that ...”. Since several programs may make the same erroneous assumption, this approach is dangerous, and it is by far better to put some effort to understand one program thoroughly. This allows to assess its validity for the problem to be solved, and have greater confidence in the results.

If programs are compared one should also take into account the different definitions used. Frequent sources of misunderstandings include:

- Different numbering of multipole components in Europe and in the USA.
- Different sign conventions for magnetic fields.
- Including or excluding numeric denominators like $k!$ in multipole coefficients.
- Use of different variables: (x, x', y, y') versus (x, p_x, y, p_y) .
- Dropping factors like $\beta = v/c$ or ignoring terms containing $\gamma^{-1} = m/E$ in programs for electron machines.

Convergence Problems

Truncated Taylor maps are often applied without giving consideration to the convergence problem.⁴⁴ At best it is observed that a certain optimum order gives the best accuracy, but a precise convergence criterion is not stated.

Knowing the poles of the developed function, the convergence radius of a Taylor series in one variable is defined by the magnitude of the pole closest to the origin. A Taylor map with one degree of freedom uses two variables, and the convergence radius in the first variable depends on the value of the second variable. Finding its convergence domain requires the study the locus of the poles in a plane with two complex coordinates. This is only feasible when the exact map is known. The paper by Abell¹ is an example of such a study, it gives good estimates for an example.

A Taylor map in n degrees of freedom yields a series in $2n$ variables. One would expect one has to study the locus of the poles in a $2n$ -dimensional space with complex coordinates, which may be a formidable task. A lot of research remains to be done in this domain.

Measuring Transfer Maps

It has been shown^{3,4,10,15} that it is possible to determine most terms in the coupled linear one-turn map from measurement on the machine. It is important to use the symplecticity of the map together with the results of Fourier analysis. This allows to check the consistency of measurements and to compute some coefficients which are otherwise not available. Even with large measurement errors a reasonable accuracy can be obtained and used for correcting coupling effects.

Object-Oriented Programming

For large machines it is vital that both the computer programs and their data are properly structured. Only under these conditions can the computations be organized in a flexible way, permitting to analyze the whole machine and parts of the machine with equal ease. It has been shown by various authors^{11,33,34,41} that an object-oriented approach promises the most flexible organization.

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