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The Minimal Supersymmetric Standard Model and
Electroweak Precision Data

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Abstract

Our general approach for a model independent analysis of precision electroweak data, based on the variables $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_b , is applied to the particularly interesting case of the Minimal Supersymmetric Standard Model. The model is in at least as good an agreement with the data as the Standard Model. Actually the fit to especially ϵ_3 and ϵ_b becomes better than for the Standard Model if the chargino and the scalar top are light, (that is, close to their experimental limits and within reach of LEP200).

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Recently we have proposed [1, 2] a general strategy for the analysis of precision electroweak tests in view of the search for new physics beyond the Standard Model. Our method is more complete and less model dependent than the similar approach based on the variables S , T and U , [3-5] which, from the start, necessarily assumes dominance of the vacuum polarisation diagrams from new physics and truncation of the q^2 expansion of the corresponding amplitudes. In a completely model independent way we define [1] four variables, called[†] $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_b , that are precisely measured and can be compared with the predictions of different theories. In the present note we apply our general approach to the particularly interesting case of the Minimal Supersymmetric Standard Model (MSSM) [6] [7]. Actually, since the MSSM is a completely specified, well defined and computable theory one could imagine to directly compare the data with the predictions of the theory. The problem is that the MSSM depends on many more unknown input parameters than the Standard Model, so that a direct fitting procedure of the present data would not lead to a significant determination of the individual parameters. Rather we will show that the comparison of the experimental values of the epsilons with, on the one hand, the Standard Model and, on the other hand, the MSSM, for the same values of the top quark mass m_t , leads to very interesting indicative trends. In particular, we conclude that the MSSM is at least in as good an agreement with the data as the Standard Model. Indeed the fit to especially ϵ_3 and ϵ_b becomes much better than for the Standard Model if some definite and restrictive conditions on the spectrum of the MSSM are fulfilled.

The quantities $\epsilon_1, \epsilon_2, \epsilon_3$ and ϵ_b are defined in ref.1 in one to one correspondence with the set of observables $m_W/m_Z, \Gamma_Z, A_{FB}^l$ and Γ_b . The four epsilons are defined without need of specifying m_t (and m_H). In the Standard Model, for all observables at the Z pole, the whole dependence on m_t arising from one-loop diagrams only enters through the epsilons. The same is true for any extension of the Standard Model such that all possible deviations only occur through vacuum polarisation diagrams and/or the $Z \rightarrow b\bar{b}$ vertex. As discussed in detail in ref.1, for such a model one can compare the theoretical predictions with the experimental determination of the epsilons as obtained from the whole set of LEP data. If a model does not satisfy this requirement then the comparison is to be made with the epsilons determined from the defining variables only, or with some more limited enlargement of the same set, depending on the particular case (for example, if lepton universality is maintained, then the data on A_{FB}^l can be replaced by the combined result on g_V/g_A from all lepton asymmetries). The epsilons are defined in such a way that they are exactly zero in the Standard Model in the limit of neglecting all pure electroweak loop corrections (i.e. when only the predictions from the tree level Standard Model plus pure QED corrections are taken into account). Thus the epsilons represent an efficient parameterisation of the small deviations from what is solidly established in a way that is unaffected by our ignorance of m_t .

The MSSM is a well motivated extension [6] of the Standard Model. It provides the possibility of a natural escape route from the worst aspects of the hierarchy problem [8] while keeping fundamental scalar Higgses in the basic lagrangian. The measured values of

[†] Here we resume the notation ϵ_i for exactly the same quantities as defined in ref.1, where they were denoted ϵ_{N_i} (the index N_i for "new", had been inserted to signal some small differences with respect to the original definitions in ref.2).

$\sin 2\theta_W$ and of $\alpha_s(m_Z)$, the absence of observable proton decay and, to some extent, the observed value of m_b/m_τ provide some empirical support [9] for supersymmetric SU(5) GUTS [10] as opposed to ordinary SU(5) [11]. The MSSM is a consistent perturbative theory, just as the Standard Model with a not too heavy Higgs, so that the predictions for precision electroweak tests can be completely specified in terms of the, unfortunately numerous, input parameters of the theory. The MSSM is defined from the properties of generalising the Standard Model with broken supersymmetry, of minimal particle content and of supersymmetry breaking being generated in a hidden sector and transmitted to standard matter by universal supergravity couplings [12]. In fact the additional requirement of R-parity conservation, which is usually assumed, does not play a great role in the present discussion. The MSSM has the same dimensionless couplings as the Standard Model except for one: the quartic Higgs self coupling is not a free parameter but is related to the gauge couplings. But there are five parameters with dimensions of mass in the MSSM, $m_i=1,\dots,5$, in place of the single mass parameter appearing in the quadratic term of the Standard Model scalar potential. In a standard notation [6], $m_i=m, M, \mu, m_A, m_B$, all of them being related to the scale of effective supersymmetry breaking (even the μ parameter which, in the low energy theory, appears in a supersymmetric term, may well originate, in a more complete theory, as a result of supersymmetry breaking, as many examples show).

In the following, in order to simplify an otherwise inextricable complexity, we shall essentially concentrate on two limiting situations, denoted "heavy" MSSM and "light" MSSM. In the heavy case, we assume $m_i \gg m_Z$. Then the light spectrum of the MSSM is identical to that of the Standard Model with a light Higgs of mass smaller or comparable with the Z mass (as well known by now, the exact upper limit on the lightest Higgs mass critically depends on the top mass [13]). All the remaining particles predicted by the MSSM can be made arbitrarily heavy, although at the price of an increasingly unnatural fine tuning which would progressively destroy one of the main motivations of the model, i.e. the understanding of the weak scale of mass. A crucial property of the MSSM is that the mass terms of the heavy particles are compatible with unbroken SU(2) $\otimes$ U(1) symmetry. This property not only explains the observed light spectrum as the one formed by all the states whose mass is forbidden in the SU(2) $\otimes$ U(1) symmetric limit, but also implies that the heavy particles decouple from radiative corrections (unlike the top quark in the Standard Model). For the electroweak precision tests the heavy MSSM tends to reproduce the predictions of the Standard Model with a light Higgs [14].

In the light MSSM option some of the superpartners have a relatively small mass, close to their experimental lower bounds. In this case the pattern of radiative corrections may sizeably deviate from that of the Standard Model. The most interesting effects occur in vacuum polarisation amplitudes and/or the Z- $b\bar{b}$ vertex and therefore are particularly suitable for a description in terms of the epsilons (because in such a case, as explained in ref.1, the predictions can be compared to the experimental determination of the epsilons from the whole set of LEP data). They are:

- i) a threshold effect in the Z wave function renormalisation [14] mostly due to the vector coupling of charginos and (off-diagonal) neutralinos to the Z itself. Defining the vacuum polarisation functions by $\Pi_{\mu\nu}(q^2) = -ig_{\mu\nu}[A(0)+q^2F(q^2)]+q_\mu q_\nu$ terms, this is a positive contribution to $\epsilon_5 = m_Z^2 F'_{ZZ}(m_Z^2)$, the prime denoting a derivative with respect to q^2

(i.e. a contribution to a higher derivative term not included in the usual S,T,U formalism). The ϵ_5 correction shifts $\epsilon_1, \epsilon_2, \epsilon_3$ by $-\epsilon_5, -\epsilon_5^2$ and $-\epsilon_5^3$ respectively, where $\epsilon^2 = \cos^2 \theta_W$, so that all of them are reduced by a comparable amount. Correspondingly all the Z widths are reduced without affecting the asymmetries. This effect can be significant but requires the lightest chargino to have a mass close to the experimental lower limit of 45 GeV.

ii) a positive contribution to ϵ_1 from the virtual exchange of the scalar top and bottom superpartners [15], analogous to the contribution of the top-bottom quark doublet. The needed isospin splitting requires one of the two scalars (in the MSSM the s-top) to be light.

iii) a negative contribution to ϵ_b due to the virtual exchange of a charged Higgs [16]. If one defines, as customary, $\tan\beta = v_2/v_1$ (v_1 and v_2 being the vacuum expectation values of the Higgs doublets giving masses to the down and up quarks, respectively), then, for negligible bottom Yukawa coupling or $\tan\beta \ll m_t/m_b$, this contribution is proportional to $m_t^2/\tan^2\beta$.

iv) a positive contribution to ϵ_b due to virtual chargino-s-top exchange [17] which in this case is proportional to $m_t^2/\sin^2\beta$. This effect again requires the chargino and the s-top to be light in order to be sizeable.

All the above effects are cumulatively displayed in figs.1-4, where we give the pair correlations $\epsilon_3 - \epsilon_1, \epsilon_3 - \epsilon_b, \epsilon_b - \epsilon_1$ and $\epsilon_2 - \epsilon_3$ in the MSSM in the form of scatter plots. The ellipses are the 1σ contours obtained from the present combined LEP experimental data (with the addition of the measurements of m_W/m_Z) [18]. The theoretical points in each plot are for fixed $m_t = 130$ or 190 GeV. The Standard Model prediction as a function of m_H is shown by two black stars corresponding to $m_H = 50$ GeV and $m_H = 1000$ GeV connected by a line. All Standard Model input parameters are fixed to the same values as in ref.1. In particular $\alpha(m_Z) = (128.87 \pm 0.12)^{-1}$ [19] and $\alpha_s(m_Z) = 0.118 \pm 0.007$ [20]. The MSSM scatter plot are obtained for $\tan\beta > 1$ (which is relevant to the effects described in iii) and iv) above), the charged Higgs mass $m_{H^\pm} > 100$ GeV (relevant for iii)) the lightest s-top and the approximately degenerate s-bottom masses (relevant to ii) and iv)), $m_{stop} > 50$ GeV and $m_{bottom} > 150$ GeV. This last constraint requires an additional qualification. In the mass matrix of the two top superpartners we limit the off diagonal term by taking $m_A < 3m_{bottom}$. This constrains the relative s-top-s-bottom splitting when m_{bottom} gets large and consequently the size of the effect on ϵ_1 . A particularly important role is played by the lightest chargino mass $m_{\chi^\pm}$, relevant to i) and iv). The white bullets refer to very light charginos: $60 > m_{\chi^\pm} > 48$ GeV, while the white stars, that by superposition form a dark area, refer to $m_{\chi^\pm} > 60$ GeV. The figures are actually realised by taking $\tan\beta = 1$ and 4, $m_H^\pm = 100$ GeV or ∞ (i.e. its contributions set to zero), $m_{stop} = 50$ or 200 GeV, $m_{bottom} > 150$ GeV and varying the chargino masses in the mentioned ranges. The fact that only few representative values of $\tan\beta, m_{stop}, \dots$ were plotted produces the filaments which are visible for light charginos. In this case the most important effects are those from ϵ_3 and, if m_{stop} is also light, from ϵ_b via the mechanism iv) which strongly depends on $\tan\beta$. For example, ϵ_5 , as already mentioned, gives proportional contributions to ϵ_1, ϵ_2 and ϵ_3 and therefore produces lines in the planes of any two of those variables. The empty spaces between the lines would be filled up by taking intermediate values of the parameters.

In figs. 1-4 the dark area leans towards the heavy MSSM while the white bullets are from the light MSSM. At fixed m_t the lowest values of e_2 and e_3 , which are correlated, are obtained for a light chargino (note that the predicted value of e_3 is not larger in the MSSM than in the Standard Model); the lowest values of e_1 are realised for a light chargino and a heavy s-top; the largest values of e_b are obtained for the chargino and the s-top both light, whereas, on the contrary, small values of e_b require a light charged Higgs and small $t\bar{g}\bar{b}$. As apparent from figs. 1-4, the differences between the theoretical predictions of the Standard Model and of the MSSM are not large in comparison with the present accuracy of the experiments, but they will become more important with the increase of statistics at LEP.

A defect of figs. 1-4 is that from them it is not clear that the same set of values of the parameters can simultaneously fit all four of the ellipses shown. This drawback is remedied in figs. 5-8 where for each e_i ($i=1,2,3,b$) we plot two curves, as functions of m_t , and compare them with the data and with the Standard Model predictions. Both curves refer to the case of the light MSSM, where the effects are large, but with values of the parameters chosen in such a way as to approach the experimental data or to diverge from them. In figs. 5-8 all the curves that come close to the data (denoted as A) refer to $m_{\chi^+} = 55$ GeV, $m_{H^+} \rightarrow \infty$ (i.e. its contribution is set to zero) and $m_{\text{stop}} = 150-200$ GeV, while those that are far from the data (B) refer to $m_{H^+} = 100$ GeV, $m_{\chi^+} \rightarrow \infty$ and $m_{\text{stop}} = 350-450$ GeV (in both cases A and B, the values of m_{bottom} are actually chosen so that e_1 is an extremum). For both sets of curves $t\bar{g}\bar{b}=1$ and $m_{\text{stop}}=50$ GeV.

In the MSSM there is an upper limit on m_t that arises from the existence of an infrared fixed point in the associated Yukawa coupling [21]. This limit reads $m_t < (190 \text{ GeV}) \sin\beta$. Thus 190 GeV is the maximum allowed value of m_t in the MSSM (which is obtained for $t\bar{g}\bar{b}$ large). As the upper limit on m_t is derived from different physical assumptions than those that determine the structure of radiative corrections at the weak scale, in figs. 1-8 we did not impose the constraints induced by the correlation between $t\bar{g}\bar{b}$ and m_t , e.g. that $m_t < 135$ GeV for $t\bar{g}\bar{b} = 1$. From figs. 1-8 we see that a value of m_t as large as 190 GeV can actually be compatible with the data at about the 1σ level in the MSSM, if a light chargino is present.

In conclusion, the present precision electroweak data are well consistent with the MSSM. As the present data are very constraining this statement is highly non trivial. For example, simple technicolor models face serious problems [3,4,22]. In its heavy version, the pattern of radiative corrections predicted by the MSSM is practically indistinguishable from that obtained from the Standard Model with a light Higgs. The agreement with data improves considerably in the MSSM if a light gaugino and a light s-top exist, with masses close to their experimental bounds. The resulting effects can be important to bring e_3 and e_b in closer agreement with the data. If this is the case then the gaugino and the s-top should be visible at LEP200.

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Figure Captions

- [1] ϵ_3 vs ϵ_1 at fixed m_t : $m_t=130$ GeV (a) or 190 GeV (b). The ellipse is the 1σ contour from all LEP data. The Standard Model prediction as a function of m_H is shown by two black stars corresponding to $m_H=50$ GeV and $m_H=1000$ GeV connected by a line. The MSSM scatter plot are obtained for $\tan\beta>1$, the charged Higgs mass $m_{H^\pm}>100$ GeV, the lightest s-top and the approximately degenerate s-bottom masses $m_{stop}>50$ GeV and $m_{sbottom}>150$ GeV. The white bullets refer to very light charginos: $60 > m_{\chi^\pm} > 48$ GeV, while the white stars, that by superposition form a dark area, refer to $m_{\chi^\pm}>60$ GeV. The figures are actually realised by taking $\tan\beta=1$ and 4, $m_H^\pm=100$ GeV or ∞ (i.e. its contributions set to zero), $m_{stop}=50$ or 200 GeV, $m_{sbottom}>150$ GeV and varying the chargino masses in the mentioned ranges.
- [2] The same as fig.[1], but for ϵ_3 vs ϵ_b .
- [3] The same as fig.[1], but for ϵ_b vs ϵ_1 .

- [4] The same as fig.[1], but for ϵ_2 vs ϵ_3 .
- [5] ϵ_1 vs m_t . The data from the combined LEP experiments are compared with the Standard Model prediction for $m_H=50$ -1000 GeV (dashed band) and two representative curves from the light MSSM. The line denoted by A refers to $m_{\chi^\pm}=55$ GeV, $m_H^\pm \rightarrow \infty$ (i.e. its contribution is set to zero) and $m_{sbottom}=150$ -200 GeV, while the line B refers to $m_H^\pm=100$ GeV, $m_{\chi^\pm} \rightarrow \infty$ and $m_{sbottom}=350$ -450 GeV (in both cases A and B, the values of $m_{sbottom}$ are actually chosen so that ϵ_1 is an extremum). For both sets of curves $\tan\beta=1$ and $m_{stop}=50$ GeV.
- [6] The same as fig. [5] but for ϵ_2 vs m_t . Note that the lower edge of the Standard Model band almost coincides with the curve B in this case.
- [7] The same as fig. [5] but for ϵ_3 vs m_t . Note that the lower edge of the Standard Model band almost coincides with the curve B in this case.
- [8] The same as fig. [5] but for ϵ_b vs m_t . Note that the Standard Model band (obtained for $\alpha_s(m_Z)=0.125$) reduces to a line in this case because the Higgs mass dependence of ϵ_b is negligible.

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig.3a

Fig. 3b

Fig. 4a

Fig. 4b

Fig. 5

Fig. 6

Fig. 7

Fig. 8