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EHS/NA22 Collaboration

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Abstract. The correlation of negative particles at small momentum difference and its dependence on multiplicity and on kinematical cuts is studied in π^+p -interactions at 250 GeV/c. In terms of the Kopylov-Podgoretskiĭ parametrization, an average radius of the pion emitting region of $r_K = 1.59 \pm 0.14$ fm and a life-time (or emission depth) $\tau = 0.83 \pm 0.25$ fm are found. The Lorentz invariant parametrization of Goldhaber gives $r_G = 0.85 \pm 0.04$ fm. Assuming two different sources of pions, their radii are estimated as $r_1 = 1.75 \pm 0.25$ fm and $r_2 = 0.60 \pm 0.08$ fm. An angular and multiplicity dependence of the space-time size of the source is observed. The source is elongated along the collision axis and has larger size r_K at higher multiplicities. The radius r_K decreases with increasing pion pair momentum. The size of the emitting region appears to be larger for low rapidity pions than for pions from the fragmentation region. No evidence is found for a unique reference frame, where the pion source is motionless for each π^+p collision, i.e. where the space-time size of the source is definitely smaller than in any other frame.

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1 Introduction

The processes of elastic and quasi-elastic scattering of hadrons can serve as a source of information on the impact parameter distribution, i.e. on the transverse size of the interaction region (see, for example, [1]). Some restriction on this size can be deduced also from inelastic interactions [2,3]. Another important characteristic of a multiple production process is the space-time size of the particle emitting region, which can noticeably exceed the size of the incident hadron interaction region. As established in refs. [4-10], the space-time extension of the emitting region can give rise to definite correlation in the spectra of two (or more) identical particles.

Let us consider a radiating source consisting of two point-like one-particle emitters. Then, the amplitude of double counts consists of two terms corresponding to the two indistinguishable cases of emitting one or the other particle by each of the emitters. If the emitters radiate independently (with random relative phase), the probability of double counts contains an interference term which can play an important role at small relative momentum of the detected particles. This interference is constructive for particles obeying Bose-Einstein statistics and destructive for the case of Fermi-Dirac statistics. The character of the two-particle correlation is determined by the Fourier transform of the space-time distribution of point-like emitters in the radiating region.

The experimental investigation of interference effects allows, in principle, to measure the space-time size of the emitting region in an arbitrary direction and for specific classes of interactions, differing by the multiplicity of secondaries, their transverse momentum, their rapidity and other characteristics.

The interference method has first been used by Hanbury-Brown and Twiss for the determination of the angular size of stars, by measuring the intensity correlations between separated telescopes [4]. The first experimental evidence for interference effects between identical pions (the Bose-Einstein correlations) in hadronic interactions has been reported by Goldhaber et al [5].

After that, a large number of experiments has been performed concerning the study of interference effects in interactions of different types of projectile and target for a wide range of incident energies. Recent experimental results are published in [11-20], recent reviews are given in [21-24].

In this work, we report on the investigation of Bose-Einstein correlations in π^+p -collisions at 250 GeV/c in the NA22 experiment, performed at the CERN SPS with the help of the European Hybrid Spectrometer EHS. The results are based on the full statistics of the π^+p -sample, allowing a detailed study of the space-time structure of multihadron production. Earlier results, based on one third of the present statistics are published in [25].

In the following section we present the parametrizations commonly used for the description of the interference effects in the probability distribution of two identical pions. The experimental procedure is described in Sect. 3. The space-time characteristics of the multihadron production process and the dependence on multiplicity, on kinematical cuts and on the reference frame are presented in Sect. 4. Conclusions are

summarized in Sect. 5.

2 The Parametrization

The coincidence rate for the observation of two identical pions normalized to an uncorrelated “background” is

$$R = 1 + \lambda f_1(\mathbf{q}) f_2(q_0) \quad (1)$$

where $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ is the three momentum difference and $q_0 = |E_1 - E_2|$ is the energy difference of the two pions. The functions $f_1(\mathbf{q})$ and $f_2(q_0)$ are Fourier transforms of the space and life-time distributions of the emitters, respectively. These distributions are assumed to be independent.

For example, if emitters are distributed uniformly on a disk or a spherical surface of radius r , then (in units $\hbar = c = 1$) [7,9]:

$$f_1(q_T) = 4J_1^2(q_T r) / (q_T r)^2, \quad (2)$$

$J_1(x)$ being the first-order Bessel function and q_T the modulus of the component of the vector $\mathbf{q}$ transverse to $\mathbf{p}_1 + \mathbf{p}_2$.

If emitters have a spherically-symmetric Gaussian distribution then [5,7,9]:

$$f_1(\mathbf{q}) = \exp(-|\mathbf{q}|^2 r^2), \quad (3)$$

where r is related to the r.m.s. radius of the distribution by $r(r.m.s.) = \sqrt{3}r$.

If emitters are excited simultaneously and decay exponentially with life-time τ , then [7]:

$$f_2(q_0) = (1 + q_0^2 \tau^2)^{-1}. \quad (4)$$

Alternatively, the parameter τ can be interpreted as the thickness of the layer from which pions produced at the same instant inside the source volume can escape [9,26].

The parameter λ in (1) characterizes the strength of the interference effect [27], with a maximum possible value of $\lambda = 1$. The experimentally observed values are practically always $\lambda < 1$. Different reasons can lead to a value $\lambda < 1$, for example a contamination from coherently radiating emitters for which interference effects are absent [26,28], from point-like emitters radiating two or more like pions [29]; the presence of two pion sources of different size [29,30], or simply detector induced biases (as misidentification of particles, wrong charge assignment or track losses).

Expressions (1)-(4) are not Lorentz-invariant. In general, the variables are, therefore, calculated in the centre of mass of the initial collision.

A possible Lorentz-invariant parametrization (known as Goldhaber parametrization) is

$$R = 1 + \lambda \exp(-r^2 Q^2), \quad (5)$$

with $Q^2 = -(p_1 - p_2)^2 = M^2 - 4m_\pi^2$, where M is the invariant mass of the pion pair and m_π is the pion mass. It corresponds to a Gaussian shape of the source in the centre of mass of the pair, where $q_0 = \Delta E = 0$.

In this paper, the experimental data are analyzed in terms of the following parametrizations:

i) The one-dimensional Kopylov-Podgoretzkiĭ parametrization

$$R(q_T) = \gamma_K[1 + \lambda_K(4J_1^2(r_K q_T)/(r_K q_T)^2)](1 + \delta_K q_T). \quad (6)$$

ii) The two-dimensional Kopylov-Podgoretzkiĭ parametrization

$$R(q_T, q_o) = \gamma_K[1 + \lambda_K(4J_1^2(r_K q_T)/(r_K q_T)^2)(1 + \tau^2 q_o^2)^{-1}]. \quad (7)$$

iii) The Lorentz-invariant parametrization of Goldhaber

$$R(Q^2) = \gamma_G[1 + \lambda_G \exp(-r_G^2 Q^2)](1 + \delta_G Q^2). \quad (8)$$

iv) Parametrizations corresponding to two sources with different size r_1 and r_2 ,

a) the Lorentz-invariant parametrization with two Gaussians

$$R(Q \text{ or } Q^2) = \gamma[1 + \lambda_1 \exp(-r_1^2 Q^2) + \lambda_2 \exp(-r_2^2 Q^2)]; \quad (9)$$

b) the case, where both sources are present in each event [29]

$$R(|\mathbf{q}|) = \gamma[1 + \{\lambda_1 \exp(-r_1^2 |\mathbf{q}|^2/2) + \lambda_2 \exp(-r_2^2 |\mathbf{q}|^2/2)\}^2] \quad (10)$$

for pion pairs with very small q_0 in the overall cms;

c) the case of a mixture of two subsamples of events, each subsample corresponding to one of the sources [29]

$$R(|\mathbf{q}|) = \gamma[1 + \lambda_1 \exp(-r_1^2 |\mathbf{q}|^2) + \lambda_2 \exp(-r_2^2 |\mathbf{q}|^2)] \quad (11)$$

for pion pairs with very small q_0 in the overall cms.

The parameters r_1 and r_2 in (10) and (11) are related to the r.m.s. radii of the spherical-symmetric Gaussian distributions by $r_1(r.m.s.) = \sqrt{3}r_1$ and $r_2(r.m.s.) = \sqrt{3}r_2$. The parameters γ_K, γ_G and γ in (6)-(11) are normalization coefficients, and δ_K and δ_G are introduced to account for a possible slow variation of the ratio R outside the interference peak.

Expressions (6)-(11) are compared to the experimentally observed like-pion coincidence rate after normalization to a “background” rate for which the like-pion interference effects are absent. We apply two methods to form this “background” (reference) sample:

a) The reference sample is formed by the so-called mixed-event technique, i.e. a pion from one event is combined with pions randomly chosen from different events of the same multiplicity class (e.g. $n=6$, $n=8$, $n=10$, $n=12$, $n \geq 14$). Energy and momentum are not conserved in this reference sample.

b) The reference sample is formed from pairs of unlike charged pions in the same physical event and resonances like ρ^0 are excluded. This reference sample typically

suffers from the reflection of non-excluded resonances and from contamination by Dalitz decay and nearby γ -conversion.

Differences observed in the results obtained with the two different reference samples reflect some of the uncertainties inherent to this type of interferometry. In special cases, these differences can be explained (see e.g. Fig.10 below), but generally they are used as an estimate for possible systematic effects.

3 Data sample and reconstruction procedure

The full experimental set-up of EHS, exposed to a positive meson enriched beam with momentum 250 GeV/c, is described in detail in [31,32,33]. It consists of an active vertex detector (Rapid Cycling Bubble Chamber filled with H_2) and a down-stream two-lever-arm spectrometer. Tracks of secondary charged particles are reconstructed from hits in the wire and drift chambers of the spectrometer and from measurement in RCBC. The momentum resolution varies from (1-2)% for tracks reconstructed in RCBC, to (1-2.5)% for tracks reconstructed in the first lever arm and to 1.5% for tracks reconstructed in the full spectrometer.

We accept π^+p events for which the measured and reconstructed charge multiplicities are consistent, charge balance is satisfied, no electron is detected and the number of badly reconstructed (and, therefore, rejected) tracks is at most 1, 2 and 3 for events with charge multiplicity $n=6, 8$ and >8 , respectively [33]. Only events with charge multiplicity $n \geq 6$ are accepted, the minimum required to yield a negative pair. After these cuts, the total number of accepted events is 76 195.

In the present analysis two additional track cuts are applied:

1. the track momentum error is required to be below 4%;
2. each accepted track is required to lie in the region of the Feynman variable $|x_F| < 0.5$.

The latter selection minimizes biases due to the violation of energy and momentum conservation imminent to the mixed-event technique. For positive tracks it, furthermore, reduces the contamination from protons. These two cuts reject additional 15% and 2% of tracks, respectively, but improve the credibility of the final results.

Since the charge of both incident particles is positive, Bose-Einstein correlations are studied for negative particles in this paper. All charged particles except identified protons are assumed to have pion mass. The contamination from other particles is estimated to be $(7 \pm 3)\%$ [33]. The resolution in Q^2 and q_T is estimated to be $\sigma_{Q^2} = 8 \cdot 10^{-4} \text{ (GeV/c)}^2$ at $Q^2 < 0.04 \text{ (GeV/c)}^2$ and $\sigma_{q_T} = 5 \cdot 10^{-3} \text{ GeV/c}$ at $q_T < 0.05 \text{ GeV/c}$.

4 Results

4.1 The average space and time extension of the pion source

Fig.1 shows the ratio R as a function of Q^2 compared to a fit of the Goldhaber parametrization (8). Fits are shown with mixed-event and unlike-pair reference samples. For the latter, the range $0.36 < Q^2 < 0.60(\text{GeV}/c)^2$ is contaminated by the ρ^0 decay products and, therefore, excluded from the fit (Fig.1b). The long tail of the distribution does not influence the parameter r , but is used to determine γ and δ .

The fit results are presented in Table 1 for two different binnings of Q^2 , $\Delta Q^2=0.02$ and $0.01 (\text{GeV}/c)^2$, and for the two different reference samples leading to slightly (4-10%) different values of radius r_G and correlation strength λ_G . As an average radius of the pion source, observed in the dipion rest frame, we obtain $r_G = 0.85 \pm 0.04$ fm. The quoted error includes the dispersion of r_G due to the choice of reference sample and binning. Further results from (8) will be limited to the $\Delta Q^2=0.02 (\text{GeV}/c)^2$ binning.

Fig.2 shows the ratio R as a function of q_T , for pairs with energy difference limited to $q_0 < (q_0)_{cut} = 0.2 \text{ GeV}/c$ in the overall cms, compared to a fit by the Kopylov-Podgoretskiĭ parametrization (6). Fits are shown with mixed-event and unlike-pair reference samples. For the latter (Fig.2b), the first point $q_T \leq 0.025 \text{ GeV}/c$ is biased by unidentified Dalitz pairs in the reference distribution and excluded from the figure and the fit procedure. This demonstrates clearly that results obtained with the unlike-pair reference sample have to be considered with particular care.

The fit results are presented in Table 2, with free δ_K and with δ_K fixed to 0. In all cases, the radius r_K is of order 1.3-1.6 fm. In what follows, results from the Kopylov-Podgoretskiĭ parametrization will be limited to the case of $\delta = 0$.

We have verified, that the fit parameters presented in Tables 1 and 2 are within errors in agreement with the results obtained from our preliminary sample [25].

It is expected that Bose-Einstein correlations become stronger for smaller q_0 . Except for the first point ($(q_0)_{cut} = 0.03 \text{ GeV}$) where the fit is bad (too low) at small q_T , this is confirmed in Table 3, where the fit results (with mixed-event reference sample) are presented for different values of $(q_0)_{cut}$. Indeed, the “limiting” values ($q_0 \rightarrow 0$) of $r_K \sim 1.9$ fm and $\lambda_K \sim 0.4$ are larger than the more “conservative” ones presented in Table 2.

Due to the different assumption on the shape of the source, the spatial dimensions r_K and r_G used in parametrizations (6) and (8) have a different meaning. Note also, that r_K and r_G are related to different reference frames, the overall cms and the centre of mass of the pair, respectively. At small q_T and for the case of $q_0 \rightarrow 0$, when $Q^2 \rightarrow q_T^2$, expression (2) acquires an exponential form

$$f_1(q_T) \rightarrow f_1(Q^2) \approx \exp(-r^2 Q^2/4) ,$$

so that r_K and r_G satisfy an approximate relation $r_K \approx 2r_G$. Within errors, our data are in agreement with this relation.

More complete information on the emitting region can be obtained from the two-dimensional distribution $R(q_T, q_0)$ using the Kopylov-Podgoretskiĭ parametrization (7). Fig. 3 shows the ratio R based on the mixed-event technique, as a function of the two variables q_T and q_0 in the overall cms, for pairs with $q_0 < 0.15$ GeV. The fit results according to (7) are presented in Fig. 3 and Table 4, in the latter for $q_0 < 0.15$, 0.16 and 0.20 GeV. Within errors, the values obtained for r_K and λ_K are equal to those obtained from the one-dimensional distribution $R(q_T)$ at the corresponding values of $(q_0)_{cut}$ (cf. Table 3).

Our values of $r_K = 1.59 \pm 0.14$ fm and $\tau = 0.83 \pm 0.25$ fm (obtained by averaging over the values given in Table 4 and including errors from binning) are close to those obtained in the high-statistics experiments [12,27,34], where data are analysed with the same parametrization (7) (in ref [34], the energy difference q_0 in (7) is replaced by an equivalent variable q_L , the longitudinal component of $\mathbf{q}$).

It is important to note that the fit by (7) cannot reproduce the sharp peak for $q_T \rightarrow 0$ at small q_0 -values.

4.2 The size of two different sources

In Fig.4 and in Tables 5 and 6 we present the results of an analysis under the assumption that pions are generated by two sources of different size. In Fig.4a we show $R(Q)$ based on the mixed-event technique, compared to a fit by parametrization (9). The results of the fits by R as a function of Q and Q^2 are presented in Table 5. The extracted radii $r_1 = 1.79 \pm 0.20$ fm, $r_2 = 0.56 \pm 0.04$ fm and correlation strengths $\lambda_1 = 0.44 \pm 0.06$, $\lambda_2 = 0.18 \pm 0.03$ are similar to those obtained for $\sqrt{s}=63$ GeV pp interactions ($r_1 = 2.1 \pm 0.5$ fm, $r_2 = 0.66 \pm 0.09$ fm and $\lambda_1 = 0.48^{+0.18}_{-0.19}$, $\lambda_2 = 0.28 \pm 0.05$) [35], where (9) was used for $R(Q)$. Note, that the fit by the one-exponential parametrization (8) to $R(Q^2)$ leads to “effective” values $r_2 < r_G < r_1$ and $\lambda_2 < \lambda_G < \lambda_1$ (cf. Table 1).

In Fig.4b we show the ratio $R(|\mathbf{q}|)$, based on the mixed-event technique, for pairs with $(q_0)_{cut}=0.05$ GeV in the overall cms. The fit results for parametrizations (10) and (11) are presented in Table 6. Within the large errors, the extracted parameters r_1 and r_2 are similar for parametrizations (10) and (11) and, because of the similarity of the two curves, only the result for parametrization (10) is given in Fig.4b. The r.m.s. radii of Gaussian distributed sources ($r(\text{r.m.s.}) = \sqrt{3}r$) are $r_1(\text{r.m.s.})=10.6 \pm 3.1$ fm and $r_2(\text{r.m.s.})=2.1 \pm 0.8$ fm for parametrization (10), and $r_1(\text{r.m.s.})=8.5 \pm 1.7$ fm and $r_2(\text{r.m.s.})=1.3 \pm 0.1$ fm for parametrization (11). Note, that the values of the radius r_K obtained with the assumption of a spherical surface distribution (parametrization (6)) lie closer to $r_2(\text{r.m.s.})$. The r.m.s. radii obtained with parametrization (10) are larger than the values $r_1(\text{r.m.s.})=5.5 \pm 2.0$ fm and $r_2(\text{r.m.s.})=1.1 \pm 0.5$ fm obtained for π^-p interactions at 40 GeV/c [36].

In all cases, the radius r_1 is by ~ 5 -7 times larger than r_2 . The larger radius has been interpreted as connected with decay of short-lived resonances (mainly ρ -mesons), while the smaller one as due to direct production of pions [30].

Despite the large errors, the correlation strength seems to be larger for the larger radius source than for the smaller radius one.

4.3 Charge multiplicity dependence

Results on the charge multiplicity dependence of the Bose-Einstein correlation effects are presented in Figs.5 and 6. Parameters are given in Tables 7, 8 for the Goldhaber parametrization (8) and in Tables 9, 10 for the Kopylov-Podgoretskiĭ parametrization (6). As can be seen there, the radius r_G is noticeably smaller for reactions in which only one negative pair is produced ($n=6$) than for higher multiplicities. For $n \geq 8$, no definite n -dependence of r_G is found at our energy. Also the radius r_K is larger (by a factor ~ 1.5 - 1.9) for the highest multiplicities ($n \geq 14$) than for the lowest. The errors on the correlation strength are too large to establish an n -dependence of that parameter. These trends are very similar for the mixed-event and unlike-pair reference samples.

Table 11 indicates an n -dependence for the extension of the pion production process according to parametrization (7). The n -dependence of r_K for the Kopylov-Podgoretskiĭ parametrizations (6) and (7) is similar. Because of the large errors, no definite trend can be given for τ and λ_K , but the life-time at highest multiplicities ($n \geq 16$) seems larger than at lower ones ($n \leq 10$).

A significant n -dependence of the space-time size of the emitting region is, however, observed to set in for $\sqrt{s} \geq 50$ GeV [11,19,34,37]. The effect becomes very clear in a comparison of hh [11] and AA [38] collisions and is now also observed in e^+e^- collisions at 91 GeV [15,24].

A dependence on multiplicity is expected from hydrodynamical models of nuclear interactions [39] or from a dependence on the impact parameter [40]. This would, however, not explain a multiplicity dependence in e^+e^- collisions. As has been pointed out in [41], a non-negligible multiplicity dependence has to be expected already from biases introduced by the cuts in multiplicity, themselves. These and other biases introduced by kinematical cuts will have to be studied with the help of Monte Carlo models including Bose-Einstein interference.

4.4 Angular dependence

As mentioned in Sect.2 above, the correlation method makes it possible to measure the size in any direction given by the vector $\mathbf{q}$. We here report on a study of the dependence of correlation effects on the angle θ between $\mathbf{q}$ and the collision axis evaluated in the overall cms.

The fit results according to parametrization (8) are given in Fig.7 and Tables 12 and 13 (for three regions of $|\cos \theta|$), and in Fig.8 (for five regions of $|\cos \theta|$). For the unlike-pair technique for parametrization (8), we observe that the longitudinal size of the source (at $|\cos \theta| > 0.7$) is larger than the transverse one (at $|\cos \theta| < 0.3$). The results for the angular dependence of λ_G are inconclusive.

An angular dependence of the space-time size of the source is also observed in the overall cms, when using the Kopylov-Podgoretskiĭ parametrizations (6) and (7). As can be seen from Tables 14 and 15, the longitudinal size of the source is larger than the transverse one. This is more significant for the two dimensional parametrization (7) than for the one-dimensional (6). Also the correlation strength λ_K is larger at smaller

angles. An elongation of the source along the collision axis has also been observed with parametrization (6) for pp and $\bar{p}p$ -interactions at $\sqrt{s}=53$ and 63 GeV [35].

Because of the large errors, the angular dependence of the parameter τ is less significant (Table 15). Note, that expression (7), in which τ has the meaning of a life-time of emitters, is derived under the assumption $\tau \gg r_K$. This is not confirmed by our data. So, more probably, the parameter τ has to be interpreted as the depth of the pion emitting region [9, 26].

4.5 Dependence on pion rapidity and transverse momentum and on pair momentum

It is interesting to ascertain whether pions of different rapidities, transverse or total momenta are produced by the same or by different sources. The dependence of the source radius and the correlation strength on $|y_{max}|$, the maximal absolute overall cms rapidity of accepted pions, is presented in Figs.9 and 10 according to parametrizations (6) and (8), respectively. Within errors, the data based on the unlike-pair technique do not show any dependence of r_G and r_K on $|y_{max}|$, while those based on the mixed-event technique show a decrease of r_G with increasing $|y_{max}|$ (see Fig.10). The radius r_G extracted for $|y_{max}|=0.5$ by the mixed-event technique is about 25% larger than for the unlike-pair technique. Probably, this difference is due to the contamination from Dalitz decay and γ -conversion near the interaction vertex in the unlike-pair technique.

A similar effect is seen for pions produced in other restricted rapidity intervals. One can infer from Fig.11, that central pions ($-1 < y < 0$ and $0 < y < 1$) are produced by an apparently more extended source than pions with $|y| > 1$. Again, the effect is smaller in the unlike-pair technique due to correlation and/or contamination in the reference sample.

Our data show no definite rapidity dependence of the correlation strength.

The dependence of r_G and λ_G on p_T^{min} , the minimal transverse momentum of accepted pions, is presented in Fig.12. For the unlike-pair method, r_G decreases more or less continuously as p_T^{min} increases from 0.1 to 0.5 GeV/c, while for the mixed-event method no significant decrease of r_G is observed. No significant dependence of λ_G is found on p_T^{min} .

Using the mixed-event technique and parametrization (6) at $q_0 < 0.2$ GeV, we observe a dependence of r_K on the pion pair momentum $|\vec{p}| = |\vec{p}_1 + \vec{p}_2|$ in the overall c.m.s.. In Fig.13 we present the radius r_K and the strength λ_K for pion pairs in the momentum intervals $|\vec{p}| = 0.0 - 0.4, 0.4 - 0.8, 0.8 - 1.3$ and $1.3 - 2.0$ GeV/c. It can be seen, that the radius decreases with increasing pion momentum, and the strength is noticeably smaller for the lowest momentum pions ($|\vec{p}| < 0.4$ GeV/c) than for higher momentum ones. Momentum dependence of the source size as observed in Fig.13a is predicted by Pratt [42], who has shown that the correlation function of a pion pair emitted from the surface of a thermally excited (with a temperature T) and spherically expanding (with a radial velocity v) source leads to the effective radius

$$r(\vec{p}) = r_0[(z \tanh z)^{-1} - (\sinh z)^{-2}]^{1/2} \quad (12)$$

where $z = 1/2|\vec{p}|\gamma v/T$ and $\gamma = (1 - v^2)^{1/2}$ is the Lorentz factor. The physical explanation of the monotonically decreasing behavior of $r(\vec{p})$ is that faster pions are more

likely to be emitted near a point on the shell expanding with a velocity in the direction of $\vec{p}$, whereas pairs with smaller momenta can come from more widely separated points [42]. The curves in Fig.13a are the fits by (12) leading to the parameter values

	r_0	T/γ	χ^2/NDF
mixed-event:	2.07 ± 0.16 fm	0.30 ± 0.11 GeV	0.9/2
unlike-pair:	2.09 ± 0.20 fm	0.20 ± 0.10 GeV	2.3/2

A decrease of the source size with increasing pion momentum was observed experimentally in heavy-ion collisions [43,44], and recently in pp -collisions at 400 GeV/c [12], for which the extracted parameters $r_0 = 3.67 \pm 1.37$ fm and $T/\gamma v = 0.12 \pm 0.10$ GeV.

4.6 The influence of trigger particles

Besides its dependence on multiplicity and kinematical characteristics of the produced pions, the space-time structure of the multiparticle production process may depend also on other “triggering” conditions, as high elasticity of the collision or the production of a high p_T particle.

Tables 16 and 17 present the fit results according to parametrizations (8) and (6), respectively, for the sample of events containing a leading hadron (a proton with Feynman $x_p < -0.5$ or a π^+ -meson with $x_{\pi^+} > 0.5$). For the mixed-event technique the radius tends to be smaller and the correlation strength to be larger in the triggered events than in the non-triggered ones (cf. with Tables 1 and 2, respectively). No significant difference is found as compared with fit results for non-triggered events for the unlike-pair technique.

Figs.14 and 15 present the fit results according to parametrizations (8) and (6), respectively, for different samples of events containing a charged trigger particle with transverse momentum greater than a given value $(p_T^{trig})_{min}$. The size of the pion source tends to increase with increasing $(p_T^{trig})_{min}$. For $(p_T^{trig})_{min} > 1$ GeV/c, the radius r_G is by $\sim 30\%$ larger than for non-triggered events, for both reference samples (cf. Fig.14 with Table 1).

4.7 Bose-Einstein correlations in different reference frames

As mentioned in Sect.2 above, expressions (1)-(4) are not Lorentz-invariant. They are derived for the reference frame in which the pion source is motionless. As is shown in [45], the observed values of r and τ must be *minimal* in this so-called “symmetric” reference frame. One should stress (see [10,45]) that the correlation method for the size measurement differs from the usual method of simultaneous measurement of the end coordinates of a moving rod. In the latter method, the *maximal* length of the rod is achieved in its rest frame (proper length).

Usually, the overall c.m.s. is assumed to be the “symmetric” frame. In [46], where negative particle correlations are studied in $\pi^- N$ -interactions at 40 GeV/c with parametrization (6), an indication is obtained (however, with relatively large statistical errors) that the “symmetric” frame coincides with the c.m.s. of the colliding quarks,

in which the momenta of the incident hadrons satisfy the relation $p_N = \frac{3}{2}p_\pi$ (the so-called “quark reference frame”).

In Table 18 and Fig.16 we present the fit results corresponding to parametrization (6), for pairs with an energy difference $q_0^* = |E_1^* - E_2^*| < 0.2$ GeV measured in reference frames with different values of the incident momentum ratio $\alpha = p_p^*/p_{\pi^+}^*$. Results of [46] are also presented in Fig.16. All fits by the form (6) (and by (7) below) lead to acceptable χ^2 values. All results shown in this subsection are obtained using the mixed-event technique. Results from the unlike-pair technique lead to similar conclusions.

Our data on r_K do not show the minimum at $\alpha=1.5$ (“quark reference frame”) seen in [46] and indicated by the arrow in Fig.16. In our experiment, relatively low values ($r_K \sim 1.3 - 1.5$ fm) are achieved over a wide interval $1 \leq \alpha \leq 10$, including the overall c.m.s. ($\alpha=1$), the “quark reference frame” ($\alpha=1.5$) and the reference frame of equal rapidities of the incident hadrons ($\alpha = m_p/m_\pi \approx 6.7$).

In other words, there is no unique “symmetric” reference frame in which the pion source is motionless for each π^+p -collision. This is probably due to the fact that the colliding constituents (quarks or gluons) can carry very different fractions of the initial hadron momentum and not the constant fraction of 1/2 for pion and 1/3 for nucleon, as assumed in [46].

It is also necessary to bear in mind that in different reference frames we deal with somewhat different samples of pions with small relative momentum. For example, the average lab frame rapidity of pions satisfying the conditions $q_T^* < 0.2$ GeV/c and $q_0^* < 0.2$ GeV, changes from $\langle y_{\pi^-}^{lab} \rangle = 2.2$ for $\alpha=0$ to 3.2 for $\alpha=1$ and 4.2 for $\alpha=100$. Consequently, a variation of r with α can already be caused by the change of the relevant pion sample from one reference frame to another.

Note from Table 18, that the fitted value of the correlation strength λ practically does not change in the interval $1 \leq \alpha \leq 10$, and slightly increases at the lower values of α . The normalization factor γ varies between 0.87 ± 0.02 and 0.94 ± 0.01 for all fits (not shown).

More complete information on the space-time extension of the emitting region can be obtained from the two-dimensional ratio $R(q_T, q_0)$, according to parametrization (7). The fit results are presented in Table 18 and Fig.17. Practically no dependence on α is found.

As mentioned above, the correlation method, in principle, allows to measure the extent of the source in any direction given by the vector $\mathbf{q}$. We divide our data into three subsamples covering different regions of the angle θ between $\mathbf{q}$ and the collision axis: $0.7 < |\cos \theta| < 1$, $0.3 < |\cos \theta| < 0.7$ and $0 < |\cos \theta| < 0.3$. The correlations in these subsamples measure the size in the longitudinal (collision axis), “intermediate” and transverse directions, respectively.

The fit results according to (7) are presented in Table 19 and in Fig.18. The results at $\alpha = 0$ and $\alpha > 4$ are omitted due to large errors. Again, no significant minimum is observed for r_K at $\alpha=1.5$ or for any other direction. In all reference frames, the longitudinal size is larger than the transverse and “intermediate” sizes, i.e. the emitting region has a prolate shape. Note, that in π^-N interactions at 40 GeV/c an *oblate* form ($r_L < r_T$) of the emitting region has been observed in [46].

The correlation strength λ is larger for the longitudinal direction in all reference frames (Table 19). No trend can be observed for τ .

It is also interesting to ascertain what size of source is “seen” by pions produced in the backward and forward hemispheres of various reference frames. These sizes should be identical in a “symmetric” reference frame. We find that the “backward” and “forward” radii practically coincide and are constant ($r_B \approx r_F \approx 1.5$ fm) for all $1 \leq \alpha \leq 4$. Also this is an indication for the absence of a definite “symmetric” reference frame.

5 Summary and conclusions

A study of Bose-Einstein correlations and their dependence on multiplicity and on kinematical cuts has been performed in π^+p -interaction at 250 GeV/c with the help of the EHS spectrometer. In order to estimate the uncertainties inherent to the definition of an “uncorrelated” reference sample, results obtained from the mixed-event and unlike-pair techniques are compared to each other.

- From the observed correlations of negatives we obtain, using the Lorentz invariant parametrization (8) of Goldhaber, an average radius of the pion source (as observed in the dipion rest frame) of $r_G = 0.85 \pm 0.04$ fm. Using the Kopylov-Podgoret-skiĭ parametrizations (6) and (7) (in the overall cms) we find for the average radius $r_K \sim 1.59 \pm 0.14$ fm and for the life-time (or emission depth) $\tau = 0.83 \pm 0.25$ fm. However, a sharp peak observed at small q_t and small q_0 remains unexplained.
- A more detailed study shows that the pion source can be characterized by two different radii. Obtained with parametrization (9), these radii are $r_1 = 1.75 \pm 0.25$ fm and $r_2 = 0.60 \pm 0.08$ fm in the dipion rest frame, very close to those observed in higher energy pp -interactions. In the centre of mass of the initial collision, the r.m.s. radii of the two Gaussian distributed sources extracted according to parametrizations (10) and (11) are $r_1(\text{r.m.s.}) = (10.6 \pm 3.1) \div (8.5 \pm 1.7)$ fm and $r_2(\text{r.m.s.}) = (2.1 \pm 0.8) \div (1.3 \pm 0.1)$ fm. The correlation strength seems to be larger for the larger radius source.
- Our data allow to find a multiplicity dependence of the correlation effects, but smaller than observed for higher energy pp and $\bar{p}p$ -interactions, nucleus-nucleus collisions and highest energy e^+e^- annihilation. For the cases of parametrizations (6) and (7), the radius r_K increases with increasing multiplicity, being a factor 1.6-1.9 larger at $n \geq 14$ than at $n = 6 - 8$. For the case of parametrization (8), the radius r_G is significantly smaller at multiplicity $n=6$ than at higher ones, but at $n \geq 8$ no significant n -dependence is observed.
- A significant angular dependence of the correlation effects is observed both in the overall cms (both techniques) and in the dipion rest frame (unlike-technique only). The source is found to be elongated along the collision axis.
- From the rapidity dependence, an indication is obtained that the source of pions produced in the central region has on average a larger size than that produced in the fragmentation region.
- The source size “seen” by pions with different momenta in the overall cms decreases

with increasing momentum, as is expected for a thermally excited and spherically expanding source with $T/\gamma v = 0.3 \pm 0.1$ GeV.

- Our data indicate that the source size tends to increase as the transverse momentum of a “trigger” particle is increased.
- As for the correlation strength, no definite dependence on the multiplicity, rapidity and transverse momentum of pions, and on the “trigger” conditions is observed. However, λ_K is larger at smaller angles and is smaller at the lowest momenta of pions in the overall cms.
- In our study of Bose-Einstein correlation in different reference frames of π^+p interaction, no evidence is found for a unique frame in which the pion source is motionless for each π^+p collision, i.e. where the spatial size and the life-time (the emission depth) of the pion source are definitely smaller than in other reference frames moving along the beam direction. The emitting region has a prolate shape in any frame.

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Table 1. Fit results according to parametrization (8) to the ratio R as a function of Q^2 (data for binning $\Delta Q^2=0.02(\text{GeV}/c)^2$ are shown in Fig. 1).

Reference sample, bin width $\Delta Q^2(\text{GeV}/c)^2$	r_G (fm)	λ_G	γ_G	δ_G $(\text{GeV}/c)^{-2}$	χ^2/NDF
Mixed-event					
$\Delta Q^2 = 0.02$	0.81 ± 0.01	0.34 ± 0.02	0.966 ± 0.005	0.006 ± 0.006	110/96
$\Delta Q^2 = 0.01$	0.83 ± 0.02	0.38 ± 0.02	0.968 ± 0.005	0.003 ± 0.006	227/196
Unlike-pair					
$\Delta Q^2 = 0.02$	0.88 ± 0.01	0.38 ± 0.02	0.978 ± 0.005	0.009 ± 0.006	99/84
$\Delta Q^2 = 0.01$	0.87 ± 0.02	0.41 ± 0.02	0.977 ± 0.005	0.008 ± 0.006	218/172

Table 2. Fit results for (6) to ratio R as a function of q_T (data are shown in Fig. 2).

Reference sample	r_K (fm)	λ_K	γ_K	δ_K $(\text{GeV}/c)^{-1}$	χ^2/NDF
Mixed-event	1.58 ± 0.10	0.27 ± 0.03	0.961 ± 0.020	-0.036 ± 0.031	54/36
Unlike-pair	1.35 ± 0.06	0.37 ± 0.04	0.873 ± 0.025	0.112 ± 0.045	57/35
Mixed-event	1.50 ± 0.07	0.29 ± 0.02	0.938 ± 0.005	fixed to 0	55/37
Unlike-pair	1.51 ± 0.07	0.30 ± 0.02	0.938 ± 0.005	fixed to 0	63/36

Table 3. Fit results according to parametrization (6) to the ratio R as a function of q_T , for different values of $(q_0)_{cut}$ (data are normalized to the mixed-event reference sample).

$(q_0)_{cut}$ (GeV)	r_K (fm)	λ_K	γ_K	χ^2/NDF
0.03	1.49 ± 0.15	0.35 ± 0.04	0.922 ± 0.013	49/37
0.04	1.85 ± 0.14	0.43 ± 0.04	0.931 ± 0.010	51/37
0.05	1.90 ± 0.13	0.43 ± 0.04	0.935 ± 0.009	48/37
0.10	1.81 ± 0.09	0.39 ± 0.03	0.936 ± 0.006	57/37
0.15	1.68 ± 0.08	0.33 ± 0.02	0.940 ± 0.006	61/37
0.20	1.50 ± 0.07	0.29 ± 0.02	0.938 ± 0.005	55/37

Table 4. Fit results according to parametrization (7) to the ratio R as a function of q_T and q_0 , for different intervals and binnings of q_0 . Data, are normalized by the mixed-event reference sample (shown in Fig.3 for the case of $q_0 < 0.15$ GeV).

Interval of q_0 (GeV)	binning Δq_0 (GeV)	r_K (fm)	τ (fm)	λ_K	γ_K	χ^2/NDF
0-0.15	0.03	1.65 ± 0.08	$0.87^{+0.20}_{-0.27}$	0.37 ± 0.03	0.936 ± 0.006	209/196
0-0.16	0.04	1.61 ± 0.08	$0.82^{+0.18}_{-0.24}$	0.36 ± 0.03	0.937 ± 0.005	168/156
0-0.20	0.05	1.52 ± 0.07	$0.76^{+0.14}_{-0.17}$	0.33 ± 0.02	0.938 ± 0.005	192/156

Table 5. Fit results according to parametrization (9) to R as a function of Q and Q^2 . Data are normalized by the mixed-event reference sample (shown in Fig.4 for R as a function of Q).

Vari- able	binning	r_1 (fm)	λ_1	r_2 (fm)	λ_2	γ	χ^2/NDF
Q	0.02 GeV/c	$1.79^{+0.17}_{-0.20}$	0.44 ± 0.06	0.56 ± 0.04	0.18 ± 0.03	0.954 ± 0.004	34/ 45
Q^2	0.01 (GeV/c) ²	$1.77^{+0.22}_{-0.25}$	0.38 ± 0.06	0.62 ± 0.06	0.19 ± 0.03	0.965 ± 0.003	189/195

Table 6. Fit results according to (10) and (11) to R as a function of $|q|$. Data are normalized by the mixed-event reference sample (shown in Fig.5).

Paramet- rization	$(q_0)_{cut}$ (GeV)	r_1 (fm)	λ_1	r_2 (fm)	λ_2	χ^2/NDF
(10)	0.05	$6.1^{+1.4}_{-1.8}$	0.87 ± 0.22	$1.24^{+0.33}_{-0.46}$	0.51 ± 0.14	48/35
(11)	0.05	$4.9^{+0.9}_{-1.0}$	1.47 ± 0.53	0.73 ± 0.07	0.43 ± 0.05	52/35

Table 7. Fit results for (8) to the ratio R as a function of Q^2 , for selections on charge multiplicity n . Data are normalized by the mixed-event reference sample (shown in Fig. 5).

n	r_G (fm)	λ_G	γ_G	δ_G (GeV/c) $^{-2}$	χ^2/NDF
6	0.53 ± 0.05	0.33 ± 0.09	0.920 ± 0.052	0.001 ± 0.052	113/96
6-8	0.85 ± 0.04	0.29 ± 0.06	0.983 ± 0.013	-0.019 ± 0.013	105/96
8	0.91 ± 0.05	0.30 ± 0.07	0.992 ± 0.014	-0.025 ± 0.014	94/96
10	0.70 ± 0.02	0.32 ± 0.04	0.964 ± 0.013	-0.004 ± 0.013	91/96
12	0.68 ± 0.02	0.32 ± 0.03	0.935 ± 0.013	0.052 ± 0.013	85/96
14	0.82 ± 0.03	0.33 ± 0.04	0.958 ± 0.011	0.021 ± 0.011	104/96
16-18	1.09 ± 0.04	0.43 ± 0.06	0.963 ± 0.008	0.028 ± 0.008	88/96
≥ 20	1.05 ± 0.07	0.34 ± 0.09	0.976 ± 0.017	-0.022 ± 0.017	106/96

Table 8. Fit results for (8) to the ratio R as a function of Q^2 , for selection on the charge multiplicity n . Data are normalized by the unlike-pair reference sample

n	r_G (fm)	λ_G	γ_G	δ_G (GeV/c) $^{-2}$	χ^2/NDF
6	0.60 ± 0.06	0.36 ± 0.11	0.908 ± 0.058	0.071 ± 0.061	110/84
6-8	0.92 ± 0.04	0.37 ± 0.06	0.964 ± 0.014	0.058 ± 0.017	98/84
8	1.08 ± 0.06	0.41 ± 0.09	0.970 ± 0.014	0.055 ± 0.017	81/84
10	0.90 ± 0.04	0.31 ± 0.05	0.983 ± 0.012	0.015 ± 0.013	85/84
12	0.83 ± 0.03	0.37 ± 0.04	0.955 ± 0.012	0.048 ± 0.014	69/84
14	0.90 ± 0.03	0.34 ± 0.05	0.976 ± 0.012	0.009 ± 0.013	83/84
16-18	1.05 ± 0.04	0.38 ± 0.05	0.980 ± 0.010	0.008 ± 0.012	91/84
≥ 20	0.89 ± 0.06	0.40 ± 0.08	0.995 ± 0.022	-0.095 ± 0.023	92/84

Table 9. Fit results for (6) to the ratio R as a function of q_T , for selections on the charge multiplicity n . Data are normalized by the mixed-event reference sample.

n	r_K (fm)	λ_K	γ_K	χ^2/NDF
6-8	0.97 ± 0.15	0.27 ± 0.04	0.903 ± 0.022	45/37
10	1.26 ± 0.13	0.32 ± 0.04	0.916 ± 0.014	59/37
12	1.36 ± 0.14	0.28 ± 0.03	0.930 ± 0.012	63/37
≥ 14	1.86 ± 0.14	0.28 ± 0.03	0.953 ± 0.007	38/37

Table 10. Fit results for (6) to the ratio R as a function of q_T , for selections on the charge multiplicity n . Data are normalized by the unlike-pair reference sample.

n	r_K (fm)	λ_K	γ_K	χ^2/NDF
6-8	1.18 ± 0.16	0.30 ± 0.05	0.915 ± 0.018	58/36
10	1.33 ± 0.15	0.29 ± 0.04	0.931 ± 0.013	45/36
12	1.48 ± 0.14	0.32 ± 0.04	0.932 ± 0.011	58/36
≥ 14	1.78 ± 0.14	0.26 ± 0.03	0.954 ± 0.007	31/36

Table 11. Fit results for (7) to the ratio R as a function of q_T and q_0 , for selections on the charge multiplicity n . Data correspond to the case of $q_0 < 0.15$ GeV, normalized by the mixed-event reference sample.

n	r_K (fm)	τ (fm)	λ_K	γ_K	χ^2/NDF
6-10	1.22 ± 0.12	$0.54^{+0.36}_{-0.54}$	0.32 ± 0.04	0.907 ± 0.012	203/196
12-14	1.64 ± 0.14	$0.92^{+0.31}_{-0.49}$	0.34 ± 0.04	0.935 ± 0.009	200/196
≥ 16	1.87 ± 0.18	$1.22^{+0.37}_{-0.53}$	0.38 ± 0.06	0.935 ± 0.010	196/196

Table 12. Fit results for (8) to the ratio R as a function of Q^2 , for selections on $|\cos \theta|$. Data, normalized by the mixed-event reference sample, are shown in Fig.7.

cut	r_G (fm)	λ_G	γ_G	δ_C (GeV/c) $^{-2}$	χ^2/NDF
$0.7 < \cos \theta < 1.0$	0.81 ± 0.01	0.37 ± 0.03	0.959 ± 0.006	0.025 ± 0.006	86/96
$0.3 < \cos \theta < 0.7$	0.94 ± 0.03	0.30 ± 0.04	0.975 ± 0.009	-0.006 ± 0.009	86/96
$0.0 < \cos \theta < 0.3$	0.87 ± 0.03	0.35 ± 0.04	0.952 ± 0.013	0.016 ± 0.013	92/96

Table 13. Fit results for (8) to the ratio R as a function of Q^2 , for selections on $|\cos \theta|$ (see text). Data are normalized by the unlike-pair reference sample.

cut	r_G (fm)	λ_G	γ_G	δ_C (GeV/c) $^{-2}$	χ^2/NDF
$0.7 < \cos \theta < 1.0$	1.03 ± 0.03	0.28 ± 0.03	0.977 ± 0.006	0.032 ± 0.007	111/84
$0.3 < \cos \theta < 0.7$	0.85 ± 0.03	0.41 ± 0.03	0.999 ± 0.011	-0.072 ± 0.012	76/84
$0.0 < \cos \theta < 0.3$	0.79 ± 0.03	0.47 ± 0.04	1.000 ± 0.016	-0.131 ± 0.017	82/84

Table 14. Fit results for (6) to the ratio R as a function of q_T , for selections on $|\cos \theta|$. Data are normalized by the mixed-event reference sample.

cut	r_K (fm)	λ_K	γ_K	χ^2/NDF
$0.7 < \cos \theta < 1.0$	1.82 ± 0.12	0.39 ± 0.03	0.940 ± 0.008	45/37
$0.3 < \cos \theta < 0.7$	1.34 ± 0.12	0.24 ± 0.02	0.937 ± 0.010	38/37
$0.0 < \cos \theta < 0.3$	1.35 ± 0.14	0.24 ± 0.03	0.938 ± 0.011	33/37

Table 15. Fit results for (7) to the ratio R as a function of q_T and q_0 , for selections on $|\cos \theta|$ (see text). Data correspond to the case of $q_0 < 0.15$ GeV, normalized by the mixed-event reference sample.

cut	r_K (fm)	τ (fm)	λ_K	γ_K	χ^2/NDF
$0.7 < \cos \theta < 1.0$	1.85 ± 0.13	$0.63^{+0.32}_{-0.63}$	0.46 ± 0.06	0.927 ± 0.009	201/196
$0.3 < \cos \theta < 0.7$	1.33 ± 0.13	$0.70^{+0.33}_{-0.70}$	0.28 ± 0.04	0.919 ± 0.011	221/196
$0.0 < \cos \theta < 0.3$	1.04 ± 0.12	$1.24^{+0.33}_{-0.46}$	0.29 ± 0.04	0.896 ± 0.016	244/196

Table 16. Fit results for (8) to the ratio R as a function of Q^2 for a sample of events with a leading hadron of $|x_{lead}| > 0.5$.

Reference sample, x_{lead}	r_G (fm)	λ_G	γ_G	δ_G (GeV/c) $^{-2}$	χ^2/NDF
Mixed-event					
$x_p < -0.5$	0.64 ± 0.03	0.38 ± 0.05	0.984 ± 0.020	-0.083 ± 0.019	87/96
$x_{\pi^+} > 0.5$	0.82 ± 0.05	0.53 ± 0.08	1.011 ± 0.017	-0.122 ± 0.017	113/96
Unlike-pair					
$x_p < -0.5$	0.87 ± 0.04	0.40 ± 0.06	0.961 ± 0.018	0.024 ± 0.020	91/84
$x_{\pi^+} > 0.5$	0.88 ± 0.05	0.48 ± 0.08	0.952 ± 0.020	0.034 ± 0.023	90/84

Table 17. Fit results for (6) to the ratio R as a function of q_T , for the sample of events with a leading hadron ($|x_{lead}| > 0.5$).

Reference sample, x_{lead}	r_K (fm)	λ_K	γ_K	χ^2/NDF
Mixed-event				
$x_p < -0.5$	1.25 ± 0.14	0.39 ± 0.05	0.892 ± 0.019	40/37
$x_{\pi^+} > 0.5$	1.15 ± 0.11	0.55 ± 0.06	0.846 ± 0.021	40/37
Unlike-pair				
$x_p < -0.5$	1.59 ± 0.19	0.38 ± 0.06	0.920 ± 0.016	52/36
$x_{\pi^+} > 0.5$	1.43 ± 0.17	0.43 ± 0.06	0.900 ± 0.018	40/36

Table 18. Fit results for (6) to the ratio $R(q_T)$ and for (7) to the ratio $R(q_T, q_0)$ at different α .

α $(p_p^*/p_{\pi^+}^*)$	Parametrization (6)		Parametrization (7)		
	r (fm)	λ	r (fm)	τ (fm)	λ
0.0	1.76 ± 0.18	0.43 ± 0.02	1.59 ± 0.16	$0.40^{+0.31}_{-0.40}$	0.44 ± 0.07
0.25	1.60 ± 0.07	0.33 ± 0.02	1.57 ± 0.07	$0.64^{+0.14}_{-0.19}$	0.36 ± 0.03
0.5	1.68 ± 0.07	0.31 ± 0.02	1.67 ± 0.08	$0.92^{+0.14}_{-0.17}$	0.38 ± 0.03
0.75	1.54 ± 0.07	0.29 ± 0.02	1.53 ± 0.07	$0.72^{+0.14}_{-0.18}$	0.33 ± 0.02
1.0	1.50 ± 0.07	0.29 ± 0.02	1.52 ± 0.07	$0.76^{+0.14}_{-0.17}$	0.33 ± 0.02
1.25	1.46 ± 0.07	0.28 ± 0.02	1.49 ± 0.07	$0.73^{+0.14}_{-0.17}$	0.32 ± 0.02
1.5	1.36 ± 0.07	0.27 ± 0.02	1.36 ± 0.07	$0.75^{+0.13}_{-0.16}$	0.31 ± 0.02
1.75	1.50 ± 0.07	0.30 ± 0.02	1.50 ± 0.07	$0.66^{+0.14}_{-0.17}$	0.34 ± 0.02
2.0	1.49 ± 0.07	0.29 ± 0.02	1.52 ± 0.07	$0.84^{+0.13}_{-0.15}$	0.34 ± 0.02
2.5	1.35 ± 0.07	0.27 ± 0.02	1.35 ± 0.07	$0.79^{+0.13}_{-0.16}$	0.31 ± 0.02
3.0	1.37 ± 0.07	0.26 ± 0.02	1.39 ± 0.07	$0.64^{+0.14}_{-0.20}$	0.28 ± 0.02
4.0	1.35 ± 0.07	0.26 ± 0.02	1.38 ± 0.07	$0.74^{+0.13}_{-0.18}$	0.31 ± 0.02
5.0	1.51 ± 0.08	0.25 ± 0.02	1.63 ± 0.09	$1.10^{+0.16}_{-0.20}$	0.34 ± 0.03
10.0	1.33 ± 0.08	0.25 ± 0.02	1.33 ± 0.08	$0.49^{+0.17}_{-0.28}$	0.27 ± 0.02
100.0	1.79 ± 0.13	0.31 ± 0.03	1.78 ± 0.13	$0.56^{+0.22}_{-0.45}$	0.34 ± 0.04

Figure Captions

- Fig.1 The ratio R as a function of Q^2 , as obtained with the a) mixed-event and b) unlike-pair reference sample. The curves correspond to the best fit according to parametrization (8).
- Fig.2 The ratio R as a function of q_T , as obtained with the a) mixed-event and b) unlike-pair reference sample. The curves correspond to the best fit according to parametrization (6). The point $q_T \leq 0.025$ GeV/c in Fig.2b is excluded from the fit.
- Fig.3 The ratio R as a function of q_T and q_0 as obtained with the mixed-event reference sample. The curve corresponds to the best fit according to parametrization (7).
- Fig.4. a) The ratio R as a function of Q . The data are normalized by the mixed-event reference sample. The curve corresponds to the best fit according to parametrization (9).
b) The ratio R as a function of $|\vec{q}|$ for pairs with $q_0 < 0.05$ GeV. The data are normalized by the mixed-event reference sample. The curve corresponds to the best fit according to parametrization (10).
- Fig.5 The ratio R as a function of Q^2 with selections on the charge multiplicity n , as indicated. The data are normalized by the mixed-event reference sample. The curves correspond to the best fit according to parametrization (8).
- Fig.6 The correlation parameters as a function of the charge multiplicity n , for mixed-event (full circles) and unlike-pair (open circles) reference samples.
- Fig.7 The ratio R as a function of Q^2 with the $|\cos \theta|$ selections as indicated. The data are normalized by the mixed-event reference sample. The curves correspond to the best fit according to parametrization (8).
- Fig.8 The radius r_G (circles) and correlation strength λ_G (triangles), based on parametrization (8) as a function of $|\cos \theta|$.
- Fig.9 The radius r_K and correlation strength λ_K , based on parametrization (6), for different symmetric rapidity intervals ($-y_{max} < y < y_{max}$).
- Fig.10 The radius r_G and correlation strength λ_G , based on parametrization (8), for different symmetric rapidity intervals.
- Fig.11 The radius r_G and correlation strength λ_G , based on parametrization (8), for different rapidity intervals.
- Fig.12 The radius r_G and correlation strength λ_G , based on parametrization (8), for the sample of pions with transverse momentum $p_T > p_T^{min}$.
- Fig.13 The radius r_K and correlation strength λ_K , based on parametrization (6), for pion pairs in four momentum intervals $|\vec{p}| = |\vec{p}_1 + \vec{p}_2|$ in the overall cms. The lines correspond to parametrization (12) in the text.
- Fig.14 The radius r_G and correlation strength λ_G , based on parametrization (8), as a function of $(p_T^{trig})_{min}$ (see text).
- Fig.15 The radius r_K and correlation strength λ_K , based on parametrization (6), as a function of $(p_T^{trig})_{min}$ (see text).

- Fig.16 The radius r_K (full circles) based on parametrization (6), as a function of α . The results of [46] for r (empty triangles) are also shown.
- Fig.17 The radius r_K (full circles), the life-time or the emission depth τ (empty circles) and strength λ (triangles), based on parametrization (7), as a function of α .
- Fig.18 The size r_K and life-time (emission depth) τ in the three directions θ indicated, based on parametrization (7), as a function of α .

Fig. 1

Fig. 2

Fig. 3

Fig.4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig.9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Fig. 18