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The Spectral Distortions of the Cosmic Microwave Background Radiation, LEP and Heavy Neutrinos

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ABSTRACT : In the early Universe, energy may have been released into the primordial plasma, generating spectral distortions of the cosmic background radiation (CBR) which, incidentally, would be today the precious signatures of the primeval epochs. In the first three sections, the delicate interplay between radiation and plasma is investigated. When thermal contact is established between photons and electrons, the radiation spectrum relaxes towards a Bose-Einstein distribution as a consequence of inverse Compton scatterings. The spectrum latterly achieves a Planck distribution as bremsstrahlung and double Compton emissions proceed. The μ and y spectral distortions are discussed. Then, the primordial behaviour of particles, such as the neutrino, is generically presented in order to analyse the implications of the recent LEP and COBE results as regards the cosmological rôle of heavy neutrinos. The latter are no longer viable candidates for cosmological dark matter. Finally, in the last section, the hypothetical 17 keV neutrino recently reported in some β -decay experiments faces the COBE constraints on the CBR spectrum.

1 - The Kompaneets equation

As a result of the linearity of Maxwell's equations, electromagnetic waves do not interact with each other. Photons achieve thermal equilibrium because they merely collide with free electrons which, therefore, act as the seeds of the radiation thermalization. The Thomson-Compton cross section may be expressed as a function of the classical radius of the electron $r_0 = e^2/4\pi m_e c^2 = 2.818 \times 10^{-13}$ cm

$$\sigma_T = \frac{8\pi}{3} r_0^2 \simeq 6.67 \times 10^{-25} \text{ cm}^2 . \quad (1)$$

Note that when an incident photon with energy $h\nu \ll m_e c^2$ collides with an electron at rest, the angular distribution of the scattered photon favours the forward and backward directions

$$\frac{1}{\sigma_T} \frac{d\sigma_T}{d\Omega} = \frac{3}{16\pi} (1 + \cos^2\theta) , \quad (2)$$

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where θ denotes the angle between the initial and final photon momenta. Energy is transferred from the photon to the electron

$$\nu - \nu' \simeq \frac{h\nu^2}{m_e c^2} (1 - \cos\theta) . \quad (3)$$

As soon as recombination occurs and electrons neutralize with protons into hydrogen atoms, the cosmic background radiation (CBR) drops out of equilibrium and its features get quenched.

This section is devoted to the intricate interplay between the radiation and the plasma of free electrons. Our aim is to derive the Kompaneets equation [1] which accounts for the relaxation of the photon distribution $\eta(\nu)$ towards a Bose-Einstein spectrum in thermal equilibrium at temperature $T_\gamma = T_e$. We focus on non-relativistic electrons and consider photons with energy $h\nu \ll m_e c^2$.

1.1 - A heuristic approach

To commence, we assume a hot electron gas. Photon energies $h\nu$ are small with respect to the electron temperature T_e . This condition is met for instance in the Rayleigh-Jeans region of the spectrum. We specially focus on an electron which, in the cosmological frame $\mathcal{R}_C$, has velocity $\vec{\beta} = \beta \vec{U}$, with $\beta \ll 1$. When a photon with initial momentum $(h\nu_i/c) \vec{U}$ collides with that electron, its frequency undergoes, on average, a Doppler shift.

In the rest frame $\mathcal{R}_e$ of the electron, the frequency of the incident photon is

$$\nu'_i = \sqrt{\frac{1-\beta}{1+\beta}} \nu_i . \quad (4)$$

Since $h\nu'_i \ll m_e c^2$, relation (3) implies that the frequency ν' is not changed by the scattering in the rest frame $\mathcal{R}_e$. In addition, the final momentum may be crudely assumed, with equal probabilities, either aligned with, or opposite to the incident direction $\vec{U}$. In the first case, the scattered photon points towards the same direction as the electron, and its frequency, in $\mathcal{R}_C$, is not modified. In the opposite situation, the scattered photon and the electron are back to back so that, in $\mathcal{R}_C$, the frequency ν_f is red-shifted with respect to the initial value ν_i . In the cosmological frame $\mathcal{R}_C$, the situation may therefore be summarized by

$$\nu_f(\rightarrow) = \nu_i(\rightarrow) \quad \text{while,} \quad \nu_f(\leftarrow) = \left(\frac{1-\beta}{1+\beta}\right) \nu_i(\rightarrow) . \quad (5)$$

If the photon scatters from the forward direction, the same reasoning applies and we readily infer a blue-shift of the frequency

$$\nu_f(\leftarrow) = \nu_i(\leftarrow) \quad \text{while,} \quad \nu_f(\rightarrow) = \left(\frac{1+\beta}{1-\beta}\right) \nu_i(\leftarrow) . \quad (6)$$

Photons with incident momenta along the same direction as the electron undergo an average frequency shift

$$\langle \nu_f \rangle (\text{after collision}) \simeq \left(2 + \frac{1-\beta}{1+\beta} + \frac{1+\beta}{1-\beta} \right) \left(\frac{\nu_i}{4} \right) , \quad (7)$$

which, once expanded up to second order in the electron velocity β , leads to

$$\frac{\Delta \nu}{\nu} \simeq \beta^2 . \quad (8)$$

Photons with incident momenta transverse to the electron velocity are also, on average, scattered perpendicularly so that their frequencies are not shifted. That possibility has two chances out of three of occurring in each collision.

Since every photon undergoes $\sigma_T c n_e$ collisions each second, a third of which only are not sterile and lead to the frequency shift (8), we infer the average increase

$$\langle \Delta \nu \rangle \simeq \frac{1}{3} (\sigma_T c n_e) \langle \beta^2 \rangle \nu \Delta t , \quad (9)$$

during the time interval Δt . Since electrons, whose density is denoted by n_e , behave as a perfect gas, the mean value $\langle \beta^2 \rangle$ may be expressed as a function of T_e and m_e . The parameter y is defined as

$$dy = \frac{d\nu}{\nu} = (\sigma_T c n_e) \left(\frac{k T_e}{m_e c^2} \right) dt = \frac{dt}{t_C} , \quad (10)$$

where t_C is the typical time scale for the inverse Comptonization of the photons. As is clear from expressions (9) and (10), thermal contact between a hot electron gas and the radiation leads to the so-called y spectral distortion [2]

$$\nu(t) = \nu(0) e^y , \quad (11)$$

where the parameter y evolves according to

$$y = \int \frac{dt}{t_C} . \quad (12)$$

Since the frequency of the distorted spectrum is rescaled upwards by the factor e^y , the number of photons per logarithmic interval $d \text{Log} \nu$ is constant as is clear on the upper panel of figure 1. The solid line stands for a pure black-body spectrum at temperature T_0

$$\frac{dn}{d \text{Log} \nu} = \frac{\nu^3}{e^{h\nu/kT_0} - 1} , \quad (13)$$

while the short dash curve exhibits a $y = 0.5$ distortion and is shifted towards the high energy region of the diagram.

Figure 1

Upper panel : A Planck distribution (solid curve) experiences a $y = 0.5$ spectral distortion which results into a shift of the spectrum (dashed curve) towards higher frequencies.

Lower panel : In the Rayleigh-Jeans region, the effective black-body temperature $T_{\text{eff}}(\nu)$ of the distorted spectrum (dashed curve) is shifted downwards by an e-fold with respect to the initial temperature T_0 (solid line).

The lower panel displays the effective black-body temperature T_{eff} . For the pure Planck spectrum (solid line), the temperature has constant value $T_{eff} = T_0$. Since photons collide with hot electrons and gain energy, high frequencies are populated at the expense of low energies. If on the one hand, the distorted spectrum (short dash curve) exhibits an excess in the Wien region, on the other hand, the Rayleigh-Jeans part is deficient. Note that T_{eff} is indeed shifted downwards with respect to the pure black-body curve. For low frequencies, the spectrum merely reduces to $\propto \nu^2 T$, so that the effective Rayleigh-Jeans temperature decreases by a factor e^{-2y}

$$T_{RJ}(\nu) = e^{-2y} T_0 . \quad (14)$$

Since in our example $y = 0.5$, the Rayleigh-Jeans temperature is shifted downwards by an e-fold, as is clear in figure 1.

We have analysed the spectral distortion generated by the thermal contact between hot electrons and low energy photons with $h\nu \ll kT_e$. We have neglected the frequency shift which photons undergo in the electron rest frame in which, moreover, the radiation is *not* isotropic. We have also disregarded stimulation effects which may be important during Thomson collisions. Notice, however, that the y -distortion effect is typical of the Rayleigh-Jeans region of the spectrum for which the condition $h\nu \ll kT_e$ is always fulfilled.

1.2 - The energy transfer between electrons and photons

We first assume a pure black-body radiation with temperature T_0 . In the cosmological frame $\mathcal{R}_C$, the radiation is isotropic. However, in the rest frame $\mathcal{R}_e$ of an electron moving with velocity $\vec{\beta}$, photons from the forward direction are blue-shifted, while those from the backward direction are red-shifted. In $\mathcal{R}_C$, the photon distribution is

$$\eta(\nu) = \frac{1}{e^{h\nu/kT_0} - 1} . \quad (15)$$

Since that distribution is a Lorentz invariant, it may be expressed as

$$\eta(\nu', \theta) = \frac{1}{e^{h\nu'/kT_\theta} - 1} \quad (16)$$

in the electron rest frame $\mathcal{R}_e$. The angle between the electron and photon momenta is denoted by θ , and ν' stands for the frequency. We readily infer an anisotropy of the temperature distribution in $\mathcal{R}_e$

$$T_\theta \simeq (1 - \beta \cos \theta) T_0 , \quad (17)$$

for non-relativistic $\beta \ll 1$ electrons. In $\mathcal{R}_e$, photons with initial direction θ transfer to the electron an average momentum $(h\nu'/c) \cos \theta$ per collision. Since there are many more

photons coming from the forward ($\theta = \pi$) than from the backward ($\theta = 0$) directions, the net transfer of momentum results into the drag force

$$F_{drag} = \int \left(\frac{h\nu'}{c} \cos\theta \right) (\sigma_T c) \left(\eta(\nu', \theta) \frac{2\nu'^2 d\nu' d\Omega}{c^3} \right) . \quad (18)$$

Once frequencies have been integrated out, that expression simplifies

$$F_{drag} = \left(\frac{8\pi^5}{15} \frac{k^4}{h^3 c^3} \right) T_0^4 \sigma_T \int \left(\frac{T_\theta}{T_0} \right)^4 \cos\theta \left(\frac{d\Omega}{4\pi} \right) , \quad (19)$$

and the photon energy density ρ_γ may be factored out. The drag force is therefore a mere function of the electron velocity β and of the energy density ρ_γ

$$\vec{F}_{drag} = -\frac{4}{3} \sigma_T \rho_\gamma \vec{\beta} . \quad (20)$$

Radiation acts as a viscous medium with respect to the electrons which experience a damping of their propagation. That friction results into the heating of the photon gas at the expense of the electrons, whose kinetic energy is transferred to the radiation with the rate

$$\epsilon_{e \rightarrow \gamma} = \langle F_{drag} \beta c \rangle n_e , \quad (21)$$

per unit volume. For non-relativistic free electrons, that energy transfer simplifies finally into

$$\epsilon_{e \rightarrow \gamma} = 4 \rho_\gamma (\sigma_T c n_e) \left(\frac{kT_e}{m_e c^2} \right) = 4 \rho_\gamma \frac{dy}{dt} . \quad (22)$$

The careful reader will have noticed that stimulation is absent from our discussion. However, we have not neglected that important effect. In $\mathcal{R}_e$, if a photon with momentum $\vec{p}_1$ scatters into the state $\vec{p}_2$, it gives to the electron a momentum proportional to $\eta_1 (1 + \eta_2) (\vec{p}_1 - \vec{p}_2)$. The reverse reaction is possible and results in the momentum transfer $\eta_2 (1 + \eta_1) (\vec{p}_2 - \vec{p}_1)$. The stimulation factors vectorially cancel because $d\sigma_T/d\Omega$ depends only on the angle $(\vec{p}_1, \vec{p}_2)$ between the photon momenta. In addition, since $h\nu \ll m_e c^2$, frequencies in the initial and final states are identical and $\nu_1 = \nu_2$.

For a hot electron gas, i.e., $T_\gamma \ll T_e$, the energy flow from radiation to electrons is negligible. The energy density ρ_γ evolves therefore as

$$d\rho_\gamma \simeq \epsilon_{e \rightarrow \gamma} dt = 4 \rho_\gamma dy , \quad (23)$$

and exponentially increases with the y parameter

$$\rho_\gamma(t) = \rho_\gamma(0) e^{4y} . \quad (24)$$

As a matter of fact, we could have readily guessed that result. Since $T_\gamma \ll T_e$, a y -distortion obtains and frequencies are shifted according to the exponential growth (11).

Remember that ρ_γ is an integral over $\nu^3 d\nu$, and is proportional to the fourth power of the average frequency which increases as e^ν .

Before COBE, the X-ray background was understood as the thermal emission of very hot electrons with $T_e \sim 300$ keV, heated up by, say, quasars at a red-shift $z \sim 5$. An exciting signature of this model is the resulting y spectral distortion of the CBR which was predicted to be

$$y = \frac{\sigma_T c n_e^0}{H^0} \Omega_{IGM} h \frac{kT_e}{m_e c^2} \int_0^5 \sqrt{1+z} dz , \quad (25)$$

for a flat Universe. The parameter h is the Hubble constant expressed in units of $H^0 = 100$ km/s/Mpc while Ω_{IGM} stands for the density of the inter-galactic medium. For $\Omega_{IGM} \sim 0.2$ and $h = 1/2$, the distortion was inferred to be $y \simeq 0.07 (kT_e/m_e c^2)$. COBE [3] has placed the stringent constraint $y \leq 10^{-3}$, so that the inter-galactic medium temperature cannot exceed ~ 10 keV. Hot electrons are therefore ruled out as a plausible source of the X-ray background.

In order to derive the energy transfer $\epsilon_{\gamma \rightarrow e}$ from the radiation to the electrons, we will assume the latter at rest in the cosmological frame $\mathcal{R}_C$. When a photon with energy $h\nu$ collides with an electron, it loses an average energy

$$h\Delta\nu \simeq \left(\frac{h\nu}{m_e c^2} \right) h\nu , \quad (26)$$

where the factor $\cos\theta$ of relation (3) has been integrated out. That energy transfer is second order in β since, if we grossly assume $T_\gamma \sim T_e$, the fraction $(h\nu/m_e c^2) \sim (kT_\gamma/m_e c^2)$ and is of order $(kT_e/m_e c^2) \sim \beta^2$. Stimulation effects do not cancel now and, after integration over the photon distribution $\eta(\nu)$, the rate $\epsilon_{\gamma \rightarrow e}$ may be expressed as

$$\epsilon_{\gamma \rightarrow e} = \sigma_T c n_e \int \frac{8\pi\nu^2}{c^3} \frac{(h\nu)^2}{m_e c^2} \eta(\nu) [1 + \eta(\nu)] d\nu . \quad (27)$$

1.3 - The Kompaneets equation

A proper derivation of the Kompaneets equation [1] may be found, for instance, in [4] and is fairly lengthy. We will obtain that equation quite crudely, still rapidly. Frequencies may be conveniently expressed in units of the electron temperature so that we define

$$x = \frac{h\nu}{kT_e} . \quad (28)$$

The energy density of the photon bath is

$$\rho_\gamma = 8\pi \frac{(kT_e)^4}{(hc)^3} \int x^3 \eta(x) dx , \quad (29)$$

and varies according to the energy balance between radiation and electrons

$$\frac{d\rho_\gamma}{dt} = \epsilon_{e \rightarrow \gamma} - \epsilon_{\gamma \rightarrow e} = 8\pi \frac{(kT_e)^4}{(hc)^3} \int x^3 dx (4 - x - x\eta) \frac{\eta}{t_C} . \quad (30)$$

The variation of ρ_γ is directly related to the time derivative of the photon distribution so that, after integration by part of the various integrals, the Kompaneets equation obtains

$$t_C \left. \frac{\partial \eta}{\partial t} \right|_K = \frac{1}{x^2} \frac{\partial}{\partial x} \left\{ x^4 \left(\frac{\partial \eta}{\partial x} + \eta + \eta^2 \right) \right\} , \quad (31)$$

where $\partial \eta / \partial t$ is expressed as a function of the first and second derivatives of the photon distribution η with respect to the frequency x .

As an illustration, we derive directly from the Kompaneets equation the time evolution of the Rayleigh-Jeans temperature, *i.e.*, relation (14). The distribution of low-frequency photons at temperature T_γ simplifies into

$$\eta(x) \simeq \frac{kT_\gamma}{h\nu} = \frac{\alpha(t)}{x} , \quad (32)$$

where the time-dependent ratio T_γ/T_e is denoted by α , and is assumed to be very small with respect to 1. In the Rayleigh-Jeans region of the spectrum, the ordering $x \ll \alpha(t) \ll 1$ obtains, and we may neglect η and η^2 with respect to $\partial \eta / \partial x$ in equation (31). The evolution of α is readily inferred to be

$$t_C \frac{d\alpha}{dt} = -2\alpha(t) , \quad (33)$$

and leads to relation (14) with an exponential decrease of the Rayleigh-Jeans temperature.

Note finally that, under the action of Thomson-Compton collisions, a photon spectrum evolves until it reaches the equilibrium configuration for which

$$\frac{\partial \eta}{\partial x} + \eta + \eta^2 = 0 . \quad (34)$$

The previous relation is satisfied for a Bose-Einstein spectrum

$$\eta_{BE}(x) = \frac{1}{e^{(x+\mu)} - 1} , \quad (35)$$

where the constant of integration μ of the differential equation (34) may be translated into the chemical potential $-(\mu \times kT_e)$ of the Bose-Einstein distribution.

2 - The relaxation of the chemical potential $\mu \rightarrow 0$

When heat is injected into radiation, the average energy per photon increases and a spectral distortion develops. After a time $\sim t_C$, Thomson collisions redistribute the additional

energy, and the spectrum relaxes towards a Bose-Einstein distribution with chemical potential $\mu > 0$ and temperature $T_\gamma = T_e$. Thomson collisions conserve the number of photons. If they operate alone, the spectrum (35) does not evolve towards the pure black-body shape

$$\eta_{PI}(x) = \frac{1}{e^x - 1} . \quad (36)$$

In fact, as a result of the combined action of bremsstrahlung and double Compton reactions, for which the number of photons may increase, μ relaxes towards 0.

2.1 - The bremsstrahlung process

The bremsstrahlung reaction $e p \rightarrow e p \gamma$ replenishes the radiation with additional photons. If an electron is accelerated by an ion of charge Z_i , the corresponding electric dipole moment $\vec{d}$ of the electron-ion pair radiates a power

$$P = \frac{2}{3c^3} \frac{\partial^2 d}{\partial t^2} , \quad (37)$$

so that the total energy emitted at frequency $\nu = \omega/2\pi$ during the passage of the electron near the ion is

$$\frac{dW}{d\omega} = \frac{8\pi}{3c^3} \omega^4 \hat{d}^2(\omega) . \quad (38)$$

Note that $\hat{d}$ stands for the Fourier transform of the dipole moment

$$\omega^2 \hat{d}(\omega) = \frac{Z_i e^3}{\pi m_e v b} , \quad (39)$$

where an undeflected electron trajectory with velocity v and parameter of impact b , has been assumed. Each electron radiates an energy dW per second

$$\frac{dW}{d\omega} = \frac{16}{3c^3} \frac{Z_i^2 e^6}{m_e^2} \frac{n_i}{v} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} , \quad (40)$$

where n_i is the density of ions. Above $b_{\max} \sim v/\omega$, the period associated to the frequency ω is so small with respect to the duration of the passage of the electron in the vicinity of the ion, that the Fourier transform of $\vec{d}$ is completely chopped out and vanishes. On the other hand, if b is too small, deflection may be important, and quantum effects come into play as soon as b gets smaller than the de Broglie wavelength $\hbar/m_e v$ of the electron. Integration over those electrons whose kinetic energy exceeds $\hbar\omega$ leads to the bremsstrahlung evolution equation

$$t_{\gamma e} x^3 \left. \frac{\partial \eta}{\partial t} \right|_B = Q e^{-x} g(x) [1 - \eta(x, t)(e^x - 1)] , \quad (41)$$

with the Q factor of the form

$$Q = \frac{\alpha}{(2\pi)^{7/2}} \left(\frac{\hbar c}{kT_e} \right)^3 \left(\frac{m_e c^2}{kT_e} \right)^{1/2} \sum n_i Z_i^2 . \quad (42)$$

The Gaunt factor $g(x)$ may be expressed as

$$g(x) = \text{Log} \left(\frac{2.2}{x} \right) \quad x \leq 1$$

while,

$$g(x) = \frac{\text{Log}(2.2)}{\sqrt{x}} \quad x \geq 1 . \quad (43)$$

The typical collision time of photons with electrons is denoted by $t_{\gamma e} = (n_e c \sigma_T)^{-1}$. Note that the bremsstrahlung production and absorption processes are in balance as soon as a Planck distribution obtains.

2.2 - The Double Compton reaction

The differential cross section of the *double* Compton reaction $e \gamma(\nu') \rightarrow e \gamma(\nu_1) \gamma(\nu_2)$ is of order α with respect to the Thomson-Compton cross section

$$\frac{d\sigma_{DC}}{d\nu_2} = \frac{4\alpha}{3\pi} \left(\frac{h\nu'}{m_e c^2} \right)^2 (1 - \cos\theta_1) \frac{1}{\nu_2} d\sigma_T . \quad (44)$$

One of the final photons has approximately the same energy as its progenitor, i.e., $\nu_1 \simeq \nu'$. When integrated over the photon and electron distributions, the evolution of the radiation spectrum owing to double Compton emission and absorption takes the form [5]

$$t_{\gamma e} x^3 \frac{\partial \eta}{\partial t} \Big|_{DC} = \frac{4\alpha}{3\pi} \left(\frac{kT_e}{m_e c^2} \right)^2 \mathcal{I}(t) [1 - \eta(x, t)(e^x - 1)] . \quad (45)$$

The integral $\mathcal{I}(t)$ over the field distribution of photons is the driving term for the double Compton process

$$\mathcal{I}(t) = \int x'^4 \eta(x', t) [1 + \eta(x', t)] dx' , \quad (46)$$

where $x' = h\nu'/kT_e \simeq h\nu_1/kT_e$. Should not there be a single photon, double Compton would be completely ineffective.

2.3 - The relaxation of a Bose-Einstein distribution towards a Planck spectrum

Suppose that at some initial time $t = 0$, a pure black-body radiation is thermally coupled to a hot plasma of ions and electrons at temperature $T_e > T_\gamma$. The photon distribution $\eta(x, t)$ evolves under the combined action of Thomson collisions, and bremsstrahlung and double Compton reactions

$$\frac{\partial \eta}{\partial t} \Big|_{tot} = \frac{\partial \eta}{\partial t} \Big|_K + \frac{\partial \eta}{\partial t} \Big|_B + \frac{\partial \eta}{\partial t} \Big|_{DC} , \quad (47)$$

where $x = h\nu/kT_e$. This equation must be integrated numerically. The main features of the evolution of $\eta(x, t)$ may nevertheless be sketched. Comptonization stops as soon as a Bose-Einstein spectrum is established, while bremsstrahlung and double Compton drive the photon distribution towards a Planck spectrum.

If on the one hand, Comptonization occurs on the time scale t_C , on the other hand, bremsstrahlung and double Compton absorptions are respectively associated with the time scales

$$t_B^{abs} = \frac{x^2}{Q g(x)} t_{\gamma e} \quad (48)$$

and

$$t_{DC}^{abs} = \frac{3\pi}{4\alpha} \left(\frac{m_e c^2}{kT_e} \right)^2 \mathcal{I}^{-1}(t) x^2 t_{\gamma e} , \quad (49)$$

in the low energy part of the spectrum. Provided its frequency is lower than the critical value

$$x_B = \left\{ g(x_B) \frac{Q}{8} \left(\frac{m_e c^2}{kT_e} \right) \right\}^{1/2} , \quad (50)$$

for which $t_B^{abs}(x_B) = t_C/8$, a low energy photon is more likely to be absorbed by inverse bremsstrahlung than to migrate towards higher frequencies. Below x_B , bremsstrahlung is therefore more efficient than electron scattering, and a Planck spectrum readily obtains. For the double Compton process, that critical frequency is

$$x_{DC} = \left\{ \frac{\alpha}{6\pi} \left(\frac{kT_e}{m_e c^2} \right) \mathcal{I}(t) \right\}^{1/2} . \quad (51)$$

As shown in [5], double Compton dominates bremsstrahlung whenever

$$\frac{n_\gamma}{n_e} \geq 0.1 \left(\frac{m_e c^2}{kT_e} \right)^{5/2} , \quad (52)$$

i.e., for a hot and diluted gas. For a colder and denser medium, bremsstrahlung takes over.

Lightman [5] has numerically investigated the evolution of an initial radiation spectrum embedded inside a homogeneous plasma at temperature $kT_e = m_e c^2/10$ and density $n_e = n_i = 10^{23} \text{ cm}^{-3}$, under the combined action of electron scattering and bremsstrahlung (figure 2) or double Compton emissions (figure 3).

In figure 2, no photons are present initially. After $t \sim 0.4 t_{\gamma e}$, the low energy part of the spectrum has already relaxed towards a pure black-body distribution. Below x_B , photons are created and, due to electron scatterings, they migrate from the Rayleigh-Jeans region to higher frequencies. The spectrum eventually achieves a Planck form (dashed line) at $t \sim 1.5 \times 10^6 t_{\gamma e}$. Note that x_B is essentially constant during the evolution.

Figure 2

Time evolution of the radiation spectrum in a homogeneous plasma under the processes of bremsstrahlung emission and absorption and of electron scattering. No photons are present initially. Note that x_B is nearly constant. The photon distribution eventually achieves a Planck spectrum (dashed curve) at $t \sim 1.5 \times 10^6 t_{re}$.

Figure 3

Time evolution of the radiation spectrum in a homogeneous plasma under the processes of double Compton emission and absorption and of electron scattering. A small number of seed photons is assumed initially at $x = 1$. Note the increase of x_{DC} and the self-similar evolution of the photon distribution which rapidly relaxes towards a Planck spectrum.

Under the action of double Compton processes, the radiation spectrum of figure 3 evolves self-similarly. The initial condition is a small number of seed photons at $x = 1$. Since field photons are scarce at early times, the evolution starts slowly. The larger $\mathcal{I}(t)$, the faster the relaxation of the photon distribution. The Planck spectrum obtains a hundred times faster than for pure bremsstrahlung emission. As may be guessed from relation (51), the critical value x_{DC} increases dramatically with $\mathcal{I}(t)$, in contrast with the evolution of figure 2 where x_B is constant.

3 - The cosmological distortions of the microwave background spectrum

3.1 - The various time scales

As discussed above, the typical collision time of photons with free electrons is $t_{\gamma e} = (n_e c \sigma_T)^{-1}$. When recombination proceeds, the electron density n_e drops down while $t_{\gamma e}$ increases.

The redistribution of energy among photons, described by the Kompaneets equation (31), is due to the inverse Compton scatterings of low frequency photons by energetic electrons, and is associated with the typical time scale

$$t_C \simeq \left(\frac{m_e c^2}{k T_e} \right) t_{\gamma e} . \quad (53)$$

The evolution of the electron temperature is due to the energy balance between matter and radiation

$$3n_e k \frac{dT_e}{dt} = \epsilon_{\gamma \rightarrow e} - \epsilon_{e \rightarrow \gamma} . \quad (54)$$

Note that protons contribute an additional factor $3kn_e/2$ to the heat capacity of the plasma. The electron temperature evolves according to the differential equation

$$\frac{dT_e}{dt} + \frac{4\sigma_T \rho_\gamma}{3m_e c} (T_e - T_e^{equi}) = 0 , \quad (55)$$

and relaxes towards its kinetic equilibrium value

$$T_e^{equi} = \int \frac{8\pi\nu^2}{c^3} \frac{(h\nu)^2}{4k\rho_\gamma} \eta(\nu) [1 + \eta(\nu)] d\nu , \quad (56)$$

with the typical time scale

$$t_{e\gamma} = \frac{3m_e c}{4\sigma_T \rho_\gamma} . \quad (57)$$

As shown by Lightman [5], the typical time scale on which bremsstrahlung may replenish a deficient photon population can be expressed as

$$t_B \simeq \frac{2\zeta(3)}{Q} g^{-2}(x_B) t_{\gamma e} , \quad (58)$$

while the restoration of a Planck distribution by double Compton emission occurs on the time scale

$$t_{DC} \simeq \frac{\pi}{8\alpha} \left(\frac{m_e c^2}{kT_e} \right)^2 t_{\gamma e} \quad (59)$$

In figure 4, those various time scales are displayed as a function of the red-shift $1 + z = T_\gamma(t)/2.375$ K, and are compared with the corresponding age $t_U(z)$ of the Universe [6]. The expansion equation has been integrated numerically with two light neutrino species, radiation and a baryon density of $n_B/n_\gamma = 3 \times 10^{-10}$, in agreement with the primordial nucleosynthesis estimates.

3.2 - The γ and μ distortions

In figure 4, the solid line stands for the age t_U of the Universe. Note that $t_{e\gamma}$ (dotted curve) is always much smaller than the other relevant time scales, so that the electron temperature T_e relaxes rapidly towards its equilibrium value at any given time and for any given photon distribution.

Photons stop to collide with electrons after recombination. For a red-shift $z \geq 10^3$, $t_{\gamma e}$ (short dashed curve) is smaller than the typical expansion time t_U . However, thermalization of the photon bath is only effective above a red-shift $z_{BE} \sim 2 \times 10^5$ below which the inverse Comptonization time scale t_C (long dashed curve) exceeds the expansion time t_U . The red-shift z_{BE} corresponds to an age of $\sim 10^9$ sec.

For the low baryon abundance which we have adopted, double Compton emission is more efficient than bremsstrahlung. Indeed, the time scale t_B (dotted - short dashed curve) always exceeds t_U . On the other hand, double Compton processes restore the Planck form of the radiation spectrum above the critical red-shift $z_{PI} \sim 1 - 2 \times 10^7$, at which t_{DC} (short dashed - long dashed curve) crosses t_U . The red-shift z_{PI} is associated with an age of $\sim 10^5$ sec.

We may therefore sketch the scenario of the spectral distortions of the microwave background radiation. Four possibilities may occur, depending on when the energy is released into the primordial plasma.

Figure 4

The age of the Universe t_U (solid) is presented as a function of the red-shift $(1 + z)$ and is compared to the relevant time scales $t_{e\gamma}$ (dotted), $t_{\gamma e}$ (short dashed), t_C (long dashed), t_{DC} (short dashed - long dashed) and finally t_B (dotted - short dashed).

$$1) z \geq z_{PI}$$

Double Compton emission acts efficiently and any spectral distortion is erased during an expansion time. No matter how large the energy release is, the microwave background is not affected.

$$2) z_{BE} \leq z \leq z_{PI}$$

Any heat release generates a μ -distortion. The radiation spectrum relaxes towards a Bose-Einstein distribution faster than the Universe expands ($t_C \leq t_U$). However, a pure black-body radiation cannot be achieved since double Compton emission is slower than expansion ($t_{DC} \geq t_U$). The photon bath is still in thermal equilibrium, but with an average photon energy larger than for a Planck spectrum, which translates into a non-zero chemical potential μ . In units where $\hbar = k = c = 1$, the energy ρ_{BE} , number n_{BE} and entropy σ_{BE} densities may be expressed as

$$\rho_{BE} = \frac{T_e^4}{\pi^2} I_3(\mu), \quad n_{BE} = \frac{T_e^3}{\pi^2} I_2(\mu) \quad \text{and} \quad \sigma_{BE} = \frac{T_e^3}{\pi^2} \left\{ \frac{4}{3} I_3(\mu) + \mu I_2(\mu) \right\}, \quad (60)$$

where the integral $I_n(\mu)$ is defined as

$$I_n(\mu) = \int_0^\infty x^n (e^x + \mu - 1)^{-1} dx. \quad (61)$$

If $\Delta\rho_\gamma$ (or Δn_γ) denotes the increase of the energy (or number) density due to the heat release, the plasma temperature T_e and chemical potential μ evolve according to

$$\begin{aligned} \Delta n_\gamma &= \left(\frac{\partial n_{BE}}{\partial T_e} \right) dT_e + \left(\frac{\partial n_{BE}}{\partial \mu} \right) d\mu \\ \text{while} & \\ \frac{\Delta \rho_\gamma}{T_e} &= \left(\frac{\partial \sigma_{BE}}{\partial T_e} \right) dT_e + \left(\frac{\partial \sigma_{BE}}{\partial \mu} \right) d\mu. \end{aligned} \quad (62)$$

Note that

$$\frac{dI_n}{d\mu} = -n I_{n-1}(\mu). \quad (63)$$

Since in the practical cases of interest μ is small – COBE [3] sets the stringent constraint $\mu \leq 0.009$ – the previous integrals may be evaluated at $\mu = 0$ and simplify into $I_1(0) = \pi^2/6$, $I_2(0) = 2\zeta(3)$ and $I_3(0) = \pi^4/15$.

The electron temperature T_e relaxes towards the radiation temperature T_γ much faster than expansion. In general, high energy photons are injected into the plasma so that $\Delta n_\gamma \ll n_\gamma$. This leads to a unique relation between T_e and μ , and we may therefore express the variation of the chemical potential parameter μ as a function of the relative increase of the photon energy

$$\frac{\Delta \rho_\gamma}{\rho_\gamma} = \left\{ \left(\frac{2\pi^2}{9\zeta(3)} \right) - \left(\frac{90\zeta(3)}{\pi^4} \right) \right\} d\mu \simeq 0.714 d\mu. \quad (64)$$

$$3) \ z_{rec} \leq z \leq z_{BE}$$

Both t_{DC} and t_C exceed the expansion time t_U . Photons may still collide with electrons so that a y -distortion ensues, which corresponds to the small value $y \sim t_U/t_C$. If energy is released in the primordial plasma, electrons and protons are heated up to the temperature $T_e \gg T_\gamma$. As a result of rare inverse Compton reactions, the Rayleigh-Jeans region is slightly depleted by a factor e^{-2y} with respect to the rest of the spectrum. Remember that COBE [3] sets the bound $y \leq 10^{-3}$.

$$4) \ z \leq z_{rec}$$

Below $z_{rec} \sim 10^3$, free electrons combine with protons. Figure 4 has assumed a constant ionization fraction $x_e = 1$. In fact, as soon as recombination drives the population of free electrons to extinction, $t_{\gamma e}$ exceeds t_U and the primordial radiation decouples from the neutralized matter. The microwave spectrum gets quenched and its features are fossilized until now. The cosmic background radiation is therefore of crucial importance as regards the thermal history of the Universe. It provides us with a unique probe of its recent evolution.

4 - The primordial behaviour of particles

As observed by the COBE satellite, the CBR is very close to a pure black body radiation. We would like to investigate now the constraints set by the CBR observations upon the properties of particles such as neutrinos. What we need to understand is the generic behaviour of particles in the early Universe. As we shall see, species decouple from equilibrium and get quenched. The possibility of predicting the present abundance of their remnants is quite exciting. As an example, the CBR is the fossil radiation of the primordial photon gas which decoupled from baryons at recombination.

4.1 - The equilibrium density

A denotes a generic particle with g_A helicity states. Collisions of A particles with other species imply their thermalization at temperature T . Moreover, A may annihilate with antipartners $\bar{A}$ in, say, electron-positron pairs. If the following set of reactions

$$2\gamma \rightleftharpoons e^+e^- \rightleftharpoons A + \bar{A} \rightleftharpoons e^+e^- \rightleftharpoons 3\gamma \quad (65)$$

is in chemical equilibrium, the various chemical potentials are given by

$$\mu_A + \mu_{\bar{A}} = \mu_{e^+} + \mu_{e^-} = 2\mu_\gamma = 3\mu_\gamma . \quad (66)$$

We readily conclude that the equilibrium of reactions (65) implies a vanishing μ_γ and, therefore, a Planck spectrum for the primordial fire-ball. Moreover, the chemical potentials of matter and antimatter are related to each other by

$$\mu_A = -\mu_{\bar{A}} . \quad (67)$$

If we assume furthermore that there is no asymmetry between the species A and $\bar{A}$, which is indeed the case for baryons whose density is small, $n_B/n_\gamma = 3 \times 10^{-10}$, we infer $\mu_A = \mu_{\bar{A}} = 0$. The number density of A 's at equilibrium may therefore be expressed as

$$n_A^e = \int \frac{d^3\vec{p}}{8\pi^3} g_A \frac{1}{e^{E/T} + \epsilon} , \quad (68)$$

where $\epsilon = 1$ (fermions) or -1 (bosons). The energy E is related to the momentum p since $E = \sqrt{M^2 + p^2}$, where M denotes the mass of the particle. Depending on the ratio M/T , two regimes may be distinguished.

• *The ultra-relativistic limit*

When $M/T \rightarrow 0$, i.e., for high temperatures and momenta, the A population is ultra-relativistic. Since $E \simeq p$, the previous expression simplifies into

$$n_A^e = \frac{g_A}{\pi^2} \zeta(3) T^3 \left\{ 1 \text{ (B)} \text{ or } \frac{3}{4} \text{ (F)} \right\} . \quad (69)$$

Since at early times $T \propto R^{-1}$, the codensity of an ultra-relativistic population, i.e., the density of particles per volume which expands with the expanding Universe, is constant

$$n_A^e R^3 \propto \frac{n_A^e}{T^3} = f_A^e = \frac{g_A}{\pi^2} \zeta(3) \left\{ 1 \text{ (B)} \text{ or } \frac{3}{4} \text{ (F)} \right\} . \quad (70)$$

Since $(k/\hbar c)^3 = 83.22 \text{ cm}^{-3} \text{ K}^{-3}$, the codensity is $\simeq 20 \text{ cm}^{-3} \text{ K}^{-3}$ for bosons with two helicity states – as photons – and $\simeq 15 \text{ cm}^{-3} \text{ K}^{-3}$ for fermions. We readily infer from the value of the CBR temperature $T_\gamma^0 = 2.735 \text{ K}$ measured by COBE [3] that the photon density, today, is $\sim 400 \text{ cm}^{-3}$.

• *The non-relativistic regime*

For large values of M/T , the codensity is exponentially suppressed by a factor e^{-a} .

$$f_A^e = \frac{g_A}{(2\pi)^{3/2}} a^{3/2} e^{-a} , \quad (71)$$

where $a = M/T$. In the non-relativistic regime, bosons and fermions have the same behaviour.

4.2 - The thermal decoupling

Thermalization is achieved by the numerous collisions of A 's with the other species. By numerous, we mean that the collision rate exceeds the expansion rate so that the temperature T_A has plenty of time to relax towards the temperature of the other populations. In illustration of this idea, we present the thermal behaviour of neutrinos. The rate of their collisions with electrons is given by

$$\Gamma_C = c\sigma_C n_e \simeq (G_F^2 T^2) \times \left(\frac{3}{\pi^2} \zeta(3) T^3\right) \simeq 4.9 \times 10^{-23} \text{ MeV} \left(\frac{T}{1 \text{ MeV}}\right)^5, \quad (72)$$

where the Fermi constant is $G_F = 1.16 \times 10^{-11} \text{ MeV}^{-2}$. At early times, the curvature of space may be neglected so that the Hubble parameter may be expressed as

$$H = \left\{ \frac{8\pi}{3} \frac{\rho}{M_{Pl}^2} \right\}^{1/2} = \left\{ \frac{8\pi^3}{45} g_{eff} \right\}^{1/2} \frac{T^2}{M_{Pl}} \simeq 4.5 \times 10^{-22} \text{ MeV} \left(\frac{T}{1 \text{ MeV}}\right)^2. \quad (73)$$

The Planck mass is $M_{Pl} = 1.22 \times 10^{22} \text{ MeV}$ while the energy density ρ has been expressed in units of the photon energy density $\rho_\gamma = \pi^2 T^4/15$, hence an effective number of degrees of freedom $g_{eff} = 43/8$. The collision rate can be compared now with the expansion rate

$$\frac{\Gamma_C}{H} \simeq \left(\frac{T}{2 \text{ MeV}}\right)^3, \quad (74)$$

and we readily infer a decoupling temperature of $\sim 2 \text{ MeV}$ below which collisions are so rare that neutrinos no longer see their surroundings and become a fossil radiation.

4.3 - Annihilation and the chemical freeze-out

If neutrinos have a mass M larger than the decoupling temperature of $\sim 2 \text{ MeV}$, they may substantially annihilate before they freeze out. As an illustration, we focus on heavy neutrinos with a mass $M = 2 \text{ GeV}$.

At high temperature, for $T > 2 \text{ GeV}$, heavy neutrinos are in chemical equilibrium. They steadily annihilate into $f\bar{f}$ pairs while the reverse process is also very active. The annihilation - production reaction

is in equilibrium and the A 's density relaxes towards its equilibrium value (68).

Below 2 GeV , A and $\bar{A}$ annihilate. As long as the chemical reaction (75) is in equilibrium, the A 's density is given by relation (68). As the temperature decreases, the A 's are significantly depleted by annihilation and their density drops down. The antiparticles $\bar{A}$ with which A 's annihilate, become rare also.

At $T \simeq 100$ MeV, the density n_A is so low, particles and antiparticles are so much depleted that reaction (75) ceases to be in equilibrium. The probability for an A to encounter an antipartner becomes less than 1 per typical expansion time. Annihilations stop under the combined action of the dilution due to the expansion of space, and of the severe depletion of the A species which has occurred between $T = 2$ GeV and $T = 100$ MeV.

Below 100 MeV, annihilations are inhibited and the codensity f_A remains constant. Around $T \simeq 2$ MeV, heavy neutrinos stop colliding with the other species and become mere fossils of the early stages of the Universe. If they are stable, they pervade the intergalactic medium until the present epoch, and may even contribute a significant fraction to the closure mass.

The density n_A evolves according to the differential equation

$$\frac{dn_A}{dt} = -3Hn_A - \langle \sigma_a v \rangle n_A^2 + \langle \sigma_a v \rangle n_A^e{}^2, \quad (76)$$

where the first expression on the right-hand side refers to the dilution resulting from expansion. The second term accounts for the A 's annihilations while the last expression describes the retro-creation of $A\bar{A}$ pairs from light fermions and assumes detailed balance. In terms of the codensity, the evolution equation simplifies into

$$\frac{df_A}{dt} + (\langle \sigma_a v \rangle n_A) f_A = \langle \sigma_a v \rangle T^3 f_A^e{}^2. \quad (77)$$

In order to solve the differential equation (77), two typical time scales may be defined.

1) When equilibrium is reached, *i.e.*, when $f_A = f_A^e$, the time derivative $df_A/dt \simeq 0$. The characteristic time scale of the relaxation of f_A towards its kinetic equilibrium value f_A^e is merely related to the annihilation rate

$$\tau_{rel}^{-1} = \langle \sigma_a v \rangle n_A. \quad (78)$$

2) The time scale of the variations of the equilibrium f_A^e itself may be expressed as

$$\tau_{eq}^{-1} = -\frac{d}{dt} \text{Log} \{ f_A^e{}^2 T^3 \} \simeq 2aH, \quad (79)$$

in the non-relativistic regime. As is clear in figure 5, two phases may be distinguished :

- At high temperature, as long as $\tau_{rel} < \tau_{eq}$, f_A has plenty of time to relax towards the equilibrium f_A^e which evolves at a much slower pace. As a consequence, the annihilation reaction (75) is in chemical equilibrium so that $f_A = f_A^e$. On the upper panel of figure 5, the solid line f_A cannot be distinguished from the dashed curve f_A^e . As is clear from the lower panel, as T decreases and $a = M/T$ increases, relaxation becomes progressively less efficient until the ratio τ_{rel}/τ_{eq} eventually reaches 1 where the decoupling from equilibrium occurs.

Figure 5

Upper panel : The codensity f_A (solid curve) follows the equilibrium value f_A^e (dashed curve) until the mass-to-temperature ratio a reaches the freeze-out point at $a \simeq 18$. For $a > 18$, f_A decouples from equilibrium and decreases slowly towards its present value.

Lower panel : The ratio of the relaxation time τ_{rel} to the equilibrium time τ_{eq} is plotted as a function of a . Decoupling occurs as soon as τ_{rel} exceeds τ_{eq} .

• Below the freeze-out temperature T_F , the relaxation time τ_{rel} exceeds τ_{eq} . Whilst f_A^e drops down and vanishes, f_A still decreases a little bit. The freeze-out point may be derived from the equality $\tau_{rel} = \tau_{eq}$ which, for heavy neutrinos whose annihilation cross-section is

$$\sigma_a v (A\bar{A} \rightarrow f\bar{f}) = \frac{G_F^2}{2\pi} M^2 N_A , \quad (80)$$

translates into

$$\sqrt{a_F} e^{a_F} = \frac{3\sqrt{5}}{(2\pi)^4} G_F^2 M^3 M_{Pl} N_A g_{eff}^{-1/2} \simeq 0.7 \times 10^7 \left(\frac{M}{1 \text{ GeV}} \right)^3 \frac{N_A}{\sqrt{g_{eff}}} . \quad (81)$$

For a 2 GeV neutrino, the effective number of annihilation channels is $N_A \sim 14$, whilst the effective number of degrees of freedom is $g_{eff} = 57/8$ at a temperature $T_F \sim 100$ MeV. We infer from the previous equation a freeze-out point at $a_F \simeq 18.06$. In figure 5, the freeze-out is indicated by a heavy vertical line at $a = a_F$. Note that as soon as the ratio τ_{rel}/τ_{eq} exceeds unity, the codensity f_A (solid line of the upper panel) decouples from the equilibrium value f_A^e (dashed curve). From decoupling until now, f_A has been slightly decreasing according to the relation

$$\frac{df_A}{dt} = - \langle \sigma_a v \rangle n_A f_A \quad (82)$$

which, in terms of the parameter $x = 1/a$, simplifies into

$$\frac{df_A}{f_A^2} = \sqrt{\frac{45}{8\pi^3}} g_{eff}^{-1/2} \langle \sigma_a v \rangle M M_{Pl} dx . \quad (83)$$

Integration of the previous differential equation from $x_F = 1/a_F$ where the codensity decouples from equilibrium, i.e., $f_A(a_F) = f_A^e(a_F) = f_F$, until the present epoch $x = 0$, where f_A reaches its asymptotic value f_A^{asy} , yields

$$f_A^{asy} = \frac{f_F}{1 + 2a_F} \simeq \sqrt{\frac{8\pi^3}{45}} g_{eff}^{1/2} (\langle \sigma_a v \rangle M M_{Pl} x_F)^{-1} . \quad (84)$$

At present, the codensity of a *stable* 2 GeV neutrino is $f_A^{asy} \sim 4 \times 10^{-9}$ which translates into $\sim 3.3 \times 10^{-7} \text{ cm}^{-3} \text{ K}^{-3}$. Note finally the dependence of f_A^{asy} on the annihilation cross section : the larger $\langle \sigma_a v \rangle$, the lower the relic abundance f_A^{asy} .

5 - Heavy neutrinos confront LEP and COBE

The recent results [7] of the Large Electron Positron collider (LEP) at CERN, Geneva, as well as the COBE measurements [3] of the CBR spectrum have given heavy neutrinos quite a turn as regards their cosmological rôle.

5.1 - The relic density of heavy neutrinos

Slightly after the thermal freeze-out of neutrinos at a temperature ~ 2 MeV, electrons and positrons annihilate. Radiation is reheated and, as a consequence, the photon to neutrino temperature ratio T_γ/T_ν increases by a factor $(11/4)^{1/3} \simeq 1.4$. We infer a neutrino temperature $T_\nu^0 \simeq 2$ K at present. The careful reader may feel uncomfortable with a neutrino temperature which does not make any *thermodynamical* sense below 2 MeV where, indeed, T_ν is more a typical scale factor than a temperature per se, and varies as R^{-1} . Depending on the mass of the neutrino, two regimes may be discussed in order to derive the relic abundance ρ_ν .

- For $M < 2$ MeV, neutrinos decouple before they may annihilate. The resulting fossil population is ultra-relativistic at decoupling so that the present neutrino codensity is merely given by expression (70). If neutrinos are stable, we infer a relic abundance

$$\rho_\nu = \frac{3\zeta(3)}{2\pi^2} T_\nu^{0,3} M \simeq 110 \text{ keV cm}^{-3} \left(\frac{M}{1 \text{ keV}} \right), \quad (85)$$

to be contrasted with the closure density

$$\rho_C \simeq 2 \times 10^{-29} h^2 \text{ g cm}^{-3} \simeq 11.25 h^2 \text{ keV cm}^{-3}, \quad (86)$$

which corresponds to a *flat* Friedmann-Lemaître cosmology. If the mass density exceeds ρ_C , the Universe is spherical and will recontract in the future while, in the opposite case, its geometry is hyperbolic and the expansion will last for ever. The Hubble constant h is expressed in units of 100 km/s/Mpc. The relic density ρ_ν may be conveniently expressed in units of the closure density

$$\Omega_\nu h^2 \simeq \frac{M}{100 \text{ eV}}. \quad (87)$$

The larger M , the larger the relic abundance $\Omega_\nu h^2$ which, as a matter of fact, cannot reasonably exceed ~ 1 under the penalty of overclosing the Universe, whose age should be larger, at least, than the age of our galactic disk, i.e., ~ 9 billion years. We readily infer a cosmological upper bound on the mass of light stable neutrinos of ~ 100 eV.

- If $M > 2$ MeV, neutrinos may significantly annihilate before reaction (75) gets quenched. The freeze-out temperature below which annihilations stop as a result of the severe depletion of the neutrino density may be derived from equation (81) with appropriate values for N_A and g_{eff} .

Figure 6

The cosmological relic abundance $\Omega_\nu h^2$ of stable heavy neutrinos is presented as a function of their mass, for Majorana (solid) and Dirac (dashed) species. The heavier the neutrino, the lower its contribution to the present mass density of the Universe.

As may be guessed from (81), the larger M , the larger $a_F = M/T_F$. The relic density of heavy neutrinos may be expressed as

$$f_A^{asy} \simeq 9 \times 10^{-9} \frac{g_{eff}^{1/2}}{N_A} \frac{a_F}{M^3} , \quad (88)$$

which translates into the ratio

$$\Omega_\nu h^2 \simeq 0.98 \left(\frac{g_{eff}^{1/2} a_F}{N_A} \right) \left(\frac{1 \text{ GeV}}{M} \right)^2 , \quad (89)$$

to which both neutrinos and antineutrinos contribute. For a neutrino mass $M \sim 2 \text{ GeV}$, we may take $g_{eff} = 57/8$ and $N_A = 14$ so that we infer a fossil abundance

$$\Omega_\nu h^2 \sim \left(\frac{1.8 \text{ GeV}}{M} \right)^2 , \quad (90)$$

in fairly good agreement with figure 6 where the annihilation cross sections of heavy *Majorana* (solid curve) and *Dirac* (dashed line) neutrinos have been carefully computed. As is obvious in relation (84), the relic abundance $\Omega_\chi h^2$ of any stable species χ is inversely proportional to the associated annihilation cross section. As a consequence of expression (80), the fossil density $\Omega_\nu h^2$ decreases approximately as M^{-2} as soon as the neutrino mass exceeds 2 MeV (see both curves of figure 6). The neutrino relic abundance $\Omega_\nu h^2$ is peaked around $M \sim 2 \text{ MeV}$ and is fairly well described by expression (87) for $M < 2 \text{ MeV}$ whilst, above 2 MeV, relation (90) is valid.

Since Majorana neutrinos are their own antipartners, their annihilation is suppressed with respect to the Dirac case. As a result, the relic abundance $\Omega_\nu h^2$ of Majorana species is slightly larger than for Dirac particles. Provided a *stable* heavy Dirac neutrino has a mass in the range $\sim 2 - 10 \text{ GeV}$, it may contribute a significant fraction to the closure mass and may be a plausible candidate to the cosmological dark matter. For a Majorana neutrino, the relevant mass range is between 3 and 20 GeV.

5.2 - LEP, heavy neutrinos and dark matter

Recent experiments at LEP [7] have observed the reaction

$$e^+ e^- \rightarrow Z^0 \rightarrow f \bar{f} , \quad (91)$$

where an electron and a positron annihilate into a pair of light fermions. As the center of mass energy $\sqrt{s}$ of the electron-positron pair varies, the cross section of the previous process exhibits a characteristic Breit-Wigner resonance, with a peak at $\sqrt{s} = M_{Z^0}$ which corresponds to the production of the Z^0 boson. The width of this lineshape is merely the

total decay rate of the Z^0 boson. From the direct observation of the Z^0 decays into quarks (the constituents of nucleons) and charged leptons (there are three charged leptons : the electron, the muon and the tau), and from the determination of the total decay rate, the invisible width, *i.e.*, the decay rate into invisible neutrinos, has been for the first time measured to be ~ 0.49 GeV. Note that the Z^0 decay rate into a neutrino-antineutrino pair is given by

$$\Gamma_{Z^0} = \frac{G_F}{12\sqrt{2}\pi} M_{Z^0}^3 \kappa(\beta) \simeq 165 \text{ MeV } \kappa(\beta) , \quad (92)$$

where β denotes the velocity of the final neutrinos in the rest frame of the decaying Z^0 boson. The kinematic function $\kappa(\beta) = \beta^3$ for Majorana neutrinos whilst, for Dirac species, $\kappa(\beta) = \beta(\beta^2 + 3)/4$. For massless particles, $\beta = 1$ and $\kappa = 1$, so that the decay width is a mere 165 MeV. The invisible width corresponds to three light neutrino species which were already known to exist in association with the above mentioned charged leptons. A fourth family heavy neutrino may exist but should not contribute to the Z^0 decay width, *i.e.*, it should not be produced in Z^0 decays. We readily infer the lower bound $M > M_{Z^0}/2 \sim 45$ GeV, so that should such a species exist and be stable, its remnants would not contribute significantly to the closure mass density of the Universe as may be readily inferred from figure 6. A heavy neutrino cannot be a good dark matter candidate as was incidentally advocated before the LEP results. LEP has shown that the conventionnal model of the electro-weak interactions is distressingly valid.

5.3 - The 17 keV neutrino

There is new evidence, in β -decay experiments [8], that the electron neutrino oscillates in one percent of the cases into another neutrino of mass 17 keV. According to our present knowledge, the mass of the muon neutrino cannot exceed 10 keV whilst the number of light neutrino generations, N_ν , is three as measured by the LEP experiments [7]. Therefore, the 17 keV neutrino may be either the tau neutrino, or an additional species with vanishing hypercharge. The properties of this neutrino are severely constrained by astrophysics and cosmology.

1) If the 17 keV neutrino is stable, its mass may overclose the Universe by a factor $\Omega_\nu h^2 \sim 170$ as derived from expression (87). A value of the Hubble constant H^0 as low as 50 km/s/Mpc associated with a large closure density $\Omega_\nu \sim 700$ implies a Universe not older than ~ 1 billion years. Such a low value is in conflict with current cosmological observations. In particular, the age of the Universe is known to be at least larger than the age of the earth, *i.e.*, 5 billion years. From the luminosity function of white dwarves and its peculiar cut-off at low luminosities, the age of the galactic disk has been inferred [9] to be ~ 9 billion years.

Figure 7

Astrophysical and cosmological arguments constrain the 17 keV neutrino to lie in specific domains of this (τ_ν, B_γ) diagram. The region on the right-hand side of the closure bound (a) (heavy line) is excluded. The domain above curves (b) and (c) is also forbidden. The quasi-Planckian spectrum of the CBR implies the constraint (b). Finally, the dashed curve (c) is derived from the solar maximum mission data.

Finally, the magnitude shift between the main-sequence turn-off and the horizontal branch of the Hertzsprung-Russell diagram of globular clusters is compatible [10] with an age of population II stars greater than 15 billion years. We readily conclude that the 17 keV neutrino must be unstable.

Should the decay products be non-relativistic, they would merely dominate the expansion of the Universe just as if the neutrino were stable. That species must therefore decay into relativistic products whose energy density may be red-shifted. If decays occur at temperature T_γ^D , the decay products contribute today a factor

$$\Omega_{dp} = 170 h^{-2} \left(\frac{2.735 \text{ K}}{T_\gamma^D} \right) , \quad (93)$$

to the closure density, so that the earlier the decay, the larger T_γ^D and the lower Ω_{dp} . Provided that the 17 keV neutrino decays before the temperature reaches down ~ 500 to 2000 K , its decay products do not overclose the Universe. We infer an upper bound on the neutrino lifetime τ_ν of $\simeq 10^{12}$ seconds, which corresponds to the heavy vertical line (a) of figure 7.

2) Spectral distortions of the CBR may be generated by the radiative decays of the 17 keV neutrino ν_H . As a result of the process $\nu_H \rightarrow \gamma \nu_e$, whose branching ratio is B_γ , the electromagnetic energy density increases with time

$$d\rho_\gamma = n_\nu(t) \frac{m_\nu}{2} B_\gamma \frac{dt}{\tau_\nu} . \quad (94)$$

In what follows, we focus only on massless final states. Since the temperature of interest is much less than 17 keV, massive neutrinos may be considered at rest so that the photon energy is $m_\nu/2$ on average. The density of the massive species ν_H exponentially decays in time while the resulting μ -distortion evolves as

$$d\mu = 7.07 \times 10^{-2} \left(\frac{m_\nu}{T_\gamma} \right) B_\gamma e^{-t/\tau_\nu} \frac{dt}{\tau_\nu} . \quad (95)$$

We have grossly assumed that a μ -distortion was the only possible spectral alteration of the CBR. As mentioned in section 3.2, a μ -distortion is only possible between 10^5 and 10^9 seconds. After 10^9 seconds and until recombination, which occurs around 10^{11} seconds, a y -distortion ensues. In order to estimate the implications of the 17 keV neutrino on the CBR spectrum, we have numerically integrated equation (95) and set the constraint [3] $\mu \leq 9 \times 10^{-3}$. The resulting upper bound on B_γ is presented in figure 7 as a function of the neutrino lifetime τ_ν and results into the solid line (b).

3) Finally, data from the Gamma-Ray Spectrometer on the Solar Maximum Mission (SMM) satellite [11] show no excess γ -ray emission coincident with the ~ 10 second neutrino burst from supernova 1987A. From the absence of a detectable signal, an upper

bound on the flux of MeV photons from the supernova has been found to be ~ 1 photon cm^{-2} during a period of observation ~ 10 seconds encompassing the neutrino burst. If neutrinos decay radiatively in flight between the supernova and the solar system, a lower bound on their radiative lifetime may be placed [12] from the SMM observations, which translates into the limit

$$\tau_\nu \geq (1.49 \times 10^{14} \text{ sec}) B_\gamma , \quad (96)$$

and the dashed curve (c) of figure 7 above which the excluded region lies.

The constraints of figure 7 may be summarized [13] by

$$B_\gamma \leq 4 \times 10^{-5} \quad \text{and} \quad \tau_\nu \leq 10^{12} \text{ sec} . \quad (97)$$

Should the 17 keV neutrino decay into three light neutrinos, the corresponding lifetime τ_ν would exceed $\sim 10^{16-17}$ seconds. Moreover, the radiative branching ratio in such a model would merely be $B_\gamma \sim \alpha \sim 10^{-2}$, well above the constraint (97). The 17 keV neutrino should therefore decay into scalar species such as the axion or the majoron. We hope to have convinced the reader of the importance of COBE as regards the properties of particles. The discovery of a 17 keV neutrino, if confirmed, would be an exciting clue for the existence of a new scalar species, and would revive the interest in axion, majoron or familon theories. Thanks to the COBE results, such a discovery would definitely point towards unconventional physics.

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