On a class of finite sigma-models and string vacua: a supersymmetric extension

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Abstract

Following a suggestion made by Tseytlin, we investigate the case when one replaces the transverse part of the bosonic action by an $n = 2$ supersymmetric sigma-model with a symmetric homogeneous Kählerian target space. As conjectured by Tseytlin, the metric is shown to be exactly known since the beta function is known to reduce to its one-loop value.

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It is currently of the greatest interest to analyze the most possible models leading to classical string vacua, in particular those with exact solutions. This, with the hope that some of them will provide interesting enough vacua such that they may have some possible physical interpretations.

Such an example has been discovered within some gauged $WZNW$ coset models [1] and exhibits blackhole-type solutions in two dimensional target space.

Recently a new class of models has been introduced by Tseytlin [2]. These are $\sigma$-models with Minkowskian signature and the key idea was to consider such models which have symmetric target space metric with covariantly constant null Killing vector and are therefore finite.

In this note we plan to investigate a suggestion made by Tseytlin himself [2] and concerning the case when the so-called "transverse" part\(^1\) of the model possesses an $n = 2$ supersymmetry with an homogeneous Kähler target space.

Ref.[2] considers the line element:

\[
ds^2 = G_{\mu\nu} dx^\mu dx^\nu = -2dudv + f(u)\gamma_{ij}(x)dx^i dx^j
\]

with $\mu, \nu = 0, 1, ..., N, N+1$; $i, j = 1, ..., N$

and the $\gamma_{ij}$ corresponding to a symmetric space (constant curvature). It is then shown that, with a specific choice of $f(u)$, the $\sigma$-model with (1) as target space metric is ultra-violet finite. The part of (1) proportional to $f(u)$ is referred to as the "transverse" part and $u$ and $v$ are light cone coordinates. Furthermore, only the case of a curvature of the form

\[
R_{ijkl} = K/(N-1)(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}); K \equiv R/N,
\]

qualified of maximally symmetric, is considered in order to make easier the perturbative expansions of the various quantities necessary to the interpretation of the model.

We shall not discuss any further Tseytlin's model except for the parts relevant to our purpose. The finiteness of the model on a flat 2d background needs the condition

\[
\beta^{G}_{\mu\nu} + D(\mu M_{\nu}) = 0
\]

with $\beta^{G}_{\mu\nu}$ concerning the full $\sigma$-model with target space metric (1) beta function and $M_{\nu}$ a vector to be determined for each separate case in order to satisfy (3).

The analysis of ref.[2] leads, among others, to the following basic results:

\[
\beta^{G}_{ij} = \beta(f)\gamma_{ij}, \quad \text{with}
\]

\[
\beta(f) = a + (N - 1)^{-1}a^2f^{-1} + \frac{N+3}{4}(N-1)^{-2}a^3f^{-2} + 0(a^4f^{-3})
\]

\[
a \equiv \alpha'K.
\]

\(^1\) All along this note, we shall refer to the terminology, to the notations and results, as published in [2]. In particular, formulas (4), (5), (6) and (7) are the formulas (11), (12), (19) and (20) of ref.[2], respectively.

\(^\S\) For renormalization techniques in non flat spaces, see for instance references [4]
$f^{-1}$ plays the role of a coupling of the symmetric space $\sigma$-model and satisfies

\begin{equation}
\frac{df}{d\tau} = \beta(f),
\end{equation}

with $\tau$ a kind of RG "time" parameter defined just after (18, ref.[2]). Of course one has

\begin{equation}
f(u) = a (\tau + (N - 1)^{-1} \log \tau + O(\tau^{-1})) ; \tau = \tau(u)
\end{equation}

when $\beta(f)$ is given by (5) above.

Now we come to the transverse part, which we recall is $N$-dimensional in the target space.

One knows from previous studies that if this transverse part is Kählerian, it enjoys an $n = 2$ supersymmetry. Moreover we know from a particular example [3] that its beta function is exactly given by its first term (1-loop). The example we refer to requires in addition that the transverse space not only be Kählerian but also homogeneous. Appropriate Kähler manifolds of this type can be found in the literature (ref.[3] and ref. therein).

Suppose our choice of space is such a manifold, in this case (5) and (7) reduce to

\begin{equation}
\beta(f) = \text{Const} \equiv C
\end{equation}

\begin{equation}
f(u) = C.\tau(u).
\end{equation}

$\tau$ is well defined in each case in terms of $u$, as said before and therefore the metric (1) is given exactly by

\begin{equation}
ds^2 = -2du dv + C.\tau \gamma_{ij}(x) dx^i dx^j
\end{equation}

We conclude that the results (8)–(10) confirm, in the specific case considered, the conjecture made by Tseytlin in the footnote 3 of ref.[2]. At this point we have not yet identified any string vacua. However, depending on whether the vector $M_\nu$ in (3) is an exact gradient, the appropriate dilaton field can be exactly determined in order to satisfy Weyl invariance of the model which then leads to exact string tree-level vacua represented by the resulting backgrounds.

**REFERENCES**

E. Witten, Phys. Rev. D44 (1991) 314; 

