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ON RELATION BETWEEN DISTRIBUTION
FUNCTIONS IN HARD AND SOFT PROCESSES

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ON RELATION BETWEEN DISTRIBUTION FUNCTIONS IN HARD AND SOFT PROCESSES

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ABSTRACT

It is shown that in the particle-exchange model the hadron-hadron scattering amplitude admits parton-like representation with the distribution functions coinciding with those extracted from deep inelastic processes.

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1 Introduction

All high-energy scattering processes may be subdivided into two classes, soft and hard processes. Besides energy, hard collisions (deep inelastic lepton-hadron scattering, Drell-Yang process, etc.) are characterized by large momentum transfer. In such processes, final states are formed through the interaction of the constituents of colliding particles at small space-time scales.

The cross-sections of hard processes are expressed in terms of quark and gluon distributions. These, in turn, allow rigorous quantum field definitions in terms of the matrix elements of composite operators.

Hadron-hadron collisions go mainly at small momentum transfers. These processes are, generally, related with the coherent scattering of a large number of partons at the periphery of colliding hadrons.

To describe hadronic amplitudes, the eikonal parametrization is used, in whose terms the unitarity of the amplitude T_{AB} in the s -channel is easily attainable. In a number of papers it is assumed (see, e.g., [1]) that the rising part of the eikonal, χ_{AB} , is represented in the form

$$\chi_{AB} \sim \sum_{i,j} F_A^i \otimes F_B^j \otimes \hat{\sigma}_{ij}. \quad (1)$$

Here F_A^i means the distribution of the parton of sort i inside the hadron A , $\hat{\sigma}_{ij}$ is the cross-section for the partons i and j , and the symbol $\otimes$ denotes integration over the momentum fractions and convolution in the impact parameter space.

The functions F_A^i , entering formula (1), are thought to be *the same* as those describing quark and gluon distributions in deep inelastic process. This raises a number of questions:

- are F_A^i really related with the parton distributions revealed in hard processes?
- why the RHS of formula (1) should contain the cross-section of active partons but not, say, their scattering amplitude?
- for which of the two quantities, the eikonal or the amplitude, the parton-like representation (1) is valid?

In the present paper we will show that it is the hadronic scattering amplitude that admits the representation in terms of parton characteristics in the particle-exchange approximation [2]. Schematically, the representation has the following form:

$$T_{AB} \sim \sum_{i,j} f_A^i \otimes f_B^j \otimes \hat{a}_{ij}. \quad (2)$$

Here f_A^i are "hard" parton distributions, and $\hat{a}_{ij}$ may be assigned the meaning of the $\hat{\mathcal{Q}}PI$ part of the parton scattering amplitude.

In this, the eikonal is expressed through the "truncated" distributions that are not directly related with the quark and gluon distributions measured in deep inelastic lepton-hadron scattering.

2 Hadron-Hadron Scattering

The hadron-hadron scattering amplitude of particles A and B in the eikonal approach is parametrized in the following way:

$$T_{AB}(s, t) = 4s \int d^2b \exp(i\vec{q}\vec{b}) \tilde{T}_{AB}(s, \vec{b}), \quad (3)$$

$$\tilde{T}_{AB}(s, \vec{b}) = \frac{\exp[i\tilde{\chi}_{AB}(s, \vec{b})] - 1}{2i}, \quad (4)$$

where $s = (p_A + p_B)^2$, $t = q^2$, $q = p'_A - p_A = p_B - p'_B$.

In quantum mechanics, the eikonal $\tilde{\chi}$ is energy-independent and related with the potential $\tilde{V}(\vec{r})$:

$$\tilde{\chi}(\vec{b}) \sim \int_{-\infty}^{\infty} dz \tilde{V}(\vec{b} + \vec{n}z), \quad (5)$$

where $\vec{n} = (\vec{p}_A + \vec{p}_A)/|\vec{p}_A + \vec{p}_A|$, $\vec{b}\vec{n} = 0$. Then

$$\chi(t) \sim V(\vec{q}) \Big|_{\vec{q}\vec{n}=0}^{\vec{q}\vec{n}=t}. \quad (6)$$

In the quantum field theory, the generalization of the potential is the quasi-potential [3], related with particle exchanges in the t or u -channel.

The account of all possible exchanges with the quantum numbers allowed and arbitrary (integer) spins leads to [2]

$$T_{AB}(s, t) = i \sum_{\pm}^{\infty} \gamma_J^A(t) \frac{1}{M_J^2 - t} \gamma_J^B(t) P_J(z_t). \quad (7)$$

Here the signature index $(\pm)$ points to that the summation is either over even, or over odd spins. In (7), the $P_J(z)$ is the Legendre polynomial, while z_t means the cosine of the scattering angle in the t -channel in the c.m.s. of colliding particles:

$$z_t = \frac{4p_A p_B + t}{\sqrt{(4m_A^2 - t)(4m_B^2 - t)}}. \quad (8)$$

The series (7) converges in the physical region of the t -channel. Further on we will understand that it is analytically extended to the physical region of the s -channel via the Sommerfeld-Watson transformation.

Suppose now that the exchange quanta lie on the Regge trajectory, so that

$$J = \alpha(M_J^2), \quad J = 0, 1, \dots \quad (9)$$

The function $M_J^2 = \alpha^{-1}(J)$ is unambiguously defined and may be extended from integer points onto the complex plane J , provided the following conditions are satisfied:

- J is not bounded from above ($\alpha(t) \rightarrow \infty$ at $t \rightarrow \infty$)
- the growth of $\alpha(t)$ is not slower than $(1/\pi) \ln t$ at $t \rightarrow \infty$.

The uniqueness of the analytic continuation is guaranteed by the Carlson theorem (see [4]). Hence, the function $(M_J^2 - t)^{-1}$ has a pole at $J = \alpha(t)$ with the residue $\alpha'(t)$.

Assuming the pole $J = \alpha(t)$ to be the leading singularity, we get the Regge asymptotics [2]

$$T_{AB}(s, t) \simeq \frac{\pi}{2} \frac{1 \pm \exp(-i\pi\alpha(t))}{\sin \pi\alpha(t)} \alpha'(t) \times \\ \times \gamma_{\alpha(t)}^A(t) \gamma_{\alpha(t)}^B(t) \left[\frac{s - u}{\sqrt{(4m_A^2 - t)(4m_B^2 - t)}} \right]^{\alpha(t)}. \quad (10)$$

The expression for the amplitude (7) may be rewritten to be:

$$T_{AB}(s, t) = \int_0^1 dx \int_0^1 dy F_A(x, t) F_B(y, t) \hat{A}(s, xy, t). \quad (11)$$

Here we have introduced the "distributions"

$$F_A(x, t) = \oint_C \frac{dJ}{2\pi i} x^{-J} F_J^A(t), \quad (12)$$

where

$$F_J^A(t) = \gamma_J^A(t) [\alpha'(t)]^{1/2} \frac{1}{J - \alpha(t)}. \quad (13)$$

The function $\hat{A}(s, xy, t)$ in (11) is given by the formula

$$\hat{A}(s, xy, t) = i \sum_{J=0}^{\infty} (xy)^J [J - \alpha(t)] P_J(z_t). \quad (14)$$

The Regge asymptotics of the amplitude (10) corresponds to the following singular behavior of the distributions at small x

$$F_A(x, t)|_{x \ll 1} \sim x^{-\alpha(t)}. \quad (15)$$

In perturbative QCD, the Regge behavior of the distributions is thought to be associated with the t -channel 2PR parton ladder diagrams [5]. In fact, the asymptotics $F_A(x, t)$ at $x \rightarrow 0$ is determined by the more complicated, so-called fan diagrams that take into account the parton recombination [6]. This means that in the place of the residues $\gamma_J^A(t)$, the full vertices with multi-particle intermediate states should enter formula (7). The corresponding cuts in the J -plane lie, generally speaking, to the right of the pole $J = \alpha(t)$.

In accordance with the above-said, we obtain

$$T_{AB}(s, t) = i \sum_{J=0}^{\infty} (\pm) < p_A + q | I_{\mu_1 \dots \mu_J}(0) | p_A > D_{\mu_1 \dots \mu_J \nu_1 \dots \nu_J}(q) \times \\ \times < p_B - q | I_{\nu_1 \dots \nu_J}(0) | p_B >, \quad (16)$$

where $I_{\mu_1 \dots \mu_J}$ is the source of the spin- J Heisenberg field (further denoted as $H_{\mu_1 \dots \mu_J}$) and $D_{\mu_1 \dots \mu_J \nu_1 \dots \nu_J}$ is its propagator. To achieve a Lorentz-invariant

description of the spin- J hadron (J being an integer), we take the symmetric tensor representation complemented by the conditions

$$g^{\mu_1 \mu_2} I_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_J} = 0, \quad (17)$$

$$\partial^{\mu_1} I_{\mu_1 \dots \mu_i \dots \mu_j \dots \mu_J} = 0. \quad (18)$$

It is clear from equation (17) that we are dealing with the irreducible Lorentz group representations. Eq. (18) entails that the fields $I_{\mu_1 \dots \mu_J}$ are transformed under the irreducible representation of the rotation group with the dimension equal to $2J + 1$.

The matrix element of the operator $I_{\mu_1 \dots \mu_J}$ has the following Lorentz-structure

$$< p_A + q | I_{\mu_1 \dots \mu_J}(0) | p_A > = I_J^A(t) \frac{1}{(2J - 1)!!} \sum_{k=0}^{[J/2]} (-1)^k (2J - 2k - 1)!! \times \\ \times (4m_A^2 - t)^k S [B_{\mu_1 \mu_2} \dots B_{\mu_{2k-1} \mu_{2k}} A_{\mu_{2k+1}} \dots A_{\mu_J}], \quad (19)$$

where we denote

$$A_\mu = 2p_\mu^A + q_\mu, \quad B_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \quad (20)$$

and $[a]$ is the integer part of a . Here and below the operator S means the complete symmetrization over the indices without averaging. For instance, $S[g_{\mu_1 \mu_2} p_{\mu_3}] = g_{\mu_1 \mu_2} p_{\mu_3} + g_{\mu_1 \mu_3} p_{\mu_2} + g_{\mu_2 \mu_3} p_{\mu_1}$.

Substituting the expression for the matrix element $I_{\mu_1 \dots \mu_J}$ (19) into formula (16), we obtain that

$$T_{AB}(s, t) = i \sum_{J=0}^{\infty} (\pm) \frac{J!}{2^J (2J - 1)!!} [(4m_A^2 - t)(4m_B^2 - t)]^{J/2} \times \\ \times I_J^A(t) D_J(t) I_J^B(t) P_J(z_t), \quad (21)$$

where $D_J(t)$ is the Lorentz-invariant part of the propagator of the spin- J hadron field:

$$D_{\mu_1 \dots \mu_J \nu_1 \dots \nu_J}(q) = D_J(t) \frac{1}{J!} S \prod_{i=1}^J g_{\mu_i \nu_{m_i}} + \dots \quad (22)$$

The Lorentz-structures, omitted in the RHS of Eq. (22), do not contribute to the amplitude (16) in virtue of the conditions (17) and (18).

As a result, we come to the expression

$$T_{AB}(s, t) = i \sum_J^{\infty} \frac{(J!)^2}{(2J)!} [(4m_A^2 - t)(4m_B^2 - t)]^{J/2} [b_J(t; \mu^2)]^{-1} \times \quad (23)$$

$$\times \Gamma_J^A(t) \frac{d_J(t)}{J - \alpha(t)} \Gamma_J^B(t) P_J(z_t).$$

In (23), the $d_J(t)$ stands for the residue of the propagator

$$d_J(t) = D_J(t)(M_J^2 - t), \quad (24)$$

and $\Gamma_J^A(t; \mu^2)$ are invariant vertices, analogous to the residues in the Regge field theory:

$$\Gamma_J^A(t; \mu^2) = [b_J(t; \mu^2)]^{1/2} I_J^A(t) [\alpha'(t)]^{1/2}, \quad (25)$$

with $\alpha'(t)$ being the Regge trajectory slope. In (25) the function $b_J(t; \mu^2)$ is introduced in order to make $\Gamma_J^A(t; \mu^2)$ dimensionless at all t . It will be defined below (see Eq. (71)).

Define the function

$$f_A(x, t; \mu^2) = \oint_C \frac{dJ}{2\pi i} x^{-J} f_J^A(t; \mu^2), \quad (26)$$

where

$$f_J^A(t; \mu^2) = \Gamma_J^A(t; \mu^2) D_J(t) [\alpha'(t)]^{-1}. \quad (27)$$

The integration contour C in (26) is to the right of the singularities of the function $f_J^A(t; \mu^2)$. If at infinity it rises with J slower than the linear exponential, then we find that $f_A(x, t; \mu^2)$ is equal to zero at $x > 1$ and

$$f_J^A(t; \mu^2) = \int_0^1 dx x^{J-1} f_A(x, t; \mu^2). \quad (28)$$

The compactness of the support of the function $f_A(x, t; \mu^2)$ makes it possible to represent T_{AB} in the form allowing of its parton interpretation

$$T_{AB}(s, t) = \int_0^1 dx \int_0^1 dy f_A(x, t; \mu^2) f_B(y, t; \mu^2) \hat{a}(s, xy, t; \mu^2), \quad (29)$$

where we denote

$$\hat{a}(s, xy, t; \mu^2) = i \sum_J^{\infty} \frac{(J!)^2}{(2J)!} [(4m_A^2 - t)(4m_B^2 - t)]^{J/2} \times \quad (30)$$

$$\times (xy)^J [d_J(t) b_J(t; \mu^2)]^{-1} [J - \alpha(t)] P_J(z_t).$$

Note, at $s \gg m_A^2, m_B^2$ the function $\hat{a}(s, xy, t; \mu^2)$ depends on the kinematic variables $\hat{s} = sxy$ and t only. It is also important that the asymptotics of $\hat{a}$ with respect to $\hat{s}$ is independent of the sorts of colliding hadrons, A and B .

Eq. (29) may be interpreted in terms of interacting partons if $f_A(x, t; \mu^2)$ is understood as the generalized distribution of the parton inside the hadron A . Its Fourier transform

$$\tilde{f}_A(x, \vec{b}; \mu^2) = \int d^2q \exp(i\vec{q}\vec{b}) f_A(x, t; \mu^2). \quad (31)$$

should be given the meaning of the distribution of the parton inside the hadron A in the momentum fraction x and the impact parameter $\vec{b}$.

If more than one Regge trajectory contribute to the amplitude, formula (29) becomes

$$T_{AB}(s, t) = \sum_{n=1}^N \int_0^1 dx \int_0^1 dy f_A^n(x, t; \mu^2) f_B^n(y, t; \mu^2) \hat{a}_n(s, xy, t; \mu^2), \quad (32)$$

where n numbers the Regge trajectories, and N stands for the total number of the trajectories. Let us define the distribution of the parton of sort i inside the hadron A as the linear combinations of the quantities f_A^n :

$$f_A^n(x, t; \mu^2) = \sum_{i=1}^M c_i^n f_i^n(x, t; \mu^2), \quad n = 1, \dots, N. \quad (33)$$

Here c_i^n are some constants. Then from Eq. (32) we find

$$T_{AB}(s, t) = \sum_{i,j} \int_0^1 dx \int_0^1 dy f_A^i(x, t; \mu^2) f_B^j(y, t; \mu^2) \hat{a}_{ij}(s, xy, t; \mu^2), \quad (34)$$

with the "parton subprocess amplitude" looking like ($s \gg m_A^2, m_B^2, -t$)

$$\hat{a}_{ij}(s, xy, t; \mu^2) = i \sum_{n=1}^N c_i^n c_j^n \sum_{J=0}^{\infty} s^J [d_J^n(t) b_J(t; \mu^2)]^{-1} [J - \alpha_n(t)]. \quad (35)$$

As follows from formulas (25), (27) and (33), at $M = N$ the moments of the distributions are set by the expression

$$f_J^{A,i}(t; \mu^2) = \sum_{n=1}^N (c^{-1})_n^i [b_J(t; \mu^2)]^{1/2} [\alpha_n'(t)]^{-1/2} I_{f_J}^{A,n}(t) D_J^n(t), \quad (36)$$

where $(c^{-1})_n^i$ is an inverse matrix.

While $f_A^i(x, t; \mu^2)$ in (34) may be understood as the distribution of the parton i inside the hadron A , the dimensionless quantity $\hat{a}_{ij}(\hat{s}, t; \mu^2)$ (30) cannot be interpreted as the total amplitude of the parton subprocess, since it has no pole contributions. It should be treated as the *2PI part* of the scattering amplitude of constituents with the invariant energy $\hat{s}$ (see comment after Eq. (15)).

It has been stated above that the moments of the distributions f_J^A (27) have multi-particle singularities in the t -channel. In the eikonal model (3), the corresponding singularities of the amplitude T_{AB} are to be generated when the eikonal exponential is expanded in a series. This means that the representation for χ_{AB} should look like the RHS of (11) with the "distributions" having no multi-particle singularities and thus not coinciding with conventional parton distributions.

3 Deep Inelastic Scattering

Remember that the moments of the distributions, appearing when describing deep inelastic lepton-hadron scattering, are defined in terms of the forward matrix elements of the composite operators:

$$f_J^{A,i}(\tilde{\mu}^2) = \frac{n_{\mu_1} \dots n_{\mu_J}}{(p_A n)^J} \langle p_A | O_{\mu_1 \dots \mu_J}^i(0; \tilde{\mu}^2) | p_A \rangle, \quad (37)$$

where $n_\mu^2 = 0$, i is the sort of the operator, and $\tilde{\mu}$ is its renormalization scale.

Composite operators enter the light-cone operator product expansion of the electromagnetic hadronic currents (for the sake of simplicity, we are considering scalar currents)

$$TJ(x)J(0)|_{x^2 \approx 0} \simeq \sum_i \sum_{J=0}^J i^J C_J^i(x^2 + i0; \tilde{\mu}^2) O_{\mu_1 \dots \mu_J}^i(0; \tilde{\mu}^2) x^{\mu_1} \dots x^{\mu_J}. \quad (38)$$

The terms in the RHS of (38) may be regrouped so that the expansion would be in *traceless* symmetric tensor operators. The one-particle matrix element of the traceless symmetric operator $O_{\mu_1 \dots \mu_J}^i$ has the form (further on we consider hadron A to be scalar)

$$\langle p_A | O_{\mu_1 \dots \mu_J}^i(0; \tilde{\mu}^2) | p_A \rangle = f_J^{A,i}(\tilde{\mu}^2) \Delta_{\mu_1 \dots \mu_J}, \quad (39)$$

with

$$\begin{aligned} \Delta_{\mu_1 \dots \mu_J} &= \frac{1}{J!} \sum_{k=0}^{[J/2]} (-1)^k 2^{-k} (J-k)! m_A^{2k} \times \\ &\times S[g_{\mu_1 \mu_2} \dots g_{\mu_{2k-1} \mu_{2k}} p_{\mu_{2k+1}}^A \dots p_{\mu_J}^A]. \end{aligned} \quad (40)$$

Consider the Green's function

$$T(p_A, k) = \int d^4x \exp(ikx) \langle p_A | T J(x) J(0) | p_A \rangle. \quad (41)$$

The account of the relation $T(p_A k, Q^2) = T(-p_A k, Q^2)$ allows one to find from formulas (38)–(41) that

$$\begin{aligned} T(p_A, k) = T(Q^2, z) &= \frac{1}{2} \sum_i \sum_{J=0}^{\infty} \tilde{C}_J^i(Q^2; \tilde{\mu}^2) \left(\frac{m_A}{Q} \right)^J \times \\ &\times [C_J^1(z) + C_J^1(-z)] f_J^{A,i}(\tilde{\mu}^2). \end{aligned} \quad (42)$$

Here $C_n^1(z)$ is the Gegenbauer polynomial possessing the symmetry property $C_n^1(-z) = (-1)^n C_n^1(z)$, $Q \equiv \sqrt{-k^2}$ and

$$z = \frac{p_A k}{m_A Q}. \quad (43)$$

The Fourier transform of the coefficient function $\tilde{C}_J^i(Q^2; \tilde{\mu}^2)$ is defined by the formula

$$\tilde{C}_J^i(Q^2; \tilde{\mu}^2) = (k^2 \frac{\partial}{\partial k^2})^J \int d^4k \exp(ikx) C_J^i(x^2 + i0; \tilde{\mu}^2). \quad (44)$$

Write down the dispersion relation for the function $T(Q^2, z)$ in terms of the variable z

$$T(Q^2, z) = \int_{(Q/m_A)^2}^{\infty} dz'^2 \frac{W(Q^2, z')}{z'^2 - z^2} \quad (45)$$

4 Composite Operator Expansion in Hadronic Fields. Relation between Distributions

The structure functions of deep inelastic lepton–nucleon scattering are expressed in terms of the parton distributions inside the nucleons. In their turn, the distributions are determined by the matrix elements of the composite operators (37). As for hadron–hadron collision, they are represented by the generalized distributions (28), related with the matrix elements of the local hadronic fields:

$$f_f^{A,i}(t; \mu^2) = (2J-1)!! [b_f(t; \mu^2)]^{1/2} \sum_{n=1}^N (c^{-1})_n^i [\alpha_n'(t)]^{-1/2} \times \\ \times \frac{m_{\mu_1} \dots m_{\mu_J}}{(Pm)^J} < p_A + q | H_{\mu_1 \dots \mu_J}^n(0) | p_A >, \quad (53)$$

where $P = (p_A + p_A')/2 \equiv p + q/2$, $m_\mu = 2\sqrt{-q^2}p_\mu + (\sqrt{-q^2} - \sqrt{4m_A^2 - q^2})q_\mu$ is a light-like vector. In the reference frame $p_\mu = (m_A, \vec{0})$, the vector m_μ is proportional to the 4-vector $(1, 0, 0, -1)$.

The distributions in hard processes and hadron–hadron collisions may be mutually related with the help of the hypothesis formulated in Refs. [8], [9].¹ In accordance with that hypothesis, any local (in the sense of the local commutativity) composite operator $O_{\mu_1 \dots \mu_J}^i$, acting on the physical space and realizing some definite Lorentz group representation, can be represented as a linear combination of the local fields, associated with physical hadrons [9]

$$O_{\mu_1 \dots \mu_J}^i(x; \tilde{\mu}^2) = \sum_{j=0}^J S \left[\Lambda_{\mu_1 \dots \mu_{n_{j-1}}}^j (i\partial_x; \tilde{\mu}^2) \sum_{n=1}^N \delta_n^i H_{\mu_{n_{j-1}+1} \dots \mu_{n_J}}^n(x) \right]. \quad (54)$$

In the general case, the tensors $\Lambda_{\mu_1 \dots \mu_L}^j$ are integro-differential operators. Their symmetry properties agree with the symmetry properties of the tensor operators $O_{\mu_1 \dots \mu_J}^i$ and $H_{\mu_1 \dots \mu_J}^n$, whence it follows that

$$\Lambda_{\mu_1 \dots \mu_L}^j(i\partial; \tilde{\mu}^2) = a_{j-L}^j(\tilde{\mu}^2) \frac{i^L}{(2J)!} \frac{1}{(\partial^2)^{L/2}} \sum_{k=0}^{[L/2]} (2J-2k)! (\partial^2)^k \times$$

¹ Long ago an attempt has been made to use the operator product expansion for hadron–hadron scattering in order to obtain large distance behaviour of the hadronic current correlation function [10].

$$M_n(Q^2) = 4 \left(\frac{Q}{m_A} \right)^n \int_{Q/m_A}^{\infty} dz' [z' + \sqrt{z'^2 - 1}]^{-n-1} W(Q^2, z'). \quad (46)$$

The generating functional for the Gegenbauer polynomial is of the form [7]

$$(1 - 2hz + h^2)^{-\nu} = \sum_{n=0}^{\infty} C_n^\nu(z) h^n. \quad (47)$$

Putting in (47) $\nu = 1$ and $h = z' - \sqrt{z'^2 - 1}$, we come to

$$\frac{1}{z' - z} = 2 \sum_{n=0}^{\infty} C_n^1(z) [z' + \sqrt{z'^2 - 1}]^{-n-1}. \quad (48)$$

The set of Eqs. (42), (45), (46) and (48) reveals the connection of the moments M_n with the matrix elements of the composite operators:

$$M_n(Q^2) = \sum_i \tilde{C}_n^i(Q^2; \tilde{\mu}^2) f_n^{A,i}(\tilde{\mu}^2). \quad (49)$$

As a rule, in the matrix element of the operator $O_{\mu_1 \dots \mu_J}^i$ (39) one confines himself to the single Lorentz-structure $p_{\mu_1} \dots p_{\mu_J}$ (i.e., to only one term in the sum (40) which corresponds to $k = 0$). Then, instead of relation (42), one gets the equality

$$T(Q^2, \omega) = \sum_i \sum_{n=0}^{\infty} \tilde{C}_{2n}^i(Q^2; \tilde{\mu}^2) \omega^{2n} f_{2n}^{A,i}(\tilde{\mu}^2), \quad (50)$$

where

$$\omega = \frac{2p_A k}{Q^2}. \quad (51)$$

Correspondingly, Eq. (49) holds for the even moments in the Bjorken variable ω (51), specified by the formula

$$M_{2k}'(Q^2) = 2 \int_1^{\infty} d\omega \omega^{-2k-1} W(Q^2, \omega). \quad (52)$$

The Bjorken asymptotics ($Q^2 \gg m_A^2$, ω being fixed) of the RHS of Eqs. (42) and (50) coincide. However, the representation (42) is preferable, since (50) bears no information about the $O(4)$ -symmetry of the structure function (for details concerning the $O(4)$ -symmetry, see Sect. 5).

$$\times S[g_{\mu_1, \mu_2} \dots g_{\mu_{2k-1}, \mu_{2k}} \partial_{\mu_{2k+1}} \dots \partial_{\mu_L}]. \quad (55)$$

For the following, it is convenient to take the dimension of b_n^i to be equal to one: ($\dim[b_n^i] = 1$). Then the dimensions of the coefficients a_j^i in (55) are equal to

$$\dim[a_j^i] = \begin{cases} J, & j = J, J-2, \dots, J-2[J/2] \\ J-1, & j = J-1, J-3, \dots, J-1-2[(J-1)/2] \end{cases} \quad (56)$$

The power of the operator ∂^2 in the RHS of Eq. (55) are chosen so as to make agree the finitenesses of the matrix elements $< p_A + q | H_{\mu_1 \dots \mu_j} | p_A >$ and

$$\begin{aligned} < p_A + q | O_{\mu_1 \dots \mu_J}^i(0; \tilde{\mu}^2) | p_A > = \sum_{j=0}^J S[\tilde{\Lambda}_{\mu_1 \dots \mu_{n_j-j}}^j(q; \tilde{\mu}^2) \times \\ & \times \sum_{n=1}^N b_n^i < p_A + q | H_{\mu_{n_j-j+1} \dots \mu_{n_j}}^n(0) | p_A >]. \end{aligned} \quad (57)$$

at $q_\mu \rightarrow 0$.

At $L \geq 2$ the Fourier transforms of the integro-differential operators $\tilde{\Lambda}_{\mu_1 \dots \mu_L}(q, \tilde{\mu}^2)$ have poles at $q^2 = 0$. As follows from formula (19), the matrix elements $< p_A + q | H_{\mu_1 \dots \mu_j}^n(0) | p_A >$ also contain singular terms of the form $q_{\mu_1} \dots q_{\mu_k} / (q^2)^{[k/2]}$, where $k = 0, 1, \dots, j$. One should require that the singular terms in the expression for the nonforward matrix elements of the composite operator (57) be mutually cancelled. This is equivalent to imposing the constraints on the quantities

$$(\Omega_A^i)^j(t, \tilde{\mu}^2) = a_j^i(\tilde{\mu}^2) \sum_{n=1}^N b_n^i I_{f_j}^{A,n}(t) D_j^n(t). \quad (58)$$

Consider this constraints for even and odd J separately. Let $J = 2m$, m being an integer. Denote

$$(\Omega_A^i)^{2m}_{2k} = (\alpha_A^i)^m_k, \quad k = 0, 1, \dots, m. \quad (59)$$

In the RHS of Eq. (57) we isolate singular terms of the form

$$\frac{q_{\mu_1} \dots q_{\mu_{2m-2p}}}{(q^2)^{m-p}} p_{\mu_{2m-2p+1}}^A \dots p_{\mu_{2m}}^A, \quad p = 0, 1, \dots, m-1. \quad (60)$$

The term, corresponding to $j = 2k$ in the sum (57), contains (60) with the factor

$$2^{p-k} m_A^{2(k-p)} \frac{(2k+2p-1)!! (2m-2p)!}{(4k-1)!! (2m-2k)! (k-p)!}. \quad (61)$$

Therefore, the requirement that the singular terms be absent from the matrix element (57), reduces to the system of $m-1$ homogeneous linear equations for m quantities $(\alpha_A^i)^m_k(0; \tilde{\mu}^2)$:

$$\begin{aligned} & \sum_{k=p}^m 2^{-k} m_A^{2k} \frac{(2k+2p-1)!!}{(4k-1)!! (2m-2k)! (k-p)!} (\alpha_A^i)^m_k(0; \tilde{\mu}^2) = 0, \\ & p = 0, 1, \dots, m-1. \end{aligned} \quad (62)$$

The solutions to the system of Eqs. (62) are of the form ($k = 1, 2, \dots, m$)

$$\begin{aligned} (\alpha_A^i)^m_k(0; \tilde{\mu}^2) &= (-1)^{m-k} 2^{2k} m_A^{2(m-k)} \frac{(4k+1)!! (2m-2k)! (m+k)!}{(2m+2k+1)! (m-k)!} \times \\ &\times (\Omega_A^i)^{2m}_{2m}(0; \tilde{\mu}^2). \end{aligned} \quad (63)$$

In App. A it is shown that Eq. (62) is satisfied under the substitution therein of expressions (63).

Provided $J = 2m+1$, the consideration of the singular terms

$$\frac{q_{\mu_1} \dots q_{\mu_{2m-2p}}}{(q^2)^{m-p}} p_{\mu_{2m-2p+1}}^A \dots p_{\mu_{2m+1}}^A, \quad p = 0, 1, \dots, m-1. \quad (64)$$

leads to the system of equations

$$\begin{aligned} & \sum_{k=p}^m 2^{-k} m_A^{2k} \frac{(2k+2p+1)!!}{(4k+1)!! (2m-2k)! (k-p)!} (\beta_A^i)^m_k(0; \tilde{\mu}^2) = 0, \\ & p = 0, 1, \dots, m-1, \end{aligned} \quad (65)$$

with the notation

$$(\Omega_A^i)^{2m+1}_{2k+1} = (\beta_A^i)^m_k, \quad k = 0, 1, \dots, m. \quad (66)$$

Correspondingly, the solutions of Eqs. (65) are

$$\begin{aligned} (\beta_A^i)^m_k(0; \tilde{\mu}^2) &= (-1)^{m-k} 2^{2k} m_A^{2(m-k)} \frac{(4k+3)!! (2m-2k)! (m+k+1)!}{(2m+2k+3)! (m-k)!} \times \\ &\times (\Omega_A^i)^{2m+1}_{2m+1}(0; \tilde{\mu}^2), \quad k = 1, 2, \dots, m. \end{aligned} \quad (67)$$

Eqs. (62), (65) relate the characteristics of the hadronic fields, whose spin has the same parity as the "spin" J of the composite operator $O_{\mu_1 \dots \mu_J}^i$.

The particular cases $J = 3$ and $J = 4$ are scrutinized in App. B.

Solving in (57) the constraints which result from the requirement that the poles be absent at $q^2 = 0$ and taking the limit $q_\mu \rightarrow 0$, we find

$$\langle p_A | O_{\mu_1 \dots \mu_J}^i(0; \tilde{\mu}^2) | p_A \rangle = (\Omega_A^i)^J(0; \tilde{\mu}^2) \Delta_{\mu_1 \dots \mu_J}, \quad (68)$$

The conclusion drawn from formulas (39) and (68) is that the "hard" distributions, describing deep inelastic scattering, are given by the expression

$$f_J^{A,i}(\tilde{\mu}^2) = (\Omega_A^i)^J(0; \tilde{\mu}^2) = a_J^J(\tilde{\mu}^2) \sum_{n=1}^N b_n^i I_J^{A,n}(t) D_J^n(t). \quad (69)$$

Compare now formula (69) with that for the "soft" distributions (36), arising in the particle-exchange model of hadronic amplitudes. They bring us to the following *relation between "hard" and "soft" distributions*:

$$\begin{aligned} \text{hard } f_J^{A,i}(\tilde{\mu}^2) &= a_J^J(\tilde{\mu}^2) [b_J(0; \mu^2)]^{-1/2} \sum_{j=1}^M \omega_j t f_J^{A,j}(0; \mu^2) \times \\ &\times [\alpha_j'(0)]^{1/2} \sum_{n=1}^N b_n^i c_j^n, \quad i = 1, \dots, M. \end{aligned} \quad (70)$$

Therefore, in order that soft and hard scatterings be described by universal distributions, the mass scales of the given processes are to be identified $\mu = \tilde{\mu}$. An l besides, one should put

$$[b_J(0; \mu^2)]^{1/2} = C a_J^J(\mu^2), \quad (71)$$

$$[\alpha_j'(0)]^{1/2} \sum_{n=1}^N b_n^i c_j^n = C \delta_{ij}, \quad i, j = 1, \dots, M, \quad (72)$$

where C is a dimensionless constant. Eq. (72) can always be satisfied by the proper choice of the elements of the matrix c_i^k at $M \leq N$ (i.e., if the number of the sorts of the partons does not exceed the number of the Regge trajectories).

5 $O(4)$ -Symmetry of Structure Functions

According to formula (42), the integer moments of the hard distributions $f_n^{A,i}$ appear as the coefficients in expanding the structure functions of deep inelastic scattering in Gegenbauer polynomials $C_n^1(z)$. The Gegenbauer polynomials are the base functions of the $O(4)$ -group representations. In turn, the $O(4)$ -group arises (formally) from the entire Lorentz group for spinless scattered particles in the limit $q_\mu \rightarrow 0$ (see, for instance, [11]).

The $O(4)$ -group representations contain one each $O(3)$ -group representations with $0 \leq l \leq n$. Correspondingly, the polynomials $C_n^1(z)$ can be expanded in the Legendre polynomials $P_l(z)$. For $n = 2m$ this expansion looks like [12]:

$$C_{2m}^1(z) = (2m+1) \sum_{k=0}^m \gamma_k^m P_{2k}(z), \quad (73)$$

where

$$\gamma_k^m = \frac{1}{2m+1} 2^{4k} (4k+1) \frac{(2m-2k)! [(m+k)!]^2}{(2m+2k+1)! [(m-k)!]^2}. \quad (74)$$

For $n = 2m+1$ it takes the form [12]:

$$C_{2m+1}^1(z) = (2m+2) \sum_{k=0}^m \delta_k^m P_{2k+1}(z), \quad (75)$$

where

$$\delta_k^m = \frac{1}{2m+2} 2^{4k+2} (4k+3) \frac{(2m-2k)! [(m+k+1)!]^2}{(2m+2k+3)! [(m-k)!]^2}. \quad (76)$$

Let us demonstrate that the expansion of the composite operators in the local hadronic fields (54) is in accord with the $O(4)$ -symmetry of the structure function (remember that we consider deep inelastic scattering of scalar "photon" off scalar "nucleon"). To this end, calculate the relative contribution of hadronic fields to the forward matrix element of the composite operator (68).

The forward matrix element $\langle p_A | O_{\mu_1 \dots \mu_J}^i | p_A \rangle$ is the limit value of the nonforward matrix element $\langle p_A + q | O_{\mu_1 \dots \mu_J}^i | p_A \rangle$ at $q_\mu \rightarrow 0$. Since we are dealing with the *one-particle* initial and final states, for which $p^2 = (p_A + q)^2 = m_A^2$, the 4-vector q_μ tends to zero not arbitrarily but obeying the condition $q^2 + 2p_A q = 0$. Thus, for one-particle matrix elements the limit $q_\mu \rightarrow 0$ is equivalent to the limit $q^2 \rightarrow 0$.

In order that the limit would not break the Lorentz-invariance, one also has to take an average over the directions of the 4-vector q_μ . As a result, we get [9]

$$\begin{aligned} < \lim_{\substack{q \rightarrow 0 \\ 2pq+q^2=0}} > \frac{q_{\mu_1} \dots q_{\mu_{2k}}}{(q^2)^k} \equiv \frac{\int d^4u \delta(2pu) \delta(1-u^2) u_{\mu_1} \dots u_{\mu_{2k}}}{\int d^4u \delta(2pu) \delta(1-u^2)} = \\ = \frac{1}{(2k+1)!} S \prod_{i=1}^k \left[g_{\mu_{2i-1} \mu_{2i}} - \frac{p_{\mu_{2i-1}} p_{\mu_{2i}}}{m^2} \right]. \end{aligned} \quad (77)$$

Taking in both parts of (57) the limit $q_\mu \rightarrow 0$, with an account of formula (77), we obtain

$$\begin{aligned} < p_A | O_{\mu_1 \dots \mu_J}^i(0; \mu^2) | p_A > = \Delta_{\mu_1 \dots \mu_J} \frac{1}{J+1} \sum_j (-1)^{(J-j)/2} 2^j m_A^{J-j} \times \\ \times \frac{[(J+j)/2]!}{(2j-1)! [(J-j)/2]!} (\Omega_A^i)_j^J(0; \mu^2). \end{aligned} \quad (78)$$

Here the summation is taken over the spins $j = J, J-2, \dots, J-2[J/2]$. Therefore, for even J , $J = 2m$, the contribution of the spin $j = 2k$ ($k = 0, 1, \dots, m$) to the forward one-particle matrix element of the composite operator is

$$u_k^m = \frac{1}{2m+1} (-1)^{m-k} 2^{2k} m_A^{2(k-m)} \frac{(m+k)!}{(4k-1)! [(m-k)!]} (\alpha_A^i)_k^m, \quad (79)$$

while for $J = 2m+1$ the contribution of spin $j = 2k+1$ ($k = 0, 1, \dots, m$) is

$$v_k^m = \frac{1}{2m+2} (-1)^{m-k} 2^{2k+1} m_A^{2(k-m)} \frac{(m+k+1)!}{(4k+1)! [(m-k)!]} (\beta_A^i)_k^m. \quad (80)$$

With an account of the expressions for $(\alpha_A^i)_k^m$ (63) and $(\beta_A^i)_k^m$ (67) and formulas (68), (69), from (78) we find

$$f_{2m}^{A,i}(\mu^2) = \sum_{k=0}^m u_k^m, \quad u_k^m = \gamma_k^m (\Omega_A^i)_{2m}^{2m}(0; \mu^2), \quad (81)$$

$$f_{2m+1}^{A,i}(\mu^2) = \sum_{k=0}^m v_k^m, \quad v_k^m = \delta_k^m (\Omega_A^i)_{2m+1}^{2m+1}(0; \mu^2). \quad (82)$$

Thus, the relative contributions of the hadronic fields with the same parity as J to the distributions (u_k^m and v_k^m) coincide with the coefficients in the expansion of the Gegenbauer polynomials in the Legendre polynomials (cf. formulas (81), (82) with formulas (73) and (75)).

Such a relationship is not accidental. The poles in the complex plane of the *four-dimensional* representation index J , the Toller poles α_T [11], contain all the Regge poles like $\alpha_R = \alpha_T - l$, where $l = 0, 1, \dots$.² That is why the poles of the distribution $f_J^{A,i}$ in the complex J -plane at $J = \alpha_T$ should be understood not as Regge poles, but as Toller poles.

For $\alpha_T = J$, J being an integer, the sequence $\alpha_R = \alpha_T - l$, $l = 0, 1, \dots$ stops at $\alpha_R = 0$. It corresponds to the fact that the forward matrix element $< p_A | O_{\mu_1 \dots \mu_J}^i | p_A >$ accumulates the contributions from the hadronic fields with the spins $j \leq J$, involved in the expansion of the composite operator (54). The total contribution of the hadronic fields to $< p_A | O_{\mu_1 \dots \mu_J}^i | p_A >$ (i.e., to the distribution function $f_J^{A,i}$), is expressed through the characteristics of the field with a highest spin J (68) (cf. also (81), (82)).

6 Conclusions

We have explored high energy scattering amplitude of spinless hadrons in the particle-exchange model (arbitrary integer spins are allowed) [2]. In the given approximation, the hadronic amplitude is expressed by means of the quantities that may, respectively, be interpreted as the "soft" distributions of the colliding hadrons constituents and the 2PI part of the scattering amplitude of these constituents. The universality of "hard" distributions, extracted from deep inelastic scattering, and "soft" ones has been demonstrated on the basis of the composite operator expansion in local hadronic fields.

In Refs. [13] a connection of hard and soft processes was studied on the basis of the leading logarithmic approximation for the scattering amplitude. An expression has been obtained which looks formally similar to our Eq. (23). However, the meaning of entries is different. Also in [13] the main contri-

²In the full amplitude Regge propagator corresponding to supercritical trajectory should be screened by vertices at nonpositive t in order to respect unitarity. Only for t above the threshold in the t -channel the Regge pole appears from the second Riemannian sheet and represents normal resonance in a physical region of the t -channel.

bution was estimated as due to twist-2 operators both for hard and soft processes.

In our general approach which does not refer to the perturbation theory arbitrary operators describing multiparton interactions will contribute into soft scattering that reflects the failure of simple parton picture for large distances.

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Appendix A

Let us prove the quantities $(\alpha_A^i)^m$, specified in (63), to be the solutions of the system of Eqs. (62). Substituting expression (63) into the LHS of (62) and using the relation

$$(2k + 2p - 1)!! = \frac{(2k + 2p)!}{2^{k+p}(k + p)!} \quad (83)$$

and the identity $4k + 1 \equiv 2(2m + 2k + 1) - (4m + 1)$, represent the LHS of Eq. (62) as follows:

$$I = (-1)^m m_A^{2m} 2^{-2p} (I_1 - I_2), \quad (84)$$

where

$$I_1 = 2 \sum_{k=p}^m (-1)^k \frac{(2k + 2p)! (m + k)!}{(2m + 2k)! (m - k)! (k + p)! (k - p)!}, \quad (85)$$

$$I_2 = (4m + 1) \sum_{k=p}^m (-1)^k \frac{(2k + 2p)! (m + k)!}{(2m + 2k + 1)! (m - k)! (k + p)! (k - p)!}. \quad (86)$$

Next, we have [7]

$$(2p + 2k)! = 2^{2p+2k} \frac{1}{\sqrt{\pi}} (p + k)! \Gamma(p + k + \frac{1}{2}). \quad (87)$$

After simple calculations, we come to

$$I_1 = (-1)^p 2^{-2m+1} \frac{1}{(m - p - 1)! (m - p)!} \times \\ \times \sum_{l=0}^{m-p} B(2p + l + \frac{1}{2}, m - p) \binom{m - p}{l}, \quad (88)$$

where $B(x, y)$ is the beta function, and $\binom{a}{b}$ are binomial coefficients.

The summation in (88) may be done using the integral representation for the beta function [7]

$$B(x, y) = \int_0^1 dt t^{x-1} (1 - t)^{y-1} \quad (89)$$

and fulfilling the summation over l and integration over dt in a reverse order. Finally, for I_1 we get the expression

$$I_1 = (-1)^p 2^{-2m+1} \frac{1}{(m - p - 1)! (m - p)!} B(2p + \frac{1}{2}, 2m - 2p). \quad (90)$$

Quite analogously, we obtain

$$I_2 = (-1)^p 2^{-2m-1} (4m + 1) \frac{1}{((m - p)!)^2} B(2p + \frac{1}{2}, 2m - 2p + 1). \quad (91)$$

Formulas (84), (90), (91) let us conclude that

$$I = 0, \quad (92)$$

Q.E.D.

By the same procedure the quantities $(\beta_A^i)^m$ (66) may be proven to be the solutions of the system of Eqs. (65).

Appendix B

Now let us illustrate the general formulas of Sect. 4 taking as an example the matrix elements of the spin-4 and spin-3 composite operators. For simplicity, assume that there is only one type of the composite operators and only one Regge trajectory on which the hadrons, whose Heisenberg fields enter expansion (54), are located. Dropping in (54) the index i , as well as the summation over n , for $J = 4$ we find

$$\begin{aligned} O_{\mu\nu\sigma\lambda} = & \alpha_4^4 H_{\mu\sigma\lambda} + i\alpha_3^4 [\partial_\mu H_{\nu\sigma\lambda} + \partial_\nu H_{\mu\sigma\lambda} + \partial_\sigma H_{\mu\nu\lambda} + \partial_\lambda H_{\mu\nu\sigma}] - \\ & - \alpha_2^4 \left[\left(\frac{\partial_\mu \partial_\nu}{\partial^2} - \frac{1}{8} g_{\mu\nu} \right) H_{\sigma\lambda} + \left(\frac{\partial_\mu \partial_\sigma}{\partial^2} - \frac{1}{8} g_{\mu\sigma} \right) H_{\nu\lambda} + \left(\frac{\partial_\mu \partial_\lambda}{\partial^2} - \frac{1}{8} g_{\mu\lambda} \right) H_{\nu\sigma} + \right. \\ & + \left. \left(\frac{\partial_\nu \partial_\sigma}{\partial^2} - \frac{1}{8} g_{\nu\sigma} \right) H_{\mu\lambda} + \left(\frac{\partial_\nu \partial_\lambda}{\partial^2} - \frac{1}{8} g_{\nu\lambda} \right) H_{\mu\sigma} + \left(\frac{\partial_\sigma \partial_\lambda}{\partial^2} - \frac{1}{8} g_{\sigma\lambda} \right) H_{\mu\nu} \right] - \\ & - i\alpha_1^4 \left\{ \left[\frac{\partial_\mu \partial_\nu \partial_\sigma}{\partial^2} - \frac{1}{8} (g_{\mu\nu} \partial_\sigma + g_{\mu\sigma} \partial_\nu + g_{\nu\sigma} \partial_\mu) \right] H_\lambda + \left[\frac{\partial_\nu \partial_\sigma \partial_\lambda}{\partial^2} - \right. \right. \\ & - \left. \frac{1}{8} (g_{\mu\nu} \partial_\lambda + g_{\mu\lambda} \partial_\nu + g_{\nu\lambda} \partial_\mu) \right] H_\sigma + \left[\frac{\partial_\mu \partial_\sigma \partial_\lambda}{\partial^2} - \frac{1}{8} (g_{\mu\sigma} \partial_\lambda + g_{\mu\lambda} \partial_\sigma + \right. \\ & + \left. g_{\sigma\lambda} \partial_\mu) \right] H_\nu + \left[\frac{\partial_\mu \partial_\sigma \partial_\lambda}{\partial^2} - \frac{1}{8} (g_{\nu\sigma} \partial_\lambda + g_{\nu\lambda} \partial_\sigma + g_{\sigma\lambda} \partial_\nu) \right] H_\mu \left. \right\} + \\ & + \alpha_0^4 \left\{ \frac{\partial_\mu \partial_\nu \partial_\sigma \partial_\lambda}{(\partial^2)^2} - \frac{1}{8} \left[\frac{\partial_\mu \partial_\nu \partial_\lambda}{g_{\mu\nu}} + \frac{\partial_\nu \partial_\sigma \partial_\lambda}{g_{\mu\sigma}} + \frac{\partial_\mu \partial_\sigma \partial_\lambda}{g_{\nu\sigma}} + \frac{\partial_\mu \partial_\nu \partial_\sigma}{g_{\mu\lambda}} + \frac{\partial_\nu \partial_\sigma \partial_\sigma}{g_{\mu\sigma}} + \frac{\partial_\mu \partial_\sigma \partial_\sigma}{g_{\nu\sigma}} + \frac{\partial_\mu \partial_\nu \partial_\sigma}{g_{\mu\lambda}} + \frac{\partial_\nu \partial_\sigma \partial_\sigma}{g_{\mu\sigma}} + \frac{\partial_\mu \partial_\sigma \partial_\sigma}{g_{\nu\sigma}} \right] \right\} H. \quad (93) \end{aligned}$$

In accordance with formula (19), the one-particle matrix elements of the Heisenberg fields have the following Lorentz-structures:

$$\begin{aligned} < p + q | H_{\mu\nu\sigma\lambda} | p > = I_4(t) D_4(t) \frac{1}{560} \left\{ 35 A_\mu A_\nu A_\sigma A_\lambda - 5(4m^2 - t) \times \right. \\ & \times [B_{\mu\nu} A_\sigma A_\lambda + B_{\mu\sigma} A_\nu A_\lambda + B_{\mu\lambda} A_\nu A_\sigma + B_{\nu\sigma} A_\mu A_\lambda + B_{\nu\lambda} A_\mu A_\sigma + \\ & + B_{\sigma\lambda} A_\mu A_\nu] + (4m^2 - t)^2 [B_{\mu\nu} B_{\sigma\lambda} + B_{\mu\sigma} B_{\nu\lambda} + B_{\mu\lambda} B_{\nu\sigma}] \left. \right\}, \quad (94) \end{aligned}$$

$$\begin{aligned} < p + q | H_{\mu\nu\sigma} | p > = I_3(t) D_3(t) \frac{1}{40} \left\{ 5 A_\mu A_\nu A_\sigma - (4m^2 - t) \times \right. \\ & \times [B_{\mu\nu} A_\sigma + B_{\mu\sigma} A_\nu + B_{\nu\sigma} A_\mu] \left. \right\}, \quad (95) \end{aligned}$$

$$< p + q | H_{\mu\nu} | p > = I_2(t) D_2(t) \frac{1}{12} \left\{ 3 A_\mu A_\nu - (4m^2 - t) B_{\mu\nu} \right\}, \quad (96)$$

$$< p + q | H_\mu | p > = I_1(t) D_1(t) \frac{1}{2} A_\mu, \quad (97)$$

$$< p + q | H | p > = I_0(t) D_0(t), \quad (98)$$

with the A_μ and $B_{\mu\nu}$ defined by Eq. (20).

Requiring the cancellation of the singular terms like $q_\mu q_\nu q_\sigma q_\lambda / (q^2)^2$ and $q_\mu q_\nu p_\sigma p_\lambda / q^2$ in the matrix element $< p + q | O_{\mu\nu\sigma\lambda} | p >$, we come to the system of equations for the quantities $\alpha_i^4(0; \tilde{\mu}^2)$, $i = 0, 2, 4$

$$\begin{aligned} \frac{3}{35} (m^2)^2 \alpha_4^4(0; \tilde{\mu}^2) + 2m^2 \alpha_2^4(0; \tilde{\mu}^2) + \alpha_0^4(0; \tilde{\mu}^2) &= 0, \\ \frac{1}{7} m^2 \alpha_4^4(0; \tilde{\mu}^2) + \alpha_2^4(0; \tilde{\mu}^2) &= 0, \end{aligned} \quad (99)$$

whose solution is

$$\alpha_2^4(0; \tilde{\mu}^2) = -\frac{1}{7} m^2 \alpha_4^4(0; \tilde{\mu}^2), \quad \alpha_0^4(0; \tilde{\mu}^2) = \frac{1}{5} (m^2)^2 \alpha_4^4(0; \tilde{\mu}^2). \quad (100)$$

The forward matrix element $< p | O_{\mu\nu\sigma\lambda} | p >$ is derived from the matrix element $< p + q | O_{\mu\nu\sigma\lambda} | p >$ in the limit $q_\mu \rightarrow 0$ that has been described in detail in Sect. 5. The calculations by means of formula (77) give us the result

$$\begin{aligned} < p | O_{\mu\nu\sigma\lambda} | p > = \frac{1}{5} \left[\frac{128}{35} \alpha_4^4(0; \tilde{\mu}^2) - \frac{8}{m^2} \alpha_2^4(0; \tilde{\mu}^2) + \frac{1}{m^4} \alpha_0^4(0; \tilde{\mu}^2) \right] \times \\ & \times \left\{ p_\mu p_\nu p_\sigma p_\lambda - \frac{1}{8} m^2 [g_{\mu\nu} p_\sigma p_\lambda + g_{\mu\sigma} p_\nu p_\lambda + g_{\mu\lambda} p_\nu p_\sigma + g_{\nu\sigma} p_\mu p_\lambda + \right. \\ & + \left. g_{\nu\lambda} p_\mu p_\sigma + g_{\sigma\lambda} p_\mu p_\nu] + \frac{1}{48} (m^2)^2 [g_{\mu\nu} g_{\sigma\lambda} + g_{\mu\sigma} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\sigma}] \right\}. \quad (101) \end{aligned}$$

The account of (100) makes Eq. (101) look like

$$\begin{aligned} < p | O_{\mu\nu\sigma\lambda} | p > = \alpha_4^4(0; \tilde{\mu}^2) \left\{ p_\mu p_\nu p_\sigma p_\lambda - \frac{1}{8} m^2 [g_{\mu\nu} p_\sigma p_\lambda + g_{\mu\sigma} p_\nu p_\lambda + \right. \\ & + \left. g_{\mu\lambda} p_\nu p_\sigma + g_{\nu\sigma} p_\mu p_\lambda + g_{\nu\lambda} p_\mu p_\sigma + g_{\sigma\lambda} p_\mu p_\nu] + \frac{1}{48} (m^2)^2 \times \right. \\ & \times \left. [g_{\mu\nu} g_{\sigma\lambda} + g_{\mu\sigma} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\sigma}] \right\}. \quad (102) \end{aligned}$$

For $J = 3$ the composite operator expansion will be:

$$\begin{aligned} O_{\mu\nu\sigma} = & \alpha_3^3 H_{\mu\nu\sigma} + i\alpha_2^3 [\partial_\mu H_{\nu\sigma} + \partial_\nu H_{\mu\sigma} + \partial_\sigma H_{\mu\nu}] - \\ & - \alpha_1^3 \left[\left(\frac{\partial_\mu \partial_\nu}{\partial^2} - \frac{1}{6} g_{\mu\nu} \right) H_\sigma + \left(\frac{\partial_\mu \partial_\sigma}{\partial^2} - \frac{1}{6} g_{\mu\sigma} \right) H_\nu + \left(\frac{\partial_\nu \partial_\sigma}{\partial^2} - \frac{1}{6} g_{\nu\sigma} \right) H_\mu \right] - \\ & - i\alpha_0^3 \left[\frac{\partial_\mu \partial_\nu \partial_\sigma}{\partial^2} - \frac{1}{6} (g_{\mu\nu} \partial_\sigma + g_{\mu\sigma} \partial_\nu + g_{\nu\sigma} \partial_\mu) \right] H. \quad (103) \end{aligned}$$

The matrix element $\leq p + q | O_{\mu\nu\sigma} | p >$ does not include singular terms like $q_\mu q_\nu p_\sigma / q^2$, provided

$$\frac{1}{5} m^2 \beta_3^3(0; \tilde{\mu}^2) + \beta_1^3(0; \tilde{\mu}^2) = 0. \quad (104)$$

In the limit $q_\mu \rightarrow 0$ we get

$$\begin{aligned} \leq p | O_{\mu\nu\sigma} | p > &= \frac{1}{4} \left[\frac{16}{5} \beta_3^3(0; \tilde{\mu}^2) - \frac{4}{m^2} \beta_1^3(0; \tilde{\mu}^2) \right] \times \\ &\times \left\{ p_\mu p_\nu p_\sigma - \frac{1}{6} [g_{\mu\nu} p_\sigma + g_{\mu\sigma} p_\nu + g_{\nu\sigma} p_\mu] \right\}. \end{aligned} \quad (105)$$

With an account of Eq. (104), it leads to

$$\leq p | O_{\mu\nu\sigma} | p > = \beta_3^3(0; \tilde{\mu}^2) \left\{ p_\mu p_\nu p_\sigma - \frac{1}{6} [g_{\mu\nu} p_\sigma + g_{\mu\sigma} p_\nu + g_{\nu\sigma} p_\mu] \right\}. \quad (106)$$

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