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LEPTON FLAVOUR VIOLATION INDUCED BY HEAVY MAJORANA NEUTRINOS

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ABSTRACT

We investigate the reaction $e^-e^- \rightarrow W^-W^-$ in the centre-of-mass energy regime far above threshold for W -pair production. We assume lepton flavour violation to be mediated by a Majorana mass matrix for six neutrino flavours as is natural in $SO(10)$ assignments. We show that the cross-section compatible with constraints from low energy may well be detectable at $\sqrt{s} = 0.5$ TeV under conservative assumptions on neutrino mixing angles and masses. The dependence on centre-of-mass energy $\propto (\sqrt{s})^4$ may thus lead to sizeable rates at 1-2 TeV.

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1. Introduction

The prospect of the emergence of linear colliders in the energy range $\sqrt{s} \gg m_W$ makes the evaluation of certain extensions of the Standard Model which might lead to outstanding experimental signatures at these machines, reasonable. In particular, neutrino mass phenomena may find unique manifestations at such lepton colliders.

In this note, we wish to further this aim by studying the potential use of linear colliders in the NLC energy range as electron-electron machines. In addition to their rôle as tools for the investigation of novel hard-scattering interactions that may escape detection in the presence of the annihilation graphs dominating e^+e^- interactions, they may serve for the exploration of massive Majorana neutrino-mediated processes, such as (see Fig. 1) [1],[2]

$$e^-e^- \rightarrow W^-W^- . \quad (1.1)$$

This interaction can lead to such signature final states as

$$\begin{aligned} e^-e^- &\rightarrow e^- \mu^- + \text{missing momentum} , \\ &\rightarrow \mu^- \tau^- + \text{missing momentum} , \\ &\rightarrow e^- \tau^- + \text{missing momentum} , \\ &\rightarrow \text{lepton} + \text{jet} + \text{missing momentum} . \end{aligned} \quad (1.2)$$

It is likely that these two-prong (or one-prong-plus jet) final states at large momentum transfers can be singled out with high resolution by appropriate detector configurations. We therefore wish to investigate the question whether an NLC (TLC) operating at a luminosity of some $10^{33} \text{ (cm}^{-2}\text{sec}^{-1})$ and at an energy $\sqrt{s} = 0.5(1) \text{ TeV}$ has a chance to lead to detectable event rates for reasonable ranges of Majorana neutrino masses.

Fig. 1. The doubly weak graph $e^-e^- \rightarrow l^-l^- + \text{neutrinos}$ mediated by heavy Majorana neutrino exchange.

We note that the process inverse to Eq. (1.1),

$$(W^-W^-)^* \rightarrow e^-e^- \quad (1.3)$$

mediates neutrinoless double beta decay. Hence, time reversal will connect the two inverse reactions that are capable of testifying to the Majorana nature of massive neutrinos and to the action of their mixing matrices. The formalism we develop below will show that the mass suppression factors that may well put heavy neutrinos beyond practically attainable detection limits in " $\beta\beta$ decay" experiments are less severe for the $e^-e^- \rightarrow W^-W^-$ amplitudes, which contain a term strongly increasing with energy.

In a separate study, we expect to investigate QED generated backgrounds and detailed aspects of detection in a realistic laboratory environment.

At any rate, the experimental programs of an e^-e^- version of the NLC—motivated by such speculative aims as the search for doubly charged leptons[3],[4] and doubly charged Higgs and/or gauge bosons[5] in addition to their more run-of-the-mill usefulness as photon-photon and electron-photon colliders—should include a sensitive search for lepton flavor violation. It is important to remember that the Majorana-neutrino-mediated processes that are capable of producing this final-state feature are sensitive to the specific chiral couplings assumed in Figs. 1 and 2. The experimental possibilities of obtaining highly polarized electron beams for the initial state add a decisive advantage for several signal - vs. - background recognition strategies.

In the following, we start by exploring the feasibility of obtaining detectable rates in a framework where we permit mixing both of chiral gauge couplings and of neutrino masses. It will be seen that presently available limits on $\beta\beta$ decay preclude the observation of signals due to this process, which involves right-handed currents. We will then show that a general treatment of mass mixing possibilities in a purely left-handed framework points up a mechanism that leads us to amplitudes with a strong energy dependence, capable of producing measurable signals in the NLC or TLC scenarios.

2. Lepton-flavour violating amplitudes with left- and right-handed W couplings

We choose a particular model for a calculation of rates: we assume incoming electrons of opposite helicity, such that the produced W^- also have opposite, i.e., non-standard, chiral couplings. The couplings involve two distinct vertices, $(\nu_e W_{LeL})$ and $(N_R W_{ReR})$.

Fig. 2. Graphs for the amplitude $A((e_L^-, p_1; e_R^-, p_2) \rightarrow W^-(\epsilon_1, p_3) W^-(\epsilon_2, p_4)) = A_{LR}(\epsilon_1 \epsilon_2 P)$.

We also assume $g_{WL} = g_{WR}$, as suggested by minimal extensions of the Standard Model. In the symmetry limit, they equal g_W , and in the $SU_2 \times U_1$ limit,

$$g_W^2/2 = 2\sqrt{2}G_F m_W^2.$$

This implies the assumption of a unifying gauge group which enforces a common coupling in the symmetry limit. The transformation from weak interaction eigenstates

$$\nu_i = \begin{cases} \nu_i = e, \mu, \tau; & i = 1 \dots 3 \\ N_i = E, M, T; & i = 4 \dots 6 \end{cases} \quad (i = 1 \dots 6)$$

to the mass eigenstates

$$\hat{\nu}_\alpha = \begin{cases} \alpha = 1, 2, 3 & \text{light } \nu \\ \alpha = 4, 5, 6 & \text{heavy } N \end{cases} \quad (\alpha = 1 \dots 6)$$

is effected by the unitary 6×6 matrix U , described in detail in Appendix 1,

$$\nu_i = \bar{U}_{i\beta} \hat{\nu}_\beta \quad \text{and} \quad \hat{\nu}_\alpha = U_{j\alpha} \nu_j \quad \begin{matrix} (i, j = 1 \dots 6) \\ (\alpha, \beta = 1 \dots 6) \end{matrix} \quad (2.1)$$

The matrix $U_{i\alpha}$ is obtained from the mass matrix m_{ij} in the quadratic Lagrangian

$$\begin{aligned} \mathcal{L}_M &= \frac{1}{2} \left\{ (\nu_i)_\delta \mathcal{M}_{ij} (\nu_j)^\delta + h.c. \right\} \\ &= \left\{ (N_I)_\delta \mu_{IJ} (\nu_j)^\delta + h.c. + \right. \\ &\quad \left. + \frac{1}{2} \left[(N_I)_\delta M_{IJ} (N_J)^\delta + h.c. \right] \right\} \quad (2.2) \\ (\nu_i)_\delta &= (\epsilon)_\delta \hat{\nu}_i (\nu_i)^\dagger, \quad \delta, \hat{\gamma} = 1, 2 \\ \epsilon &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{aligned}$$

by way of the substitution

$$\mathcal{M} = U m_D U^T,$$

so that

$$\begin{aligned} U_{i\alpha} (m_D)_\alpha U_{j\alpha} &= \mathcal{M}_{ij} \\ (m_D)_{\alpha\beta} &= \delta_{\alpha\beta} (m_D)_\alpha \end{aligned} \quad (2.3)$$

and

$$(m_D)_\alpha = m_\alpha \geq 0, \quad \alpha = 1 \dots 6.$$

m_D is the diagonal mass matrix. The major term in the Majorana mass matrix is understood to be M , the eigenvalues of which are expected to be in the TeV range:

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}. \quad (2.4)$$

Then the amplitudes A_{LR} involve the following combinations of mixing matrix elements:

$$\eta_\alpha = U_{e\alpha} \bar{U}_{E\alpha}, \quad \text{with} \quad \sum_{\alpha=1 \dots 6} \eta_\alpha = 0. \quad (2.5)$$

Here, the indices e, E refer to left-handed and right-handed electron neutrinos, respectively. The left- and right-handed gauge bosons $W_{L,R}$ are supposed to mix so as to

generate the physical (light) W boson plus a heavy, mainly right-handed boson W' :

$$\begin{aligned} W_{\mu L} &= \cos \phi W_{\mu} - \sin \phi W'_{\mu}, \\ W_{\mu R} &= \sin \phi W_{\mu} + \cos \phi W'_{\mu}. \end{aligned} \quad (2.6)$$

We now consider the production of two W bosons with this mixing angle ($m_W = 80$ GeV) as in Fig. 2, restricted to opposite chirality leptonic couplings:

$$A_{LR}(\varepsilon_1, \varepsilon_2, P) = \frac{1}{2} g_W^2 (\sin \phi \cos \phi) A^{red}.$$

Here

$$A^{red} = F(t) \sum \eta_{\alpha} \frac{1}{f_{\alpha} - z} + F(u) \sum \eta_{\alpha} \frac{1}{f_{\alpha} + z}, \quad (2.7)$$

with $z = \cos \theta$.

For kinematical definitions, see Appendix 2. Here, the factors f_{α} tell light from heavy neutrinos

$$\begin{aligned} f_{\alpha} &\sim 1 + \frac{m_W^4}{2Eq(E+q)^2} \quad (\text{for } \alpha = 1 \dots 3 \text{ light neutrinos}) ; \\ f_{\alpha} &\sim 1 + \binom{0(1)}{\gg 1} \quad (\text{for } \alpha = 4 \dots 6 \text{ heavy neutrinos}). \end{aligned}$$

In the above estimate of f_{α} for $\alpha = 4, 5, 6$, the symbol $0(1)$ refers to $m_N = 0(\sqrt{s})$; $\gg 1$ refers to $(m_N \gg \sqrt{s})$. The A^{red} are distinguished by the spin alignment of the outgoing W bosons. Specifically, we define four amplitudes:

$$\begin{aligned} \text{for } \varepsilon_1 = \varepsilon_{1tr}, \quad \varepsilon_2 = \varepsilon_{2tr} \quad &\rightarrow A^{red} = A_{tr, tr}, \\ \text{for } \varepsilon_1 = \varepsilon_{1tr}, \quad \varepsilon_2 = \varepsilon_{2long} \quad &\rightarrow A^{red} = A_{tr, long}, \\ \text{for } \varepsilon_1 = \varepsilon_{1long}, \quad \varepsilon_2 = \varepsilon_{2tr} \quad &\rightarrow A^{red} = A_{long, tr}, \\ \text{for } \varepsilon_1 = \varepsilon_{1long}, \quad \varepsilon_2 = \varepsilon_{2long} \quad &\rightarrow A^{red} = A_{long, long}. \end{aligned} \quad (2.8)$$

As functions of the angle between incoming electrons and produced W , and after

Bose symmetrization, they take the form

$$\begin{aligned} A_{tr, tr}^{red} &= \sum_{\alpha} \left\{ a_{\alpha}^{\sigma_1 \sigma_2} \sin \theta \frac{1}{f_{\alpha} - z} + b_{\alpha}^{\sigma_1 \sigma_2} \sin \theta \frac{1}{f_{\alpha} + z} \right\}; \\ A_{long, tr}^{red} &= \frac{E}{\sqrt{2} m_W} \sum_{\alpha} \eta_{\alpha} \left\{ C_{\alpha} + \left(\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right) S_{\alpha} + \right. \\ &\quad \left. + \left(\frac{1}{f_{\alpha} - z} - \frac{1}{f_{\alpha} + z} \right) D_{\alpha} \right\}; \\ \begin{cases} C_{\alpha} &= 4 \frac{E}{q} (f_{\alpha} - 1); \\ S_{\alpha} &= -\frac{E}{q} (f_{\alpha}^2 - 1) + \sigma_2 \left[f_{\alpha}^2 - \left(\frac{E}{q} + \frac{E}{2} \right) f_{\alpha} + 1 \right]; \\ D_{\alpha} &= \frac{E}{q} \left(f_{\alpha}^2 - \frac{q^2}{E^2} \right) - \sigma_2 (f_{\alpha}^2 - 1), \end{cases} \\ A_{tr, long}^{red} &= A_{long, tr}^{red}(-z); \\ A_{long, long}^{red} &= \frac{E^2}{m_W^2} \sum \eta_{\alpha} \left(\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right) \left(1 + \frac{q^2}{E^2} - 2 \frac{q}{E} f_{\alpha} \right). \end{aligned} \quad (2.9)$$

Using Eq. (2.7), we then rewrite the latter amplitude

$$A_{LR}(long, long) = \frac{GF}{\sqrt{2}} sV \sum_{\alpha} \eta_{\alpha} \left(\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right) \left(1 + \frac{q^2}{E^2} - 2 \frac{q}{E} f_{\alpha} \right),$$

where we have used $V = \sin \phi \cos \phi$.

For all kinematical details, see again Appendix 2.

These amplitudes show that the contributions from the light neutrinos ($\alpha = a = 1, 2, 3$) are suppressed, for high energies ($\sqrt{s} \gg m_W$), since $f_{\alpha} \rightarrow 1$. For $\sqrt{s} \gg m_W$, the above-mentioned estimate applies:

$$f_{\alpha} - 1 = \binom{0(1)}{\gg 1};$$

this excludes ultimately high energies, where the lowest-order electroweak approximation for these amplitudes will fail. $A_{long, long}$ will then be dominant, and will occur via the case involving heavy neutrinos ($\alpha = 4, 5, 6$; $A = \alpha - 3 = 1, 2, 3$).

Let us now restrict our cross-section estimates to the case where $\sqrt{s} \ll m_N$ (N_A heavy neutrinos). This gives the approximate leading amplitude for $A_{long, long}$.

$$A_{long, long} \simeq -2\sqrt{2} G_F s (V_{\eta H}). \quad (2.10)$$

Recall that $V = \sin \phi \cos \phi$, $\eta_H = \sum_{\alpha=4 \dots 6} \eta_{\alpha}$. Here, $\sqrt{s}$ is the centre-of-mass energy of the two produced W^- .

Then, for initial electrons e_L^- , e_R^- ,

$$\begin{aligned}\sigma_{long, long.}(e_L^-, e_R^-) &\simeq \frac{1}{2} \frac{4\pi}{64\pi^2 s} |A_{long, long.}|^2, \\ &= \sigma_0(s) |V\eta_H|^2.\end{aligned}\quad (2.11)$$

In the above,

$$\begin{aligned}\sigma_0(s) &= \frac{G_F^2 s}{4\pi} = \sigma^* \left(\frac{\sqrt{s}}{500 \text{ GeV}} \right)^2, \\ \sigma^* &= 1.059 \text{ nb}.\end{aligned}\quad (2.12)$$

$\sigma_{long, long.}$ in eq. (2.11) is dominated by contributions from heavy neutrinos which, when expanded in powers of $m_{N_A}^2/s \ll 1$ contain only a finite number of partial waves n_{eff}^{LR} ; unitarity then constrains the range of applicability of the perturbative expansion involving, to lowest order, the Feynman diagrams shown in Fig. 2. This imposes the constraint

$$\begin{aligned}\frac{G_F^2 s}{4\pi} |V\eta_H|^2 &< \frac{16\pi}{s} n_{eff}^{LR} \\ &\rightarrow \sqrt{s} < (8\pi \sqrt{n_{eff}^{LR}} |V\eta_H|)^{1/2} \frac{1}{(G_F)^{1/2}}; \\ G_F^{-1/2} &= 292.8 \text{ GeV}.\end{aligned}$$

The cross-section for $(e_{1R}, p_1; e_{2L}, p_2)$ is the same as for $(e_{1L}, p_1; e_{2R}, p_2)$.

For unpolarized beams, we have

$$\underbrace{\sigma_{long, long.}(e_1, e_2)}_{\text{unpolarized}} = \frac{1}{2} \sigma_{long, long.}(e_L^- e_R^-), \quad (2.13)$$

but

$$\begin{aligned}\sigma_{long, long.}(e_{1L}, e_{2L}) &= 0, \\ \sigma_{long, long.}(e_{1R}, e_{2R}) &= 0.\end{aligned}$$

We now have to estimate the mixing parameters; explicitly,

$$V = \sin \phi \cos \phi; \quad \eta_H = \sum_{A=1\dots 3} U_{eA} \bar{U}_{EA}.$$

To estimate the mixing angle between right- and left-handed W gauge bosons, recall that, together with the neutrino mass pattern, it will lead to the breakdown of lepton universality relations.

In the quark sector, the addition of a right-handed current in a non-strange system of the first generation will yield, for non-strange weak β decays, an effective Fermi constant

$$G_\beta^{eff} = G^{SM} \cos \phi \cos \theta_c [1 + (\tan \phi / \cos \theta_c) X_{ud}^R]. \quad (2.14)$$

Here, θ_c is the Cabibbo angle, and X_{ud}^R denotes the right-handed mixing matrix element in analogy to the left-handed V_{ud} . Taking the best value for V_{ud} from present evidence (weak β decay) in the particle data book,

$$\begin{aligned}|V_{ud}| &= 0.9747 \pm 0.0011, \\ |V_{us}| &= 0.220 \pm 0.003, \\ |V_{ub}| &< 0.01,\end{aligned}\quad (2.15)$$

and assuming that there are no accidental cancellations at work, we conclude

$$|R(X_{ud}^R)| |V| \leq 0.004. \quad (2.16)$$

In an SO(10)-based gauge theory, we would expect $\text{Re} X_{ud}^R = 1$; in an extended gauge theory $G \supseteq E_6$, a suppression of the right-handed current couplings beyond the value of $|V|$ is possible.

The product $V\eta_H$ of the left-right mixing angle ($V = \sin \phi \cos \phi$) and the neutrino mass matrix parameter η_H , which determine the high-energy behaviour of $\sigma_{long, long.}$ in Eq. (2.11), is most severely restricted in modulus by the limits on neutrinoless double β decays [6], when combined with calculations of the relevant nuclear matrix elements [7]-[10].

We quote two limits: L_1 is based on the calculations of Refs. [7],[8], and $L_2 \ll L_1$ based on Ref. [9],[10], where the Tellurium-Xenon complex is taken into account:

$$\begin{aligned}L_1 : |V\eta_H| &\leq (2.1)10^{-7}, \quad [7][8] \\ L_2 : |V\eta_H| &\leq (1.8 \div 2.1)10^{-8}. \quad [9][10]\end{aligned}\quad (2.17)$$

From the limit L_1 in Eq. (2.16), we infer the bound

$$\sigma_{long, long.}(e_L^-, e_R^-) \leq (4.7)10^{-8} \text{ fb} \left(\frac{\sqrt{s}}{500 \text{ GeV}} \right)^2 \quad (2.18)$$

for $|\eta V| \leq (2.1)10^{-7}$ (and $\sqrt{s} \ll m_N$).

The bound in Eq. (2.18) eliminates the possibility to observe effects due to the lepton number violating amplitude A_{LR} at any foreseeable e^+e^- facility operating at $\sqrt{s} = 500 \text{ GeV}$ and considerably beyond.

3. Lepton-flavour violating amplitudes with left-handed W couplings only: generic and non-generic estimation of mixing parameters

We now turn our attention to lepton-flavour violating amplitudes induced specifically through the mass matrix $\mathcal{M}$ given in Eq. (2.3); we limit our considerations to left-handed W couplings of the three weak lepton doublets ν_i, ℓ_i :

$$A_i = \frac{g_W}{\sqrt{2}} W_\mu^+ (\nu_i)^* \gamma^\mu \frac{1 + \gamma_5}{2} \ell_i^- \quad (i = 1 \dots 3). \quad (3.1)$$

Here, the ℓ_i denote the mass eigenstates of the charged lepton e^-, μ^-, τ^- (for $i = 1, 2, 3$, respectively). If we now neglect, as we did in Section 2 above, the electron mass, we need to consider only negative electron helicities; the graphs corresponding to Fig. 2 are then shown in Fig. 3:

Fig. 3. Graphs for the amplitude $B[(e_L^-, p_1; e_L^-, p_2) \rightarrow W^-(e_1, p_3) W^-(e_2, p_4)] = B_{LL}(e_1, e_2, P)$.

Again, in parallel to the A_{LR} amplitudes of Section 2, we then write a new amplitude

$$B_{LL}(e_1, e_2, P) = \frac{g_W^2}{2} B^{red}. \quad (3.2)$$

with

$$B^{red}. = (\epsilon_1^* \epsilon_2^*) [G(t) + G(u)] + \{(\epsilon_1^*)^0 (\epsilon_2^*)^3 - (\epsilon_1^*)^3 (\epsilon_2^*)^0\} [G(t) - G(u)].$$

We maintained the kinematic notations from Appendix 2; the helicity decomposition of the functions $G(t), G(u)$ is detailed in Appendix 3. Note that the B_{LL}^{red} amplitudes are again denoted by the helicities of the outgoing W^- , just as in Eq. (2.8) above. For details, see Appendix 3.

Explicitly, Eqs. (A3.5)-(A3.7) then yield for B_{LL}^{red} :

$$\begin{aligned} B_{tr, tr}^{red}. &= B_{\sigma_1 \sigma_2} \quad \text{for } \sigma_1 = \sigma_2 = \pm \quad B_{+-} = B_{-+} = 0 \\ B_{++(-)} &= \frac{1}{q} \Sigma_\alpha \xi_\alpha (1 \mp f_\alpha) \left(\frac{1}{f_\alpha - z} + \frac{1}{f_\alpha + z} \right) \\ B_{tr, tr}^{red}. &= B_{\sigma_2} = B_\pm \\ B_{tr, long}^{red}. &= B'_\sigma \quad \text{with } B'_\sigma(z) = B_\sigma(-z) \\ B_\pm &= \frac{1}{q} \frac{E \mp q}{\sqrt{2} m_W} \sqrt{1 - z^2} \Sigma_\alpha \xi_\alpha \left(\frac{1}{f_\alpha - z} - \frac{1}{f_\alpha + z} \right) \\ B_{long, long}^{red}. &= B_t \\ &= -\frac{1}{q} \Sigma_\alpha \xi_\alpha \frac{m_W^2}{2} \left(\frac{1}{f_\alpha - z} + \frac{1}{f_\alpha + z} \right) \end{aligned} \quad (3.3)$$

In these equations, ξ_α is the neutrino mass mixing parameter

$$\xi_\alpha = (\mathcal{U}_{e\alpha})^2 m_\alpha, \quad (3.4)$$

with the condition

$$\sum_{\alpha=1}^6 \xi_\alpha = 0.$$

We can now write down the cross-sections for the process $e_L^- e_L^- \rightarrow W^-(e_1) W^-(e_2)$; e_1, e_2 are the final-state helicities. Averaging over the centre-of-mass scattering angle, we obtain

$$\sigma_{++(-)}^* = \frac{1}{2} \int_{-1}^{+1} dz |q B_{++(-)}|^2, \quad \text{etc.} \quad (3.5)$$

Expressing the squared B amplitudes in terms of neutrino mass mixing parameters

$$W_{\alpha\beta} \equiv R e \xi_\alpha \xi_\beta^*; \quad \sum_\alpha W_{\alpha\beta} = 0 \quad (3.6)$$

and the angular integrals in terms of

$$I(J)_{\alpha\beta} = \int_{-1}^{+1} dz \frac{1}{f_\alpha - z} \frac{1}{f_\beta \mp z}, \quad (3.7)$$

the cross-sections turn out to be of the form

$$\begin{aligned}
\sigma_{++(-)}^* &= \sum_{\alpha\beta} W_{\alpha\beta} (f_\alpha \mp 1) (f_\beta \mp 1) \{I_{\alpha\beta} + J_{\alpha\beta}\}, \\
\sigma_{00}^* &= \sum_{\alpha\beta} W_{\alpha\beta} \frac{m_\alpha^2}{m_W^2} \frac{m_\beta^2}{m_W^2} \{I_{\alpha\beta} + J_{\alpha\beta}\}, \\
\sigma_{+0(-)}^* &= \frac{(E \mp q)^2}{2m_W^2} \sum_{\alpha\beta} W_{\alpha\beta} ((1 - f_\alpha f_\beta) I_{\alpha\beta} - (f_\alpha - 1)(f_\beta - 1) J_{\alpha\beta}).
\end{aligned} \tag{3.8}$$

In these expressions, the integrals I, J (see Appendix 4) are elementary functions of kinematic quantities, e.g.,

$$f_\alpha = 1 + \frac{m_\alpha^2}{2Eq} + \frac{m_W^2}{2Eq} \frac{m_\alpha^2}{(E+q)^2}. \tag{3.9}$$

As in Section 2 above, we restrict our considerations to the kinematic domain

$$2m_W \ll \sqrt{s} \ll (m_N)_A.$$

The leading terms for the quantities $\sigma_{\lambda_1\lambda_2}^*$ in Eq. (3.8), when expanded in the parameter $E_A \equiv \frac{s}{2(m_N)_A}$, are determined in Appendix 4: Eqs. (A4.16) and (A4.19) give both these and the sub-leading terms (of order $E_A E_B$). The leading terms are the first terms on the right-hand side of Eq. (A4.3)

$$\begin{aligned}
\sigma_{--}^* &\rightarrow \frac{s^2}{m_W^4} 2|\xi^{(\nu)}|^2, \\
\sigma_{00}^* &\rightarrow \frac{s^2}{m_W^4} \{|\xi^{(\nu)}|^2 + |\sum_A E_A \xi_A|^2\}.
\end{aligned} \tag{3.10}$$

Recalling the definition of the ξ parameters in Eq. (3.4), we now have

$$\begin{aligned}
\xi_A &= (U_{eA})^2 (m_N)_A & A = 1 \dots 3, & \alpha = 4 \dots 6 \\
\xi^{(\nu)} &= \Sigma (U_{e\alpha})^2 (m_\nu)_\alpha & \alpha = 1 \dots 3.
\end{aligned} \tag{3.11}$$

Note that Eq. (3.5) contains only the leading helicity projections: the $\sigma_{--}^*, \sigma_{00}^*$ terms both entail the production of a W -pair with equal left-circular or longitudinal polarization, respectively. From these, we can now write down the full helicity-projected

cross-sections as induced by standard left-handed lepton- W couplings:

$$\begin{aligned}
\sigma_{\lambda_1\lambda_2}^{LL} &\equiv \frac{1}{4} \sigma(e_L^- e_L^- \rightarrow W_{\lambda_1}^- W_{\lambda_2}^-) \\
&= \frac{G_F^2}{4\pi} \frac{m_W^4}{s(s-4m_W^2)} \sigma_{\lambda_1\lambda_2}^* \\
&\approx \frac{G_F^2}{4\pi} \frac{m_W^4}{s^2} \sigma_{\lambda_1\lambda_2}^*.
\end{aligned} \tag{3.12}$$

Here, we have averaged over the incoming electron helicities, hence the factor $\frac{1}{4}$; the B_{LL} amplitude will contribute only to this helicity combination.

Combining Eqs. (3.10)-(3.12), we thus find:

$$\begin{aligned}
\sigma_{--}^{LL} &\rightarrow \frac{G_F^2}{4\pi} 2|\xi^{(\nu)}|^2, \\
\sigma_{00}^{LL} &\rightarrow \frac{G_F^2}{4\pi} (|\xi^{(\nu)}|^2 + |\xi^{(N)}|^2).
\end{aligned} \tag{3.13}$$

In this equation, $\xi^{(\nu)}$ is given by Eq. (3.6), and

$$\begin{aligned}
\xi^{(N)} &= \sum_A E_A \cdot \xi_A \\
&= \frac{s}{2} \sum_A U_{eA}^2 \frac{1}{(m_N)_A}
\end{aligned} \tag{3.14}$$

This latter expression turns out to be of pivotal importance, because it introduces an explicit s -dependence into the cross-section. Summing over final-state helicities, we obtain two characteristic contributions, one due to light neutrino (ν) exchanges, the other to that of heavy (N) neutrinos:

$$\begin{aligned}
\sigma^{LL} &= \sum_{\lambda_1\lambda_2} \sigma_{\lambda_1\lambda_2}^{LL} = \frac{G_F^2}{4\pi} \left\{ 3|\xi^{(\nu)}|^2 + |\xi^{(N)}|^2 \right\} \\
&= \sigma^{(\nu)} + \sigma^{(N)}
\end{aligned} \tag{3.15}$$

We will discuss these two terms separately.

The light-neutrino-exchange term, $\sigma^{(\nu)}$, is severely restricted from existing limits on neutrinoless double β decay [6]-[11], just as the LR amplitudes discussed in the

previous section [Eq. (2.18)]:

$$\begin{aligned}\sigma^{(\nu)} &= \frac{3}{4\pi} G_F^2 |\xi^{(\nu)}|^2 \\ &= \sigma_{1e\nu} \left| \frac{\xi^{(\nu)}}{1\text{eV}} \right|^2,\end{aligned}\quad (3.16)$$

with $\sigma_{1e\nu} \equiv \frac{3}{4\pi} G_F^2 (1\text{eV})^2 = 1.26 \times 10^{-17} \text{fb}$. Note that this minuscule cross-section corresponds to the case involving the three light neutrinos just as in neutrinoless 2β decay, and that $|\xi^{(\nu)}|^2$ corresponds to the bounds quoted from the literature for $|V\eta_H|$:

$$|\xi^{(\nu)}| \leq \left\{ \begin{array}{l} 1.6 \\ 0.5 \end{array} \frac{\text{eV}}{\text{eV}} \right\}, \quad (3.17)$$

and leaves us with an estimate

$$\sigma^{(\nu)} \leq 2 \times 10^{-17} \text{fb}, \quad (3.18)$$

independent of $\sqrt{s}$ in the kinematical region considered. There is, obviously, no experimental scheme thinkable to look for this effect at the predicted level. We therefore turn to the

“SUBLEADING” CROSS – SECTION $\sigma^{(N)}$

$\sigma^{(N)}$ is exclusively mediated by heavy Majorana neutrino exchange, and involves only the term $|\xi^{(N)}|^2$. From Eqs. (3.10) and (3.14), we conclude

$$\xi^{(N)} = \sum_A (U_{eA})^2 (m_N)_A E_A. \quad (3.19)$$

Then, Eq. (3.10) together with the condition $\sum_{\alpha=1}^6 \xi_\alpha = 0$ gives us, together with the definition of our kinematic regime,

$$E_A = \frac{s}{2(m_N^2)_A} \ll 1,$$

the possibility to define as a “generic” estimate the validity of the order-of-magnitude relation

$$|(U_{eA})^2 (m_N)_A| \approx \left| \sum_B (U_{eB})^2 (m_N)_B \right| = |\xi^{(\nu)}| \leq 1.6 \text{ eV} \quad (3.20)$$

If we now apply this generic criterium to $\sigma^{(N)}$ as defined in Eqs. (3.15) and (3.17), it follows that

$$\sigma^{(N)} \ll \sigma^{(\nu)}. \quad (3.21)$$

Given the small values for $\sigma^{(\nu)}$, we need not further pursue this generic case.

Instead, we can choose the limiting case of vanishing light neutrino masses

$$(m_\nu)_\alpha \equiv 0 \quad (\alpha = 1, 2, 3)$$

to define a minimal mass-generating scenario, where the mixing matrix between light and heavy neutrinos

$$U_{eA} \neq 0 \quad (A = 1, 2, 3; \quad \alpha = 4, 5, 6) \quad (3.22)$$

is non-negligible. In this case, which we will call the special non – generic scenario, the estimates for the cross-section turns out to be quite different from the one proffered above.

For the heavy neutrinos, assumed to have masses in the 1 TeV range, let us introduce the reduced mass

$$\frac{1}{m_{red}} = \sum_A \frac{1}{(m_N)_A}. \quad (3.23)$$

Using the dimensionless quantities

$$\begin{aligned}\eta^{(N)} &= \sum_A (U_{eA})^2 \frac{m_{red}}{(m_N)_A} = \frac{2m_{red}}{s} \xi^{(N)}, \\ \eta^{(\nu)} &= \sum_A (U_{eA})^2 \frac{(m_N)_A}{m_{red}} = \frac{1}{m_{red}} \xi^{(\nu)},\end{aligned}\quad (3.24)$$

we can then formulate the order-of-magnitude relation

$$\begin{aligned}|\eta^{(N)}| \gg |\eta^{(\nu)}| &= \frac{|\xi^{(\nu)}|}{m_{red}} \\ &= 0 \left(\frac{1\text{eV}}{1\text{TeV}} \right) \\ &\simeq 10^{-12}.\end{aligned}$$

Note that $|\eta^{(N)}|$ satisfies the inequality

$$|\eta^{(N)}| \leq \sum_A |U_{eA}|^2, \quad (3.25)$$

which leads to an estimate for the cross-section involving heavy-neutrino exchange

$$\begin{aligned}\sigma^{(N)} &= \frac{G_F^2 s}{16\pi} \left(\frac{s}{m_{red}^2} \right) |\eta^{(N)}|^2 \\ &= \sigma_0(s) \frac{s}{4m_{red}^2} |\eta^{(N)}|^2,\end{aligned}$$

with $\sigma_0(s)$ as defined in Eq. (2.12). The important new feature is that $\sigma^{(N)}$ is propor-

tional to s^2 . While this is strictly true only as long as $\sqrt{s} \ll 2m_{red}$, the strong energy dependence remains approximately the same up to energies $\sqrt{s} \approx (2m_{red})$.

As in the case of the left-right amplitudes in Chapter 2, there is a constraint imposed by unitarity on the perturbative contributions, shown in Fig. 3, to be dominant for $\sigma^{(N)}$

$$\begin{aligned} \frac{G_F^2 s}{16\pi} |\eta_N^{(N)}|^2 &\leq \frac{16\pi}{s} n_{eff}^{LL} \\ &\rightarrow \sqrt{s} \leq (16\pi \frac{\sqrt{n_{eff}^{LL}}}{|\eta(N)|})^{1/2} \frac{1}{(G_F)^{1/2}} \end{aligned}$$

The subsequent expansion of the amplitude dominating $\sigma^{(N)}$, in terms of the quantities $m_{red}^2/s \ll 1$, contains only a finite number n_{eff}^{LL} of partial waves.

With this, our estimate yields

$$\sigma^{(N)} = \sigma^* \left(\frac{\sqrt{s}}{0.5TeV} \right)^4 \left(\frac{1TeV}{m_{red}} \right)^2 \left| \frac{\eta^{(N)}}{2} \right|. \quad (3.26)$$

Having chosen $m_{red} = 1$ TeV, we then obtain the cross-section

$$\sigma^{(N)} = 1fb \left(\frac{\sqrt{s}}{0.5TeV} \right)^4 \left| 10^3 \frac{\eta^{(N)}}{2} \right|^2. \quad (3.27)$$

Present limits on $\eta^{(N)}$ typically are in the range

$$|\eta^{(N)}| \leq (2-50)10^{-4},$$

which leads to a final cross-section estimate of

$$\begin{aligned} \sigma^{(N)} &\leq (0.01 - 6.25)fb \text{ at } \sqrt{s} = 0.5TeV, \\ &\leq (0.16 - 100)fb \text{ at } \sqrt{s} = 1TeV, \end{aligned}$$

for the NLC and TLC energy ranges, respectively.

4. Minimal neutrino-mass generation - Illustration of deviation from lepton-flavour universality delimiting the parameter $\eta^{(N)}$

The parameter $\eta^{(N)}$ determining the lepton-flavour violating cross-section $\sigma^{(N)}$ [Eqs. (3.26), (3.27)], if non-vanishing, implies additional lepton-flavour violating features.

In particular, deviations from lepton-flavour universality of electroweak couplings will be evident in the reaction pairs

$$\begin{aligned} \text{i)} \quad & \mu^- \rightarrow e^- \bar{\nu}_e \bar{\nu}_\mu \quad \text{versus} \quad (A, Z) \rightarrow (A, Z+1) e^- \bar{\nu}_e; \\ \text{ii)} \quad & \pi^\pm \rightarrow \mu^\pm \bar{\nu}_\mu \quad \text{versus} \quad \pi^\pm \leftrightarrow e^\pm \bar{\nu}_e \\ \text{iii)} \quad & \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \quad \text{versus} \quad \tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu \text{ and } \nu_\tau e^- \bar{\nu}_e \end{aligned} \quad (4.1)$$

Besides the above, lepton flavour violating reactions will occur

$$\begin{aligned} \text{iv)} \quad & \mu^\pm \rightarrow e^\pm \gamma, \quad \mu^\pm \rightarrow e^\pm e^+ e^-, \quad \tau^\pm \rightarrow \mu^\pm \gamma, \quad e^\pm \gamma, \text{ etc.} \\ \text{v)} \quad & \mu^-(A, Z) \rightarrow e^-(A, Z) \end{aligned} \quad (4.2)$$

We propose to discuss aspects pertaining to reactions i)-iii), adopting a specific neutrino mass matrix for which the (three) light neutrino masses exactly vanish

$$(m_\nu)_a = 0 \quad a = 1, 2, 3 \quad (4.3)$$

This case is presented in detail in Section 4 of Appendix 1.

We recall here the special structure of the $N_f \times N_f (3 \times 3)$ blocks forming the mass matrix $\mathcal{M}$ in Eq. (1.4). The minimal, special mass-generating case, m.s.c., is defined by the relation

$$\mu^T M^{-1} \mu = 0. \quad (A1.13)$$

The unitary matrix $U_{2N_f \times 2N_f}$ that transforms the diagonal form to $\mathcal{M}$ as defined in Eq. (2.3), then becomes [Eq. (A1.24)]:

$$U = \left(\frac{\delta_{ij} - 2 \sin^2 \frac{\alpha}{2} \Lambda_i \Lambda_j^*}{-\sin \alpha \tau_a^* \Lambda_j^*} \middle| \frac{\sin \alpha \Lambda_i \rho_\beta}{(V_0)_{\alpha\beta} - 2 \sin^2 \frac{\alpha}{2} \tau_a^* \rho_\beta} \right). \quad (4.4)$$

In Eq. (4.4), $(\Lambda_\alpha, \tau_\alpha)$, and ρ_α with $\alpha = 1, 2, 3$ denote (unitary) unit vectors V_0 stands for a unitary $N_f \times N_f$ transformation matrix with

$$\rho_\beta = \tau_\alpha (V_0)_{\alpha\beta}. \quad (4.5)$$

In the following, we will neglect the admixture of right-handed couplings discussed in Chapter 2; i.e., we set the angle ϕ defined in Eq. (2.6), back to 0.

We first consider muon decay, reaction i) in Eq. (4.1). Since in this decay the heavy neutrino-flavours are inactive, we have effectively, from Eq. (2.1):

$$\nu_\mu \rightarrow \bar{U}_{2b}\bar{\nu}_b \quad \nu_e \rightarrow \bar{U}_{1b}\bar{\nu}_b \quad (\nu_\tau \rightarrow \bar{U}_{3b}\bar{\nu}_b) \quad (4.6)$$

In the present discussion, we neglect radiative corrections for simplicity. The effective muon decay constant $|G_\mu|^2$ then becomes

$$|G_\mu|^2 = G_F^2 (1 - \sin^2 a |\Lambda_\mu|^2) (1 - \sin^2 a |\Lambda_e|^2),$$

whereas the nuclear β -decay constant is

$$|G_\beta|^2 = G_F^2 \cos^2 \theta_c (1 - \sin^2 a |\Lambda_e|^2).$$

We thus obtain

$$\left| \frac{G_\beta}{G_\mu} \right| \cong \cos \theta_c \left(1 + \frac{1}{2} \sin^2 a |\Lambda_\mu|^2 \right), \quad (4.7)$$

with $\cos \theta_c = V_{cd}$.

Similarly, as in Eq. (2.18), we obtain the bound

$$\sin^2 a |\Lambda_\mu|^2 \leq 8 \cdot 10^{-3} \quad (4.8)$$

from reactions i) in Eq. (4.1).

The reactions ii) in charged pion decay lead to the ratio

$$\begin{aligned} \frac{\Gamma_{\pi^+ \rightarrow \ell^+ \bar{\nu}_\ell}}{\Gamma_{\pi^+ \rightarrow \ell^+ \bar{\nu}_i}} &= 1 - \sin^2 a |\Lambda_i|^2, \quad \text{with } i = \mu \text{ or } e \\ \Gamma_{\pi^+ \rightarrow \ell^+ \bar{\nu}_i}^{st} &= \frac{1}{4\pi} G_F^2 f_\pi^2 \cos^2 \theta_c m_{\ell_i}^2 \frac{m_\pi^2 - m_{\ell_i}^2}{m_\pi} \end{aligned} \quad (4.9)$$

The ratio

$$R_{e\mu}^\pi = \frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e, \pi\gamma)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu, \pi\gamma)}$$

is then changed from its Standard Model value (Ref. [12]),

$$\begin{aligned} R^{st} &= (1.2345) 10^{-4} \pm (0.0010) \cdot 10^{-4}, \\ \text{to a new value } R_{e\mu}^\pi &= R^{st} \frac{1 - \sin^2 a |\Lambda_e|^2}{1 - \sin^2 a |\Lambda_\mu|^2}. \end{aligned} \quad (4.10)$$

The quantity $R_{e\mu}^\pi$ was recently measured, with high precision (Ref. [13]), yielding the value

$$R_{e\mu}^\pi = (1.235 \pm 0.005) 10^{-4}. \quad (4.11)$$

From Eqs. (4.10), (4.11), we then infer

$$\sin^2 a \left| |\Lambda_e|^2 - |\Lambda_\mu|^2 \right| \leq 5 \cdot 10^{-3}. \quad (4.12)$$

Finally, we turn to a comparison of τ and μ decays [reactions iii) in Eq. (4.1)].

The deviations of the leptonic widths from the quantities computed in the Standard Model are of the form

$$\begin{aligned} R_\mu^\tau &= \frac{\Gamma(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu)}{\Gamma^{st}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu)} = (1 - \sin^2 a |\Lambda_i|^2) \times (1 - \sin^2 a |\Lambda_\mu|^2)^2, \\ R_e^\tau &= \frac{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)}{\Gamma^{st}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = (1 - \sin^2 a |\Lambda_\tau|^2) \times (1 - \sin^2 a |\Lambda_e|^2)^2. \end{aligned} \quad (4.13)$$

The leptonic τ -widths were analyzed by Marciano (Ref. [14]). For a τ -mass

$$m_\tau = 1776.9 \pm 0.4 \pm 0.3 \text{ (MeV Ref. [15])},$$

the following theoretical leptonic widths were computed (Ref [16])

$$\begin{aligned} \Gamma^{st}(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) &= (4.11_{-0.04}^{+0.03}) 10^{-13} \text{ GeV}, \\ \Gamma^{st}(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_\mu) &= (4.00_{-0.04}^{+0.03}) 10^{-13} \text{ GeV}. \end{aligned} \quad (4.14)$$

We compare these values with the results of the ALEPH Collaboration (Ref. [17])

(for the results of all LEP collaborations, see Ref. [16]):

$$\begin{aligned}\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) &= (4.04 \pm 0.21) 10^{-13} \text{ GeV}, \\ \Gamma(\tau^- \rightarrow \nu_\tau \mu^- \bar{\nu}_e) &= (3.87 \pm 0.19) 10^{-13} \text{ GeV}.\end{aligned}\quad (4.15)$$

Comparing Eqs. (4.14) and (4.15), we infer

$$\begin{aligned}\sin^2 a \left| |\Lambda_\tau|^2 - |\Lambda_\mu|^2 \right| &\leq 0.05, \\ \sin^2 a \left| |\Lambda_\tau|^2 - |\Lambda_e|^2 \right| &\leq 0.05.\end{aligned}\quad (4.16)$$

The bounds in Eq. (4.16) correspond to one standard (experimental) deviation.

Let us now study the parameter $\eta^{(N)}$ from Eq. (3.24). In the m.s.c., it is given by

$$\eta^{(N)} = \sin^2 a \Lambda_e^2 \sum_A \rho_A^2 p_A, \quad (4.17)$$

where we have used

$$p_A = \frac{m_{red}}{(m_N)_A}; \quad \sum_A p_A = 1; \quad \sum_A |\rho_A|^2 = 1.$$

From Eq. (4.17), we then obtain inequality

$$|\eta^{(N)}| \leq \sin^2 a |\Lambda_e|^2. \quad (4.18)$$

The bounds in Eqs. (4.12) and (4.16) do not limit $|\eta^{(N)}|$ or $\sin^2 a |\Lambda_e|^2$ severely, and define the upper bound in (4.19). Nevertheless, we will assume the bounds

$$|\eta^{(N)}| \leq (2 - 50) 10^{-4}. \quad (4.19)$$

Finally, we turn to the decay $\mu \rightarrow e \gamma$ [Eq. (4.1), iv)]. The branching fraction of this mode can be calculated in the Standard Model, since the mass - and mixing properties of all neutrinos contribute only through their properties of ordinary $SU(2)_L$ doublets.

The corresponding expression to order α is (Refs. [18]-[20]):

$$\begin{aligned}B(\mu \rightarrow e \gamma) &= \frac{3\alpha}{32\pi} \left| \sum_\alpha U_{e\alpha}^* U_{\mu\alpha} F\left(\frac{m_\alpha^2}{m_W^2}\right) \right|^2, \\ \text{with} \\ F\left(\frac{m^2}{m_W^2}\right) &= 6 \int_0^1 d\tau (1-\tau)^2 \frac{\tau m^2}{\tau m^2 + (1-\tau)m_W^2}.\end{aligned}\quad (4.20)$$

The function $F\left(\frac{m^2}{m_W^2}\right)$ turns out to have characteristic values

$$F \rightarrow \begin{cases} \frac{m^2}{m_W^2} & \text{for } m \ll m_W, \\ \frac{1}{2} & \text{for } m = m_W, \\ 2 & \text{for } m \gg m_W. \end{cases} \quad (4.21)$$

If we neglect the contribution proportional to $\left((m_\nu)_d/m_W\right)^2$, the branching fraction is then dominated by heavy neutrinos:

$$B(\mu \rightarrow e \gamma) \simeq \frac{3\alpha}{8\pi} \left| \sum_{A=1}^3 U_{eA}^* U_{\mu A} \right|^2 = \frac{3\alpha}{8\pi} Y^2. \quad (4.22)$$

From Eq. (4.22), we obtain the bounds

$$Y \leq \begin{cases} (2.4) 10^{-4} & \text{for } B(\mu \rightarrow e \gamma) \leq 5.10^{-11} \\ (3.4) 10^{-5} & \text{for } B(\mu \rightarrow e \gamma) \leq 10^{-12}, \end{cases} \quad (4.23)$$

where the first branching fraction refers to the 1992 PDB, the second to the sensitivity level of presently active experiments.

Combining the inequalities in Eq. (3.25) with Eq. (4.23), we obtain no direct consequence for $|\eta_N|$, even though we may be tempted to assume

$$\sum |U_{eA}|^2 = O\left(\sum_A |U_{eA}^* U_{\mu A}|\right). \quad (4.24)$$

In the minimal special case, we have

$$Y = |\Lambda_e \Lambda_\mu| \sin^2 \alpha \quad (4.25)$$

and

$$\eta^{(N)} = \Lambda_e^2 \sin^2 \alpha \sum_A \rho_A^2 p_A; \quad |\eta^{(N)}| \leq \left| \frac{\Lambda_e}{\Lambda_\mu} \right| \cdot Y$$

Then the inequality

$$|\eta^{(N)}| \leq \left| \frac{\Lambda_e}{\Lambda_\mu} \right| (2.4 \cdot 10^{-4}) \quad (4.26)$$

follows if we adopt the present branching fraction $B(\mu \rightarrow e\gamma) \leq 5 \cdot 10^{-11}$ in eq. (4.23).

So, unless $|\Lambda_e| \gg |\Lambda_\mu|$, Eq. (4.25) implies

$$|\eta^{(N)}| \leq O(2.4 \cdot 10^{-4}) \quad (4.27)$$

Note, however, that the relative magnitude of the mixing parameters Λ_e, Λ_μ is a completely open question.

5. Experimental aspects of the search for massive neutrino exchange in the process $e^-e^- \rightarrow \ell_i^- \ell_j^-$ ($i, j \neq e$)

Let us now consider the question of detectability of a signal for massive Majorana neutrino exchanges in the reaction $e^-e^- \rightarrow W^-W^-$. For the total signal level, we assume the upper and lower limits estimated in Eq. (3.26) above, for the mixing parameters $|\eta^{(N)}|$, to be 2×10^{-4} and 5×10^{-3} , respectively. We do this for the practical cases of the so-called Next Linear Collider (NLC) with luminosity $10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ at $\sqrt{s} = 0.5 \text{ TeV}$ as well as for the 1 TeV collider (TLC), with a luminosity of $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$.

In so doing, let us first assume that we can use an appropriate detector to see the final states $W^-W^- \rightarrow \mu^-\mu^- + \text{neutrinos}$, $\mu^-\tau^- + \text{neutrinos}$, $\tau^-\tau^- + \text{neutrinos}$ with full efficiency, in addition to the somewhat less distinctive experimental signatures involving electrons and quarks in the W^- decays. We will later see that some background rejection can be gained only by virtue of angular cuts and resulting restrictions in efficiency, so that the realistic numbers are somewhat, but not by much, reduced from those shown below.

For our special non-generic scenario, with heavy neutrino masses estimated to be in the 1 TeV range, we approximate counting rates by inserting the numerical values for the lower and upper reaches of the limiting values of $\eta^{(N)}$, the neutrino mass mixing parameter. Table 5.1 mentions the resulting numbers for both the $\sqrt{s} = 0.5 \text{ TeV}$ NLC and the 1 TeV TLC machines at their design luminosities of 10^{33} and $10^{34} \text{ cm}^{-2} \text{ sec}^{-1}$, respectively. In all cases, the standard 10^7 seconds/year duty cycle is used for the integrated luminosity.

	$\sqrt{s} \text{ (TeV)}$	$ \eta^{(N)} $	$ \eta^{(N)} $
NLC	0.5	2×10^{-4}	5×10^{-3}
TLC	1.0	0.01	6
		2	1000

Table 5.1

Number of detected events for the process
 $e^-e^- \rightarrow W_L^-W_L^- \rightarrow \ell_i^- \ell_j^- + \text{neutrinos}$
under the assumptions given in the text.

While by no means overwhelming, experimental signals due to the upper reaches of these estimates are well within the realm of detectability, given their spectacular two-negative-prong signatures at high $p_\perp$.

It now remains to be determined which backgrounds are liable to create difficulties if a few events of this topology and particle content are found, and also which cuts on the data will be needed to exclude, as much as possible, ambiguous events of different physical origin.

Preliminary detector design for the NLC incorporates a 10° open cone in the forward and backward directions. Given the sizeable $p_\perp$ of the outgoing leptons, we do not foresee a serious degradation of detection efficiency due to this lack of small-angle coverage; rather, as we stress below, this cut helps reduce certain QED-related backgrounds.

Backgrounds: While quantitative estimates of relevant background processes are relegated to a separate study, we will address here salient arguments for our belief that they will not vitiate the detection process if due precaution is taken.

A - QED radiative pair production: This is expected to be a large background, given the high currents needed for the required luminosity. However, the graph in Fig. 4

shows that

Fig. 4. Background process: QED pair production for γ, Z^0 exchange.

to be fooled by it, we would have to lose four out of six charged leptons, and detect only two negative μ, τ leptons. While the electrons may well run down the beam pipe, we see no possibility of observing the like-sign non-electron leptons at high p_L , while their partners escape detection. The p_L requirement by itself eliminates all but the remotest tail of final-state QED-generated charged leptons.

B - Multihadronic events with decay $\mu(\tau)$: Two-photon physics will generate $c\bar{c}$ and $b\bar{b}$ pairs that could yield like-sign $\mu^- \mu^-, \mu^- \tau^-, \tau^- \tau^-$ pairs according to the sequence (cf. Fig. 5) $\gamma\gamma \rightarrow b\bar{b}$, with two detectable final-state charged leptons:

Fig. 5. $\gamma\gamma$ collisions can lead to like-sign lepton pairs from $W^-, \bar{c}$ decays.

These leptons, however, will be embedded in jets that are not likely to escape detection easily. Also, we do not expect a sizeable part of this yield to display the p_L characteristics and the back-to-back topologies of the signal process.

C - W^-W^- production with and without Z^0, γ exchange: There is a large amount of Standard Model graphs that yields W^-W^- and neutrino pairs in the final state; as we show generically in Fig. 6, they are characterized by the emission of four (instead of two) neutrinos. This will tend to soften their p_L characteristics, and the back-to-back topologies. In addition,

Fig. 6. Standard Model graphs leading to the $W^-W^- + \text{neutrino}$ signature of the signal process.

the large Q^2 to be exchanged from e^- to e^- that is needed for W^- pair production will depress the cross-section. The ultimate experimental test for the presence of such backgrounds may have to lie in a comparison of e^-e^- runs with $e_L e_R$ vs. $e_L e_L$ polarizations: only the Standard Model processes will allow these backgrounds in both polarization combinations. The signal is possible for $e_L e_L$ only. This argument does not apply to the case of W^-W^- rescattering, which we are in the process of evaluating.

D - Beyond-the-Standard-Model processes: Of the various novelties that can be called upon to yield potentially similar final-state particles as our signal process, we mention the production and decay of a doubly-charged Higgs boson H^{--} as suggested by certain extended symmetry groups. This process (Fig. 7) will show s -channel structure due to the production of $e^-e^-, \mu^-\mu^-, \tau^-\tau^-$ pairs, and its characteristic isotropic angular

Fig. 7. s -channel production and decay of H^{--} .

distributions will set it apart just as much as its possible preferential coupling to $\tau^-\tau^-$ (rather than $\mu^-\mu^-$ or $\mu^-\epsilon^-$).

There are many more "new physics" scenarios due to, say, "neutralino" exchanges in supersymmetric extensions of the Standard Model. We do not wish to address such speculative issues here. We will, however, mention that they will tend to soften the p_L

characteristic of the final state, and will be susceptible to certain helicity tests we plan to address in a separate note.

6. CONCLUSIONS

In the foregoing, we have shown that the production and subsequent decay of W^-W^- pairs in electron-electron colliders is a signature process for the discovery of the existence of heavy Majorana neutrinos. Under reasonable assumptions for the masses and the mixing parameters of these states, we arrive at rates that are accessible to experimentation at linear colliders of the NLC and TLC varieties, and do not need especially designed detectors. It appears quite reasonable to expect that, for e^-e^- operation, the luminosity parameters for the (lower-energy) NLC machine can be raised (w.r.t. e^+e^-) by a factor that easily makes up for the absence of the luminosity-enhancing beam-beam pinch effect. This factor makes e^-e^- operations even more attractive, because it enhances the usefulness of an e^-e^- collider for $\gamma\gamma$ and γe physics and other, more rate-limited, studies of exotic effects.

While the present paper contains no information on production angles and decay kinematics, we have estimated both final-state acceptance of the signature channels and signal-to-background ratios after straightforward cuts, and find them quite favourable in the framework of presently foreseen detector characteristics. A few events will stand out such as to make a positive identification reasonably straightforward.

Our treatment of the W^- pair production process by means of massive neutrino exchange differs from previous work [1],[2] in a variety of ways. To wit, we assume the light neutrinos to mix with heavy N states through the see-saw mechanism to all orders; this gives us a prescription for recursive reduction of transformation matrices from weak interaction eigenstates to mass eigenstates in block diagonal form. It permits us to include exactly massless neutrinos in our considerations.

Whereas previous estimates were predicated on specific restrictive assumptions ($s \gg m_N^2$, specific couplings, and convenient choices of the neutrino mass matrix), we extend our calculations to all presently imaginable mass and mixing parameters. This leads to a result that stresses the increasingly promising nature of an NLC/TLC machine with centre-of-mass energies well into the TeV range, but also shows that a spectacular discovery potential exists at $\sqrt{s} = 0.5 - 1$ TeV in e^-e^- collisions, well beyond that of e^+e^- colliders at similar energies. Their purpose is mostly an investigation of Standard Model phenomena such as W^+W^- scattering, top quark physics, etc.

A unique feature of our treatment is the implication of a sensitive search for the exchange of heavy neutrinos of the Majorana variety with the very distinct experimental signatures

$$\begin{aligned} e^-e^- &\rightarrow W^-W^- \rightarrow \mu^-\mu^- + \text{neutrinos} , \\ &\rightarrow \mu^-\tau^- + \dots , \\ &\rightarrow \tau^-\tau^- + \dots , \\ &\rightarrow e^-\mu^- + \dots , \\ &\rightarrow e^-\tau^- + \dots . \end{aligned}$$

(in addition to quasi-collinear jet-jet and charged-lepton-plus-jet configurations of an equally spectacular topology) involving only left-handed chiral gauge couplings. Permitting the experiment with highly polarized beams then permits such checks and balances as a determination whether any newly observed signals are enhanced or suppressed by individual chiral gauge couplings. In particular, the above final states are expected to be heavily suppressed in $e_L e_R$ interactions. This feature, together with the strong energy dependence of the cross-section, should make for a clear-cut set of convincing experimental observables.

APPENDIX 1 - The neutrino mass matrix

A1 - Reduction of the $2N_f \times 2N_f$ mass matrix $\mathcal{M}$ to $N_f \times N_f$ submatrices

The block diagonalization of the mass matrix $\mathcal{M}$ [Eq. (2.4)],

$$\mathcal{M} = \begin{pmatrix} 1 & \dots & N_f & \dots & \dots & 2N_f \\ \vdots & & \vdots & & & \vdots \\ N_f & \begin{pmatrix} 0 & \vdots & \vdots & \vdots & \mu^T \end{pmatrix} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 2N_f & \begin{pmatrix} \mu & \vdots & \vdots & M \end{pmatrix} \end{pmatrix} \quad (A1.1)$$

$$\nu_i = \tilde{U}_{i\beta} \hat{\nu}_\beta ; \quad \hat{\nu}_\alpha = U_{J\alpha} \nu_J ; \quad i, j, \alpha, \beta = 1, \dots, 2N_f$$

will be performed in this section.

In Eq. (A1.1), the ν_i denote the neutrino fields in the weak interaction basis, whereas the $\hat{\nu}_\alpha$ denote mass eigenstates. We will now align the weak interaction basis with the mass eigenstates of charged leptons.

$\tilde{m}_D$ in Eq. (A1.1) denotes a diagonal, non-negative matrix:

$$(\tilde{m}_D)_{\alpha\beta} = \tilde{m}_\alpha \quad \delta_{\alpha\beta} ; \quad \tilde{m}_\alpha \geq 0, \quad \alpha = 1, \dots, 2N_f$$

Here, we denote with a tilde $2N_f \times 2N_f$ matrices that are of $N_f \times N_f$ block-diagonal form:

$$\tilde{m}_D = \begin{pmatrix} m_D & 0 \\ 0 & M_D \end{pmatrix} ;$$

$$\begin{matrix} m_\alpha & \alpha = 1, \dots, N_f & \text{(light neutrino - masses)} \\ M_A & A = 1, \dots, N_f & \text{(heavy neutrino - masses)} \end{matrix}$$

The reduction of $\mathcal{M}$ [Eq. (A1.1)] to $N_f \times N_f$ block diagonal form is achieved by

the following steps:

$$U = T \tilde{U}_0 ; \quad \tilde{U}_0 = \begin{pmatrix} U_0 & 0 \\ 0 & V_0 \end{pmatrix} \quad (A1.2)$$

$$\tilde{U}_0 \tilde{m}_D \tilde{U}_0^T = \begin{pmatrix} U_0 m_D U_0^T & 0 \\ 0 & V_0 M_D V_0^T \end{pmatrix} = T^{-1} \mathcal{M} (T^{-1})^T$$

The unitary $2N_f \times 2N_f$ matrix T , not to be confused with the suffix T denoting transposition, is decomposed as follows:

$$T = \tilde{U}^T A (\tilde{U}^T)^{-1},$$

$$A = \begin{pmatrix} \cos a_D & \sin a_D \\ -\sin a_D & \cos a_D \end{pmatrix}, \quad (A1.3)$$

$$\tilde{U}^T = \begin{pmatrix} u & 0 \\ 0 & w \end{pmatrix}.$$

In Eq. (A1.3), $\tilde{U}^T$ denotes a block diagonal unitary matrix, whereas a_D stands for a diagonal, non-negative $N_f \times N_f$ matrix:

$$(a_D)_{rt} = a_r \delta_{rt} ; \quad a_r \geq 0, \quad r, t = 1, \dots, N_f$$

Equation (A1.3) yields the block structure of T :

$$T = \begin{pmatrix} u \cos a_D u^{-1} & u \sin a_D w^{-1} \\ -w \sin a_D u^{-1} & w \cos a_D w^{-1} \end{pmatrix} \quad (A1.4)$$

With the help of Eqs. (A1.3)-(A1.4), we obtain for the expression on the right-hand side of Eq. (A1.2):

$$T^{-1} \mathcal{M} (T^{-1})^T = \begin{pmatrix} \{11\} & \{12\} \\ \{12\}^T & \{22\} \end{pmatrix},$$

where

$$\begin{aligned} \{11\} &= u \cos a_D u^{-1} & [11] &= \left((u^{-1})^T \cos a_D u^T \right) \\ \{12\} &= u \cos a_D u^{-1} & [12] &= \left((w^{-1})^T \cos a_D w^T \right) \\ \{22\} &= u \cos a_D u^{-1} & [22] &= \left((w^{-1})^T \cos a_D w^T \right) \\ \{21\} &= \{12\}^T ; & [21] &= [12]^T \end{aligned} \quad (A1.5)$$

$T^{-1}\mathcal{M}(T^{-1})^T$ becomes block diagonal precisely if $\{12\} = [12] = 0$.

The three independent $N_f \times N_f$ matrices $[11]$, $[12]$, $[22]$ are:

$$\begin{aligned} [11] &= -t\mu - \mu^T t^T + tM\bar{t}^T, \\ [12] &= \mu^T - tM - t\mu\bar{t}, \\ [22] &= \mu\bar{t} + t^+ \mu^T + M. \end{aligned} \quad (A1.6)$$

In these expressions,

$$t = u \tan a_D w^{-1}, \quad \bar{t} = (u^{-1})^T \tan a_D w^T.$$

From Eq. (A1.6), the condition $[12] = 0$ for the matrix $T^{-1}\mathcal{M}(T^{-1})^T$ to be in block diagonal form can be formulated as follows:

$$t = \mu^T M^{-1} - t\mu\bar{t}M^{-1}. \quad (A1.7)$$

Equation (A1.7) is solved by iteration, which is equivalent to a power expansion in the quantity λ ; $\mu \rightarrow \lambda\mu$:

$$\begin{aligned} \mu^T M^{-1} &\rightarrow \lambda\mu^T M^{-1}; \quad \lambda \rightarrow 1 \\ t_{n+1} &= \mu^T M^{-1} - t_n \mu\bar{t}_n M^{-1} \\ t_0 &= 0, \quad t_1 = \mu^T M^{-1}, \quad t_2 = \mu^T M^{-1} - \mu^T M^{-1} \mu\mu^+ \bar{M} M^{-1}, \dots \\ t &= \lim_{n \rightarrow \infty} t_n. \end{aligned} \quad (A1.8)$$

A2 - Construction of T from the $N_f \times N_f$ matrix t

The full unitary matrix T [Eqs. (A1.1)-(A1.2)] is constructed from $t = \tan a_D w^{-1}$ as determined by Eq. (A1.8):

$$T = \begin{pmatrix} (1+tt^+)^{-1/2} & (1+tt^+)^{-1/2}t \\ -t^+(1+tt^+)^{-1/2} & (1+t^+t)^{-1/2} \end{pmatrix}, \quad (A1.9)$$

$$t = \mu^T M^{-1} - t\mu\bar{t}M^{-1}.$$

The block structure of T [Eq. (A1.4)] obtains from t together with a set of identities:

$$\begin{aligned} u \cos a_D u^{-1} &= (1+tt^+)^{-1/2}, \\ w \cos a_D w^{-1} &= (1+t^+t)^{-1/2}, \\ u \sin a_D w^{-1} &= (1+tt^+)^{-1/2}t = t(1+t^+t)^{-1/2}, \\ w \sin a_D u^{-1} &= (1+t^+t)^{-1/2}t^+ = t^+(1+tt^+)^{-1/2}. \end{aligned} \quad (A1.10)$$

The remaining diagonal blocks of $T^{-1}\mathcal{M}(T^{-1})^T$ in Eq. (A1.2),

$$T^{-1}\mathcal{M}(T^{-1})^T = \begin{pmatrix} \{11\} & 0 \\ 0 & \{22\} \end{pmatrix} \quad (A1.2)$$

with

$$\{11\} = U_0 m_D U_0^T, \quad \{22\} = V_0 M_D V_0^T,$$

become:

$$\begin{aligned} \{11\} &= (1+tt^+)^{-1/2} \left[-t\mu - \mu^T t^T + tM\bar{t}^T \right] (1+t\bar{t}^T)^{-1/2} \\ \{22\} &= (1+t^+t)^{-1/2} \left[\mu\bar{t} + t^+ \mu^T + M \right] (1+t^T \bar{t})^{-1/2} \end{aligned} \quad (A1.11)$$

From Eqs. (A1.2) and (A1.11), we infer that, in general, U_0 and V_0 differ arbitrarily from the unit matrix. The lowest-order approximation for $\{11\}$, $\{22\}$ is:

$$\{11\} \cong -\mu^T M^{-1} \mu; \quad \{22\} \cong M.$$

APPENDIX 2 - The amplitudes $A_{LR}(\epsilon_1^* \epsilon_2^*, P)$

Kinematics

As in Eq. (1.2), we use spinor notation with the following chiral conventions

$$\begin{aligned} \gamma_5 &= i\gamma_0\gamma_1\gamma_2\gamma_3 = (\gamma_5)_L \\ \frac{1+\gamma_5}{2} u(p, \lambda) &= u^{\dot{\gamma}}(p, \lambda), \quad \frac{1-\gamma_5}{2} u(p, \lambda) = u_a(p, \lambda). \end{aligned} \quad (A2.1)$$

$u^{\dot{\gamma}}$ = left chiral components u_{γ} = right chiral components
(upper) (lower)

$(\alpha, \dot{\gamma}) = (1, 2)$; $\lambda_{1,2} = \pm 1$ denote the helicities of the incoming electrons.

We also assign the labels 1, 2 to the incoming electron momenta, neglecting their mass (see Fig. 2):

$$\begin{aligned} \text{electron 1: } p_1 &\cong E(1, 0, 0, 1); & \lambda_1 &= -1 \quad (L) \\ \text{electron 2: } p_2 &\cong E(1, 0, 0, -1); & \lambda_2 &= +1 \quad (R). \end{aligned} \quad (A2.2)$$

In the above, $E = \frac{\sqrt{s}}{2}$, $s = (p_1 + p_2)^2$. In the centre-of-mass frame, the 2-2 invariants are

$$\begin{aligned} s &= (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2 = - \left[q^2 + E^2 - 2Eqz \right] \\ u &= (p_1 - p_4)^2 = - \left[q^2 + E^2 + 2Eqz \right] \end{aligned} \quad (A2.3)$$

The labels 3, 4 are assigned to the momenta of the outgoing W -bosons as follows

$$\begin{aligned} p_3 &= (E, q \sin \theta, 0, q \cos \theta) \\ p_4 &= (E, -q \sin \theta, 0, -q \cos \theta) \\ q &= (E^2 - m_W^2)^{1/2}, \quad z = \cos \theta \end{aligned} \quad (A2.4)$$

Their helicities are ϵ_1, ϵ_2 , respectively.

In Eq. (A2.4), θ denotes the centre-of-mass scattering angle (from e_1 to W_1).

With the notation of Eqs. (A2.1)-(A2.4), the reduced amplitudes in Eq. (2.7) are of the form, for the LR-case

$$\begin{aligned} F(t) &= \frac{1}{2Eq} \left\{ \begin{array}{c} u^{\rho}(p_2, \lambda_2) (\epsilon_2^*)_{\rho\dot{\delta}}^* \quad (p_1 - p_3)^{\dot{\delta}\beta} \times \\ \times (\epsilon_1^*)_{\beta\dot{\gamma}} \quad u^{\dot{\gamma}}(p_1, \lambda_1) \end{array} \right\} \\ F(u) &= \frac{1}{2Eq} \left\{ \begin{array}{c} u^{\rho}(p_2, \lambda_2) (\epsilon_1^*)_{\rho\dot{\delta}} \quad (p_1 - p_4)^{\dot{\delta}\beta} \times \\ \times (\epsilon_2^*)_{\beta\dot{\gamma}} \quad u^{\dot{\gamma}}(p_2, \lambda_2) \end{array} \right\} \end{aligned} \quad (A2.5)$$

In Eq. (A2.5), the association of a spinor 2×2 matrix to a four-vector is used:

$$\begin{aligned} a^{\mu} &= (a^0, \vec{a}) \rightarrow (a)_{\alpha\dot{\beta}} = (a^0 + \vec{a}\vec{\sigma})_{\alpha\dot{\beta}} \\ &\rightarrow (\vec{a})^{\dot{\delta}\delta} = (a^0 - \vec{a}\vec{\sigma})_{\dot{\delta}\delta} \end{aligned} \quad (A2.6)$$

t and u -channel exchanges of (massive) neutrinos give rise to the pole terms [Eq. (1.7)]:

$$\begin{aligned} \frac{1}{(m_D^2)_a - t} &= \frac{1}{2Eq} \frac{1}{f_a - z} \\ \frac{1}{(m_D^2)_a - u} &= \frac{1}{2Eq} \frac{1}{f_a + z} \\ f_a &= \frac{1}{2Eq} \left[(m_D^2)_a + E^2 + q^2 \right] \\ \text{For } \sqrt{s} \gg m_W, \quad f_a &\rightarrow 1 + \frac{2(m_D^2)_a}{s} + 2 \left(\frac{m_W^2}{s} \right)^2. \end{aligned} \quad (A2.7)$$

The expressions for the reduced amplitude in Eq. (2.8) correspond to the following set of helicities $\lambda_{1,2}$ for the incoming electrons, $\epsilon_{1,2}^*$ for the outgoing W -bosons:

$$\begin{aligned} \lambda_1 &= -1(L), \quad \lambda_2 = +1(R) \\ \epsilon_1^*, \epsilon_2^* &\Rightarrow (\sigma_1, \sigma_2) \quad \sigma_{1,2} = \pm 1 \text{ for } A_{tr, tr} \cdot \\ (\epsilon_1^*, \epsilon_2^*) &\Rightarrow (\sigma_1^{long}, \sigma_2) \quad \sigma_2 = \pm 1 \text{ for } A_{long, tr} \cdot \\ (\epsilon_1^*, \epsilon_2^*) &\Rightarrow (\sigma_1, \epsilon_2^{long}) \quad \sigma_1 = \pm 1 \text{ for } A_{tr, long} \cdot \\ (\epsilon_1^*, \epsilon_2^*) &\Rightarrow (\epsilon_1^{long}, \epsilon_2^{long}) \quad \text{for } A_{long, long} \cdot \end{aligned} \quad (A2.8)$$

The (complex) polarization vectors $(\epsilon_1^*, \epsilon_2^*)$ for the outgoing W -bosons are (in the

centre-of-mass system):

$$\begin{aligned}
(\epsilon_1^*)_t &= (0, \cos \theta, -\sigma_1 i, -\sin \theta) \frac{1}{\sqrt{2}} \\
(\epsilon_2^*)_t &= (0, -\cos \theta, -\sigma_2 i, \sin \theta) \frac{1}{\sqrt{2}} \\
\sigma_{1,2} &= \begin{cases} +1 & (R) \\ -1 & (L) \end{cases} \\
(\epsilon_1^*)_{long} &= \frac{E}{m_W} \left(\frac{q}{E}, \sin \theta, 0, \cos \theta \right) \\
(\epsilon_2^*)_{long} &= \frac{E}{m_W} \left(\frac{q}{E}, -\sin \theta, 0, -\cos \theta \right)
\end{aligned} \tag{A2.9}$$

Transversely polarized outgoing W 's are right-handed (left-handed) for $\sigma_{1,2} = +1(-1)$, respectively.

In terms of the quantities defined in Eqs. (A2.6)-(A2.9), the functions $F(t, u)$, which we mean to indicate $F(t)$ or $F(u)$, in Eq. (2.7) become, for the LR-case:

$$\begin{aligned}
F(t) &= -\frac{1}{q} \left[\epsilon_2^*(p_1 - p_3) \epsilon_1^* \right]_{1,2}, \\
F(u) &= -\frac{1}{q} \left[\epsilon_1^*(p_1 - p_4) \epsilon_2^* \right]_{1,2}.
\end{aligned} \tag{A2.10}$$

In Eq. (A2.10), $\epsilon_{1,2}^*$, $(p_1 - p_{3,4})$ denote spinor matrices as defined in Eq. (A2.6).

The functions $F(t, u)$ are assigned to definite W -boson helicities according to Eq. (A2.6):

$$\begin{aligned}
F(t, u) &\rightarrow F_{\sigma_1 \sigma_2}(t, u) \text{ in } A_{tr}, \\
&\rightarrow F_{\sigma_2}(t, u) \text{ in } A_{long, tr}, \\
&\rightarrow F_{long}(t, u) \text{ in } A_{long, long}.
\end{aligned} \tag{A2.11}$$

We now obtain for the amplitude $A_{tr, tr}$ [Eq. (2.8)]:

$$\begin{aligned}
F_{\sigma_1 \sigma_2}(t) &= a_{\alpha}^{\sigma_1 \sigma_2} \cdot \sin \theta, \quad F_{\sigma_1 \sigma_2}(u) = b_{\alpha}^{\sigma_1 \sigma_2} \cdot \sin \theta, \\
a_{\alpha}^{++} &= a_{\alpha}^{--} = -b_{\alpha}^{++} = -b_{\alpha}^{--} = \left(\frac{E}{q} f_{\alpha} - 1 \right) = a_{\alpha}^{+-} = -b_{\alpha}^{-+} \\
a_{\alpha}^{-+} &= -b_{\alpha}^{+-} = \left(\frac{E}{q} f_{\alpha} + 1 \right)
\end{aligned} \tag{A2.12}$$

In going from eq. (A2.10) to Eq. (A2.12), all terms are dropped which are multiplied by $\sum_{\alpha=1}^6 \eta_{\alpha} = 0$ (due to the orthogonality relations) when substituted into $A_{tr, tr}$.

Similarly, the amplitudes $A_{long, tr} = A_{\sigma_2}$ [Eq. (2.8)] become:

$$\begin{aligned}
A_{\sigma_2} &= \frac{E}{\sqrt{2} m_W} \sum_{\alpha} \eta_{\alpha} \left\{ C_{\alpha} + \left[\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right] S_{\alpha} + \left[\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right] D_{\alpha} \right\} \\
C_{\alpha} &= 4 \frac{E}{q} (f_{\alpha} - 1), \\
S_{\alpha} &= -\frac{E}{q} (f_{\alpha}^2 - 1) + \sigma_2 \left[f_{\alpha}^2 - \left(\frac{E}{q} + \frac{q}{E} \right) f_{\alpha} + 1 \right], \\
D_{\alpha} &= \frac{E}{q} \left(f_{\alpha}^2 - \frac{q^2}{E^2} \right) - \sigma_2 (f_{\alpha}^2 - 1).
\end{aligned} \tag{A2.13}$$

The amplitudes $A_{tr, long} \equiv A'_{\sigma_1}(z)$ are obtained from $A_{long, tr} \equiv A_{\sigma_2}(z)$ by Bose symmetry [Eq. (2.8)]:

$$A'_{\sigma_1}(z) = A_{\sigma_1}(-z) \tag{A2.14}$$

Finally, the amplitude $A_e = A_{long, long}^{red}$ takes the form

$$\begin{aligned}
A_e &= \frac{E^2}{m_W^2} \sum_{\alpha=1}^6 \eta_{\alpha} \left[\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right] \left(1 + \frac{q^2}{E^2} + 2 \frac{q}{E} f_{\alpha} \right), \\
A_{long, long} &= \frac{G_{FS} V}{\sqrt{2}} \sum_{\alpha=1}^6 \eta_{\alpha} \left[\frac{1}{f_{\alpha} - z} + \frac{1}{f_{\alpha} + z} \right] \times \left(1 + \frac{q^2}{E^2} + 2 \frac{q}{E} f_{\alpha} \right).
\end{aligned} \tag{A2.15}$$

so that we obtain for

APPENDIX 3 - The amplitudes $B_{LL}(\epsilon_1^* \epsilon_2^*, P)$

The standard left-chiral W -boson couplings give rise to the following reduced amplitudes in Eq. (3.2):

$$B^{red} = (\epsilon_1^* \cdot \epsilon_2^*) (G(t) + G(u)) + [(\epsilon_1^*)^0 (\epsilon_2^*)^3 - (\epsilon_1^*)^3 (\epsilon_2^*)^0] (G(t) - G(u)) \quad (A3.1)$$

The amplitudes $G(t, u)$ in Eq. (A3.1) is determined by the quantities f_α defined in Eq. (A2.7), and by the parameters

$$\xi_\alpha = (U_{e\alpha})^2 m_\alpha \quad (\alpha = 1, \dots, 6), \quad (A3.2)$$

which involve the light and heavy neutrino masses [see Eq. (3.4)].

$G(t, u)$ takes the form

$$\begin{aligned} G(t) &= \sqrt{s} \sum_\alpha \xi_\alpha \frac{1}{m_\alpha^2 - t} \\ &= \frac{1}{q} \sum_\alpha \xi_\alpha \frac{1}{f_\alpha - z} \end{aligned} \quad (A3.3)$$

As in Eq. (A2.8), we obtain the following helicity amplitudes for the (LL) case:

$$\begin{array}{lll} B_{tr, tr} : & (\sigma_1, \sigma_2) & \sigma_1, \sigma_2 = \pm 1 \\ B_{long, tr} : & (\epsilon_1^{long}, \sigma_2) & \sigma_2 = \pm 1 \\ B_{tr, long} : & (\sigma_1, \epsilon_2^{long}) & \sigma_1 = \pm 1 \\ B_{long, long} : & (\epsilon_1^{long}, \epsilon_2^{long}) & \end{array} \quad (A3.4)$$

The neutrino mass parameters ξ_α defined in Eq. (A3.2) satisfy the constraint

$$\sum_{\alpha=1}^6 \xi_\alpha = 0. \quad (A3.5)$$

Assigning helicity amplitudes as in Eq. (A2.11),

$$\begin{aligned} G(t, u) &\rightarrow B_{\sigma_1, \sigma_2}(t, u) \text{ in } B_{tr, tr} \\ &\rightarrow B_{\sigma_1}(t, u) \text{ in } B_{tr, long} \\ &\rightarrow B_{long}(t, u) \text{ in } B_{long, long}. \end{aligned} \quad (A3.6)$$

we obtain for the LL -initial electron helicities the amplitudes [Eq. (3.3)] modulo terms

proportional to $(\sum_\alpha \xi_\alpha = 0)$:

$$\begin{aligned} B_{\pm\pm} &= \frac{1}{q} \sum_\alpha \xi_\alpha (1 \mp f_\alpha) \left(\frac{1}{f_\alpha - z} + \frac{1}{f_\alpha + z} \right), \\ B_\pm &= \mp \frac{E \mp q}{\sqrt{2} m_W} \frac{1}{q} \sqrt{1 - z^2} \times \sum_\alpha \xi_\alpha \left(\frac{1}{f_\alpha - z} - \frac{1}{f_\alpha + z} \right) \\ B_{long} &= -\frac{1}{q} \sum_\alpha \xi_\alpha \frac{m_\alpha^2}{m_W^2} \left(\frac{1}{f_\alpha - z} + \frac{1}{f_\alpha + z} \right). \end{aligned} \quad (A3.7)$$

APPENDIX 4 - The angular integrals $I_{\alpha\beta}, J_{\alpha\beta}$

In this Appendix, we assemble the relevant properties of the angular integrals in Eq. (3.7):

$$\begin{aligned}
 I_{\alpha\beta} &= \int_{-1}^{+1} dz \frac{1}{f_\alpha - z} \frac{1}{f_\beta - z} \\
 J_{\alpha\beta} &= \int_{-1}^{+1} dz \frac{1}{f_\alpha - z} \frac{1}{f_\beta + z} \\
 f_\alpha &= 1 + g_\alpha; \quad g_\alpha = 2 \frac{m_\alpha^2}{s} + f_W \\
 f_W &= \frac{m_W^4}{2Eq(E+q)^2} = 2 \left(\frac{m_W^2}{s} \right)^2 (1-2x)^{-1} (1-x)^{-2}
 \end{aligned} \tag{A4.1}$$

$$x = \frac{E-q}{2E} = \frac{1 - \sqrt{1 - \frac{4m_W^2}{s}}}{2} \simeq \frac{m_W^2}{s} \quad \text{for } s \gg 4m_W^2.$$

The integrals I, J are elementary:

$$\begin{aligned}
 I_{\alpha\beta} &= \begin{cases} \frac{1}{g_\alpha} - \frac{1}{2+g_\alpha} & \text{for } \alpha = \beta \\ \frac{1}{g_\beta - g_\alpha} \left[\log \frac{g_\alpha}{g_\beta} - \log \frac{1+g_\beta/2}{1+g_\alpha/2} \right] & \text{for } \alpha \neq \beta \end{cases} \\
 J_{\alpha\beta} &= \frac{1}{2+g_\alpha+g_\beta} \left[\log \frac{4}{g_\alpha g_\beta} + \log \left\{ (1+g_\beta/2)(1+g_\alpha/2) \right\} \right]
 \end{aligned} \tag{A4.2}$$

For the contributions involving the heavy neutrinos only, $\alpha, \beta \Rightarrow A, B$, $\alpha = A+3$, $\beta = B+3$; $A, B = 1, 2, 3$, it is useful to express I, J as functions of the inverse quantities $\epsilon_\alpha = 1/g_\alpha$, not to be confused with the polarization symbol for the W^- :

$$\begin{aligned}
 I_{\alpha\beta} &= 2\epsilon_\alpha \epsilon_\beta \begin{cases} \frac{1}{1+2\epsilon_\alpha} & \text{for } \alpha = \beta \\ \left[\frac{1}{z-y} \log \frac{1+z}{1-y} \right] & \text{for } \alpha \neq \beta \end{cases} \\
 &\quad \text{where we have used the symbols } x = 2\epsilon_\alpha, y = 2\epsilon_\beta.
 \end{aligned} \tag{A4.3}$$

$$J_{\alpha\beta} = 2\epsilon_\alpha \epsilon_\beta \left\{ \left[\frac{1}{x+y+xy} \log(1+x+y+xy) \right] \right\}_{\substack{x=2\epsilon_\alpha \\ y=2\epsilon_\beta}}.$$

The quantities in curly brackets in Eq. (A4.3) $I_{\alpha\beta}(J) = 2\epsilon_\alpha \epsilon_\beta i_{\alpha\beta}(j)$ can be ex-

panded for small arguments $\epsilon_\alpha, \epsilon_\beta$ (or x, y) in the following way:

$$\begin{aligned}
 i_{\alpha\beta} &= \begin{cases} \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n+1} & \text{for } \alpha = \beta \\ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} \frac{z^{n+1}-y^{n+1}}{z-y} & \text{for } \alpha \neq \beta \end{cases} \\
 j_{\alpha\beta} &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} (x+y+xy)^n.
 \end{aligned} \tag{A4.4}$$

Expanding the quantities i, j in Eq. (A4.4) to second order in the arguments $\epsilon_\alpha, \epsilon_\beta$ we obtain:

$$\begin{aligned}
 i_{\alpha\beta} &= 1 - (\epsilon_\alpha + \epsilon_\beta) + \frac{4}{3}(\epsilon_\alpha^2 + \epsilon_\beta^2) + \frac{4}{3}\epsilon_\alpha \epsilon_\beta + O(\epsilon^3), \\
 j_{\alpha\beta} &= 1 - (\epsilon_\alpha + \epsilon_\beta) + \frac{4}{3}(\epsilon_\alpha^2 + \epsilon_\beta^2) + \frac{4}{3}\epsilon_\alpha \epsilon_\beta + O(\epsilon^3).
 \end{aligned} \tag{A4.5}$$

The combinations of $I_{\alpha\beta}, J_{\alpha\beta}$ determining the helicity-projected angular averages in Eq. (3.8) for $\epsilon_\alpha \rightarrow 0$ are, for the convention HH to denote heavy neutrino exchange,

$$\begin{aligned}
 \sigma_{--}^{*HH} &: \epsilon_\alpha^{-1} \epsilon_\beta^{-1} (I_{\alpha\beta} + J_{\alpha\beta}) = 4 \left[1 - (\epsilon_\alpha + \epsilon_\beta) + \frac{4}{3}(\epsilon_\alpha^2 + \epsilon_\beta^2) + \epsilon_\alpha \epsilon_\beta \right] + O(\epsilon^3) \\
 \sigma_{++}^{*HH} &: \left(\frac{1}{\epsilon_\alpha} + 2 \right) \left(\frac{1}{\epsilon_\beta} + 2 \right) (I_{\alpha\beta} + J_{\alpha\beta}) = \\
 &= 4 \left[1 + (\epsilon_\alpha + \epsilon_\beta) + \frac{2}{3}(\epsilon_\alpha^2 + \epsilon_\beta^2) + \epsilon_\alpha \epsilon_\beta \right] + O(\epsilon^3) \\
 \sigma_{00}^{*HH} &: \left(\frac{s}{2m_W^2} \right)^2 \left(\frac{1}{\epsilon_\alpha} - f_W \right) \left(\frac{1}{\epsilon_\beta} - f_W \right) (I_{\alpha\beta} + J_{\alpha\beta}) = \\
 &= \left(\frac{s}{m_W^2} \right)^2 \left\{ 1 - (1 + f_W)(\epsilon_\alpha + \epsilon_\beta) + \left(\frac{4}{3} + f_W \right) (\epsilon_\alpha^2 + \epsilon_\beta^2) + \right. \\
 &\quad \left. (1 + f_W)^2 \epsilon_\alpha \epsilon_\beta + O(\epsilon^3) \right\}
 \end{aligned}$$

$$\sigma_0^{*HH} : (-2) \left[(1 + \epsilon_\alpha + \epsilon_\beta) i_{\alpha\beta} + j_{\alpha\beta} \right] = -4 \left\{ 1 - \frac{1}{2}(\epsilon_\alpha + \epsilon_\beta) + \frac{5}{6}(\epsilon_\alpha^2 + \epsilon_\beta^2) + O(\epsilon^3) \right\} \tag{A4.6}$$

The expressions in Eqs. (A4.3)-(A4.6) determine the contributions of heavy neutrino mass eigenstates in the region $m_N^2 \gg s \gg 4m_W^2$.

Next we turn to the light-light contributions, indicated by the superscript LL :

$$I_{\alpha\beta} \rightarrow I_{ab}, \quad J_{\alpha\beta} \rightarrow J_{ab}, \\ \alpha = a = 1, 2, 3 \quad \beta = b = 1, 2, 3$$

and to the heavy-light contributions

$$I_{\alpha\beta} \rightarrow I_{Ab}, \quad J_{\alpha\beta} \rightarrow J_{Ab}, \\ \alpha = a = 1, 2, 3, \quad \beta = b = 1, 2, 3, \quad A = 1, 2, 3.$$

For $\alpha = a = 1, 2, 3$, the quantities $g_\alpha \rightarrow g_a$ are :

$$g_a = 2 \frac{(m_\nu^2)_a}{s} + f_W \simeq 2 \frac{(m_\nu^2)_a}{s} + 2 \left(\frac{m_W^2}{s} \right)^2 \quad (A4.7)$$

In the region of centre-of-mass energies $\sqrt{s} = O(1 \text{ TeV})$, we are interested in, we have

$$\frac{(m_\nu^2)_a}{s} \ll \left(\frac{m_W^2}{s} \right)^2 \Rightarrow \sqrt{s} \ll m_W \cdot \frac{m_W}{(m_\nu)_a} \quad (A4.8)$$

and thus

$$g_a \simeq f_W, \text{ independent of } \alpha$$

For $4m_W^2 \ll s \ll (m_N)^2$, we obtain asymptotically:

$$\begin{aligned} I_{ab} &\rightarrow \frac{s^2}{2m_W^4} \simeq \frac{1}{f_W}, \\ I_{Ab} &\rightarrow \frac{s}{2(m_N^2)_A} \log \frac{s}{m_W^2}, \\ J_{ab} &\rightarrow 2 \log \frac{s}{m_W^2}, \\ J_{Ab} &\rightarrow \frac{s}{(m_N^2)_A} \log \frac{s}{m_W^2}. \end{aligned} \quad (A4.9)$$

From Eq. (A4.9), we infer the corresponding contribution to the helicity-projected angular averages:

$$\begin{aligned} \sigma_{--}^{*LL} &\rightarrow \frac{4}{f_W}; \\ \sigma_{++}^{*L} &\rightarrow f_W; \\ \sigma_{00}^{*LL} &\rightarrow \frac{(m_\nu^2)_a}{m_W^2} \frac{(m_\nu^2)_b}{m_W^2} \frac{1}{f_W}; \\ \sigma_{\pm 0}^{*LL} &\rightarrow (-2). \end{aligned} \quad (A4.10)$$

The interference between heavy and light neutrino exchanges yields the leading

contributions

$$\begin{aligned} \sigma_{--}^{*HL} &: 8 \log \frac{s}{m_W^2}; \\ \sigma_{++}^{*HL} &: 4 f_W \log \frac{s}{m_W^2}; \\ \sigma_{00}^{*HL} &: 2 \frac{s}{m_W^2} \frac{(m_\nu^2)_b}{m_W^2} \log \frac{s}{m_W^2}; \\ \sigma_{\pm 0}^{*HL} &: -2 \log \frac{s}{m_W^2}. \end{aligned} \quad (A4.11)$$

Combining Eqs. (A4.10), (A4.11), (3.5), we obtain the leading contribution to the quantities $\sigma_{\sigma_1 \sigma_2}^*$ defined in Eq. (3.5):

$$\begin{aligned} \sigma_{--}^{*LL} &\rightarrow \frac{2s^2}{m_W^4} |\xi^{(\nu)}|^2 \\ \sigma_{++}^{*LL} &\rightarrow \frac{2m_W^4}{s^2} |\xi^{(\nu)}|^2 \rightarrow 0 \\ \sigma_{00}^{*LL} &\rightarrow \frac{s^2}{2m_W^4} \left| \sum_a (U_{ea})^2 \frac{m_a^3}{m_W^2} \right|^2 \rightarrow 0 \\ \sigma_{-0}^{*LL} &\rightarrow -\frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0 \\ \sigma_{+0}^{*LL} &\rightarrow \frac{m_W^2}{s} |\xi^{(\nu)}|^2 \rightarrow 0. \end{aligned} \quad (A4.12)$$

with $\xi^{(\nu)} = \sum_{a=1}^3 (U_{ea})^2 m_a$

From Eq. (A4.12), we infer that the cross-sections with W helicities $++$, ± 0 , $0\pm$ are suppressed for $\sqrt{s} \gg 2m_W$, whereas σ_{00}^{*LL} is suppressed by light neutrino masses appearing in the factors

$$\frac{m_a^2}{m_W^2} \frac{m_b^2}{m_W^2}$$

The interference terms between heavy and light neutrino exchanges yield the fol-

lowing leading contributions:

$$\begin{aligned}
\sigma_{--}^{*HL} &\rightarrow -16 \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0, \\
\sigma_{++}^{*HL} &\rightarrow -16 \frac{m_W^4}{s^2} \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0, \\
\sigma_{++}^{*HL} &\rightarrow -4 \frac{s}{m_W^2} \log \frac{s}{m_W^2} \operatorname{Re} \left(\xi^{(\nu)*} \sum U_{eb}^2 \frac{m_b^2}{m_W^2} \right), \\
\sigma_{-0}^{*HL} &\rightarrow 2 \frac{s}{m_W^2} \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 (\rightarrow 0), \\
\sigma_{+0}^{*HL} &\rightarrow 2 \frac{m_W^2}{s} \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 (\rightarrow 0).
\end{aligned} \tag{A4.13}$$

All interference terms [Eq. (A4.13)] are subleading as compared to σ_{--}^{*LL} .

Relative suppression factors are of two types:

$$\begin{aligned}
&\left(\frac{m_W^2}{s} \right)^P \log \frac{s}{m_W^2} \text{ in } (\sigma_{\pm\pm}^*, \sigma_{\pm 0}^*, \sigma_{0\pm}^*)^{HL} \\
&\left(\frac{m_W^2}{s} \right) \log \frac{s}{m_W^2} \frac{(m_{\nu b})^2}{m_W^2} \text{ in } \sigma_{00}^{*HL}
\end{aligned}$$

The expressions for $\sigma_{\pm 0}^{*HL} = \sigma_{0\pm}^{*HL}$ in Eq. (A4.13) are the leading contributions for the given final state W -helicities. Thus the arrow ($\rightarrow 0$) behind the corresponding contribution is understood only relative to σ_{--}^{*LL} .

Nest, we turn to the contribution from heavy neutrino exchanges as given in Eq. (A4.6). We distinguish leading and subleading contributions, with respect to the expansion parameters

$$\epsilon_A = \frac{s}{2 m_A^2}$$

to the quantities $\sigma_{\sigma_1 \sigma_2}^{*HH}$. We use the notation

$$\sigma_{\sigma_1 \sigma_2}^{*HH} = H_{\sigma_1 \sigma_2} + \Delta H_{\sigma_1 \sigma_2}. \tag{A4.14}$$

The main contributions to $H, \Delta H$ are given separately:

$$\begin{aligned}
H_{--} &\rightarrow 4 |\xi^{(\nu)}|^2 \rightarrow 0 \\
H_{++} &\rightarrow 4 |\xi^{(\nu)}|^2 (\rightarrow 0) \\
H_{00} &\rightarrow \left(\frac{s}{m_W^2} \right)^2 |\xi^{(\nu)}|^2 \\
H_{-0} &\rightarrow -2 \frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0 \\
H_{+0} &\rightarrow -2 \frac{m_W^2}{s} |\xi^{(\nu)}|^2 \rightarrow 0
\end{aligned} \tag{A4.15}$$

Summing all leading contributions, but neglecting all $0 \left(\frac{m_W^2}{s} \right)$ corrections, we then obtain

$$\begin{array}{ll}
\sigma_{--}^* &\rightarrow 2 \frac{s^2}{m_W^2} |\xi^{(\nu)}|^2 & LL \text{ dominant} \\
\sigma_{++}^* &\rightarrow 4 |\xi^{(\nu)}|^2 \rightarrow 0 & HH \text{ dominant} \\
\sigma_{00}^* &\rightarrow \frac{s^2}{m_W^2} |\xi^{(\nu)}|^2 & HH \text{ dominant} \\
\sigma_{-0}^* &\rightarrow 2 \frac{s}{m_W^2} \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0 & HL \text{ dominant} \\
\sigma_{+0}^* &\rightarrow 2 \frac{m_W^2}{s} \log \frac{s}{m_W^2} |\xi^{(\nu)}|^2 \rightarrow 0 & HL \text{ dominant}
\end{array} \tag{A4.16}$$

Finally we turn to the subleading contributions $O(\epsilon_\alpha, \epsilon_\alpha^2, \epsilon_\alpha \cdot \epsilon_\beta)$, from heavy neutrino exchange:

$$\begin{aligned}
\Delta H_{--} &\rightarrow 4 \operatorname{Re} \left[\left(\sum_A (2\epsilon_A - \frac{8}{3} \epsilon_A^2) \xi_A^* \right) \xi^{(\nu)} \right] + 4 \left| \sum_A \epsilon_A \xi_A \right|^2, \\
\Delta H_{++} &\rightarrow -4 \operatorname{Re} \left[\left(\sum_A (2\epsilon_A - \frac{4}{3} \epsilon_A^2) \xi_A^* \right) \xi^{(\nu)} \right] + 4 \left| \sum_A \epsilon_A \xi_A \right|^2, \\
\Delta H_{00} &\rightarrow \frac{s^2}{m_W^4} \operatorname{Re} \left[\left(\sum_A (2\epsilon_A - \frac{8}{3} \epsilon_A^2) \xi_A^* \right) \xi^{(\nu)} \right] + \frac{s^2}{m_W^4} \left| \sum_A \epsilon_A \xi_A \right|^2, \\
\Delta H_{-0} &\rightarrow \frac{-2s}{m_W^2} \operatorname{Re} \left[\left(\sum_A (\epsilon_A - \frac{5}{3} \epsilon_A^2) \xi_A^* \right) \xi^{(\nu)} \right] \rightarrow 0, \\
\Delta H_{+0} &\rightarrow \frac{-2m_W^2}{s} \operatorname{Re} \left[\left(\sum_A (\epsilon_A - \frac{5}{3} \epsilon_A^2) \xi_A^* \right) \xi^{(\nu)} \right] \rightarrow 0.
\end{aligned} \tag{A4.17}$$

We assume at this point that, despite the relation

$$\sum_{A=1}^3 \xi_A = -\xi^{(\nu)}; \quad \xi^{(\nu)} = \sum_{\alpha=1}^3 \left(U_{e\alpha} \right)^2 m_\alpha,$$

some other combination of elements $\xi_A = \left(U_{eA} \right)^2$ m_A are not generically $O(m_\nu)$:

$$\left| \sum_A \epsilon_A \xi_A \right| \gg |\xi^{(\nu)}| \text{ for some } \epsilon_A = \frac{s}{2m_A^2}. \quad (\text{A4.18})$$

The order-of-magnitude relation in Eq. (A4.18) defines the non-generic case. We omit the terms proportional to $\xi^{(\nu)}$ in Eq. (A4.17) and obtain

$$\begin{aligned} \Delta H_{--} &\rightarrow 4 \left| \sum_A \epsilon_A \xi_A \right|^2 \quad (\rightarrow 0) \\ \Delta H_{++} &\rightarrow 4 \left| \sum_A \epsilon_A \xi_A \right|^2 \quad (\rightarrow 0) \\ \Delta H_{00} &\rightarrow \frac{s^2}{m_W^4} \left| \sum_A \epsilon_A \xi_A \right|^2 \\ \Delta H_{+0} &\rightarrow 0 \end{aligned} \quad (\text{A4.19})$$

Equations (A4.16) and (A4.19) contain the leading contributions to the helicity-projected cross-sections. The relation in Eq. (A4.17) defining the non-generic case is further discussed in Chapter 3.

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