We study the temperature dependence of the Chern-Simons number fluctuations in the $SU(2)$ Higgs Model on Euclidean lattices with spatial sizes up to $20^3$. Temperatures well above the Higgs phase transition $T_H$ are achieved on anisotropic lattices. Numerical results are compared to perturbative results on finite lattices as well as in continuum perturbation theory. We find qualitative agreement with perturbative estimates and see at high temperatures a tendency towards static configurations. Up to temperatures $T \simeq 2T_H$ we find indication that tunneling between vacuums with different Chern-Simons numbers are still exponentially suppressed.

The non-conservation of the baryon and lepton number in the electroweak theory is well known Hooft. It has been argued that the baryon number violating processes can be strongly enhanced at high temperature. Calculations of the corresponding rates are based on the one hand on semi-classical estimates for transitions between topologically distinct vacuums of the electroweak theory ArnMcLe, on the other hand they have been performed through Monte Carlo simulations within the framework of an effective, Hamiltonian model, which is derived from the finite temperature Euclidean theory in the high temperature limit Ambjorn. Both approaches rely on dimensional reduction, which is expected to be valid at high temperature and should allow to treat the timelike component of the $SU(2)$ gauge fields as static fields. In the vicinity of the electroweak phase transition this approach breaks down and a non-perturbative understanding of the structure of the topologically distinct vacuums and their temperature dependence will be important.

We will study here thermal fluctuations of Chern-Simons number distributions within the framework of the Euclidean formulation of the $SU(2)$ Higgs model on the lattice. From an analysis of their correlation in Euclidean time we will be able to test the validity of the static approximation in the vicinity of the electroweak phase transition. The non-perturbative results for this correlations as well as moments of the Chern-Simons numbers will be compared with high-temperature perturbation theory in the continuum and on the lattice.

A baryon number violating process is a transition from a vacuum field configuration to one in another vacuum, which can not be reached through small gauge transformations from the initial one. In the $A_0 = 0$ gauge it is possible to assign a Chern-Simons number, equation $n_{cs} = -18\pi^2 \int d^3x \epsilon_{ijk} \text{tr} (A_i (\partial_j A_k + 23A_j A_k))$, $EQ - CS$.\[92x760]