RESULTS ON QCD AND JETS FROM THE LEP EXPERIMENTS

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ABSTRACT
All LEP collaborations have studied multihadronic \( Z^0 \) decays in order to understand hadronic event shapes and to test the theory of QCD in the perturbative and non-perturbative regimes. Among the topics that are reviewed are event shapes, coherence and intermittency effects, the determination of \( \alpha_s \), and angular correlations in four-jet events. The results obtained so far (fall 1990) are in very good agreement with QCD predictions.

1. INTRODUCTION

1.1 Why does one study QCD at LEP?

There are several reasons for studying QCD at the \( Z_0 \) energy:

- The \( e^+e^- \) annihilation process into a \( q\bar{q} \) pair is well understood and relatively uncomplicated as one does not have to deal with spectator jets and structure functions.
- The event rate at the \( Z^0 \) pole is large.
- The energy of the produced system is high, leading to very collimated jets and a reduced value for \( \alpha_s \). The influence of higher order corrections is thus reduced leading to a smaller dependence on non-perturbative hadronisation effects.
- Sophisticated detectors with nearly 4\( \pi \) coverage are available, leading to analyses that require only small acceptance corrections.
- Finally, multihadronic events constitute an important background e.g. in the search for new particles or rare processes. Consequently it is imperative to have a good understanding of multihadronic \( Z^0 \) decays.

In Table 1 a comparison between various \( e^+e^- \) storage rings is made in terms of the detection of multihadronic events. The "jettiness" of the events is expressed by the ratio \( P_T/P_L \), the average transverse and longitudinal momenta in a jet. The relative values are normalised to unity for LEP.
Already now (end of 1990) $Z^0$ factories are in advantage for studying QCD.\footnote{The only possible exception is the determination of $\Lambda_{\overline{MS}}$ which, given comparable statistics, could be done more precisely at lower energies where $\alpha_s$ is more dependent on the scale parameter.}

<table>
<thead>
<tr>
<th>Storage Ring</th>
<th># Hadronic Decays</th>
<th>Relative Boost</th>
<th>Relative Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEP</td>
<td>$\approx 100000$</td>
<td>0.60</td>
<td>1.20</td>
</tr>
<tr>
<td>PETRA</td>
<td>$\approx 70000$</td>
<td>0.70</td>
<td>1.17</td>
</tr>
<tr>
<td>TRISTAN</td>
<td>$\approx 4000$</td>
<td>0.85</td>
<td>1.08</td>
</tr>
<tr>
<td>LEP</td>
<td>$\approx 180000$</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1.2 The structure of a typical hadronic event

The production of a multihadronic event can be divided into several phases according to the methods used in describing the process (Fig. 1).

- The first phase represents the annihilation of an $e^+e^-$ pairs into a virtual $Z^0/\gamma$ which then decays into a primary quark–antiquark pair. This part also contains the effects of initial state radiation and hadronic and weak loop corrections.

- The second phase describes gluon radiation off the primary $q\bar{q}$ pair as well as subsequent branching processes from the produced partons. It is assumed that strong pertubation theory can be used to describe this process as long at the momentum transfer for a branching ("virtuality" $Q^2$) is large enough.

- The third phase describes the fragmentation of colored partons into colorless hadrons and cannot be calculated perturbatively. Instead, phenomenological methods, inspired by QCD, are employed. Typically the transition between the perturbative phase and the fragmentation phase is expected to be around $Q^2 \approx 1 \text{ GeV}^2$, however, the success of the the principle of "Local Hadron Parton Duality", where partons are effectively identified with hadrons, may indicate that the perturbation phase can be carried further.

- The fourth phase describes the decay of unstable hadrons into particles observable in the detector. The input for this phase comes from experimentally determined branching ratios and
lifetimes.

Fig. 1: The structure of a multihadronic event.

1.3 The topics studied so far

A surprisingly large number of results (18 publications and preprints as of September 1990) have been published by the LEP detector groups as well as Mark II at SLC. Many analyses profited from the groundwork layed by the PETRA and PEP experiments. As can be seen from Table 2 a broad spectrum of topics has been covered so far, many of which have been studied by several experiments.

<table>
<thead>
<tr>
<th>Topic</th>
<th>Mark II</th>
<th>ALEPH</th>
<th>DELPHI</th>
<th>L3</th>
<th>OPAL</th>
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<td>MC comparison with event and single particle distributions</td>
<td>[1]</td>
<td>[2]</td>
<td>[3]</td>
<td>[4]</td>
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<td>Determination of MC parameters</td>
<td>[6]</td>
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<td></td>
<td></td>
<td>[5]</td>
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<tr>
<td>$\sigma_\phi$ from from jet rates</td>
<td>[24]</td>
<td>[30]</td>
<td>[29]</td>
<td>[28]</td>
<td>[27]</td>
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<tr>
<td>$\sigma_\phi$ from EEC, AEEC, CEEC etc.</td>
<td>[37]</td>
<td>[36]</td>
<td></td>
<td></td>
<td>[35]</td>
</tr>
<tr>
<td>Triple Gluon Vertex</td>
<td>[45]</td>
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<td>Intermittency</td>
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<td>Rapidity</td>
<td>[22]</td>
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<td></td>
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<tr>
<td>Multiplicity distributions</td>
<td>[1]</td>
<td>[13]</td>
<td>[12]</td>
<td>[5]</td>
<td></td>
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</tbody>
</table>
2. HADRONISATION

Since quarks and gluons can only be observed via their hadronisation products, the detailed knowledge of the hadronisation phase is of utmost importance. In covering the main QCD subjects I will therefore proceed "outside in", that is from the study of the hadronisation phase to the investigation of perturbative phenomena.

2.1 MC tools

An essential tool for QCD studies are MC simulation programs that describe hadronic decays in detail. These MC's differ in the way the perturbative expansion is described (matrix element or parton shower), in their implementation of spin— and nonperturbative effects, as well as in the simulation of the fragmentation (string, cluster, or independent fragmentation).

MC's based on 2nd order matrix elements [7] have a well defined dependence on the QCD scale parameter \( \Lambda_{\text{QCD}} \), but cannot describe final states of more than four partons.

Parton shower MC's are, on the other hand, based on the leading log approximation (LL) which partly describes higher order processes. On average, the parton shower MC's predict \( \approx 9 \) primary partons at \( Z^0 \) energies. Since only the first order of the leading log expansion is used, the QCD scale parameter cannot be uniquely defined and is replaced by an effective parameter \( \Lambda_{\text{eff}} \). The most commonly used implementations of the parton shower approach are:

- **JETSET 7.2**: Developed [8] by the LUND group, it is the best tested and most widely used MC. In order to combine, to a certain extend, the best of two worlds, the first branching is mapped to the \( O(\alpha_s^2) \) matrix element prediction. A string fragmentation is used.

- **HERWIG 3.4–5.0**: Developed by Marchesi and Webber [9], this MC incorporates gluon polarisation effects and coherence phenomena from the onset. It also aims at a simplified description of the fragmentation process by using the cluster concept, leading to fewer parameters to adjust.

- **ARIADNE**: Developed by Ya. Azimov et al. [10] and incorporated by the LUND group this MC provides an alternative to parton shower algorithms by identifying parts of the string with color dipoles. In this approach, the emission of a gluon corresponds to a splitting of the color dipole in two parts. Like HERWIG, this algorithm incorporates angular ordering from the beginning.

- **Next to leading log MC's**: These MC can, in principle, combine the virtues of both matrix element and parton shower based MC's. Since they involve a second order expansion, the effective scale parameter can be defined to be equal \( \Lambda_{\text{QCD}} \). HERWIG 4.6 and NNLJET [11] incorporate such an approach, in which, in principle, \( \Lambda_{\text{QCD}} \) can be directly obtained from tuning the MC to the data. However, one has to keep in mind that these MC's are based on a leading log expansion which is strictly valid for collinear kinematics only.
Historically, parton shower MC's have superseeded matrix element based MC's (using a scale $\mu^2=E_{cm}^2$) because they describe the 4-jet rate measured at PEP/PETRA, while matrix elements underestimate it. This inadequate description of the 4-jet rate, however, is understandable, as large loop corrections are expected similar to the case of the three-jet rates. While the only proper way is to calculate the next order, one may look at ways to minimise higher order corrections by a suitable choice of renormalisation scale. By using such an "optimised" scale, matrix element MC's have made a remarkable comeback. While this may sound like a "dirty trick" there is a reason behind it as discussed in the following paragraph.

2.1.1 A digression: renormalisation scale dependence

From the renormalisation group theorem follows that a physical quantity (like the 3-jet rate $R_3$), when calculated to infinite order, does not depend on the choice of scale for $\alpha_s$. However, this is not true anymore if the expansion series is truncated such as

$$R_3 = a_1 \alpha_s + a_2 \alpha_s^2 + O(\alpha_s^3) .$$

Since $\alpha_s(Q')^{-1} = (b \ln Q'/\Lambda^2) + \ldots$, a change in scale $Q'^2 = f Q^2$ leads to a relation

$$\alpha_s'(Q'^2) = \alpha_s(Q^2) - b \alpha_s^2 + O(\alpha_s^3)$$

which, in turn will lead to a different value for the observable such as

$$R_3' = a_1 \alpha_s'(Q'^2) + [a_2 + a_1 b \ln f] \alpha_s' + O(\alpha_s^3) .$$

Note that for a given expansion only the highest order term has an explicit scale factor dependence. The effect of a larger $\alpha_s'$ value is partly compensated by the second order term. The 4-jet rate, however, receives a contribution only from the highest order $O(\alpha_s^3)$ term and is therefore directly correlated with the increase in $\alpha_s'$ as no linear term in the strong coupling constant exists.

It has also been argued that the choice of scale $Q^2 = E_{cm}^2$ is much larger than the natural scale that occurs in the emission of gluons and quarks which is typically in the order of several GeV.

2.2 Event shape distributions

A frequently used family of event measures is based on the thrust

$$T = \max \left\{ \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|} \right\} ,$$

where $\vec{n}$ is a unit vector and $\vec{p}_i$ are the momenta of the particles.
where the sum is taken over all particles $i$ of the event and where the axis $\vec{n}$ for which the maximum is obtained is called the thrust axis. For ideal two-jet events one obtains $T = 1$ while perfectly spherical events lead to $T = 1/2$. The thrust major value ($T_{\text{major}}$) is defined in analogy to $T$ with the major axis orthogonal to the thrust one; the thrust minor axis ($T_{\text{minor}}$) is orthogonal to both the $T$ and the $T_{\text{major}}$ axis. Oblateness is defined as $O = T_{\text{major}} - T_{\text{minor}}$.

The sphericity tensor is defined as

$$S^{ab} = \frac{\sum_i p_i^a p_i^b}{\sum_i p_i^2}.$$ 

The three eigenvalues $\lambda_1 > \lambda_2 > \lambda_3$ can be used to construct the sphericity $S = 3(\lambda_2 + \lambda_3)/2$, with $S = 0$ corresponding to the ideal 2-jet case and $S = 1$ corresponding to spherical events. The aplanarity, $A = 3\lambda_2/2$, measures the particle flow out of the event plane. $S$ is quadratic in the momentum and cannot be calculated in perturbation theory. In order to avoid this problem one may use Sphericity

$$S_{\text{lin}}^{ab} = \frac{\sum_i p_i^a p_i^b / ||\vec{p}_i||}{\sum_i ||\vec{p}_i||},$$

a linearised version of Sphericity which is an infrared safe quantity. Variables deduced from the eigenvalues $\lambda_i$ of sphericity like $D = 27 \cdot \lambda_1 \lambda_2 \lambda_3$ or $C = 3(\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1)$ have been examined by the experiments.

Finally there is the family of Fox–Wolfram moments. The ratio of the second moment normalised to the zeroth moment

$$\frac{H_2^2}{H_0^2} = \frac{1}{2} \frac{\sum_i ||\vec{p}_i|| ||\vec{p}_j|| (3\cos^2\theta_{ij} - 1)}{\sum_i ||\vec{p}_i|| ||\vec{p}_j||}$$

is similar to the formula for energy–energy correlations.

2.2.1 Determination of MC–parameters from data at 91 GeV

As a first step, MarkII [1], ALEPH [2], OPAL [4] have compared experimental event shape distributions with the JETSET and HERWIG MC predictions using default MC parameters. Fig. 2 shows, as an example, results from the DELPHI experiment on sphericity $S$ and aplanarity $A$. The comparison with lower energy data clearly demonstrates that the events get "jettier" ($S \rightarrow 0$) and more...
planar \((A\rightarrow 0)\) with increasing center-of-mass energy.

![Graph](image)

Fig. 2: Sphericity and aplanarity as a function of center-of-mass energy.

As a second step, the large data sample has been used to tune the MC parameters to the data at 91 GeV. Both OPAL [5] and ALEPH [6] (preliminary) have taken this approach.

- OPAL determines the MC parameters by tuning to two distributions only:
  \[
  \frac{H_2}{H_0} \quad \text{and} \quad T_{\text{major}}
  \]

The modelling of the data is then tested using other variables such as
\(T, T_{\text{minor}}, O, S, A, D, \ln 1/x_p, \) and \(n_{\alpha}.

- ALEPH uses seven distributions for a simultaneous determination of MC parameters:
  \[
  x_p, P_{\text{in}}, P_{\text{out}}, S, A, T, \quad \text{and} \quad T_{\text{minor}}
  \]

Both groups then compare data from PEP/PETRA with the generators tuned at 91 GeV.

The advantage of the OPAL approach is that the fit distributions do not constrain the momentum out of the event plane. The aplanarity distribution is therefore uncorrelated with the fit and a comparison with MC can serve as a consistency check.

The advantage of the ALEPH approach is, of course, the prospect of obtaining smaller errors on the MC parameters. However, the error determination is not trivial because of correlations.
Fig. 3 shows the distribution of the aplanarity obtained at three different energies and compared to three MC models (OPAL). All MC's give a consistent description over a large energy range. The agreement is best in the case of the JETSET 72 parton shower MC ($\chi^2$/bins = 1.4 at 91 GeV).

![Graph showing aplanarity distributions at several energies.]

Fig. 3: MC comparisons with aplanarity distributions at several energies.

One may summarise the comparison of event shape distributions with MC models by observing that:

- Parton shower models based on the leading log approximation describe the global features very well over a large energy range, a result that can be considered as consistency check of QCD.

- $O(x^2)$ matrix element MC's fit the data reasonably well if an optimised scale is used (but the 4 - jet dominated region remains less well described).

- The fitted MC parameters of OPAL, ALEPH, and TASSO (34 GeV) are similar; the JETSET default parameters (Mark II at 29 GeV) however, do not describe the data at 91 GeV very well.

Given the good agreement between data and MC one may unfold the data for hadronisation effects for a direct comparison with distribution generated at the parton level.
2.3 Multiplicity and single particle distributions

2.3.1 Charged multiplicity

Charged multiplicity distributions have been studied extensively in proton and lepton induced reactions. The average, \( < n_{ch} > \), is sensitive to the emission of soft gluons and as such susceptible to higher orders or non perturbative effects such as coherence phenomena. The energy dependence of \( < n_{ch} > \), as well as the statistical distribution of \( n_{ch} \) can be compared with QCD inspired models.

Mark II [1], DELPHI [12], OPAL [5], and ALEPH [13] have determined the average charged multiplicity at the \( Z^0 \) peak. The acceptance corrected values, Table 3, agree well within errors and are also well reproduced by parton shower MC's. Matrix Element MC's also predict the correct value if an 'optimised scale' is used. The errors include tracking errors as well as unfolding and hadronisation contributions.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(&lt; n_{ch} &gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark II [1]</td>
<td>20.10 ± 1.00 ± 1.00</td>
</tr>
<tr>
<td>OPAL [5]</td>
<td>21.20 ± 0.04 ± 0.84</td>
</tr>
<tr>
<td>DELPHI [12]</td>
<td>20.71 ± 0.04 ± 0.77</td>
</tr>
<tr>
<td>ALEPH [13]</td>
<td>20.36 ± 0.06 ± 0.79</td>
</tr>
</tbody>
</table>

The energy evolution of the mean charged multiplicity can be compared to phenomenological parametrisations as well as a QCD NLL prediction of the form

\[
< n_{ch} > = a \alpha_s^b e^{c/\Lambda_{QCD}},
\]

where the energy dependence is solely due to the running of \( \alpha_s \). The value of \( \Lambda_{QCD} = 138 \pm 62 \text{ MeV} \) [12] compares well with an analysis of data at lower center-of-mass energies (\( \Lambda \approx 80 \text{ MeV} \)) predicting \( < n_{ch} > \approx 20.4 \) at 91 GeV.

DELPHI [12] and ALEPH [13] compared the actual multiplicity distribution with several statistical predictions. The DELPHI data are shown in Fig. 4 both for the whole event and single hemispheres. The shape of the distribution is well described by the original and a modified Negative Binominal Form (MNB), a Lognormal distribution, as well as the Lund parton shower MC. A QCD prediction [14], however, fails to describe the shape of the data. The data, if plotted in terms of the
KNO variable $z = \frac{n_{\text{ch}}}{\langle n_{\text{ch}} \rangle}$, can be compared to lower energy data. The data are compatible with KNO scaling.

![Diagram of DELPHI multiplicity distributions for the whole event and single hemispheres.](image)

**Fig. 4:** DELPHI multiplicity distributions for the whole event and single hemispheres.

### 2.3.2 Coherence effects

Coherence phenomena are already known from QED and are due to quantum dynamical effects. If, for example, a high energy $\gamma - \text{ray}$ converts to an almost collinear $e^+ e^-$ pair in an emulsion, there is hardly any blurring at the $e^+ e^-$ vertex where the leptons have traveled less than a typical atom radius and are still very close together. In this case the atom effectively sees a net zero charge. The origin in the case of QCD is of a similar kind, but the effect is more complex due to the self coupling of gluons.

Coherence effects arise as *interjet* phenomena where they have been shown to produce a "string effect" and as *intrajet* phenomena where they are responsible for a decrease of soft gluon emission in jets. They latter effect can be observed in the multiplicity distribution and the hadron momentum distribution at small $x_g = p/p_{\text{beam}}$. In this region soft gluon emission is expected to be suppressed due to destructive interference. The leading log approximation (LLA) predicts a roughly Gaussian distribution of $\ln 1/x_g$ with the position of the peak varying with the center–of–mass energy according to the
relation

\[ \frac{1}{x_{\text{peak}}} \propto \frac{E_{\text{cm}}}{2} \Lambda^\prime \]

where \( \Lambda \) is an effective QCD scale. The LLA approximation predicts a value of \( f = 0.5 \) while \( f = 1 \) would be expected in a world without coherence.

Fig. 5 shows the acceptance corrected \( \ln 1/x_p \) distribution from the OPAL experiment [15] compared to the Gaussian LLA prediction, a next to leading log prediction (MLLA) [16], and a Gaussian with higher moments [17]. In the region around the maximum the MLLA prediction fits the data well. The same data is also compared with JETSET and HERWIG Parton shower MC predictions. While coherence effects are intrinsic to the HERWIG MC model these can be switched on or off in case of JETSET. The string effect can be turned off by using an independent fragmentation model. Obviously, JETSET and HERWIG with coherence fit the data very well while the independent fragmentation model cannot describe the depletion of hadronic tracks at low momenta. The JETSET MC with coherence switched off fits the \( 1/x_p \) distribution surprisingly well. However, the string fragmentation, being due to coherence effects itself, mimicks partly the effect of coherence in the momentum distribution.

A clearer picture emerges once the energy dependence of the peak in the \( \ln 1/x_p \) distribution is examined. The data give an exponent of

\[ f = 0.637 \pm 0.016 \]

which compares well with the MC prediction with coherence and the MLLA prediction of \( f = 0.627 \). The result for JETSET without coherence \( (f = 0.692 \pm 0.012) \) is only marginally consistent with the data.

While coherence effects are expected from quantum mechanics, the really astonishing result of this study is the good agreement of the analytical QCD calculations (MLLA) with the data. The calculation is valid only for partons and is compared to the data asuming the concept of "Local Parton Hadron Duality". In this hypothesis the parton shower approach is continued down to very small \( Q^2 \), effectively identifying partons and hadrons at the end of the cascade.

2.3.3 Intermittency effects

The term intermittency describes nonstatistical particle density fluctuations in analogy to the hydrodynamics of turbulent liquids. Effects of this kind have both been observed in "soft" hadron–hadron collisions and hard scattering \( e^+e^- \) annihilation processes. In order to quantify the effect, factorial moments are introduced [18] by dividing a rapidity range \( \Delta \) into \( M \) bins of width \( \Delta y = Y/M \) and defining:

\[ F_q^{(\delta y)} \propto \frac{< (n) (n-1) \ldots (n-q+1) >}{< n >^q} \]
The average is taken over all bins and all events.

Obviously a bin does only contribute if $n$, the number of tracks in a given bin, is larger than the rank $q$. Consequently, factorial moments act as a filter for high multiplicity bins and are related to the probability to have more than $q$ particles in a phase space bin. For the case of purely Poissonian fluctuations and a flat rapidity distribution one expects that $F_q$ is independent on the number of bins chosen; in the case of intermittency $F_q$ will be a function of the width of the rapidity interval. In particular, self similar cascading models predict a relation of the following kind:

$$F_q \approx M^q \quad f_q > 1.$$ 

Several mechanisms may contribute to an intermittency effect:

- The effect of "self similar cascading" in the development of the parton cascade and in the fragmentation of partons (string - cluster model).
- Bose Einstein correlations as well as long range correlations.
- Hadronic resonances.
- Detector resolution effect and non - flat rapidity distributions.
Finally, intermittency could be an indicator of new physics phenomena such as the quark gluon plasma and phase transitions in general.

The first evidence for intermittency in $e^+e^-$ collisions came from an analysis of HRS data [19]; an analysis of TASSO data [20] indicated a discrepancy between the factorial moments in the data and in the MC. A disagreement was seen both in absolute magnitude and variation as a function of the binsize. A recent CELLO analysis [21] at the same energy, on the other hand, fails to reproduce the disagreement seen by TASSO and agrees with model predictions.

DELPHI [22] has presented an intermittency analysis using hadronic $Z^0$ decays. In contrast to the TASSO analysis good agreement is observed between the data and parton shower MC's (see Fig. 6). Matrix element based MC's, even if an optimised scale is used, fail to describe the data. This may indicate a large sensitivity on the tuning of the MC. The study also indicates that Bose–Einstein correlations play a minor role and that the main effect arises from the fragmentation process.

![Factorial Moments (DELPHI)](image)

**Fig. 6:** Factorial Moments (DELPHI).

### 3. THE DETERMINATION OF THE STRONG COUPLING CONSTANT

It is the ultimate goal to determine the coupling constant of QCD, $\alpha_s$, to a similar accuracy as the parameters of the standard model. Traditionally, three ways have been used to determine $\alpha_s$:

- The hadronic cross section. While to a very high degree independent on hadronisation and acceptance effects, the $\alpha_s$ dependence of $R_{QCD}$ at $Z^0$ energies is small, making a precise measurement difficult (in order to measure $\alpha_s$ with a 10% accuracy, $R_{QCD}$ has to be measured to...
better than 0.5%). Unfortunately, the calculation of the $\alpha_s^4$ contributing [23], the only calculation of that order so far, has been found to be in error and no new results are known yet.

- **Jet rates.** This is the most direct way to measure $\alpha_s$.
- **Energy-energy correlation and the corresponding asymmetry.**
- **Event shape variables.** In principle, every observable that is calculable in perturbative QCD and that has a dependence on gluon radiation effects can be used to determine $\alpha_s$.

The latter three methods are discussed in more detail below.

### 3.1 Jet rates

Determinations of $\alpha_s$ from jet rates have been presented by all LEP groups [27,28,29,30] and Mark II [24]. In all cases resolvable jets of hadrons are defined by first computing the scaled mass of all pairs of particles $i$ and $j$,

$$y_{ij} = \frac{M_{ij}^2}{E_{\text{vis}}^2} .$$

The combination with the smallest value of $y_{ij}$ is then replaced by a "pseudoparticle". The process is repeated until all pairs of particles (or pseudoparticles) satisfy the requirement $y_{ij} > y_{\text{cut}}$ where $y_{cut}$ is a resolution parameter. The remaining pseudoparticles, whose number are a function of the resolution parameter $y_{\text{cut}}$, are then called jets. This method does not only circumvent the problem of experimental resolutions, it also allows the jet rates to be calculated in perturbation theory. In $\mathcal{O}(\alpha_s^4)$ calculations one start with a system of four partons; resolvable partons are then defined similarly by demanding $y_{ab} > y$ for all partons $a,b$. There is thus a close correspondence between jet rates and partons for $y = y_{\text{cut}}$.

However, the loop corrections in these $\mathcal{O}(\alpha_s^4)$ calculations are given in terms of massless partons, while the pseudoparticles introduced by combining the 4-momenta of partons necessarily have mass. The technical problem of making these pseudoparticles artificially massless leads to a theoretical uncertainty, since the procedure (recombination scheme) is not uniquely determined.

The original JADE scheme [25], which is equivalent to the $E^3$ scheme, treats all partons as if they were massless by defining:

$$M_{ij} = 2E_iE_j(1 - \cos\theta_{ij}) ,$$

but other choices are possible.

Once a recombination scheme has been chosen, the jet rates $R_n$ can be calculated and compared to theoretical predictions [26]:

252
energies between 14 GeV and 91 GeV; the 3-jet fraction is clearly
with energy (running of
comparison, do not matter in this relative comparison. Fig. 8 shows the data for center—of-mass
diferent e⁺e⁻ colliders so that theoretical uncertainties, which would have to be included in a direct cc,
dependence on the center-—of—mass energy may be used to study the s dependence of the 3-jet
fraction. This fraction is calculated using the same theoretical model and same ym value for data from
The fact that the coefficients Cu in the calculation of the jet rates do not shown an explicit
certainty corrections (Fig. 7 b) to the parton level
distributions are very small too.

\[
R_2 = 1 + C_2(y) \alpha_s(\sqrt{s}) E_{CM}^2 + C_{22}(y) \alpha_s^2(\sqrt{s}) E_{CM}^2
\]
\[
R_3 = C_3(y) \alpha_s(\sqrt{s}) E_{CM}^2 + C_{32}(y) \alpha_s^2(\sqrt{s}) E_{CM}^2
\]
\[
R_4 = C_{43}(y) \alpha_s^3(\sqrt{s}) E_{CM}^2
\]

It can be seen that the jet rates Rₙ will be dependent on the scale µ² = E_{CM}² since the perturbative
expansion is limited to second order and the highest order coefficients show an explicit dependence on
f.

Fig. 7 shows the experimentally measured jetrates from the L3 experiment [28]. In the case of L3
the detector effects are particularly small since clusters have been used in an angular range covering
97% of 4π. Using the E⁰ scheme, the hadronisation corrections (Fig. 7 b) to the parton level

The fact that the coefficients Cₜ in the calculation of the jet rates do not shown an explicit
dependence on the center—of—mass energy may be used to study the s dependence of the 3-jet
fraction. This fraction is calculated using the same theoretical model and same y_m value for data from
different e⁺e⁻ colliders so that theoretical uncertainties, which would have to be included in a direct αₜ
comparison, do not matter in this relative comparison. Fig. 8 shows the data for center—of—mass
energies between 14 GeV and 91 GeV; the 3-jet fraction is clearly decreasing with energy (running of

Fig. 7: Jet rates as function of y_{cut} (L3).
$\alpha_s$ in agreement with the concept of asymptotic freedom.

![Graph of running of the strong coupling constant](image)

**Fig. 8:** The running of the strong coupling constant.

For the determination of $\alpha_s$, differential jet rates are studied rather than the jet rates as a function of $y_{\text{cm}}$, which are integral distributions with highly correlated errors. For the differential jet rate

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}$$

every event enters only once at the value of $y$ at which an event changes from a 2-jet to a 3-jet topology. $D_2$ is chosen because it is independent of the 4-jet rate, which is only known in leading $O(\alpha_s^2)$ order. The experimental distribution of OPAL [27], Fig. 9, is fitted with the QCD prediction as function of $\Lambda_{\text{QCD}}$ and scale $f = \mu^2/E_{\text{cm}}^2$. It can be seen that a choice of scale $f \approx 0.005$, corresponding to an energy of roughly 6.5 GeV, is preferred since it represents the data better at small values of $y$.

Using such a small scale effectively absorbs higher order corrections. Several "theoretical suggestions" have been made on how to choose a renormalisation scale in finite order that minimises higher order contributions. The predicted values for $f = 0.0024$ [31], 0.0060 [32], and 0.0081 [33] lie in the same ball park as the experimentally preferred number ($f = 0.0052_{-0.0004}^{+0.0003}$ in the $E^0$ scheme). Since there is no rigorous justification for such an approach, the uncertainty due to the scale dependence has to be included in the systematic error.
Another potential source of systematic error is the jet rate dependence on the recombination scheme. As mentioned above, the QCD calculations assume massless partons necessitating an artificial adjustment of the 4-momentum of combined parton or hadron pairs. There is an ambiguity in the choice of scheme for this adjustment and several recombination schemes have been proposed (see Table 4).

Table 4: Recombination schemes.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>( m_q^2 )</th>
<th>( \mathbf{p}_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>( (\mathbf{p}_i + \mathbf{p}_j)^2 )</td>
<td>( \mathbf{p}_i + \mathbf{p}_j )</td>
</tr>
<tr>
<td>E0</td>
<td>( 2E_i E_j (1 - \cos \theta_{ij}) )</td>
<td>( \mathbf{p}_i + \mathbf{p}_j )</td>
</tr>
<tr>
<td>P</td>
<td>( (\mathbf{p}_i + \mathbf{p}_j)^2 )</td>
<td>( \mathbf{p}_i + \mathbf{p}_j ; \ E_q =</td>
</tr>
<tr>
<td>P0</td>
<td>( (\mathbf{p}_i + \mathbf{p}_j)^2 )</td>
<td>As P scheme but ( E_{\text{凄}} ) updated</td>
</tr>
</tbody>
</table>
The effect of the choice of recombination scheme has been studied by OPAL, L3, and DELPHI, with OPAL having considered all the schemes in Table 4. The E0 scheme has the smallest dependence of the hadronisation corrections while the Lorentz-invariant E scheme is the most sensitive. This can be seen from Fig. 10 where the jet rate distributions on the parton level (dotted line) are compared with the distributions after hadronisation (solid line).

Since the complication of choosing a recombination scheme appears both in the QCD calculation and the jet-finding algorithm one has to compare theory and data using a consistent scheme if one wants to compute \( \alpha_s \). The agreement between data (symbols) and MC for each recombination scheme is excellent even though the hadronic corrections and the values for the jet fractions vary considerably between the schemes. Consequently it is assumed that there is no additional uncertainty due to the recombination scheme if the analysis is performed in a consistent way.

The results on the \( \alpha_s \) determinations from jet rates are summarised in Fig. 15. In the case of OPAL the errors contributing to the \( \alpha_s \) measurements were estimated as 2 – 3\% for each detector effect and statistics, modeling of the hadronisation, and choice of the virtuality cutoff. The dominant
error is due to the scale dependence ($\approx 6\%$).

### 3.2 Energy energy correlations

The Energy Energy Correlation (EEC) is defined as the energy−weighted histogram of the angle $\chi$ between all combinations of particles:

$$EEC(\chi) = \frac{1}{N} \sum_{i,j} \sum_{k,l} E_i E_j \delta(\chi_{ij} - \chi_{kl})$$

where $\chi_{ij}$ is the angle between particles $i,j$, $\Delta \chi$ is the histogram bin width, and $E_{ij}$ is the sum of the observed energies $E_i$. While two jet events will give rise to peaks at $\chi = 0^\circ$ and $\chi = 180^\circ$, hard gluon radiation will contribute to the central region whose area and shape are dependent on the value of $\chi$.

The energy−energy correlation asymmetry

$$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$$

removes the symmetric two jet component. It also has a smaller $O(\alpha_s^3)$ correction (15 %) as compared to EEC (30 %) and systematic errors symmetric in the angle $\chi$ are canceled.

A third observable, CEEC, has been introduced by the ALEPH collaboration and is based on jets rather than individual particles.

Fig. 11 shows the OPAL data unfolded for detector effects and compared to JETSET and HERWIG parton shower MC predictions. In order to calculate $\alpha_s$, the integrals of the parton corrected distributions between $43.3^\circ < \chi < 136.8^\circ$ for EEC and $28.8^\circ < \chi < 90^\circ$ for AEEC have been calculated using charged tracks, electromagnetic clusters, or both. A restricted $\chi$ range was chosen since hadronisation corrections are substantial e.g. at small $\chi$ angles. Fig. 12 shows the obtained values together with predictions from analytical $O(\alpha_s^3)$ calculations [34] and Matrix element MC’s as a function of the QCD scale parameter $\Lambda_{QCD}$.

Since the theoretical calculations are only known in second order, the influence of varying the renormalisation scale has been studied. The dependence of the determined value for $\Lambda_{QCD}$ on the scale parameter is shown in Fig. 13. While there is a large dependence in the case of EEC with a preferred value of $f = 0.027 \pm 0.013$, the AEEC determined value shows only a very small sensitivity for $f > 0.02$. This observation is in agreements with the notion that a large dependence on the renormalisation scale is equivalent to a large unaccounted contribution from higher orders. The results on EEC and AEEC for the OPAL [35], DELPHI [36], and ALEPH [37] are shown in Fig. 13. The results are in very good agreement.
3.3 Other ways to measure the strong coupling constant

Any variable that can be calculated in perturbative QCD (infrared safe, collinear safe) and that is sensitive to gluon radiation can in principle be used to calculate $\alpha_s$. Examples are [38]:

- thrust, oblateness, heavy jet masses etc.
Magnoli, Ratazzi, and Nason have used published OPAL data to fit $\alpha_s$ [39] and ALEPH has conducted a similar study [30]. The results are shown in Fig. 14. All determined $\alpha_s$ values, albeit with widely varying systematic errors, agree with one another. In particular the dependence on the renormalisation scale varies strongly with the various observables.

Fig. 14: Determination of the strong coupling constant from event shapes.
3.4 Summary on the strong coupling constant

A high precision $\alpha_s$ measurement (\(<\,10\%\)) is difficult to obtain because there is no one to one correspondence between partons and jets, $\alpha_s$ is relatively large (radiative effects), and the very time consuming calculation of higher order has not been done yet. The only (?) way out seems to be to study many distributions with various systematic effects (see Table 5), compare the resulting values for $\alpha_s$, and to estimate the "true" error on the $\alpha_s$ measurement from the spread of these results. The results from the LEP experiments on $\alpha_s$ are summarised in graphical form in Fig. 15. All values from the experiments are in excellent agreement independent of the methods used to determine $\alpha_s$. As average one may quote

$$\alpha_s(M_Z) = 0.119 \pm 0.008$$

$$\Lambda_{\overline{MS}}^{\text{N}^{\text{LO}} \text{\,5}} = (240^{+120}_{-90}) \text{ MeV}$$

Table 5: Systematic error sources for various methods used to determine $\alpha_s$

<table>
<thead>
<tr>
<th>method</th>
<th>renormalisation scale</th>
<th>recombination scheme</th>
<th>hadronisation dependence</th>
<th>detector effects</th>
<th>systematic error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>differential jet rates</td>
<td>√</td>
<td>√</td>
<td>- (for E0,P,P0)</td>
<td>small</td>
<td>≈8</td>
</tr>
<tr>
<td>EEC</td>
<td>√</td>
<td>-</td>
<td>fair</td>
<td>small</td>
<td>≈12</td>
</tr>
<tr>
<td>AEEC</td>
<td>-</td>
<td>-</td>
<td>small</td>
<td>small</td>
<td>≈9</td>
</tr>
<tr>
<td>CEEC</td>
<td>-</td>
<td>√</td>
<td>- (for E0,P,P0)</td>
<td>small</td>
<td>≈12</td>
</tr>
<tr>
<td>event shape variables</td>
<td>√</td>
<td>-</td>
<td>small - fair</td>
<td>small - fair</td>
<td>20 - 30</td>
</tr>
</tbody>
</table>

where (conservatively) the smallest systematic error of all measurements has been taken. These values are in good agreement with expectations from lower energy data.
4. HUNTING THE TRIPLE GLUON COUPLING

The self coupling of gluons, arising from the non-abelian character of QCD, manifests itself, for example, in the triple gluon vertex. A definite proof of existence for such a vertex would therefore be an important verification of QCD. The main diagrams contributing to 4-jet events are shown in Fig. 16.

The diagrams leading to 2 gluon + 2 quark jets can be distinguished from the 4 quark jet final state by their respective rates and different angular distribution of the partons. QCD predicts the qδ̅q̅g topology to be dominant (95.3%) while a non-abelian model, lacking the triple gluon contribution, predict a substantially lower contribution (68.7%). The angular distributions are different, because, in the case of the triple gluon vertex, one has a spin 1 gluon coupling to two spin 1 gluons, while for the 4 quark jet final state one has a spin 1 gluon coupling to two spin 1/2 quarks.
a 'QED' type model in the parton shower case is not unique. In the OPAL analysis coherence effects were constructed from the JETSET MC with modified group constants. Note, that the construction of JETSET O(c:f) matrix element MC’s have been used for the QCD prediction. Two abelian models and compared with MC predictions. JETSET and HERWIG parton shower (OPAL), as well as

The distributions of the angles listed above were then computed, corrected to the parton level, and compared with MC predictions. JETSET and HERWIG parton shower (OPAL), as well as JETSET O(a_s^2) matrix element MC’s have been used for the QCD prediction. Two abelian models were constructed from the JETSET MC with modified group constants. Note, that the construction of a "QED" type model in the parton shower case is not unique. In the OPAL analysis coherence effects

#Fig. 16: Diagrams contributing to the 4-jet final state.

Even without disentangling the triple gluon contribution (diagram 1 in Fig. 16) from the double bremsstrahlung diagram (diagram 2 in Fig. 16) one can distinguish between an abelian and an non-abelian model due to the different expected rate for the four quark-jet final state (diagram 3).

Several angles have been proposed that are sensitive to the angular correlations in four jet events, where the jets i= 1,4 with 4-momenta p_i are ordered according to their energies:

- The Nachtmann-Reiter angle [40], \( \theta_{NR} \), defined as the angle between the vectors \( p_1 - p_2 \) and \( p_3 - p_4 \).
- The Bengtson-Zerwas angle [41], \( \chi_{BZ} \), defined as the angle between the plane defined by jet 1 and 2 and the plane defined by jet 3 and 4.
- The Körner-Schierholtz-Willrodt angle [42], \( \phi_{KS\dot{W}} \).
- The angle between the two lowest energy jets, 3 and 4, \( \alpha_m \).

Note, that the two lowest energy jets tend to be the gluon jets.

Recently OPAL [43], L3 [44], and DELPHI [45] have studied the distribution of these angles. In the case of OPAL and L3 the study is based on 80200 and 49000 multihadronic events, respectively. After selecting 4 jet events using the JADE jet algorithm (\( y_{av} \) varying, \( \approx 0.01 \)) roughly 4200 (L3) and 3880 (OPAL) 4-jet events remain for the study. The lower number in the case of OPAL is explained by additional cuts on the separation of jets (to insure well defined planes) and the energy ratio for jets 2 and 3.

The distributions of the angles listed above were then computed, corrected to the parton level, and compared with MC predictions. JETSET and HERWIG parton shower (OPAL), as well as JETSET O(a_s^2) matrix element MC's have been used for the QCD prediction. Two abelian models were constructed from the JETSET MC with modified group constants. Note, that the construction of a "QED" type model in the parton shower case is not unique. In the OPAL analysis coherence effects
were disabled and the effective coupling was set to be a constant, which is a conservative assumption since in non-abelian models $\alpha_s$ should rise with $Q^2$.

Fig. 17 shows the distribution of $\theta_{NR}^*$ and $\chi_{BZ}$ from the OPAL collaboration. The MC prediction based on QCD is in good agreement with the data, while the abelian model is ruled out. Without gluon identification, $\phi_{KSW}$ is not a very sensitive observable. The shaded bands reflect uncertainties in the fragmentation parameters. From these measurements one can deduce the 95% confidence level upper limit for the $4-$ quark jet production

$$R = \frac{q\bar{q}q\bar{q}}{q\bar{q}q + q\bar{q}g} < 9.1\%$$

which should be compared to the QCD prediction of 4.7%. The abelian model ($R = 31.3\%$) is clearly ruled out.

![Fig. 17: Comparison of angular correlations with abelian and non-abelian models.](image)

The analysis was carried further by DELPHI [45] by including the angle $\alpha_M$, the angle between the lowest energy jets, in the analysis. According to DELPHI, the correlated information of $\theta_{NR}^*$ and $\alpha_M$ allows a distinction between the triple-gluon and the double-bremsstrahlungs vertex without making any assumptions on the $4-$quark jet rate. For the analysis 884 four-jet events were selected.

It is found that the data require the existence of the triple-gluon vertex contribution to the $2^{nd}$ order matrix element. In terms of the color factor $N_c$ and the fermionic Casimir operator $C_F$, DELPHI obtains the result
\[
\frac{N_c}{C_F} = 2.55 \pm 0.55 \text{ (stat)} \pm 0.4 \text{ (fragmentation + models)} \pm 0.2 \text{ (error in bias)}
\]

in agreement with the value 2.25 expected in QCD for \( N_c = 3 \) and \( C_F = 4/3 \) and significantly different from zero which is expected in an abelian model (\( N_c = 0 \)).

5. SUMMARY AND CONCLUSIONS

A large number of results on QCD and the structure of jets have been presented so far by the LEP groups. In particular it has been shown that

- Hadronic decays are accurately described by parton shower MC's over a wide energy range.
- There is evidence for gluon coherence effects in the momentum distribution of single particles.
- Intermittency effects have been observed and are in agreement with the prediction of parton shower MC's.
- The three jet rate decreases with \( E_{CM} \) consistent with the concept of asymptotic freedom.
- The determinations of \( \alpha_s \) from various experiments using different methods agree very well. The precision of \(< 8\%\) is limited by theoretical uncertainties (renormalisation scale dependence).
- The angular correlations in four-jet events support the non-Abelian nature of QCD.

ACKNOWLEDGEMENTS

I would like to thank the organisers of the 1990 CERN School of Physics for the pleasant atmosphere at the school. I would also like to thank the members of the LEP collaborations who helped provide information for this talk. In particular I am grateful to W. Gary, S. Bethke, and T. Hebekker for discussions and the provision of figures for this talk.
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