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COLOUR-COHERENCE PHYSICS AT THE Z^0

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Abstract

We review the recent results on the applications of the analytical parton-shower approach to QCD jet physics in $Z^0 \rightarrow$ hadrons. We concentrate on the studies at the Z^0 resonance of the colour-coherence phenomena, reflecting the collective nature of multihadron production.

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1 INTRODUCTION

The existence of spectacular jets of hadrons in hard processes is one of the most striking successes of Quantum Chromodynamics (QCD), the SU(3) gauge theory of coloured quarks and gluons.

Jet-like events were first seen definitely in e^+e^- collisions at SPEAR in 1975 [1]. Five years later a gluon was discovered experimentally in three-jet events at PETRA [2]. These pioneering results opened up an important new area for experimental tests of both perturbative and non-perturbative QCD.

In the last decade hadron-jet physics has been intensively studied at both e^+e^- and hadron accelerators; see, for example, lectures given by Ward [3] and Cashmore [4] at this School. It will be one of the main problems of investigation for the e^+e^- , $pp(p\bar{p})$, and ep colliders of the future.

e^+e^- annihilation into hadrons proves to be a particularly wonderful laboratory for detailed experimental tests of QCD. A vast amount of data on multihadron production has been accumulated in the experiments at PETRA, PEP, and TRISTAN. With the start of SLC and LEP activity a wealth of new experimental results has become available (see Refs. [3],[5]).

Experiments at the Z^0 resonance provide substantial advantages compared with the previous e^+e^- data: higher energy, very high statistics, abundance of the heavy-flavour (bottom) production. In addition, these experiments have, in general, better detector resolutions and efficiencies than those at lower energies.

The first experimental studies of the hadronic events from Z^0 decays [3] appear to be very useful for detailed tests of perturbative QCD and for reducing our domain of ignorance on the physics of confinement. The data show that hadronic event characteristics calculated at the parton level agree very well with the measured ones. This supports the hypothesis of local parton-hadron duality (LPHD) [6],[7]. Such local duality is naturally connected with the ‘pre-confinement’ properties [8] of the QCD parton cascades.

On the basis of this hypothesis, a detailed analysis of perturbative QCD characteristics of data on hard collisions is possible (for reviews, see Ref. [9]). These studies are important not only for testing various aspects of QCD and for the design and analysis of experiments. The characteristic QCD features of jet-like states could also provide us with a valuable additional tool, in helping to extract and to study the manifestation of New Physics.

It would be impossible to cover completely even this specialized subject of QCD jet physics at the Z^0 resonance. So I must apologize in advance for being selective in the topics discussed. In this lecture I shall review some recent results of the analytical perturbative approach (PA) [9]–[14] to jet physics in $Z^0 \rightarrow$ hadrons. The emphasis will be put on the manifestations at the Z^0 of colour-coherence phenomena.

For excellent recent reviews, covering other topics in QCD jet dynamics, see Refs. [15]–[20].

A topic closely related to the subject of this lecture is a comparison of PA with QCD-inspired Monte Carlo algorithmic models. The reader can find the presentation of the recent status of these schemes and of their development in the lectures given by Sjöstrand at this School [5] (for a review, see Ref. [16]).

Monte Carlo simulations based on a QCD parton-shower mechanism describe the existing data very successfully and are becoming better and better at building in realistic fragmentation and proper QCD evolution.

One of the approaches for the improvement of models is the most adequate incorporation of the colour coherence, reflecting the quantum mechanics of QCD. However, note that the very possibility of absorbing the interference effects into the probabilistic scheme is far from obvious, and the use of Monte Carlo generators for the evolution of the multiparton system proves to be of limited value in principle (for details, see Refs. [21],[22]). For example, under special conditions some subtle collective phenomena, breaking the standard probabilistic picture, may even become dominant and observable; see Section 6.

The outline of the lecture is as follows: In Section 1 an introduction is given. In Section 2 the parton-cascade mechanism of multihadron production is discussed. In Section 3 a guide to colour coherence is given. In Section 4 a modified leading logarithmic approximation is put forward. In Section 5 inclusive particle spectra and intrajet coherence are discussed. In Section 6 the Z^0 resonance as a laboratory to study interjet collective effects is studied. In Section 7 prospects for future experiments are given. Finally, in Section 8 the conclusions are presented.

2 PARTON-CASCADE MECHANISM OF MULTIHADRON PRODUCTION

The QCD picture of multihadron production has gone through a qualitative transformation in the last few years, reaching a mature level of sophistication, see Refs. [9]–[20] and references therein.

The milestones on the road to the present understanding of QCD jet physics were:

- i) the preconfinement idea of Amati and Veneziano [8];
- ii) the growth of particle multiplicity due to QCD showering [23],[24];
- iii) KNO scaling in QCD jets [25];
- iv) intrajet coherence and strong angular ordering (AO) in parton cascades [26],[27];
- v) the modified leading logarithmic approximation (MLLA) and strict AO [6],[28];
- vi) collective phenomena, string-effect [29], and interjet coherence [30].

Today the members¹⁾ of the international QCD community (Columbia University–Leningrad–Novosibirsk–Italy–Cambridge $\equiv$ CLIC) are able to make testable quantitative predictions for jet characteristics, with controllable accuracy in terms of analytical perturbative calculations.

The key idea is to invoke the parton-shower picture where one views the evolution of a jet as a sequence of parton branchings. The main concept is to reorganize the perturbative expansion in such a way that its zero-order approximation is systematic and involves an arbitrary number of produced particles. This approximation can be achieved through an iteration of basic $A \rightarrow B + C$ branchings. In principle, it should be possible to include higher corrections in order to systematically improve the accuracy.

The approximation mentioned above is the MLLA, which takes care of both double-logarithmic (DL) and single-logarithmic (SL) effects in a systematic way [6],[28].

In constructing the MLLA, one should pay special attention to maintaining the probabilistic interpretation of jet evolution. The existence of such an interpretation is far from trivial in the problems connected with the description of soft-particle distributions ($x \ll 1$). Here, interference contributions play an important role and prove to be unavoidable, unlike the case of the problems dealt with near $x \sim 1$. Nevertheless, it appears to be possible, by choosing an appropriate evolution parameter (jet opening angle θ_0) and

1) The names can be found among the authors of Refs. [9]–[15] and in the references therein.

Figure 1: Kinematics of gluon emission.

accounting for the specific angular dependence of soft-emission probabilities, to maintain a probabilistic interpretation of the dynamics of soft-parton cascades.

Experimental data have convincingly demonstrated the dominant role of the perturbative stage of jet evolution and have confirmed the QCD bremsstrahlung nature of multiple hadroproduction. According to the LPHD concept [6],[7], the conversion of partons into hadrons occurs at a low virtuality scale, independent of the scale of the primary hard process, and involves only low-momentum transfer, leading to a close similarity between parton and hadron distributions.

The PA attempts to describe the global features of the hadronic systems, such as the mean multiplicities and multiplicity distributions, angular patterns of energy and multiplicity flow, inclusive energy spectra and correlations of particles, etc., without making any reference at all to a fragmentation scheme. The non-perturbative effects are reduced to normalizing constants relating hadronic characteristics to partonic ones. Since there are only a few parameters to vary in connecting PA results with experiment, these predictions are simply testable. Thus, the main equation of PA reads

$$\text{PA} = \text{MLLA} + \text{LPHD}. \quad (1)$$

At the Z^0 a quark, with $E \sim m_Z/2$, starting from the annihilation time $t^{\text{ann}} \sim 1/E \sim 5 \times 10^{-3} \text{ fm}/c$ and up to its hadronization time $t^{\text{had}} \sim ER^2 \sim 10^2 \text{ fm}/c$ ($R \sim 1 \text{ fm}$ stands for the hadronization scale), behaves as if it were a true coloured object radiating gluons perturbatively without taking any care about its future confinement.

Since multiple QCD bremsstrahlung plays the key role in multihadron production, let us first discuss the basic QCD process, namely the gluon emission off a quark with momentum p , as shown in Fig. 1. The differential spectrum is given by the well-known formula

$$dw_q^g = \frac{\alpha_s(k_\perp^2)}{4\pi} 2C_F \left[1 + \left(1 - \frac{k}{p} \right)^2 \right] \frac{dk}{k} \frac{dk_\perp^2}{k_\perp^2}, \quad (2)$$

$$C_F = \frac{(N_c^2 - 1)}{2N_c} = \frac{4}{3}.$$

The effective coupling here runs with the gluon transverse momentum $k_\perp$, which comes from higher-order corrections to the Born probability.

Notice two important properties of the spectrum (2): i) the broad logarithmic distribution over transverse momentum $k_\perp$ (typical for a theory with dimensionless coupling); ii) the broad logarithmic energy distribution (specific for theories with massless vector particles).

At large emission angle and large energy one would get an extra gluon jet with a small probability.

$$\text{Multijet topology: } k_{\perp} \sim k \sim E \rightarrow w \sim \frac{\alpha_s}{\pi} \ll 1 .$$

At the same time, the bulk of radiation (quasicollinear and/or soft partons) will not lead to the appearance of additional visible jets in an event but will instead populate the original jet with secondary partons influencing the particle multiplicity and other jet characteristics.

$$\text{Intrajet activity: } k_{\perp} \ll k \ll E \rightarrow w \sim \alpha_s \log^2 E \sim 1 .$$

This DL $q \rightarrow qg$ process, together with two other basic parton splittings (DL $g \rightarrow gg$ radiation and SL decay $g \rightarrow q\bar{q}$), forms parton cascades.

To answer the question of how offspring partons influence the hadronic yield, one has to find out first what the condition is for a gluon to behave as an independent coloured object. Let us estimate the formation time, t_g^{form} , the time interval needed for a gluon to be radiated. Using the uncertainty relation to estimate the ‘lifetime’ of the virtual $(p+k)$ state, one finds:

$$t_g^{\text{form}} \sim \frac{E}{(p+k)^2} \simeq \frac{E}{kE\theta^2} \simeq \frac{k}{k_{\perp}^2} . \quad (3)$$

Comparing Eq. (3) with the hadronization time of a gluon

$$t_g^{\text{had}} \sim kR^2 , \quad (4)$$

one concludes that it is the restriction

$$k_{\perp} > R^{-1} = \text{a few hundred MeV} \quad (5)$$

which guarantees the applicability of the quark–gluon language, i.e. of the perturbative QCD.

Most dangerous (from the perturbative point of view) is the radiation of gluons with $k_{\perp} \sim R^{-1}$ [$\alpha_s(k_{\perp}^2) \sim 1$]. These partons can hardly be treated as gluons at all, since owing to Eqs. [3],[4] they are forced to hadronize just immediately after being formed. The real strong interaction comes into play here, which results in the famous *hadronic plateau* of the old parton picture. Its height, according to Eq. (2), could be estimated qualitatively as

$$dN = \left[\int_{k_{\perp} \sim R^{-1}} \frac{dk_{\perp}^2}{k_{\perp}^2} 4C_F \frac{\alpha_s(k_{\perp}^2)}{4\pi} \right] \frac{dk}{k} = \text{const} \frac{dk}{k} . \quad (6)$$

Gluons with parametrically large transverse momenta $k_{\perp} \gg R^{-1}$ will perturbatively emit, in turn, new offsprings forming cascades. If such a gluon forms its own hadronic plateau, consisting of hadrons with energies $R^{-1} \leq E_h \leq k$, one would expect that the resulting spectrum would be strongly populated with the softest hadrons: $E_h \sim m_h$. However, this common wisdom was proved to be wrong at the beginning of the eighties, when *Quantum-Mechanical Coherence* had been rediscovered in the QCD context [26],[27].

Figure 2: Bremsstrahlung radiation of a photon with momentum k after e^+e^- pair production.

The basic consequence of QCD coherence for the physics of *intrajet* cascades is AO [26],[27], which strongly reduces the multiplication of soft particles in parton cascades (see subsection 3.1).

Because of the AO, the structure of the partonic system representing the jet evolution can be treated as a tree of independent parton branchings into sequentially shrinking angular cones. The direct consequence of the AO was the prediction of the so-called *hump-backed* shape of QCD particle spectra $x dN/dx$ [31],[32].

Not the softest particles with momentum fractions $x \sim m_h/E_j$, but those with intermediate energies $x \sim (m_h/E_j)^c$ ($c \simeq 0.65$) multiply most effectively in QCD cascades (see Sections 4 and 5).

3 A GUIDE TO COLOUR COHERENCE

Our aim here is to provide the reader with an elementary introduction to the basic ideas of coherence phenomena, see Ref. [21].

Coherence problems are basic to any gauge theory. Roughly speaking, there are two types of coherence effects which occur in QCD jet dynamics.

The first manifestation of coherence is the AO of the sequential parton decays [26],[27]. Angular ordering accounts for the appropriate dependence of the soft radiation on the prehistory of parton-jet development

Coherence of the second type deals with the angular structure of particle flows when three or more partons are involved in a hard process. Here the particle angular distributions depend on the geometry and colour topology of the whole jet ensemble giving rise to *QCD radiophysics of jets* (see Refs. [13],[21],[33]).

3.1 Angular ordering in parton cascades

To elucidate the physical origin of the AO let us consider a simple model of a jet cascade, namely the radiation pattern of soft photons produced by a relativistic e^+e^- pair in a QED shower (see Fig. 2).

The question is, To what extent do the e^+ and e^- independently emit γ 's? To answer this question, let us estimate the formation time of the γ radiation from, say, e^- leg. According to Eq. (3) one has

$$t_{\text{form}} \approx \frac{1}{k\Theta_{\gamma e^-}^2} = \frac{\lambda_{\perp}}{\Theta_{\gamma e^-}} \quad (7)$$

($\lambda_{\perp}$ is the transverse wavelength of the radiated photon).

During this time the e^+e^- pair separate, transversely, by a distance

Figure 3: Wide-angle emission of a soft gluon with momentum k , off q and $\bar{q}$, acts as if the emission came off the parent gluon g imagined to be on shell.

$$\rho_{\perp}^{e^+e^-} \approx \Theta_{e^+e^-} \quad t_{\text{form}} \approx \lambda_{\perp} \frac{\Theta_{e^+e^-}}{\Theta_{\gamma e^-}}. \quad (8)$$

One concludes that for large-angle photon emissions,

$$\Theta_{\gamma e^-} \approx \Theta_{\gamma e^+} \gg \Theta_{e^+e^-},$$

the separation of two emitters, e^+ and e^- , proves to be smaller than $\lambda_{\perp}$. In this case the emitted photon cannot resolve the internal structure of the e^+e^- pair and probes only its total electric charge, which is zero. Thus for $\Theta_{\gamma e} \gg \Theta_{e^+e^-}$ we expect photon emission to be strongly suppressed²⁾. The e^+ and e^- can be said to emit γ 's independently only at $\rho_{\perp}^{e^+e^-} \gg \gamma_{\perp}$, that is when

$$\Theta_{\gamma e^+} \text{ or } \Theta_{\gamma e^-} \leq \Theta_{e^+e^-}.$$

A similar physical picture appears for QCD parton cascades, where soft-gluon radiation is governed by the conserved *colour* current. The only difference is that the coherent radiation of soft gluons by an unresolved pair of quarks (or gluons) is no longer zero but the radiation acts *as if* it were emitted from the parent gluon *imagined* to be on shell, as is illustrated in Fig. 3. This property is universal and holds true for the soft radiation accompanying $q \rightarrow qg$ and $g \rightarrow gg$ splittings as well. A remarkable fact is that one gets all leading double and single logarithmic effects correctly, for angular averaged observables, by allowing the gluon emission, independently, off line q when $\Theta_{kq} \leq \Theta_{q\bar{q}}$, off line $\bar{q}$ when $\Theta_{k\bar{q}} \leq \Theta_{q\bar{q}}$, and off the parent line g , when $\Theta_{kg} \geq \Theta_{q\bar{q}}$ (see Fig. 3).

This observation furnishes the core idea of the Marchesini–Webber model [35], the first Monte Carlo simulation that included intrajet coherence effects.

The yield of soft, wider-angle gluons remains unchanged when the number of particles inside the multiparton bunch increases, since such gluons are unable to resolve separate colour sources inside the parton jet. The soft gluon behaves as a classical probe, testing the colour charge of the jet as a whole, i.e. that of the original parton, initiating the jet. In this sense the QCD colour coherence can be said to suppress the soft radiation at large angles. As a result, to describe the jet evolution in terms of independent sequential

2) This phenomenon has been well known in cosmic-ray physics since the middle of the fifties—the so-called ‘Chudakov effect’ [34].

Figure 4: The effect of colour coherence on particle energy spectrum $\rho(k) = dn/d \ln k$. Dotted area corresponds to the contribution which is removed when turning from the incoherent model (*dashed*) to the coherent one (*solid*). Shaded area shows the old-fashioned plateau (without taking into account bremsstrahlung).

parton splittings one has to impose the AO condition—a uniform decrease of successive opening angles in the cascade.

The AO occurs not only for the time-like jet evolution but also for the space-like parton cascades determining the target fragmentation in deep-inelastic scattering and the structure of initial-state radiation in the Drell–Yan and large- $p_\perp$ scattering processes etc., see, for example, Refs. [13],[21].

3.2 Suppression of the soft intrajet radiation, hump-backed spectrum

The depletion of emission of soft particles inside a jet (hump-backed plateau in the inclusive energy spectrum) is one of the most striking predictions of perturbative QCD. It follows from the AO of the parton cascade in going from greater to lesser virtuality and is a direct manifestation of coherence. This can be understood on kinematical grounds as being the result of two conflicting tendencies: on the one hand, owing to the restriction $k_\perp > 1/R$ [see Eq. (5)] a soft particle is ‘forced out’ at large emission angle $\Theta > 1/kR$, and, on the other hand, the allowed decaying angle, after a few successive branchings, is shrunk to small values.

Let us illustrate the influence of the intrajet coherence on particle spectra with the help of the *toy model*[7], based on first-order QCD.

We start with an old-fashioned plateau $\rho(k) \equiv dn/d \ln k + \text{const}$ of hadrons with limited transverse momenta $k\Theta = k_\perp \sim R^{-1}$ for a quark jet with energy E (see shaded area in Fig. 4). Taking into account a gluon with energy ε and emission angle Θ_0 , let us use the DL expression for the radiation probability

$$dw_g \propto \alpha_s \frac{d\varepsilon}{\varepsilon} \frac{d\Theta_0}{\Theta_0} \vartheta(\varepsilon\Theta_0 - R^{-1}). \quad (9)$$

The step function ϑ restricts here the transverse momentum $p_\perp \approx \varepsilon\Theta_0 > R^{-1}$.

How does the gluon contribute to the particle yield? From the orthodox parton model one might expect the gluon to give rise to its own plateau of particles with

$$R^{-1} < k < \varepsilon$$

and limited transverse momenta with respect to the gluon: $k\Theta' \sim R^{-1}$.

Now let us verify that the coherence leads to the following reduction of this additional plateau:

$$(R\Theta_0)^{-1} < k < \varepsilon . \quad (10)$$

The distribution of particles from the gluon jet can be represented symbolically as

$$dN = \frac{dk}{k} \frac{d\Theta'}{\Theta'} \delta(k\Theta' - R^{-1}) . \quad (11)$$

This expression can be thought of as a DL spectrum of bremsstrahlung from a gluon (ε, Θ_0) ‘projected’ onto the domain of the most intensive radiation. As follows from the AO in cascade, the offspring particles are independently emitted by the gluon only inside the cone with the opening angle $\Theta' < \Theta_0$. Applying this inequality to Eq. (11), one obtains restriction (10).

Finally, one finds

$$\begin{aligned} N = & \int \frac{E}{k} \int \frac{d\Theta}{\Theta} \delta(k\Theta - R^{-1}) + \alpha_s \int \frac{E}{\varepsilon} \int \frac{d\Theta_0}{\Theta_0} \vartheta(\varepsilon\Theta_0 - R^{-1}) \\ & \times \int \frac{\varepsilon}{k} \int \frac{d\Theta'}{\Theta'} \delta(k\Theta' - R^{-1}) . \end{aligned} \quad (12)$$

The first term of this schematic expression stands for the background quark plateau, the second one is constructed from the gluon emission (9) and fragmentation (11). The parameter $\Theta_{\max}$ encodes the difference between the coherent and incoherent cases:

$$\begin{aligned} \Theta_{\max}^{\text{incoh}} &= 1 , \\ \Theta_{\max}^{\text{coh}} &= \Theta_0 . \end{aligned}$$

Using Eq. (12) one obtains for the particle energy spectrum $\rho(k)$

$$\rho^{\text{incoh}} = 1 + \frac{\alpha_s}{2} (\ln^2 ER - \ln^2 kR) , \quad (13)$$

$$\rho^{\text{coh}} = 1 + \alpha_s \ln \frac{E}{k} \ln kR .$$

The additional gluon-initiated multiplicity $\int d \ln k [\rho(k) - 1]$ appears to be twice as large for the incoherent case. So the coherence substantially depletes the soft part of energy spectra, giving rise to a hump (see Fig. 4). The maximum is increasing with energy and peaks at $k \sim \sqrt{E}$.

3.3 Radiation pattern for the ‘quark–antiquark’ antenna

Let us briefly discuss soft emission associated with a colour-singlet $q\bar{q}$ pair. This radiation pattern is interesting not only because of the process $e^+e^- \rightarrow q\bar{q}$. Neglecting the terms of the order of $1/N_c^2$, one can represent the radiation in the case of the complex hard-parton system as a sum of terms in which each external quark line is uniquely connected to an external antiquark line of the same colour ($q\bar{q}$ antennae).

In the lowest order the soft-gluon distribution takes the familiar form (classical currents) (for notations see Fig. 5):

Figure 5: Soft-gluon emission off a hard colourless $q\bar{q}$ pair.

$$dW_{q\bar{q}} = -\frac{d^3k}{(2\pi)^2 k} \alpha_s C_F \left(\frac{p_i}{p_i k} - \frac{p_j}{p_j k} \right)^2 = \frac{dk}{k} d\Omega_{\vec{n}} \frac{2C_F \alpha_s}{(2\pi)^2} (\hat{i}\hat{j}) = \frac{dk}{k} d\Omega_{\vec{n}} \frac{\alpha_s}{(2\pi)^2} W^{q\bar{q}}(\vec{n}) . \quad (14)$$

Here

$$(\hat{i}\hat{j}) = \frac{a_{ij}}{a_i a_j} , \quad a_{ij} = (1 - \vec{n}_i \vec{n}_j) , \quad a_i = (1 - \vec{n}_i \vec{n}_j) , \quad (15)$$

where $\vec{n}_i$ and $\vec{n}_j$ denote the directions of the q and $\bar{q}$ momenta respectively; $\vec{n}$ is the direction of the emitted gluon.

Let us call the distribution $(\hat{i}\hat{j})$, describing the radiation pattern of the colourless $q\bar{q}$ pair, the $q\bar{q}$ antenna. The antenna $(\hat{i}\hat{j})$ may be represented in the form

$$(\hat{i}\hat{j}) = P_{ij} + P_{ji} ,$$

where

$$P_{ij} = \frac{1}{2} \left[\frac{1}{a_i} + \frac{a_{ij} - a_i}{a_i a_j} \right] . \quad (16)$$

One can refer to the two terms in the square brackets in Eq. (16) as the *incoherent* and the *interference* terms. At fixed Θ_i , the incoherent term is independent of the azimuthal angle ϕ_i . The interference term depends on ϕ_i through the angle Θ_j . If the soft gluon lies within the cone, described by i and j , the interference is positive. If the gluon lies outside this cone the interference is negative. This term is largest when the soft gluon lies in the plane defined by i and j , $\phi_i = \phi_j$. The point about splitting the radiation pattern into two terms $P_{ij}(P_{ji})$ is that only the former (latter) has the pole at $\Theta_i = 0(\Theta_j = 0)$, so this term can be treated as ‘belonging to’ quark $i(j)$.

After integration over ϕ_i it turns out that the total contributions of the incoherent and interference terms are equal, so that after azimuthal averaging we find

$$\langle P_{ij} \rangle_{\phi_i} \equiv \int \frac{d\phi_i}{2\pi} P_{ij} = \frac{1}{a_i} \vartheta(\cos \Theta_i - \cos \Theta_{ij}) . \quad (17)$$

In other words, $\langle P_{ij} \rangle_{\phi_i}$ is just the incoherent radiation P_i^{incoh} off a quark i , effectively confined to the cone

$$\cos \Theta_i > \cos \Theta_{ij} , \quad \Theta_i < \Theta_{ij} . \quad (18)$$

For the case of a multiparton system with n hard emitters ($i = 1 \dots n$) the full soft radiation pattern may be replaced by

$$P(\Omega) \Rightarrow P_{\text{AO}} \equiv \sum_i^n P_i^{\text{incoh}} \vartheta(-\Theta_i + \Theta_{ij}) ,$$

which looks like a sum of independent emission probabilities.

Such a recipe was used, for example, to construct the Monte Carlo program HERWIG [11].

Let us make a comment concerning gluon emission from a heavy-quark pair (' $Q\bar{Q}$ antenna'), for details see Refs. [13],[36]. In such a case, the heavy-quark masses substantially reduce the phase space available for branching. As a result, after azimuthal averaging the emission of a soft gluon by a heavy quark Q of velocity β_i effectively takes place only in the following 'screened cone' [36], cf. Eq. (18),

$$\beta_j \cos \Theta_{ij} < \cos \Theta_i < \cos \Theta_i^{\min} \simeq \beta_i , \quad (19)$$

where β_j is the velocity of the parton, which is colour connected to the emitting parton.

3.4 Wave nature of drag phenomena

From the standpoint of the QCD community, the drag (or string) effect in the $q\bar{q}g$ events of e^+e^- annihilation is one of the best examples of the manifestation of the interjet colour coherence.

Let us recall that originally the string effect was predicted as the result of a Lorentz boost exerted by a gluon on the *non-perturbative* string stretched between quarks, see Refs. [29],[37]. But, in *perturbative* language, this phenomenon is a direct consequence of interference among the gluon waves radiated from the $q\bar{q}g$ composite emitter [30].

In Section 6 we shall discuss this coherence effect in more detail, but here our purpose is simply to illustrate the basic ideas.

So far, the most spectacular experimental test of the drag phenomenon is the comparison of associated hadron production in $q\bar{q}g$ events with that of $q\bar{q}\gamma$ events, with the g and γ having similar kinematics [38]. In the plane of the three jets, counting the photon as a jet, one finds a suppression of associated hadrons in the region between the q and $\bar{q}$ in $q\bar{q}g$ events as compared to $q\bar{q}\gamma$ events.

It might seem strange, at first sight, that inclusion of an additional 'emitter' (coloured gluon replacing a colourless photon) results in less QCD radiation in some direction. This is a typical quantum-mechanical interference effect, and one can illustrate its physical origin with the help of the 'QED' model, where quarks are replaced by electrons and the gluon by a collinear e^+e^- pair, as shown in Fig. 6a. The $q\bar{q}\gamma$ event is illustrated in this model by Fig. 6b.

The corresponding radiation patterns are [cf. Eq. (14)]:

$$dW_{(q\bar{q}\gamma)} = \frac{dk}{k} d\Omega_{\vec{n}} \frac{\alpha}{2\pi^2} (\hat{1}\hat{2}) , \quad (20)$$

$$\begin{aligned} dW_{(q\bar{q}g)} &= -\frac{dk}{k} d\Omega_{\vec{n}} \frac{\alpha}{4\pi^2} \left(\frac{p_1}{p_1 k} + \frac{p_2}{p_2 k} - \frac{2p_3}{p_3 k} \right)^2 \\ &= \frac{dk}{k} d\Omega_{\vec{n}} \frac{\alpha}{2\pi^2} (2[(\hat{1}\hat{3}) + (\hat{2}\hat{3})] - (\hat{1}\hat{2})) . \end{aligned} \quad (21)$$

Figure 6: A ‘QED’ model illustrating the drag effect. The gluon is represented as having double electric charge. The photon is replaced by a collinear e^+e^- pair.

For notations see Eq. (15). Note that this pattern mimics the QCD $q\bar{q}g$ sample at $N_c = \sqrt{2}$, cf. Eq. (70).

Let us turn our attention to the negative contribution of the antenna (12), connected with the ‘repulsion’ of electrons.

For qualitative analysis it is convenient to rewrite the classical current expression for the $(q\bar{q}g)$ emitter (21) in terms of three-dimensional quantities

$$W^{(q\bar{q}g)}(\vec{n}) = - \left(\frac{p_1}{p_1 k} + \frac{p_2}{p_2 k} - \frac{2p_3}{p_3 k} \right)^2 = (\vec{A} \times \vec{n})^2, \quad (22)$$

where

$$\vec{A} = \left(\frac{\vec{n}_1}{a_1} + \frac{\vec{n}_2}{a_2} - \frac{2\vec{n}_3}{a_3} \right). \quad (23)$$

One can easily see that for a symmetric configuration ($a_{13} = a_{23}$) there is no radiation emitted directly opposite to the double charge: $\vec{A} \parallel \vec{n}$. This can be understood as the vanishing of the electric field midway between the equal charges (see Fig. 6a).

For the QCD case, soft-gluon radiation in this direction is non-zero but appears to be suppressed owing to the same physical reason: replacing γ by a gluon ‘recharges’ q and $\bar{q}$ to almost opposite colour charges. The resulting ratio of multiplicity flows in symmetric $q\bar{q}\gamma$ and $q\bar{q}g$ events looks as follows:

$$\frac{dN_\gamma/d\vec{n}}{dN_g/d\vec{n}} = \frac{2(N_c^2 - 1)}{N_c^2 - 2} = \frac{16}{7}. \quad (24)$$

Note that the destructive interference is strong enough to make even the most kinematically unfavourable direction transversal to the event plane better populated as compared to the $q\bar{q}$ valley. In the case of the threefold symmetric (‘Mercedes’-like) $q\bar{q}g$ events the particle flow reads

$$\left(\frac{dN_\perp/d\vec{n}}{dN_{q\bar{q}}/d\vec{n}} \right)_g = \frac{2N_c^2 - 1}{2(N_c^2 - 2)} = \frac{17}{14}. \quad (25a)$$

Note that in the case of $q\bar{q}\gamma$ event this ratio is

$$\left(\frac{dN_\perp/d\vec{n}}{dN_{q\bar{q}}/d\vec{n}} \right)_\gamma = \frac{1}{4}. \quad (25b)$$

Figure 7: A ‘QED’ model illustrating two-soft-gluon emission in $e^+e^- \rightarrow q\bar{q}$.

Owing to the constructive interference there is a surplus of radiation in the qg and $g\bar{q}$ valleys: for the ‘Mercedes’-like $q\bar{q}g$ configuration midway between the parton directions, one gets

$$\frac{dN_{qg}}{dN_{q\bar{q}}} = \frac{5N_c^2 - 1}{2N_c^2 - 4} = \frac{22}{7}. \quad (26)$$

For the jetty final hadronic states PA predictions for the ratios of particle flows similar to Eqs. (24)–(26) should remain correct, since non-perturbative hadronization effects should cancel, at least at high energies. Studying the coherent phenomena one gets plenty of infrared stable predictions deeply rooted in the basic structure of non-Abelian gauge theory.

The experimental observations of the above coherent phenomena can be said to test very detailed features of QCD colour flows.

3.5 Characteristic properties of the two-particle correlation function

In Refs. [14] it was suggested that one should analyse the ratio of energy-multiplicity correlations, which proves to be especially sensitive to colour-flow structure in jet formation.

In the case of e^+e^- annihilation the basic quantity is the gluon–gluon correlation function characterizing the emission of two soft gluons by a hard $q\bar{q}$ antenna:

$$C^{q\bar{q}}(\vec{n}_3, \vec{n}_4) = \frac{W^{q\bar{q}}(\vec{n}_3, \vec{n}_4)}{W^{q\bar{q}}(\vec{n}_3)W^{q\bar{q}}(\vec{n}_4)}. \quad (27)$$

Here $W^{q\bar{q}}(\vec{n}_i)$ describes the radiation pattern for the $q\bar{q}$ antenna ($i = 3, 4$), see Eq. (14). $W^{q\bar{q}}(\vec{n}_3, \vec{n}_4)$ represents the angular distribution of two soft gluons with $E_1 \simeq E_2 \gg E_3 \gg E_4$. E_1 (E_2) is the quark (antiquark) energy, and $\vec{n}_1 \simeq -\vec{n}_2$.

To illustrate the specific features of the correlation function $C^{q\bar{q}}$ let us invoke once more the ‘QED’ model used in the previous subsection, see Fig. 7.

Consider, for example, the configuration when the gluons are emitted at the same polar angles relative to the direction $\vec{n}_1$ ($\Theta_3 = \Theta_4$) but in the back-to-back azimuthal directions ($\phi = \pi$). Using the three-dimensional representation for the $q\bar{q}g$ radiation pattern [see Eqs. (22) and (23)] one can easily see that in this case $\vec{A}(\vec{n}_3) \parallel \vec{n} \equiv \vec{n}_4$ and the correlation function $C^{q\bar{q}}$ vanishes. This is a result of destructive interference, which is of the same magnitude as the drag effect in a symmetric $q\bar{q}g$ configuration.

In orthogonal azimuthal directions ($\phi = \pi/2$) the gluons become uncorrelated, $C^{q\bar{q}} = 1$.

Figure 8: Genealogy of the gluon cascade.

Thus gluon-gluon correlations in the orthogonal and back-to-back azimuthal directions demonstrate the same interference phenomena as the drag effect.

The correlation function $C^{q\bar{q}}$ for the $qq\bar{g}$ case can be presented in the form

$$C^{q\bar{q}}(\vec{\eta}_3, \phi) = 1 + 2 \cos \phi \frac{\sin \Theta_3 \sin \Theta_4}{a_{34}}, \quad (28)$$

where

$$a_{34} \frac{1}{\sin \Theta_3 \sin \Theta_4} = \cosh \eta_{34} - \cos \phi. \quad (29)$$

Here $\eta_{34} = \eta_3 - \eta_4$, where η_3 and η_4 are gluon pseudorapidities, $\phi = \phi_3 - \phi_4$, and the corresponding azimuthal angles ϕ_3 and ϕ_4 are relative to the direction $\vec{n}_1 \simeq -\vec{n}_2$:

$$\cosh \eta_{34} \sin \Theta_3 \sin \Theta_4 = 1 - \cos \Theta_3 \cos \Theta_4. \quad (30)$$

4 MODIFIED LEADING LOGARITHMIC APPROXIMATION ³⁾

As has been mentioned above, the strong AO provides one with the basis for the probabilistic interpretation of soft-gluon cascades in the DL approximation:

$$k_s \ll k_f \ll k_g, \quad \Theta_{sf} \ll \Theta_{fg}, \quad (31)$$

where the subscripts denote the cascade genealogy (first introduced by V. Fadin): ‘grandpa’, ‘father’, and ‘son’, as shown in Fig. 8.

The DL approximation happens to be too rough for making reasonable predictions even for asymptotically high energies. For example, it completely ignores the energy-momentum balance (recoil effects). Therefore, it overestimates parton multiplicities in cascading processes, the characteristic energy of the most actively multiplying partons (the position of the maximum of the particle spectra), etc.

Quantitatively, in the DL case one keeps track of the $\sim \sqrt{\alpha_s}$ term in the anomalous dimension γ , disregarding $O(\alpha_s)$ contributions which give rise to the essential pre-exponential energy-dependent factors.

Thus to keep a quantitative control over parton branching processes one is forced to take into full account non-leading SL effects.

When constructing the probabilistic scheme, taking into account both DL and essential SL effects, one is faced with a dramatic growth of the number of *interference*

³⁾ This section is intended for the advanced reader. On first reading it is possible to skip it without a loss of continuity. For details see Refs. [9],[10].

contributions which must be analysed and interpreted. The interference graphs contain soft-gluon lines, connecting harder partons of quite different generations. Meanwhile, the very idea of the classical *shower picture* implies that the structure of elementary-parton decays, i.e. the ‘blocks’ for building up the parton cascade, should depend on just the nearest ‘fore-fathers’ of a considered parton. Thus the possibility of absorbing all essential interference terms into the local probabilistic scheme looks far from being obvious.

Therefore, it is even more striking that such a scheme not only exists but appears to be *a posteriori* a simple almost trivial generalization of the standard LLA scheme. That is the reason to refer to this approximation [6],[28] as the *modified* LLA (MLLA).

It is important to note that beyond the MLLA a probabilistic picture of the parton-cascade evolution simply does not exist (the so-called ‘colour monsters’: $1/N_c^2$ suppressed soft contributions, $\Delta\gamma \sim \alpha_s^2$ [6],[21]).⁴⁾

In dealing with the subleading corrections to DL asymptotics it is helpful to invoke the technique of Generating Functionals (GF), which is perfectly suited to the description of intrajet cascades, see, for example, Ref. [25].

The structure of GF can be expressed symbolically as

$$Z = C(\alpha_s(t)) \exp \left\{ \int_0^t \gamma(\alpha_a(t')) dt' \right\} . \quad (32)$$

Owing to AO the evolution parameter (‘time’ t) is connected here with the jet opening angle $dt = d\Theta/\Theta$. One can easily see that the ‘time’ derivative of Eq. (32) produces the factor $\gamma(\alpha_s(t))$. It is natural to call the symbolic quantities γ and C , the *anomalous dimension* and the *coefficient function*, respectively.

The exponent of the integrated anomalous dimension γ incorporates the Markov chains of sequential angular ordered parton decays. The regular coefficient function C , being free of collinear (or ‘mass’) singularities, could be said to describe wide-angle partonic configurations, i.e. *multijet contributions* to the evolution of the system.

Successive terms of symbolic series for $\gamma(\alpha_s)$,

$$\gamma = \sqrt{\alpha_s} + \alpha_s + \alpha_s^{3/2} + \alpha_s^2 + \dots , \quad (33)$$

correspond to the increasing accuracy of the description of elementary partonic decays at parametrically small angles $\Theta_{ij} \ll 1$ and thus of the jet evolution. Iterating the coefficient function

$$C = 1 + \sqrt{\alpha_s} + \alpha_s + \dots , \quad (34)$$

one accounts for the ensembles of increasing numbers of such jets with large relative angles $\Theta_{ij} \sim 1$.

To estimate the ‘magnitude’ of γ let us look at the simple DL evolution equation for parton multiplicity depending on the product of the energy and the opening angle of a jet

$$N(p\Theta) \approx \int_0^\Theta \frac{d\Theta'}{\Theta'} \left[\int_0^p \frac{dk}{k} 4N_c \frac{\alpha_s}{2\pi} \right] N(k\Theta') , \quad (35)$$

4) Another interesting example is connected with the interjet collective phenomena that could be reproduced in a probabilistic way only in the large N_c limit [21] (see also Section 6).

Figure 9: a) Schematic illustration of Eq. (35). b) Schematic illustration of Eq. (41).

as illustrated by (Fig 9a).

Comparing this equation with Eq. (32) one can easily see that the expression in square brackets in Eq. (35) represents the anomalous dimension. Since both the left-hand side and the right-hand side of Eq. (35) contain multiplicity factors of the same order of magnitude, the two logarithmic integrations have to compensate α_s

$$\int dt' \int \frac{dk}{k} \alpha_s \sim 1 .$$

Therefore γ can be estimated as

$$\gamma^{\text{DL}}(\alpha_s) = \int \frac{dk}{k} \alpha_s = \alpha_s \ell \sim \sqrt{\alpha_s} . \quad (36)$$

Here we have denoted by ℓ the logarithmic integral over the gluon momentum fraction, which contributes effectively as

$$\int \frac{dk}{k} = \int \frac{dz}{z} = \ell \sim \alpha_s^{-1/2} .$$

Integrating $\gamma(\alpha_s)$ estimated by Eq. (36) in Eq. (32) one comes to the characteristic exponent $\exp(c\sqrt{\ln E})$, which describes the DL behaviour of the multiplicity growth. A non-leading correction to γ , namely $\Delta\gamma \sim \alpha_s$, causes a significant energy dependence $\exp[c_1 \ln \ln(E/\Lambda)] \propto \alpha_s^{-c_1}$ as well. To describe this correctly one has to analyse the following subleading effects: influence of the $\ln(1/z)$ dependence of the running coupling argument; $g \rightarrow q\bar{q}$, $q \rightarrow qg$, and $g \rightarrow gg$ splittings with hard momenta $z \sim 1$; exact angular integration in the $g \rightarrow ggg$ (or $q \rightarrow qgg$) ‘double-soft’ emission.

Taking these effects into account in the MLLA one gets symbolically

$$\gamma^{\text{MLLA}}(\alpha_s) = \sqrt{\alpha_s} + \alpha_s . \quad (37)$$

The MLLA parton-decay probabilities look as follows:

$$dw_A^{\text{BC}} = \frac{\alpha_s(k_\perp^2)}{2\pi} \Phi_A^{\text{BC}}(z) V(\vec{n}) dz \frac{d\Omega}{8\pi}, \quad (38)$$

$$V_{f(g)}^s(\vec{n}) = \frac{a_{sg} + a_{fg} - a_{sf}}{a_{sf} a_{sg}}, \quad a_{ik} = 1 - \cos(\vec{n}_i \vec{n}_k), \quad (39)$$

where subscripts refer to the soft-gluon family as before.

Equation (38) takes into full account the SL effects. Here the Gribov–Lipatov–Altarelli–Parisi splitting functions Φ_A^{BC} include both soft-gluon emission and terms corresponding to a loss of energy logs in $A \rightarrow B + C$ decays. The exact angular kernel $V(\vec{n})$, depending on the directions of momenta of partons of three sequential generations, replaces the rough *strong* AO. To see that this angular factor is nothing but the *exact* AO, the reader is advised to check the nice property of the V -kernel [cf. Eq. (17)]:

$$\langle V_{f(g)}^s(\vec{n}) \rangle_{\text{azimuth average}} = \int_0^{2\pi} \frac{d\phi}{2\pi} V_{f(g)}^s(\vec{n}) = \frac{2}{a_{sf}} \Theta(a_{fg} - a_{sf}). \quad (40)$$

This means that the decay probability integrated over the azimuth of ‘son’ around ‘father’ results in the logarithmic Θ -distribution inside the parent cone $\Theta_{sf} \leq \theta_{fg}$ and vanishes outside. It is important to emphasize that the ‘ V -scheme’, Eq. (38), proves to eliminate not only $A \rightarrow A + g' + g''$ ($\Delta\gamma = \alpha_s$) but also $A \rightarrow A + g' + g'' + g'''$ ($\Delta\gamma = \alpha_s^{3/2}$) elementary splitting processes, factorizing them completely into the chains of two-parton decays.

When studying the global characteristics of parton systems, such as mean multiplicities and multiplicity fluctuations, energy particle spectra and correlations, etc., one is allowed to replace the full angular kernel $V(\vec{n})$ in Eq. (38) by its *azimuth averaged* analogue of Eq. (40). In this case the exact AO makes it possible to construct a simple evolution equation for jet GFs.

The system of two coupled equations for the quark (Z_F) and gluon (Z_G) functionals reads ($A, B, C = F, G$) (see Fig. 9b):

$$\begin{aligned} Z_A(E, \Theta; u(k)) &= e^{-w_A(E\Theta)} u_A(k_0 = E) + \frac{1}{2} \sum_{B,C} \int \frac{d\Theta'}{\Theta'} \int_0^1 dz e^{-w_A(E\Theta) + w_A(E\Theta')} \times \\ &\times \frac{\alpha_s(k_\perp^2)}{2\pi} \Phi_A^{\text{BC}}(z) Z_B(zE, \Theta'; u) Z_C((1-z)E, \Theta'; u), \end{aligned} \quad (41)$$

where $u(k)$ is the probing function.

The first term on the right-hand side corresponds to the form-factor damped situation when the A -jet with energy E and opening angle Θ consists of the parent parton only. The integral term describes the first splitting $A \rightarrow B + C$ with the angle Θ' between the products. The exponential factor ensures that this decay is the first one indeed: it is the probability of emitting *nothing* in the angular interval between Θ and $\Theta' \leq \Theta$. The last two factors account for the further evolution of the subjects produced having smaller energies and Θ' as the opening angles.

The MLLA form factors are the following:

$$w_F = \int \frac{d\Theta'}{\Theta'} \int_0^1 dz \frac{\alpha_s(k_\perp^2)}{2\pi} \Phi_F^F(z), \quad (F = q, \bar{q}), \quad (42)$$

$$w_G = \int \frac{d\Theta'}{\Theta'} \int_0^1 dz \frac{\alpha_s(k_\perp^2)}{2\pi} \left[\frac{1}{2} \Phi_G^G(z) + n_f \Phi_G^F(z) \right]. \quad (43)$$

Collinear and soft singularities presented in Eqs. (41)–(43) may be regularized by imposing the usual perturbative restriction, which we shall write here as

$$k_\perp \simeq Ez(1-z)\Theta' > Q_0. \quad (44)$$

Differentiating the product $Z_A(\Theta)\exp[w_A(E\Theta)]$ over Θ and using Eqs. (42) and (43) one arrives at the master equation, which is actively exploited for studying properties of QCD jets, see Refs. [6],[10],[21]:

$$\begin{aligned} \frac{d}{d \ln \Theta} Z_A(E, \Theta) &= \frac{1}{2} \sum_{B,C} \int_0^1 dz \frac{\alpha_s(k_\perp^2)}{2\pi} \Phi_A^{BC}(z) \\ &\times \{Z_B(zE, \Theta) Z_C((1-z)E, \Theta) - Z_A(E, \Theta)\} \end{aligned} \quad (45)$$

Equation (45) accumulates information about *azimuth averaged* jet characteristics in MLLA. Taking the n^{th} variational derivative of GF over the probing function $u(k_i)$ near the ‘point’ $u = 0$ one gets the *exclusive* n -parton cross-sections. Expansion of Z_A at $u = 1$ generates *inclusive* parton distributions and correlations.

To conclude this section let us present one of the most practically important and pedagogically instructive results of the MLLA technique, namely, the expression for the inclusive energy spectrum of partons B in the A-jet (for details, see Refs. [6],[21]).

The evolution equation for particle spectra following directly from Eq. (45) reads:

$$\frac{d}{d \ln \Theta} x \bar{D}_A^B(x, Y, \lambda) = \sum_{C=q,\bar{q},g} \int_0^1 \frac{dz}{z} \frac{\alpha_s(k_\perp^2)}{2\pi} \Phi_A^C(z) \left[\frac{x}{z} \bar{D}_C^B \left(\frac{x}{z}, \frac{\ln Ez\Theta}{Q_0}, \lambda \right) \right], \quad (46)$$

where

$$\begin{aligned} Y &= \ln \left(\frac{E\Theta}{Q_0} \right) = \ell + z, \quad \ell = \ln \frac{E}{k} = \ln \frac{1}{x}, \\ y &= \ln \left(\frac{k\Theta}{Q_0} \right), \quad \lambda = \ln \frac{Q_0}{\Lambda}. \end{aligned} \quad (47)$$

Here E and Θ are the energy and the opening angle of a jet A,

$$k_\perp \simeq z(1-z)E\Theta \geq Q_0.$$

Through the chain of transformations one comes to the following equation for the leading contributions $D^+(\omega, Y, \lambda)$ to the momentum representation of the spectra

$$\left(\omega + \frac{d}{dY} \right) \frac{d}{dY} D^+ = 4N_c \frac{\alpha_s}{2\pi} D^+ - a \left(\omega + \frac{d}{dY} \right) D^+. \quad (48)$$

Here $D(\omega, Y, \lambda)$ are the Mellin-transformed distributions

$$D(\omega, Y) = \int_0^1 \frac{dx}{x} x^\omega [x \bar{D}(x, Y)] = \int_0^\infty d\ell e^{-\omega\ell} D(\ell, Y) , \quad (49)$$

$$a = \frac{11}{3} N_c + \frac{2n_f}{3N_c^2} . \quad (50)$$

Introducing the anomalous dimension as follows

$$D^+(\omega, Y) = D^+(\omega, Y_0) \exp \int_{Y_0}^Y dy \gamma[\omega, \alpha_s(y)] , \quad (51)$$

one finally arrives at the MLLA expression

$$\gamma = \gamma^{\text{DL}} + \frac{\alpha_s}{2\pi} \left[-\frac{a}{2} \left(1 + \frac{\omega}{\sqrt{\omega^2 + 4\gamma_0^2}} \right) + b \frac{\gamma_0^2}{\omega^2 + 4\gamma_0^2} \right] + O(\alpha_s^{3/2}) , \quad (52)$$

where

$$\gamma_0^2 = \frac{4N_c\alpha_s}{2\pi} , \quad b = \frac{11N_c}{3} - \frac{2n_f}{3} \quad (53)$$

$$\gamma^{\text{DL}}(\omega, \alpha_s) = \frac{1}{2} \left(-\omega + \sqrt{\omega^2 + 4\gamma_0^2} \right) \quad (54)$$

corresponds to the DL approximation.

One can solve the differential equation (48) explicitly, reducing it to the confluent hypergeometric equation for the product $\alpha_s(Z)D$. Taking into account initial conditions, the solution in the physical region $\ell \leq Y$ reads:

$$x \bar{D}_q^s(x, Y, \lambda) = \frac{4C_F(Y + \lambda)}{bB(B + 1)} \int_{\varepsilon - i\infty}^{\varepsilon + i\infty} \frac{d\omega}{2\pi i} x^{-\omega} \Phi[-A + B + 1, B + 2, -\omega(Y + \lambda)] K(\omega, \lambda) ,$$

$$K(\omega, \lambda) = \frac{\Gamma(A)}{\Gamma(B)} (\omega\lambda)^B \Psi(A, B + 1, \omega\lambda) . \quad (55)$$

Φ and Ψ are the standard degenerate hypergeometric functions.

Here we have used the notations

$$A = \frac{4N_c}{b\omega} , \quad B = \frac{a}{b} . \quad (56)$$

The MLLA expression (55) is formally expected to be valid at ($\Theta \sim 1$):

$$1 \gg x > Q_0/E , \quad k \gg Q_0, \Lambda .$$

Note that the dimensional quantity Q_0 regularizes collinear divergencies. This quantity represents the minimal value of the relative $k_\perp$ of decay products in jet evolution. Q_0 also puts a bound on parton energies $E_p = xE \geq k_\perp \geq Q_0$, thus playing the role of the ‘effective mass’ of a parton.

The restored x -spectrum of partons exhibits the above-mentioned ‘hump-backed’ structure with a maximum at particle energies asymptotically approaching $\sqrt{E_{\text{jet}}/\Theta}$. At $\Theta \sim 1, Q_0 = \Lambda$,

$$\ln \left(\frac{1}{x_0} \right) = \frac{1}{2} Y \left(1 + B \sqrt{\frac{b}{4N_c Y}} \right) + O(1) \quad (57)$$

(width $\sigma \sim Y^{3/4}$).

Taking $\omega = 0$ as the moment in Eq. (55) one obtains the analytical expression for the jet-parton multiplicity through modified Bessel functions with the MLLA asymptotic

$$\ln (N^g) \simeq \sqrt{\frac{32N_c\pi}{\alpha_s(E)}} \frac{1}{b} + \left(\frac{B}{2} - \frac{1}{4} \right) \ln \alpha_s(E) + O(1) . \quad (58)$$

Note that a change of the scale parameter Λ by a factor of $O(1)$ would correspond to a correction of $O(\sqrt{\alpha_s})$ in multiplicity N [21]. Such corrections are subleading to the MLLA and therefore this parameter cannot be compared directly with the values obtained from other experiments.

The first term in Eq. (58) is reduced by a factor of $1/\sqrt{2}$, relative to the incoherent case. This reduction is a direct result of the destructive interference of soft-gluon emission.

5 INCLUSIVE PARTICLE SPECTRA AND INTRAJET COHERENCE (GOOD NEWS)

At sufficiently high energies, events in hard processes should possess the clear geometry that reflects the topology of the participating partons. The space-energy portrait represents a natural partonmeter for registering the kinematics of the energetic partons. While the *hard component* of a hadron system (a few hadrons with $z \sim 1$) determines the partonic skeleton, the *soft component* (the other hadrons with $z \ll 1$) forms the bulk of the multiplicity. This component is concentrated inside the bremsstrahlung cones of jets. The opening angle θ_0 of each cone is bounded by the nearest other jet. Even though the bremsstrahlung cones of the neighbouring jets strongly overlap, the resulting total multiplicity can be presented as the additive sum of the contributions of the individual jets.

One can study the properties of an individual quark jet when measuring the different inclusive distributions in the process $e^+e^- \rightarrow \text{hadrons}$. In spite of the great importance of coherence phenomena, the notion of an isolated jet makes sense if one does not deal with the azimuthal effects, but considers only multiplicities, energy spectra and correlations, etc.

The inclusive energy spectrum for e^+e^- annihilation is the sum of two q-jet contributions:

$$\frac{1}{\sigma} \frac{d\sigma}{d(\ln 1/x)} = 2\bar{D}_q^h(\ell, Y)|_{\Theta \simeq 1} . \quad (59)$$

Here $x = E_h/E$, $E = W/2$, where W is the total c.m. energy.

To link hadronic spectra in jets with the calculated parton distribution let us recall that the Q_0 value in Eq. (55) sets a formal boundary between two stages of jet evolution: the perturbative one and then the phase of non-perturbative transition into hadrons. If the theory of hadroproduction should exist, the final result would be independent of the quantity Q_0 . As a matter of fact, for large enough Q_0 the number of partons produced at recent energies is certainly small. So, one is forced to apply some *ad hoc* hadronization model describing the multihadron production as the evolution ‘below Q_0 ’ of a partonic

system with large invariant masses of parton pairs. Unfortunately, an experimental verification of such results looks more like a tuning of parameters of a phenomenological model rather than a real test of QCD predictions.

However, by an intent look at Eq. (55) an opportunity to make a model-independent prediction may be found. The inclusive distribution of *hadrons* could be represented by the formula similar to Eq. (55) where the K -factor is replaced by the product

$$K^h(\omega) = K(\omega, \lambda) C^h(\omega, \lambda; m_h, J^{PC}), \quad (60)$$

where $C^h(\omega)$ is a Mellin-transformed parton fragmentation function $C^h(z)$.

In the kinematical region of relatively soft particles, which we are mainly interested in, the essential values of ω under the integral are small, $\omega \ll 1$ [near the peak they are parametrically small, $\omega_0 \sim \sqrt{\alpha_{\text{rms}}(Y)}$].

To understand how hadronization affects the spectrum shape one needs to know the behaviour of $K^h(\omega)$ at $\omega \rightarrow 0$.

Let us consider two different possibilities [6],[7]:

- i) The singular behaviour $K^h(\omega) \sim 1/\omega + \dots$ corresponds to the physical picture where each coloured parton produced hadrons with a plateau-like energy distribution: $C^h(z) \propto 1/z$. In such a case the dip at small x , which is characteristic for parton spectra, would never manifest itself in hadron distributions.
- ii) The regular behaviour $K \rightarrow \text{const.}$ corresponds to a *local* in the phase-space branching and the hadronization of partons. In this case hadron and parton spectra prove to be similar.

It is perhaps surprising to see the x -dependence of $x\bar{D}^h(x)$ being given completely by means of the perturbative evolution. Non-perturbative effects can smear the distributions over a limited interval in $\ln 1/x$. This is, however, the higher-order effect from the MLLA point of view. Thus, the overall normalization factor remains the only arbitrary parameter. It may be fixed, for example, by fitting the average multiplicity (the value of Λ could be determined phenomenologically from comparison of the PA predictions for spectra with experiment).

So, in such a case, hadron distributions become similar to the parton ones, and LPHD is realized.

Whether or not present jet energies are sufficiently large for LPHD to be applied is, of course, a question to experiment. So far experimental data (see, for example, Refs. [39]–[42]) have demonstrated that the LPHD works unexpectedly well. A strong support of the PA ideas came recently from the results of the OPAL Collaboration at the Z^0 (see Ref. [40] and discussion below).

Before coming to the comparison with data let us emphasize an important property of the parton spectrum, given by Eq. (55). Namely, at very high energies, when the typical value of the product $\omega\lambda \sim \lambda/\sqrt{Y} \ll 1$, the *shape* of the spectrum appears to be insensitive to the value of Q_0 :

$$K(\omega, \lambda) \approx \frac{2}{\Gamma(B)} \left(\frac{Z_0}{2}\right)^B K_B(Z_0) = \text{const}(\omega), \quad Z_0 = \sqrt{\frac{16N_c\lambda}{b}}, \quad \lambda = \ln \frac{Q_0}{\Lambda}. \quad (61)$$

Thus when the bremsstrahlung cascade is developed enough, the shape of the resulting energy distribution of particles becomes insensitive to the processes occurring at the last

steps of evolution (at $k_{\perp} \sim Q_0$). This observation may serve to justify an attempt to provide the developed cascade not with an expensive increase of total energy E but with a decrease of Q_0 , thus enhancing the responsibility of perturbative QCD for jet evolution at recent energies, see Refs. [7].

At $\lambda = 0$ the expression (55) simplifies noticeably, since in this case $K(\omega, Q_0) = 1$ at all ω (the so-called ‘limiting’ spectrum). Decreasing λ means extending the responsibility of the perturbative stage for the jet dynamics beyond its formal range of applicability. In some sense the limiting choice $Q_0 \simeq \Lambda$ is a specific attempt to model the confinement.

Already the first comparisons [7] of the $\pi^{\pm}$ data at PETRA/PEP energies with the limiting MLLA spectrum have demonstrated a quite satisfactory agreement. The effective value of Λ appeared to be close to m_{π} . It has also been noticed that the relative yields of hadrons of different species (p, K, ...) can be reasonably simulated simply by cutting the development of the parton cascade, leading to the production of a hadron h, by the value $Q_0 = m_h$ (preserving the same value of Λ). This procedure would also influence the shape of energy spectra of massive hadrons, stiffening them via the increase of the minimal energy fraction $x = k/E_{\text{jet}} > k_{\perp}/E_{\text{jet}} \geq Q_0/E_{\text{jet}} = x_{\text{min}}$ available for the relevant partons. As a consequence, the position of the hump in the energy spectra becomes higher as the mass of the hadrons increases.

The conjecture [7] about an increase of $(Q_0)_{\text{eff}}$ with hadron mass has been supported by experiment (see Refs. [39]).

When using the limiting spectrum for comparison with the data on different types of hadrons, the effective value of Λ is expected to increase with the hadron mass [14].

In the limiting case ($Q_0 = \Lambda$) the MLLA spectrum (55) can be presented in a form [6]

$$\begin{aligned} \bar{D}_q(\ell, Y) = & \frac{4C_F}{b} \Gamma(B) \int_{-\pi/2}^{\pi/2} \frac{d\tau}{\pi} e^{-B\alpha} \left[\frac{\cosh \alpha + (1 - 2\zeta) \sinh \alpha}{(4N/b)Y(\alpha/\sinh \alpha)} \right]^{\frac{B}{2}} \\ & \times I_B \left(\sqrt{\frac{16N_c}{b} Y \frac{\alpha}{\sinh \alpha} [\cosh \alpha + (1 - 2\zeta) \sinh \alpha]} \right), \end{aligned} \quad (62)$$

which is especially convenient for numerical integration.

Here $Y = \ln(E/\Lambda)$ and $\alpha = \alpha_0 + i\tau$ with

$$\tanh \alpha_0 = 2\zeta - 1, \quad \zeta = \frac{Y - \ell}{Y}. \quad (63)$$

I_B is the modified Bessel function of order B .

Taking into account the next-to-MLLA effects leads to some change in the shape of spectra in q and g jets. The spectrum in the g jet shifts to lower x.

One may find analytically the asymptotic shape of the limiting distribution not too far from its peak by saddle-point evaluation of the integral (55)

$$\begin{aligned} \bar{D}_q(\ell, Y) = & N_q(E, \Lambda) \left(\frac{C}{\pi Y^{3/2}} \right)^{1/2} \exp \left[\frac{-c(\ell - \ln 1/x_0)^2}{Y^{3/2}} \right], \\ \ell = & \ln \frac{1}{x}, \quad C = \sqrt{\frac{36N_c}{b}}. \end{aligned}$$

(64)

It has a broad Gaussian shape with the peak position given by Eq. (57). It grows rather slowly with the jet energy E

$$\frac{E}{E_0} \frac{dE_0}{dE} = \frac{1}{2} - \sqrt{\frac{b}{64N_c Y}}, \quad E_0 = x_0 E. \quad (65)$$

A sizeable shift in $(\ln 1/x_0)$, of the order of 0.5 units, has been predicted in going from PETRA to Z^0 energy $[(\ln 1/x_0)_{Z^0} = 3.6]$

In Refs. [12] has been derived a transparent next-to-leading-order expression for the $\ln 1/x$ distribution in the soft region in the form of a distorted Gaussian

$$\bar{D}(\ell, Y) \simeq \frac{N(E, \Lambda)}{\sigma 2\pi} \exp \left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4 \right] \quad (66)$$

with

$$\delta = (\ell - \bar{\ell})/\sigma, \quad \bar{\ell} \equiv \langle \ell \rangle,$$

$$\bar{\ell} = \ln \left(\frac{1}{x_0} \right) + O(1).$$

Parameters σ , s , and k are the width, skewness, and kurtosis of the $(\ln 1/x)$ distribution [12]:

$$\begin{aligned} \sigma &= \sqrt{\frac{Y}{3}} d^{1/4} \left(1 - \frac{b}{64} d^{-1/2} \right) + O(Y^{-1/4}), \\ s &= -\frac{a}{16} \sqrt{\frac{3}{Y}} d^{-1/4} + O(Y^{-5/4}), \\ k &= -\frac{27}{5Y} \left(d^{1/2} - \frac{b}{24} \right) + O(Y^{-3/2}), \\ d &= \frac{bY}{48} = \frac{\pi}{24\alpha_s(E)}. \end{aligned} \quad (67)$$

One finds that s and k vanish asymptotically, while the so-called higher-order reduced cumulants of the distribution are $O(\alpha_s)$ or smaller and therefore should be neglected in the MLLA case.

Let us recall here that the limiting distribution (62), including $O(\sqrt{\alpha_s})$ corrections (after phenomenological fixing of Λ), is expected to be valid for relativistic particles in a region $\ln 1/x \gg 1$. It means that at high energies the range of its validity is parametrically $\Delta\ell \sim Y$.

The Gaussian-like formulae describe spectra only in the region not so far from the peak, and are formally applicable in the range $\Delta\ell \sim \sigma \sim Y^{3/4}$.

Although the underlying MLLA physics is the same in both cases, one may expect (as some compensation for a rather cumbersome form) that the limiting spectrum at high energies provides us with somewhat more information⁵⁾.

As has already been mentioned, even before the start of the activity at the Z^0 , the experimental data supported PA predictions. This nice tradition was continued by the ALEPH data [42] on the x distribution of LEP. Being replotted on a $\ln 1/x$ scale these data demonstrated a rather good agreement with the perturbative predictions⁶⁾, see also Ref. [42].

Below we shall concentrate on the new OPAL data on the inclusive momentum distribution of charged particles in e^+e^- collisions at the Z^0 energy. Being finely binned at small x , these data provide a convincing test of the analytical QCD predictions. To study the predicted spectacular energy evolution of the spectra the OPAL analysis included data at lower energies from the recent TASSO paper [41].

Figure 10 shows the OPAL data on the $\ln(1/x_p)$ distribution ($x_p = 2p/W$, p being the momentum of each hadron⁷⁾) together with the predictions of analytical formulae (62), (64), and (66). When confronted with the data, formula (62) was multiplied by the overall normalization factor $K(Y)$, connecting the numbers of hadrons and partons. Note that for the OPAL data the systematic error in the absolute normalization is 5%. The relative systematic error between the different x_p values is much smaller than 5%. LPHD-induced, this factor should be universal at very high energies. Dependence on Y effectively encodes the influence of the asymptotically vanishing perturbative effects as well as the possible deviations from the PA picture.

In good accordance with the theory, the measured spectrum has a broad maximum around $\ln(1/x_0) \simeq 3.6$. A fit performed using the limiting formula (62) around the maximum (in the interval $2.5 < \ln 1/x < 4.5$) gives the values:

$$K(m_Z) = 1.28 \pm 0.01, \quad \Lambda = 0.253 \pm 0.030 \text{ GeV}. \quad (68)$$

From Fig. 10, one can easily see that in all three cases the region around the maximum is well described. This agreement of the shape and the peak position of the measured hadron distribution with the analytical result derived for parton distribution can be considered as a strong argument in favour of the LPHD-MLLA picture of multihadron production.

The deviation from the predictions, based on the relativistic treatment in the region of non-relativistic momenta at small x_p , is surely not unexpected.

Not surprisingly, the Gaussian-like formulae break down at larger x_p . The agreement with the limiting spectrum (62) in the finite- x region looks more challenging. The question immediately arises, Is it really a coincidence?

At the present level of understanding the answer is as follows (see footnotes^{3),5)}): the agreement at finite x is *accidental* but *natural*.

To elucidate the point, the reader is reminded that at $x \ll 1$ the essential values of ω under the integral in Eq. (55) are small ($\omega \sim 1/\sqrt{Y}$). The anomalous dimension $\gamma(\omega, \alpha_s) = [\alpha_s(Y)/2\pi]/\nu_+(\omega)$ is singular at $\omega \ll 1$ [see Eq. (52)], the trajectory $\nu_+(\omega)$ is

5) I am very grateful to Yu.L. Dokshitzer for his clarifying comments on this and many other problems. I hope that sooner or later the results of the enjoyable discussions with him and S.I. Troyan will be published elsewhere. See also Ref. [10].

6) This observation was made by W. Hofmann and the author immediately after the ALEPH results appeared.

7) Recall that the analytical formulae do not discriminate between momentum and energy and are valid only for relativistic particles. The mass effects are included, for example, in Monte Carlo algorithms.

Figure 10: Single-particle inclusive $\ln(1/x_p)$ distribution at $W = 91\text{GeV}$, compared with analytical formulae [40].

Table 1: Normalization factors $K(Y)$ at different energies W (the errors do not include systematic effects) [40]

W (GeV)	91	44	35	22	14
$K(Y)$	1.28 ± 0.01	1.31 ± 0.01	1.36 ± 0.01	1.38 ± 0.02	1.46 ± 0.02

$\nu_+(\omega) = (4N_c/\omega) - a + O(\omega)$. This approximation, however, proves to simulate reasonably well the behaviour of γ also at $\omega \sim 1$, the region responsible for $x \sim 1$ ($\ell \sim 1$).

The formal extrapolation to $\omega = 1$ gives $\nu_+(1) = 12 - (101/9) = 7/9$, not far from the exact value $\nu_+ = 0$, following from the momentum conservation. Moreover, at larger $\omega > 1$, ν_+ becomes negative (as the true γ does), thus imitating the scaling violation in the fragmentation region, $x \rightarrow 1$.

The combination of the OPAL measurements with the results obtained by the TASSO Collaboration at energies between 14 and 44 GeV is shown in Figs. 11a and 11b. Figure 11a also displays the predictions of the analytical formulae (62) (full line) and (66) (dotted line). In Fig. 11b the $\ln p$ distribution is compared only with formula (62).

The Λ parameter was fixed at the value obtained at the Z^0 [see Eq. (68)]. The normalization factors $K(Y)$ were determined for each energy separately from a fit to a region around the peak (see Table 1).

From Figs. 11 one finds that the analytical MLLA formulae provide an excellent description of all the energy evolution of spectra. This result wonderfully confirms the QCD cascading picture of the multiple hadroproduction. As can be seen from Fig. 11a, the height of the hump rises with increasing energy, reflecting the increase of multiplicity. Especially spectacular is the energy evolution of the peak position $\ln 1/x_0$, which is

Figure 11: a) Single-particle inclusive $\ln(1/x_p)$ distributions at $W = 91, 44, 35, 22$, and 14 GeV, [40]; b) $\ln p$ distributions at $W = 91, 44, 35, 22$, and 14 GeV [40].

independent of the normalization factors. It is displayed in Fig. 12 together with the MLLA prediction [Eq. (57)] for values of Λ and $O(1)$ obtained from the fit of Eq. (64) to the OPAL spectrum. The confirmation of this basic prediction of the cascading picture is really exciting.

The $\ln p$ distributions, shown in Fig. 11b, give evidence in favour of the scaling behaviour at low- p values (especially for the TASSO data). This observation provided convincing confirmation of the basic idea of QCD coherence that the low-momentum particles (hadrons!) do not multiply (see also Refs. [7], [43]). To come to a final conclusion on this important problem we need a better understanding of the relative normalization between the TASSO and OPAL data in the low- p region.

Data at low p also demonstrate that with increasing energy the peak of the spectrum becomes more separated from the region where the influence of hadron masses is important.

A rigorous reader could, nevertheless, be unsatisfied with the one-sided interpretation of the new data on particle spectra solely in favour of the PA picture. The point is that the data show a systematic increase of the normalization factor $K(Y)$ with decreasing c.m. energy from 91 to 14 (22) GeV by 14% (8%) (recall that the systematic error in normalization is $\pm 7\%$).

Before expanding on the intriguing problem of the possible influence of the hadronization stage, one should better understand the non-leading perturbative corrections to the limiting spectra [see footnote⁵].

The formulae [Eqs. (55), (62)] for the MLLA spectra correspond only to the parton-gluon distributions. However, in the formation of the final hadrons the soft-sea $q\bar{q}$ pairs could participate as well. Taking them into account may result in an additional multiplica-

Figure 12: Energy evolution of the maximum of $\ln(1/x_p)$ compared with the analytical formula (57).

tive factor $[1 + O(1/\sqrt{Y})]$, which decreases with c.m. energy. Some energy dependence of $K^{\text{ch}}(Y)$ could be connected with the fact that $\bar{D}^{\text{ch}}$ is a mixture of different particles, the spectra of which prove to be not exactly similar.

In order to understand this and some other problems of the LPHD picture, the measurements of inclusive spectra of identified hadrons (π , K, p, ...) are important. They should also provide a test of the conjecture [7] about the connection of $(Q_0)_{\text{eff}}$ with the hadron mass.

Note that the inclusive momentum distributions of charged particles (Fig. 11a) can be described as the sum of MLLA spectra for π 's, K's, and p's, assuming $Q_0 = m_h$ and $\Lambda \simeq m_\pi$, see Refs. [7], [21]. This implies the natural explanation that the effective Λ , as determined from the data, is $\Lambda \simeq 0.253$ GeV, larger than the value $\Lambda \simeq m_\pi$ found in Refs. [7] from fitting only $\pi^\pm$ spectra.

To conclude this section let us mention that one can study the energy dependence of the inclusive distribution $\bar{D}_q(\ell, Y)$ by using the data at the Z^0 peak only, see Refs. [9], [14], [44]. This can be done by measuring the single inclusive distribution $\bar{D}_q^\theta(\ell, Y)$ obtained by counting only the hadrons which enter into a cone of aperture θ around the jet axis. Particles entering into this cone come from the QCD cascade of a primordial parton with virtuality of the order of $E_{\text{eff}} = E \sin \theta/2$. Therefore the inclusive distribution $\bar{D}_q^\theta(\ell, Y)$ obtained within this cone corresponds to $\bar{D}_q(\ell, Y_{\text{eff}})$. More precisely, for small x , one should find that

$$\bar{D}_q^\theta(\ell, Y) = \bar{D}_q^\theta(\ell, Y_{\text{eff}}) + [\bar{D}_q(\ell, Y) - \bar{D}(\ell, \bar{Y})], \quad (69)$$

where

$$Y_{\text{eff}} = \ln \frac{E \sin \theta/2}{\Lambda}, \quad \bar{Y} = \ln \frac{E \cos \theta/2}{\Lambda} \quad (0 < \theta < \pi/2).$$

The term in square brackets accounts for partons which are emitted in the *backward* jet. Notice that for $\theta = \pi/2$ we obtain the full distribution. Of course the value of E_{eff} should be reasonably high for perturbation theory to be applicable, so one should not take θ too small.

6 THE Z^0 RESONANCE AS A LABORATORY TO STUDY INTERJET COLLECTIVE EFFECTS

6.1 Radiophysics of particle flows

In the framework of LPHD the source of multihadron production in QCD jets is gluon bremsstrahlung, so one should expect that the produced hadrons are a consequence of the colour dynamics at small distances. Therefore, the detailed features of the parton-shower system, such as the flow of colour quantum numbers, influence significantly the distribution of colour-singlet hadrons in the final state.

Such a phenomenon has been first observed in the experiments (see Refs. [45]) studying the angular flows of hadrons in three-jet ($q\bar{q}g$) events from e^+e^- annihilation, the string [29] (or drag [30]) effect. The PETRA/PEP data have strongly supported the predicted drag of the interjet particles in the direction of the gluon jet (net destructive interference in the region between the q and the $\bar{q}$). They demonstrated that the wide-angle particles really do not belong to any particular jet, but have emission properties dependent on the overall jet ensemble.

Detailed studies of the string-like collective effects are of importance for the high-energy reactions. These effects are interesting, not only in their own right, as a test of QCD. They could also be valuable in helping to distinguish new physics signals from the conventional QCD backgrounds.

The Z^0 resonance seems to be an ideal laboratory to explore QCD collective phenomena in three-jet events for the following reasons.

- i) Jets are more energetic and collimated than at lower energies and the gluon jet is better separated (see Ref. [46]).
- ii) An essential advantage is the tagging of heavy-quark jets. It allows one not only to isolate gluon jets but also to study many subtle properties of interjet particle flows (see Refs. [21], [18]).
- iii) At SLC/LEP energies one can convincingly discriminate between the Lund (string) [29] and the PA [30] interpretation of the celebrated string effect (see Refs. [47]).

Already the first experimental studies [46] of three-jet events from Z^0 decays look very promising.

To give a QCD explanation of the drag phenomenon we present the antenna pattern for soft-gluon emission from a hard, massless $q\bar{q}g$ system (see Refs. [30], cf. subsection 3.4). Denoting the quark, antiquark, and the hard- and soft-gluon directions by $\vec{n}_1, \vec{n}_2, \vec{n}_3$, and $\vec{n}_4$, respectively, we have

$$W^{q\bar{q}g}(\vec{n}_4) \propto N_c \left(\frac{a_{13}}{a_{14}a_{43}} + \frac{a_{23}}{a_{24}a_{43}} \right) - \frac{1}{N_c} \frac{a_{12}}{a_{14}a_{42}}, \quad (70)$$

where $W^{q\bar{q}g}(\vec{n}_4)$ represents the angular distribution of the soft gluon and $a_{ij} = 1 - \vec{n}_i \cdot \vec{n}_j$. This may be compared with the pattern for a quark-antiquark system, with a hard photon replacing the gluon jet 3 to permit the directions $\vec{n}_1$ and $\vec{n}_2$ to remain the same:

$$W^{q\bar{q}\gamma}(\vec{n}_4) = W^{q\bar{q}}(\vec{n}_4) \propto 2C_F \frac{a_{12}}{a_{14}a_{42}}, \quad (71)$$

Let us emphasize here that, owing to colour coherence, the radiation of a secondary soft gluon $\vec{n}_4$ ($W \gg E_3 \gg E_4$) at angles larger than the characteristic angular size of each parton jet proves to be insensitive to the jet internal structure: g_4 is emitted by a colour current, which is conserved when a jet splits.

Thus, the colour-coherence phenomena strongly affect the total three-dimensional shape of particle flows in three-jet events, practically excluding the very possibility of representing it as a sum of three parton contributions (for a detailed analysis see Ref. [21]).

Note that if a term proportional to $1/N_c$ in Eq. (70) is dropped, the two remaining terms may be interpreted as the sum of two independent $(1\bar{3})$ and $(\hat{2}\bar{3})$ antenna patterns, boosted from their respective rest frames into the overall $q\bar{q}g$ c.m.s. This scenario literally repeats the explanation given in the Lund string model [29].

Taking, for illustration, the symmetric $q\bar{q}g$ or $q\bar{q}\gamma$ configuration, the ratio of emission in the direction opposite to the hard gluon or photon, $\vec{n}_4 = -\vec{n}_3$, in the two processes is $W^{q\bar{q}g}/W^{q\bar{q}\gamma} = 7/16$ [see Eq. (24)], showing the destructive interference in this region. This depletion was confirmed both by comparison of hadron densities of the jets [45] and by comparison of the $q\bar{q}g$ and $q\bar{q}\gamma$ final states [38].

It will be of importance to perform a comprehensive analysis at LEP energies, where, for example, the subasymptotic corrections are less essential.

A discriminative test between the string picture and PA is provided by the dependence of the multiplicity flow on the particle mass m_h or p_{out} (momentum out of the event plane).

The previous data [45] demonstrated that the string effect is enhanced by increasing m_h or p_{out} . If interpreted in terms of a Lorentz boosted string of the canonical Lund model, this enhancement will be of the same strength at all c.m. energies, given fixed angles between jets. In contrast, in the PA scenario, the effect is determined by the number of soft gluons emitted in the different interjet valleys. Therefore, at high enough energies no enhancement is expected for the subsamples with large m_h or p_{out} . While the subasymptotic effects may play an important role at low energies, ultimately the perturbative picture will win out at higher energies.

For illustration, we present in Fig. 13 the energy evolution of the particle-flow ratios in the ARIADNE program [49], which contains both soft-gluon emission and string fragmentation. One can easily see that the PA regime here becomes dominant at Z^0 energies.

6.2 Correlations of interjet particle flows

Studies of particle correlations in jets should provide an especially sensitive test of the connection between observed hadron distributions and the colour structure of an underlying hard process, see Refs. [14], [21].

For example, an interesting manifestation of the QCD wave nature of hadronic flows arises from studying the double-inclusive correlations of interjet flows in $e^+e^- \rightarrow q\bar{q}g$ events. The point is that here one faces such tiny effects as the mutual influence of different $q\bar{q}$ antennae. As a consequence ($+, - \equiv q, \bar{q}$; $1 \equiv g$) :

$$r_2 = \frac{d^2 N}{d\Omega_{(1+)}d\Omega_{(1-)}} \bigg/ \frac{d^2 N}{d\Omega_{(+-)}d\Omega_{(1-)}} < r_1 = \frac{dN}{d\Omega_{(1+)}} \bigg/ \frac{dN}{d\Omega_{(+-)}} , \quad (72)$$

(the elaborated analytical formulae may be found in Ref. [21]). Such colour-screening effects cannot be mimicked by the canonical Lund string model. But the ‘dipole formulation’ of the Lund Monte Carlo [48] reproduces them [21]. To quantify the correlation

Figure 13: The ratio of particle flow between q and g jets to that between q and $\bar{q}$ jets, for Mercedes-like configurations (using 60° sectors) [47]: a) mass dependence of the string effect at $W = 30$ and 91 GeV; b) p_{out} dependence of the string effect at $W = 30$ and 91 GeV.

Table 2: Ratios r_1 and r_2 as a function of energy W [47]

W (GeV)		30	45	75	91	750
Dipole cascade	r_1	2.74	2.65	2.43	2.38	2.13
+ string fragmentation	r_2	2.52	2.38	2.12	1.97	1.74
String fragmentation only	r_1	2.69	2.94	3.01	2.99	2.89
	r_2	2.69	2.94	2.99	2.97	2.90

effects one can compare the ratios of particle flows, projected onto the event plane, in the case of Mercedes-like symmetric events.

Table 2 illustrates the energy dependence of the ratios r_1 and r_2 , calculated using the ARIADNE program [49]. One can easily see that when the dipole cascade is used, r_2 is clearly smaller than r_1 already for $W = 30$ GeV, while $r_2 \simeq r_1$ over the whole energy range in the case when only string fragmentation is used.

Of fundamental importance is the azimuthal asymmetry of QCD jets. Recall that the treatment of the structure of final states given by the string picture qualitatively reproduced the QCD radiation pattern only up to $O(1/N_c^2)$ corrections (the large N_c limit), see Ref. [30] and discussion below Eq. (71).

However, under specific conditions $O(1/N_c^2)$ terms become sizeable or even dominant.

The simplest example is given by the azimuthal asymmetry of a quark jet in the events $e^+e^- \rightarrow q+\bar{q}-g_1$. The azimuthal distribution of particles produced inside a cone of opening half-angle θ_0 may be characterized by an asymmetry parameter:

$$A(\theta_0) = \frac{N_{\rightarrow g}(\theta < \theta_0) - N_{\rightarrow \bar{q}}(\theta < \theta_0)}{N_{tot}(\theta < \theta_0)} = \frac{(\Delta N)_{as}}{N_{tot}}. \quad (73)$$

For parametrically small θ_0 ,

Figure 14: Comparison of the QCD (solid) and large- N_c limit (dashed) predictions for the G factor, see Eq. (74).

$$A(\theta_0) \simeq \frac{2\theta_0}{\pi} G \sqrt{4N_c \frac{\alpha_s(E\theta_0)}{2\pi}}, \quad (74)$$

$$G = \frac{N_c}{2C_F} \cot \frac{\theta_{+1}}{2} + \frac{1}{2N_c C_F} \cot \frac{\theta_{+-}}{2}.$$

The first colour-flavoured term in Eq. (74) describes the asymmetry due to the ‘boosted string’ connecting the q and g directions. The corresponding asymmetry vanishes with an increase of θ_{+1} as the string straightens. Here, however, the second term enters into the game, forcing the asymmetry to increase anew, as shown in Fig. 14. This behaviour might be interpreted as an additional repulsion between particles from q and $\bar{q}$ jets.

For a symmetric configuration

$$\theta_{+1} = \theta_{-1} = \pi - \theta_{+-}/2,$$

the colour-suppressed term in Eq. (23) prevails when

$$\theta_{+1} \leq 2 \arctan(1/N_c) \approx 37^\circ. \quad (75)$$

To realize this effect, one has to select $q\bar{q}g$ events with special kinematics, in which case the hard gluon moves in the opposite direction to the quasi-collinear $q\bar{q}$ pair.

The observation of the drag (string) effect has beautifully demonstrated the connection between colour and hadronic flows. However, the ‘string’ analysis of three-jet events suffers from some inherent difficulties and weaknesses that one would prefer to avoid if possible. First of all, the necessity of selecting a three-jet event sample reduces the statistics and may introduce biases into the observed hadronic flow. The need to define jet directions introduces a dependence on the jet-finding algorithm. Discrimination between quark and gluon jets, on the basis of their relative energies, reduces the effect and prevents the use of symmetrical jet configurations.

Of course, the interjet collective phenomena could be better analysed if one could identify the quark jets. This can be done in the case of the heavy-flavour production events, in which one can tag the heavy-quark decay, see Refs. [18], [21].

Recently a new method has been proposed [14] to reveal the connection between observed hadron distributions and the colour structure of an underlying hard process. The

method is related to the ‘string effect’ analysis but has the advantage of not requiring any special event selection or jet finding. It involves measuring a ratio of energy-multiplicity correlations which is especially sensitive to colour flows in jet formation. This quantity is infrared stable and can be calculated completely perturbatively.

To illustrate the new method, consider the emission of *two* soft gluons by a hard quark–antiquark system:

$$W^{q\bar{q}}(\vec{n}_3, \vec{n}_4) \propto 2C_F N_c \left(\frac{a_{12}}{a_{13}a_{34}a_{42}} + \frac{a_{12}}{a_{23}a_{34}a_{41}} - \frac{1}{N_c^2} \frac{a_{12}^2}{a_{13}a_{32}a_{14}a_{42}} \right) \quad (76)$$

(cf. subsection 3.5). Dividing by the product of the single-gluon distributions gives the gluon–gluon correlation function

$$C^{q\bar{q}}(\eta_{34}, \phi) = \frac{W^{q\bar{q}}(\vec{n}_3, \vec{n}_4)}{W^{q\bar{q}}(\vec{n}_3)W^{q\bar{q}}(\vec{n}_4)} = 1 + \frac{N_c}{2C_F} \left(\frac{\cos \phi}{\cosh \eta_{34} - \cos \phi} \right) \quad (77)$$

(for the definition of η_{34} and ϕ see subsection 3.5).

Equation (77) provides an infrared-finite measure of the correlation between colour flows in the directions (η_3, ϕ_3) and (η_4, ϕ_4) . According to the local duality hypothesis, it can be applied directly to hadronic flows. Considering, for example, the flows in the orthogonal ($\phi = \pi/2$) and back-to-back ($\phi = \pi$) azimuthal directions, we have

$$C^{q\bar{q}}(\eta_{34}, \pi/2) = 1, \quad C^{q\bar{q}}(0, \pi) = \frac{N_c^2 - 2}{2(N_c^2 - 1)} = \frac{7}{16}. \quad (78)$$

This implies that for $\phi = \pi/2$ the hadronic multiplicities are uncorrelated, whilst for $\phi = \pi$ there is destructive interference, of the same magnitude as the string effect in symmetric jets [Eq. (24)]. Thus measurements of hadronic-flow correlations in the orthogonal and back-to-back azimuthal directions should demonstrate the same type of colour coherence as the string effect, without requiring the selection of a three-jet event sample.

To estimate the effects of finite energy corrections and hadronization [11] the Monte Carlo program HERWIG was used. The results clearly demonstrated the destructive interference at both the parton and hadron levels. The effects of hadronization are small at these energies. However, the $O(\sqrt{\alpha_s})$ corrections look rather significant. It would be interesting to compute the corrections to the correlation function analytically.

An analysis of colour flow along the above lines may be performed for other correlations. Another example is the study of azimuthal-asymmetry effects in e^+e^- events with a hard photon ($q\bar{q}\gamma$). It is interesting to observe that in correlations with an additional multiplicity flow ($q\bar{q}\gamma$), the asymmetry depends on $1/N_c^2$ corrections, which are typically neglected in Monte Carlo simulations.

7 PROSPECTS FOR FUTURE COLOUR-COHERENCE STUDIES AT THE Z^0

We present here a brief ‘shopping list’ for an experimental investigation of hadron-jet phenomena, which should reveal the colour structure of QCD and the local duality features. We concentrated mainly on the two topics: i) hadron distributions in QCD jets; ii) QCD collective effects in interjet hadronic flows (for discussions of other topics see, for example, Refs. [14], [21], [22]).

- a) Testing the various aspects of the MLLA–LPHD picture urgently requires measurements of spectra of different identified hadrons: $\pi^\pm$, π^0 , K, p, ...

- b) It will be interesting to separate contributions to particle spectra caused by the light quarks and by the heavy ones, for example, by tagging heavy-quark jets.
- c) By measuring spectra of particles restricted to lie within the particular opening angles with respect to the jet [44] one can explore the specific features of the parton multiplication processes and, in particular, sharpen the influence of angular ordering on the particle branching.
- d) Multiplicity distributions (KNO picture) in QCD jets are predicted analytically. It will be of interest to analyse these distributions in detail in the LEP energy region.
- e) One of the most interesting tasks seems to be the measurement of the two-particle inclusive distribution of hadrons (for the DL result see Refs. [31], for calculation of the next-to-leading corrections see Refs. [12]):

$$R(\ell_1, \ell_2, Y) \equiv \frac{\overline{D}^{(2)}(\ell_1, \ell_2, Y)}{\overline{D}(\ell_1, Y)\overline{D}(\ell_2, Y)}, \quad (79)$$

with

$$Y = \ln \frac{E}{\Lambda}, \quad \ell_i = \ln \frac{1}{x_i} \quad (i = 1, 2).$$

A preference is expected for $x_1 \simeq x_2$ and for both to be small. The range of these correlations is of the order of Y , so they extend over long distances in the $(\ln x_1, \ln x_2)$ plane. Thus they should be easily distinguishable from correlations due to hadronization, which would be expected to have ranges of the order of $\ln(Q_h/\Lambda)$, where the hadronization scale Q_h is at most a few GeV.

- f) To establish clear connections between theory and experiment, it is preferable [44] to work with inclusive quantities which are defined on the basis of simple jet characteristics such as energy or multiplicity flow, rather than on the basis of a given number of jets having specific directions, energies, momenta, masses, etc.
There is a direct correspondence between the jet direction and the energy-flow direction, so that we may naturally study the shapes of jets and any characteristic of the hadronic system produced in a hard interaction by introducing inclusive correlations among energy flows and multiplicity flows. In this case one does not need to apply event-selection procedures or jet-finding algorithms.
- g) The detailed tests of colour-coherence effects and their discrimination against non-perturbative dynamics require comprehensive studies of the total three-dimensional pattern of particle flows in three-jet events. Comparison with analytical results (accounting for both interjet and intrajet coherence phenomena) should make it possible to distinguish reliably the PA predictions from fragmentation schemes. Of special interest here are the energy dependence of the multiplicity flow and its dependence on m_h and p_{out} .
- h) One can study (for example by jet identification) the double-inclusive correlations r_2 of the interjet flows, where the quantum mechanical collective nature of radiation manifests itself.
- i) The ratio of energy-multiplicity correlations (77) looks very promising. This method has the advantage of not requiring any special event selection or jet finding. Since finite energy corrections depend, in general, on the particle masses, it would be useful to study these corrections for different hadron species.
- j) By studying the radiation pattern of a tagged quark jet in $q\bar{q}g$ events, one can measure the azimuthal asymmetry of a jet. An interesting vista on this problem is connected

with the $Z^0 \rightarrow c\bar{c}g$ events. To investigate the role of $1/N_c^2$ interference terms, one has to select $c\bar{c}g$ events in which the hard gluon moves in the direction opposite to the quasi-collinear $c\bar{c}$ pair, and to measure the asymmetry parameter $A(\theta_0)$ at $\theta_0 \geq 3-4^\circ$, see Ref. [21] for details. Of course, before the different behaviour in QCD prediction and its large N_c limit can be studied, the asymmetry itself must be investigated experimentally.

- k) Of special importance are the particle distributions inside jets initiated by heavy quarks, see Refs. [18], [21] for details. One of the prospective ways of fixing jet direction arises from the vertex detection.

In particular, one can study here distributions of particles inside fixed cones, energy, and angular spectra. Of special interest is the observation of the particle depletion in the forward region and the search for the manifestations of non-perturbative dynamics.

One of the interesting characteristics is the spectrum $\bar{D}_Q^h(x, W)$ of associated light particles in the heavy-quark jet. In the regime $W \gg m_Q \gg \Lambda$ (within the MLLA accuracy) this spectrum is given by [50]:

$$\bar{D}_Q^h(x, W) = \bar{D}_q^h(x, W) - \bar{D}_q^h\left(\frac{x}{\langle z \rangle}, m_Q \sqrt{e}\right), \quad (80)$$

where $\bar{D}_q^h(x, W)$ is the usual hadron inclusive spectrum in e^+e^- collisions, and $\langle z \rangle$ is the averaged scaled energy of the heavy quark.

8 CONCLUSION

With the start of activity at the Z^0 peak a wealth of experimental data on QCD jets has become available for detailed tests of PA and for reducing our domain of ignorance on the physics of confinement. These experimental studies of jets show that the gross features of hadronic event shapes calculated at the partonic level agree well with the measured ones. This strongly supports the hypothesis of ‘soft confinement’, which leads to the LPHD picture. Starting from this hypothesis it is possible within the PA picture to obtain the number of detailed predictions for hard collisions.

Hadroproduction studies within the MLLA-LPHD scheme seem to be far from being exhausted. Future experiments, in particular at the Z^0 resonance, definitely need CLIC-like PA activity. They should provide convincing tests of both perturbative and non-perturbative aspects of QCD.

The main aims of this lecture were: i) to move the experimentalists closer to the QCD-CLIC picture of multihadron production, ii) to help them to overcome some prejudices stemming from the past, iii) to present a ‘shopping list’ for colour-coherence studies at the Z^0 .

To conclude this lecture, let me try to respond to the argument of the devil’s advocate: Why, at the present high level of credibility and maturity of QCD, should we explore so thoroughly one of its inherent properties — the quantum mechanical coherence?

The answer could be as follows. The main purpose is not the proof of colour coherence: it would be inexcusable (and somewhat expensive) to test quantum mechanics at modern accelerators. Of real importance is how the coherence reveals itself in hadron spectra, i.e. how confinement disturbs the perturbative picture of multihadron production.

One should also take into account that the spectacular collective effects in the jetty final states could provide a valuable additional tool, helping to extract and to study

manifestations of New Physics.

Finally, let us mention that the most adequate incorporation of the coherence phenomena becomes now an important approach for the development of the QCD-inspired Monte Carlo algorithms.

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