Activity in 1992 related to the design of the CERN Large Hadron Collider

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Abstract

The progress on the definition of the parameters and the evaluation of the performances of the CERN Large Hadron Collider are briefly sketched. A lattice with an increased filling factor of main dipoles is now available for long-term stability studies with computer tracking simulations. Experiments on the effects of nonlinearities are pursued in the CERN-SPS. Investigations of the truncated maps as well as the Hénon maps with a modulated tune have been launched.

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Activity in 1992 related to the design of the  
CERN Large Hadron Collider  

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INTRODUCTION  
In 1992, the definition of the parameters and the evaluation of the performances of the CERN LHC progressed along three main lines:  
The magnetic structure of the lattice was modified in order to increase the filling factor of the main dipoles in the regular cells.  
The tolerances on the field quality of the guiding magnet were investigated with long-term computer tracking simulations as well as with experiments performed at the SPS.  
Additional studies were performed to speed-up the simulations using truncated maps, to propose phenomenological descriptions of the diffusive behavior in the transverse phase-spaces, and to obtain a deeper insight of the non-linear motion in the case of dynamical systems as simple as the Hénon map.  
Other activities dealing with the design of hardware components, uncorrelated to the issues of the present workshop will not be reported.  

MAGNETIC STRUCTURE OF THE LHC LATTICE.  
The magnetic structure of the LHC lattice has been modified in such a way that there are 24 regular cell per arc and three main dipoles per half-cell. The inner diameter of the dipole superconducting coils is 56 mm. The cell is 102,042 m long and the main dipoles are 13.145 m long. Near the 3.05 m long main quadrupoles there is a short straight section with beam position monitors, closed orbit dipoles, tuning quadrupoles, chromaticity sextupoles, and Landau-damping octupoles. Each main dipole contains small sextupole and decapole correctors, located in the shadow of the electrical connections, the role of which is to compensate almost locally the systematic part of the $b_3$ and the $b_5$ field-shape imperfection components (in European notation $b_3$ is a sextupole, etc.).  
The insertions have been re-matched to the modified arcs of regular cells. Each dispersion suppressor contains eight cell dipoles, one shorter dipole 8.8 m long, and four quadrupoles. Each long straight section beside the crossing points contains two triplets of quadrupoles and a set of horizontal separation-recombination dipoles. The utility straight sections for the dump and the halo cleaning have a special design.  
The orbit functions of the regular cell and of the experimental insertions are shown in Fig. 1 and 2 respectively.  

NONLINEAR CORRECTORS  
The systematic components $b_3$, $b_4$ and $b_5$ of the field-shape imperfections are particularly pronounced at injection due to the effect of the persistent currents in the superconducting coils of the main dipoles. The typical values expected for them in the LHC magnets are respectively of the order of $-2.5$, $-0.15$, and $0.5$ in $10^{-4}$ units at a reference radius of 1 cm.  
Fortunately there is no need to compensate the systematic component $b_4$, since its value changes sign at each LHC octant and therefore its effect is globally self-compensated. Such a change of sign is inherently related to the side by side design of the two-in-one magnets and to the
fact that the beam trajectory goes from the inner to the outer ring or vice-versa at each of the eight crossing points of the LHC.

The systematic components $b_3$ and $b_5$ have to be compensated using the sextupoles and the decapoles located at the ends of each main dipole. By setting the integrated strength of each of these multipoles equal to the strength of the systematic error integrated along one dipole an almost local correction can be obtained which is in general sufficient to bring the values of tuneshift close to those of an ideal machine without errors and correctors, as shown in Fig. 3.

A more sophisticated approach to set the nonlinear correctors is based on the use of the normal-forms [1]. The tuneshift functions and their dependence on the strength of the errors and of the correcting gradients are computed order by order. A merit function, defined as the average of the tuneshift integrated over the sum of the normal-form invariants, is evaluated and factorized in order to isolate the terms dependent on the sum of the invariants. The residual terms which describe the dependence of the tuneshift functions on the lattice structure, on the field-shape errors and on the correcting gradients are minimized by varying the correcting gradients. For a final confirmation, the tuneshift with the amplitude and the momentum is computed in the range of interest, by tracking the particles with a symplectic integrator. The various coefficients of the tuneshift functions are estimated making use of the Martin Berz's DA package. Typical results of the nonlinear compensation are shown in Fig. 4 and 5.

**TOLERANCES OF THE FIELD-SHAPE ERRORS**

The tolerances of the field-shape errors in the guiding magnet have been investigated with computer tracking simulations [2]. The heuristic approach proposed for the LHC is based on long-term (up to a few $10^6$ turns) simulations with thin-lens approximation and symplectic integrators. The field-shape imperfections, which can be expressed as the sum of two parts, one systematic and the other random, are represented by multipolar expansions stopped at the order 11th. The possible correlations between random multipoles of different order are neglected. The tolerances are more critical at injection, since the beam size is larger. On the other hand, the systematic errors are larger at injection due to the persistent currents.

The large value of the low-order (3rd and 5th) systematic multipoles provoke a sizeable detuning with amplitude and momentum, which can be corrected either locally or using a clever cancellation of the detuning terms with a robust minimization procedure based on the normal-forms, as discussed in the previous section.

Large high-order (7th and 9th) systematic multipoles destabilize the off-momentum particles and have to be minimized by design.

Random imperfections which vary from magnet to magnet due to the manufacturing tolerances are the main source of the resonances and the distortion functions. Statistical moments of the distributions can be easily predicted, but are insufficient for a complete knowledge of the nonlinear optics, since the resonance strengths depend on the specific sequence of the random errors around the ring, rather than on statistical properties. Therefore, criteria for magnet design are to be studied on several non-linear lattices, with different sequences of random multipoles.

Additional parameters to be considered are the residual closed orbit, the linear coupling, and the synchrotron motion.

The beam stability is influenced by the linear lattice parameters like the tune, the residual linear coupling, the peak-$β$ values in the insertion quadrupoles.

In most of the cases, the dynamic aperture is contained in the vacuum pipe of the LHC. However, sophisticated collimation systems are to be located close to the stable orbit to protect superconducting magnets from losses. For a safe operation, a careful matching of the physical aperture in presence of collimators and the stability border is needed. The budget of the physical aperture includes the space required for the beam and that required for the injection oscillations, the magnet misalignment, the residual sagitta of the dipole, which are curved to follow the beam path, the estimate of the residual closed-orbit and of the $β$-function modulations, and finally the
space required to devise a safe collimation system. At injection, the inner collimator jaw is expected to be located at 8 mm from the central orbit. In Fig. 6 the computed value of the dynamic aperture, defined as the initial amplitude of a particle surviving at least $5 \times 10^5$ turns, is plotted as a function of the inner collimator position. The dashed curve is the chaotic border above which the value of the Lyapunov exponent becomes positive. As a rule of thumb, the field-shape imperfections are considered tolerable when the chaotic limit almost coincides with the nominal position of the collimator, whilst the dynamic aperture is sufficient to accommodate three times the r.m.s. transverse beam dimension augmented by injection oscillations of a reasonable amplitude.

**EXPERIMENT WITH THE CERN-SPS**

Experiments are performed in the CERN-SPS to study the dynamic aperture and the diffusion of circulating particles in a regime as similar as possible to that expected in the LHC [3]. The protons stored in the SPS at a momentum of 120 GeV/c are excited with already existing sextupoles in order to introduce in a controlled fashion non-linearities in an otherwise linear lattice. To probe large amplitudes, a pencil beam with small emittance and momentum spread is used, to which a large enough coherent deflection is applied. In a few hundred turns, a 'hollow' distribution of charges is created around the central orbit due to nonlinear filamentation. Its behavior is observed with several instruments: current transformers record lifetime, Schottky noise detectors give tune and tune-spread, flying wires provide transverse profiles, and orthogonal pairs of position monitors are able to produce a phase space portrait.

The sextupolar excitation used minimizes the strength of the third integer resonance. Detuning compensation was experimentally tested by using existing octupoles: a 30% increase of dynamic aperture resulted from a factor ten reduction of tune-spread. This provides experimental guidance in devising correction schemes for large hadron accelerators. However most of the emphasis was put on the study of slow diffusion induced by tune modulation produced by natural power supply ripple as well as controlled modulation of a special quadrupole. The diffusion coefficient was measured as a function of the amplitude, the modulation frequency and depth, and the tune. It was obtained by scraping the beam tail with horizontal and vertical collimators, retracting them suddenly by a few mm, and observing the beam lifetime to estimate the time taken by the particles to fill the gap created by the retraction. Diffusion immediately sets in when tune modulation is turned on, and there is evidence that a ripple which leads to tune modulation of $10^{-3}$ cannot be tolerated in a machine with strong non-linearities. A simultaneous tune modulation at two frequencies is by far more destructive than a modulation at a single frequency for the same overall depth. The agreement with numerical simulations is of the order of 20%, however the dependence of diffusion on modulation depth and frequency are not yet fully explained.

A critical comparison of the experimental results at the CERN-SPS with those of a similar experiments at the FNAL-Tevatron introduced a crucial discussion [4]. The results of the latter experiment can be interpreted in terms of a fast-growing diffusion obeying to a major degree of universality: the initial transverse distribution remains unaffected for betatron amplitudes smaller than a certain time-dependent value, whilst the density is fully depleted for larger betatron amplitudes. The transient region is narrow and independent on the shape of the initial distribution. The characteristic results of the CERN experiment obtained with the scraper retraction show features which are incompatible with the fast-growing diffusion model, since the transition to losses, after collimator retraction, when protons reach the new collimator position, is rather sharp, whereas the following the pattern of intensity losses is quasi-linear. On the other hand, the features of a typical survival plot obtained with computer tracking simulations are also incompatible with the diffusion mechanisms à la Fokker-Plank, since the spread of the survival time with the initial conditions is too large. A coherent description of the previous phenomena is not yet available.
TRUNCATED TAYLOR MAPS

The conventional element-by-element tracking performed with a symplectic integrator is in general limited to about $10^5$ turns even with the computing facilities available nowadays. To overcome this limitation, the use of high-order truncated Taylor maps in the long-term stability studies for the LHC has been investigated [5]. However, with this sort of transport algorithms the volume in the phase-space is no more an invariant of the motion. The symplecticity of the one-turn matrix has been recovered using either the well-known kick factorization à la Irving, or a novel procedure called dynamic re-scaling. The latter is quite pedestrian. It consists not of adding higher-order terms to eliminate the high-order violation of symplecticity, but rather in adding a linear transformation. The transformation is staged in three different scale transformations, two in the transverse and one in the longitudinal directions respectively, characterized by three different scaling factors chosen in a suitable manner to ensure that the Liouville's theorem is obeyed, at least in average. In fact, there is an infinite set of possibilities for the choice of the three scaling factors. The additional criterion to determine their final choice is based on the following arguments. It is assumed that Taylor maps up to order 11th are sufficient for an accurate description of the beam dynamics in the LHC, at least in the region of the phase-space where the motion is regular or only weakly chaotic. The beam trajectories are computed both with element-by-element tracking and with Taylor maps with initial values of the scaling factors. The difference of amplitudes in the phase-space of the two results are estimated in few iterations by slightly varying the scaling factors. From that one can optimize the scaling factors in such a way that the amplitude differences between the direct tracking and the iteration of the Taylor map is constant as a function of the number of turns. The comparison between direct tracking and iteration of truncated maps with and without dynamic re-scaling has been made for a large number of turns: the agreement is two order of magnitude better with re-scaling that without.

HÉNON MAP

The Hénon map has been studied by numerical tracking in order to clarify the role of the trapping phenomenon in presence of a weak sinusoidal modulation of the tune and to check the possibility of defining a diffusion coefficient [5]. For small values of the frequency modulation, the well-known pulsation of the whole characteristic structure of islands and the periodic beat of the separatrices has been made evident. A critical frequency has been found numerically which has the following property: for frequency modulations of the order of the critical frequency the locking of the particles on the resonance islands can work only in one direction during the oscillation of the tune. This partial trapping acts as a very fast transport mechanism towards outer regions of the phase-space. The critical frequency has been found to be almost inversely proportional to the depth of the tune oscillation. Such a dependence has also been confirmed analytically with a Hamiltonian approach. Moreover it has been found that there is a strong dependence of the critical frequency on the initial coordinates of the particle in the resonance island.

The diffusion coefficient is heuristically defined as the increase of the particle emittance per turn. In a standard Hamiltonian treatment this parameter is expected to be inversely proportional to the tune modulation depth. Numerical simulations with the Hénon map have shown that, in a given condition, the growth of the emittance is linear with time only for a limited number of turns. In this time-scale the diffusion coefficient is in fact roughly decreasing with the modulation depth. However, the dependence is almost parabolic and shows a fine structure for small values of the modulation depth.

The domain of stability of the unperturbed Hénon map has been investigated [7] with a novel technique based on the numerical estimate of the invariant manifolds of the unstable fixed point. There are two sub-space emanating from the hyperbolic fixed point characterized one by an expanding and the other by a contracting behavior. These two manifolds are characterized by the
following features: they are both invariants of the motion, and they have at least one intersection: the unstable fixed point itself. This in fact implies that the two manifolds have an infinite number of intersections and that they can be constructed by iterating many times an ensemble of initial conditions belonging to a small part of these manifolds around the hyperbolic fixed point. It turns out that these initial conditions can be chosen on the eigenvalues of the linearized map as long as one stays within small distance from the fixed point. The invariant manifolds of the unstable fixed point have been compared to the stability regions as computed by direct tracking, and have been found to be practically identical for all values of the tune.

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REFERENCES

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[7] M. Giovannozzi "Invariant Manifolds and Stability: Some Results for 1-D Maps", these proceedings
Fig. 3 Detuning with the transverse amplitude
Solid curve: horizontal detuning
Dashed curve: vertical detuning
Curves with the symbol O: lattice without field-shape errors
Other curves: lattice with field-shape errors and local corrections
Fig. 4 Detuning with the amplitude
Fig. 5 Detuning with the momentum
Fig. 6 Dynamic aperture vs physical aperture