Gravitational Waves and γ-Ray Bursts

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Coalescing binaries in distant galaxies are one of the most promising sources of gravitational waves detectable by the LIGO project. They are also a copious source of neutrinos, however these neutrino pulses are far too weak to be detected on earth. Several years ago Eichler et al.\textsuperscript{[6]} suggested that they are also sources of γ-ray bursts (GRBs). Recently it was found\textsuperscript{[7]} that GRBs are likely to be cosmological in origin, and coalescing binary systems\textsuperscript{[6,8−11]} are probably the most promising cosmological sources. The current estimates of the burst and LIGO rates from a cosmologically distributed population are based on the systems observed in our galaxy.\textsuperscript{[12−14]} These estimates are based on only three binaries so there are large statistical uncertainties in the coalescence rate. If we accept the cosmological/coalescing binary hypothesis for GRBs, then we get a more accurate estimate of the rate at which binaries coalesce and hence of the predicted LIGO signal. The association between GRBs and binaries can significantly improve the performance of LIGO. The detection of gravitational radiation from a GRB source not only confirms the coalescing binary model, but it will also provide information on the geometry and energy generation mechanism of the burst.

GRO\textsuperscript{[7]} has shown that GRBs are located in the outer parts of the galactic halo or cosmologically. In both cases all GRB mechanisms must evolve through a pair fire ball phase,\textsuperscript{[15]} which makes it difficult to distinguish between types of sources based on burst characteristics. The association between GRBs and LIGO sources is an unambiguous, verifiable prediction of the coalescing binary model,\textsuperscript{[16]} unlike gravitational lensing\textsuperscript{[17]} and redshift effects which are generic to all cosmological models.

The ability of LIGO to find coalescing binaries is strongly limited by the random noise levels in the detectors and the need to continuously search for weak signals. There is a significant gain in sensitivity if the search for signals is limited to narrow windows of time near bright γ-ray bursts. The polarization of the gravitational waves provides evidence about the emission geometry of the GRB sources, because the relative strengths of the + and × polarizations is a function of the inclination angle. If the γ-rays are preferentially emitted along the polar axis of the binary, then the GRB sources are stronger LIGO sources than the average coalescing binary because binaries emit gravitational waves more strongly along the polar axis than in the orbital plane.

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The evidence for a cosmological origin of GRBs is the combination of isotropy on the sky and inhomogeneity in space. The \( C/C_{\text{min}} \) distribution for bright bursts has a \(-3/2\) slope characteristic of a homogeneous, Euclidean distribution. Fainter bursts show a sharp break from this slope, indicating that the distribution becomes inhomogeneous at large distances. The shape of the \( C/C_{\text{min}} \) for cosmological sources is determined by redshift effects that change the luminosity, and the effective volume.\(^{16,18-20}\) The relative sharpness of the break suggests the GRBs are good standard candles, because a wide intrinsic luminosity distribution would have smeared out the break.\(^{18}\)

We consider a burst with a differential luminosity per unit energy \( E \) equal to \( (dL/dE) = L_0 E^{-\alpha} \). In a detector like BATSE the number of counts, \( C \), during a given time interval, \( \Delta t \), from redshift \( z = x - 1 \) is

\[
C = \frac{(L_0/\alpha)E_{\text{min}}^{-\alpha}x^{(1-\alpha)}}{4\pi D_L^2(z)},
\]

where \( D_L \) is the luminosity distance and \( E_{\text{min}} \) is the minimum detectable energy. If the comoving rate of coalescences is \( r(z) \) per unit comoving volume, then the integrated rate of coalescences closer than redshift \( z = x - 1 \) is

\[
R(z) = 4\pi r_H^3 \int_1^{1+z} x^{-9/2} \tilde{D}_L^2 r(z)dx
\]

where \( D_L = r_H \tilde{D}_L = 2r_H(x - x^{1/2}) \) for the Einstein-DeSitter cosmology we use in this study, \( r_H = c/H_0 \) is the Hubble radius, and \( H_0 = 100h_0 \) km s\(^{-1}\) Mpc\(^{-1}\).

For a given spectral index \( \alpha \) and evolution model \( r(z) \), the observed distribution of \( C/C_{\text{min}} \) is fit to infer the maximal redshift, \( z_{\text{max}} \), from which GRO detects GRBs.\(^{16}\) For a constant coalescence rate, \( r \), and spectral index \( \alpha = 1 \), the best fit to the \( C/C_{\text{min}} \) distribution gives \( z_{\text{max}} = 1.45 \). Varying the spectral index from 0 to 2 changes \( z_{\text{max}} \) from 2.8 to 1. The dominant uncertainty lies in the comoving coalescence rate, \( r(z) \), since a very strong evolution of the event rate \( r(z) \propto t(z)^2 \) with cosmic time \( t(z) \) can reduce \( z_{\text{max}} \) to as low as \( z_{\text{max}} = 0.5 \).

The observed GRB rate is \( \approx 800 \) events per year\(^7\) which corresponds to a local merger rate of \( 3.6 \times 10^{-8} h_0^{-3} \) Mpc\(^{-3}\) yr\(^{-1}\) or \( 3.6 \times 10^{-6} n_2^{-1} \) mergers per galaxy per year for a galaxy density of \( n = 10^{-2} h_0^{-3} n_2 \) Mpc\(^{-3}\). The current rate increases by a factor of 10 if we assume a strong cosmological time dependence \( (r(z) \propto t^2(z)) \). The overall agreement between estimated merger rate (based on GRBs) and the estimates of \( 10^{-5 \pm 1} \) yr\(^{-1} \) mergers in the Galaxy based on galactic binary pulsars\(^{13,14}\) supports the coalescing neutron star model as sources for GRBs.\(^{16}\) The agreement is better if we assume that the event rate is independent of cosmological time. This suggests that we use the calculated distribution of GRBs to predict the distribution of gravitational wave strains \( h_c \), that could be observed by LIGO.

The characteristic strain produced by a coalescing binary with reduced mass \( \mu = 1.4\mu_{\text{NS}} M_\odot \) and total mass \( M = 2.8M_{\text{NS}} M_\odot \) scaled to the masses characteristic of NS-NS binaries, averaged over binary inclinations, and using an optimal filter\(^3\) with a knee at frequency \( f = 100f_{c2} \) Hz to detect the signal is

\[
h_c = 2.3 \times 10^{-23} h_0 \mu_{\text{NS}}^{1/2} M_{\text{NS}}^{1/3} f_{c2}^{-1/6} r_H x^{5/6} D_L^{-1}. \tag{3}
\]
This is not the same as the instantaneous strain at frequency $f$, which varies as $r_H x^{5/3} / D_L$, because the optimal filter effectively integrates the signal over many orbital periods to increase the signal to noise ratio.

In Figure 1 we show the expected rate of events stronger than a given strain $h_c$ for three different models: a constant comoving rate of $r = 10^{-7} h^{-3} \text{ Mpc}^{-3} \text{ yr}^{-1}$ based on the galactic estimates,$^{13,14}$ the rate using the fit to the $C/C_{\text{min}}$ distribution with $\alpha = 1$ and a constant comoving rate for which $z_{\text{max}} = 1.45$, and the rate using the fit to the $C/C_{\text{min}}$ distribution with $\alpha = 1$ and $r(z) \propto t(z)^2$ for which $z_{\text{max}} = 0.5$. The rapidly evolving model may be inconsistent with the local galactic estimates.

The signal processing for the LIGO project is predicated on using two or more detectors to eliminate non-Gaussian sources of noise. Under this assumption, the instantaneous noise level in the LIGO detector$^{[3,21]}$ is given by $h^2_n = f c s_n(f_c)/\langle F^2 \rangle$ where $f_c$ is the knee frequency, $\langle F^2 \rangle = 1/5$ is the angle and polarization averaged detector beam pattern, and $S_n(f_c)$ is the spectral density of the noise at the knee frequency $f_c$. This is only the average instantaneous noise level, and to estimate the detectability of a source we must take into account the fluctuations of the noise. If we are using optimal filtering, we must make trial correlations of the filter with the detector output approximately once every $f_c^{-1} = 0.1 f^{-1}c_2$ seconds, or about $3 \times 10^8 f_c^{-1}$ trials per year. For two such Gaussian detectors in which we expect 3 events per year, we must have a signal to noise ratio in the signal of $S^2 / N^2 = \ln(10^8 f_c^{-1})$ to be 90% confident the detectors are not seeing noise. This defines the detection threshold for events with an expected rate of 3 per year$^{[3]}$, $h_{3/yr} = (\ln[10^8 f_c^{-1}])^{1/2} h_n(f_c) \simeq 4.2 h_n(f_c)$. This estimate is insensitive to changes to 99% confidence rather than 90%, or 0.3 events per year rather than 3 because of the logarithmic dependence. Using the estimates from $[3]$, the initial version of LIGO should have a noise level of $h_{3/yr} \simeq 8 \times 10^{-20} f_{c3}^{3/2}$ for detecting bursts with knee frequencies near $f = 1 f_{c3}$ kHz. The higher knee frequency also reduces the strength of the signal by a factor of 1.5 because of the $f^{-1/6}$ dependence in $h_c$. The advanced versions of LIGO should have better low frequency sensitivities and noise levels of $h_{3/yr} \simeq 1.3 \times 10^{-22} f_{c2}$ for detecting bursts with knee frequencies near $f = 100 f_{c2}$ Hz. The quantum limit for detecting bursts is $h_{3/yr} \simeq 1.6 \times 10^{-25} f_{c2}^{-1/2}$.

This is not true if we use $\gamma$-ray bursts to trigger searches for gravitational waves. As we can see in Figure 1, LIGO is sensitive only to signals from the rare, bright bursts, so we should search only a short time interval near the bright bursts for gravitational waves. We expect the GRB to occur between several dynamical times (milliseconds) and several cooling times (seconds) after the coalescence of the binaries. If we are conservative and search for one minute before each of the ten brightest bursts in a given year, then we are conducting only $6000 f_{c2}^{-1}$ trials rather than $10^8 f_{c2}^{-1}$ trials, and the detection threshold for $\gamma$-ray burst events is 1.5 times larger than the $h_{3/yr}$ estimates. An increase in the detection threshold by a factor of 1.5 increases the rate by a factor of three. Events correlated with GRBs not only come at a known time, but also from a known direction. This reduces the number of trial correlations further, but it is not as important as restricting the search in time because of the poor spatial resolution of LIGO.

If the use of two detectors does not eliminate all non-Gaussian errors in the LIGO signal processing, then the use of Gaussian statistics to estimate $h_{3/yr}$ will fail catastroph-
ically for the random search mode, because the noise estimate is working way out on the tail of the Gaussian distribution. If the noise has a non-Gaussian tail, triggered searches using the GRBs may be the only way LIGO can detect coalescing binaries.

Coalescing binaries emit gravitational waves much more strongly along the polar axis than in the orbital plane, with $h_+ \propto (1 + \cos^2 i)$ and $h_\times \propto 2 \cos i$ where $i$ is the inclination angle of the binary. The characteristic amplitude $h_c \propto \langle |h_+|^2 + |h_\times|^2 \rangle^{1/2}$ includes an average over the inclination angle between the observer and the binary. If the $\gamma$-ray emission from a coalescing binary is beamed along the polar axis, we have underestimated the average gravitational wave signal by the factor $(1+11x/16+11x^2/16+x^3/16+x^4/16)^{1/2}$ where $x = \cos \psi$ and $\psi$ is the opening angle around the normal into which the bursts are beamed. If the GRB is associated with a disk formed during the merger then we would expect the emission to be beamed along the polar axis. The maximum gain is a factor of 1.6 in $h_c$, and the gain is 1.2 for $\psi = 30^\circ$. Because the detection volume increases as the cube of the detection limit on the strain, the rate increases are factors of 4 and 1.9 respectively over unbeamed bursts. If the bursts are detected by LIGO the beaming can be detected from the polarization of the bursts: unbeamed bursts tend to be dominated by the $+$ polarization, while beamed bursts are dominated by the $\times$ polarization of the gravitational waves. If the bursts are beamed along the polar axis, then the GRB rate misses fraction $\cos \psi$ of the coalescences.

The hypothesis associating GRBs with coalescing binaries has several important implications for gravitational wave searches using LIGO. The GRB rate can accurately estimate the rate of coalescences as a function of the gravitational wave strain on the earth. The LIGO sensitivity must have a strain sensitivity $h_c$ greater than $10^{-21.1}h_0$ for an event rate of one per decade, and greater than $10^{-21.4}h_0$ for a rate of once per year. The strain sensitivity of LIGO is considerably greater if the brightest GRBs serve as triggers to search for gravitational waves in a narrow window of time near the burst, and in the direction of the burst. The strain sensitivity is roughly 50% greater in a coincidence experiment compared to a random search, and the detection rate is roughly three times higher.

The presence or absence of coincidences between GRB and LIGO events may be the final proof or disproof of the coalescing binary hypothesis. Once the strain sensitivity reaches $10^{-20.7}h_0$ LIGO can begin to set limits on GRB models, initially by setting limits on the redshift distribution. If the sensitivity reaches $10^{-21.5}h_0$ then either LIGO begins to detect coalescing binaries in coincidence with GRBs, or the coalescing binary model is dead. If they are related, then polarization of the gravitational waves can be used to determine the emission geometry of the GRBs. The time delay between the merging of the binary and the GRB will help to determine the emission mechanism. The shortest possible delays will be comparable to the dynamical time (milliseconds), and longer delays might be viscous or weak interaction time scales.

The burst rate is an underestimate of the LIGO rate. For a given class of binaries, beaming of the $\gamma$-rays may mean that only a fraction of the coalescences are seen as GRBs. Moreover, not all types of coalescing compact binaries may cause bursts. The galactic rates are only estimated from NS-NS binaries, and there may be a comparable number of NS-BH mergers and a much smaller number of BH-BH mergers. It is not clear whether NS-NS or NS-BH binaries are more likely to produce bursts, although the
dynamical stability of orbits and evolution time scales favor NS-NS binaries over NS-BH binaries.[23]

Acknowledgements: The authors thank Ramesh Narayan and Shude Mao for helpful discussions.

References

Figure 1: The figure shows the integral event rate of binary coalescences with characteristic strains greater than $h_c$. The three models are a fit to the GRB rates with a constant
comoving rate and $\alpha = 1$ spectral index (solid line), a fit to the GRB rates with a comoving rate proportional to $r(z) \propto t(z)^2$ (dashed line), and a constant event rate of $10^{-7}h^{-3}$ Mpc$^{-3}$ yr$^{-1}$ (solid with points). The strain scales as $h_0\mu_{NS}^{1/2}M_{NS}^{1/3}c^{-1/6}$. The estimated LIGO sensitivity $h_{3/yr}$ is shown by the vertical solid line, and the sensitivity improvement from using coincidences with GRBs is shown by the vertical dashed line. Strains $h_c$ is shown as a function.
events per year stronger than $h_c$

$\log_{10} (h_c)$

$r(z)=r(z) = \text{constant (z}_{\text{MAX}}=1.45)$

$r(z) = r(z) \propto t(z)^2 \ (z_{\text{MAX}}=0.5)$

$r(z) = 10^{-7} h^{-3} \text{ Mpc}^{-3}$