STRING PHASE TRANSITIONS IN A STRONG MAGNETIC FIELD*

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Abstract

We consider open strings in an external constant magnetic field $H$. For an (infinite) sequence of critical values of $H$ an increasing number of (highest spin component) states lying on the first Regge trajectory becomes tachyonic. In the limit of infinite $H$ all these states are tachyons (with a common tachyonic mass) both in the case of the bosonic string and for the Neveu-Schwarz sector of the fermionic string. This result generalizes to extended object the same instability which occurs in ordinary non-Abelian gauge theories. The Ramond states have always positive square masses as is the case for ordinary QED. The weak field limit of the mass spectrum is the same as for a field theory with gyromagnetic ratio $g_S = 2$ for all charged spin states. This behavior suggests a phase transition of the string as it has been argued for the ordinary electroweak theory.

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It is well known that the QED vacuum in an environment of a constant magnetic field $H$ is stable, whatever the value of $H$ is. This is classically due to the value of the gyromagnetic ratio $g_{1/2} = 2$ for the electron and, is also true at the quantum level when $g_{1/2} - 2 \neq 0$, provided the problem is correctly handled, as shown by Schwinger long ago.\textsuperscript{1) This is in sharp contrast with the case of an environment produced by a constant electric field, where the QED vacuum suffers from an instability and pair production occurs at a calculable rate, as computed by Schwinger in 1951.\textsuperscript{2)}

The situation drastically changes when electrically charged higher spin particles are considered. Here the energy spectrum in a constant magnetic field reads:\textsuperscript{1)}

$$E_n^2 = (2n + 1)eH - g_S eH \cdot S + m^2_s,$$

where $n$ is the Landau level and $g_S$ is the gyromagnetic ratio of the particle with spin $s$, charge $e$ and mass $m^2_s$.

We see that $E_n^2 > 0$ provided $g_s S \leq 1$. The value $g_S = \frac{1}{2}$ is precisely the prescription of "minimal coupling" for higher spin particles.\textsuperscript{3) However this prescription does not hold in consistent theories coupling higher spin states. Indeed we already know that in the standard model, for $W^\pm$ bosons, $g_W = 2$ and in generalized gauge theories,\textsuperscript{4)} such as string theories, $g_S = 2$ for all excited string states.\textsuperscript{4,5)} The authors\textsuperscript{*} of Refs. [8,9] pointed out that the non-minimal e.m. coupling of $W$ bosons, implied by the non-Abelian gauge structure, causes an instability of the electroweak vacuum whenever $H$ reaches the critical value\textsuperscript{9)}

$$H_{crit} = m^2_W/e$$

as is evident from Eq. (1). For that value a phase transition may occur, characterized by $W$-condensation. Eventually a restoration of the electroweak symmetry may also take place.\textsuperscript{9)}

It is of interest nowadays to consider similar calculations ins tring theory which provides a model for consistent electromagnetic interactions of particles of arbitrary spin. In particular in open string theory, with two charges $e_1$, $e_2$ at the string end points, the

\textsuperscript{*} The special occurrence of the value $g_S = 2$ for particles of arbitrary spin was precisely discussed, in different contexts, by V. Bargmann, I. Michel, V. Telegdi\textsuperscript{6)} and S. Weinberg.\textsuperscript{7)}
electromagnetic interaction of all string states in a constant electromagnetic field strength background can be handled exactly, i.e. to all orders in the string tension $\alpha'$.\textsuperscript{10}

This treatment yields a unique set of non-minimal e.m. couplings for the charged particle states associated with the string excitations and one may consider whether the string vacuum is stable in an environment of a given e.m. field configuration.

The answer to this question was recently given in the case of a constant electric field.\textsuperscript{11} It was found that the string vacuum is unstable against pair creation with an exactly calculable rate. This rate reproduces the Schwinger formula for QED when $eE\alpha' \ll 1$ ($e = e_1 + e_2$) but deviates from the field theory result when $eE \sim O(1/\alpha')$, due to the non-minimal e.m. couplings of the charged string states.

The purpose of this letter is to consider the equally intriguing situation of a constant magnetic field $H$ where an instability is expected because of Eq. (1), due to the fact that $g_S = 2$ for the higher spin states. This fact gives further evidence that string theory is a kind of generalized gauge theory, in which the string tension parameter $\alpha'$ plays the role of an order parameter for the breaking of a huge gauge symmetry governing the unbroken phase.

Since string corrections must be small for $eH\alpha' \ll 1$, Eq. (1) certainly holds for the spin 1 massless gauge bosons, indicating that the same phenomenon of instability occurs as in ordinary gauge field theories. This result was pointed out in Ref. 10 for the open bosonic string.* More interesting is to see whether the massive string states, with masses $O(1/\alpha')$, become tachyonic for some critical value of $H$.

The answer to this question can be obtained by looking at the analog of Eq. (1) in open string theory.

We will consider both the cases of bosonic and fermionic open strings. In the case of the bosonic string the relevant formula was given in Ref. 10 and reads

* Here we have in mind a string in a constant electromagnetic (rather than chromomagnetic field so the instability will eventually occur for some central value of $H$, related to the small $(M_W^2\alpha' \ll 1)$ mass of the $W$ bosons.
\[ \alpha' E^2 = (2b_0^+ b_0 + 1) \frac{\epsilon}{2} - \frac{1}{2} \epsilon^2 \]

\[ - \epsilon \sum_{n=1}^{\infty} (a_n^+ a_n - b_n^+ b_n) + L_{\text{free}} \] (3)

where

\[ L_{\text{free}} = \sum_{n=1}^{\infty} n(a_n^+ a_n + b_n^+ b_n) - 1 + L_o^\perp. \] (4)

Here the oscillators \( a_n, b_n \), obeying canonical commutation relations, correspond to the string coordinate \( x^1 + ix^2 \), where \( H_{12} \neq 0 \) (all other components vanish). \( L_o^\perp \) denotes the Virasoro operator of the transverse coordinates with oscillators \( \alpha_n^\perp \).

The dimensionless quantity \( \epsilon \) reads

\[ \epsilon = \frac{1}{\pi} |\arctan 2\alpha' e_1 H \pi + \arctan 2\alpha' e_2 H \pi|, \] (5)

Notice that for \( \alpha' e_i H \ll 1 \)

\[ \epsilon \sim 2\alpha'(e_1 + e_2)H, \] (6)

while for \( \alpha' e_i H \gg 1, \epsilon \rightarrow 1. \)

To make contact between Eqs. (3) and (1) one identifies \( b_0^+ b_0 \) in Eq. (3) with the Landau level \( n \) in Eq. (1) and notices that

\[ \epsilon \sum_{n=1}^{\infty} (a_n^+ a_n - b_n^+ b_n) = \epsilon S \] (7)

(\( S \) is the spin component along the \( H \) direction) and

\[ L_{\text{free}} = M_2^2 \alpha'. \]

Formula (3) does reduce to Eq. (1) in the weak field limit \( \alpha' eH \ll 1, \) with \( g = 2 \) for all spin states.

Let us investigate if massive states can become tachyonic. By direct inspection of Eq. (3) one finds that only states with \( b_n^+ b_n = 0 \) \((n \geq 0)\), \( a_n^+ a_n = 0 \) \((n > 0)\) can become tachyonic. In this case we are considering states belonging to the first (parent) Regge trajectory and Eq. (5) becomes
$$\alpha' E^2 = \frac{\epsilon}{2} - \frac{\epsilon^2}{2} + (1 - \epsilon)\hat{n} - 1, \quad \hat{n} = a_1^+ a_1. \quad (8)$$

Tachyonic states appear for

$$\hat{n} < \frac{1 + \frac{\epsilon}{2}(\epsilon - 1)}{1 - \epsilon} \quad (\hat{n} > 1, \forall \epsilon). \quad (9)$$

It is important to notice that as $\alpha' eH \to \infty$ ($\epsilon \to 1$) more and more states become tachyonic. In the limit of infinite magnetic field $\epsilon = 1$ all higher helicity states in the first Regge trajectory are tachyonic with square mass $\alpha'E^2 = -1$. This result was anticipated in Ref. 10 for the first excited state $\hat{n} = 1$.

We now consider the more interesting situation of superstrings where no tachyons are present for $H = 0$, unlike the bosonic case, and spacetime fermions are also present in the spectrum. The analog of Eq. (1) takes a different form in the Neveu-Schwarz (N-S) and Ramond (R) sectors.

Let us first analyze the R-sector, containing all the spacetime fermions and in particular a light (massless) spin-$1/2$ (electron-like) state.

A simple extension of the methods of Ref. 10 gives the following expression for the energy levels

$$\alpha' E_R^2 = (2\beta_0^+ b_o + 1)\frac{\epsilon}{2} + \epsilon d_o^+ d_o - \frac{\epsilon}{2}
- \epsilon \sum_{n=1}^{\infty} (a_n^+ a_n - b_n^+ b_n + d_n^+ d_n - \tilde{d}_n^+ \tilde{d}_n) + L_{\text{free}}^R. \quad (10)$$

With respect fo formula (3), we notice that the vacuum shift is zero instead of $\frac{\epsilon}{2}(1 - \epsilon) - 1$ and that “$d$” fermionic oscillators are also present. The zero mode contribution to Eq. (10) agrees with Eq. (1) for the QED case ($S = \frac{1}{2}$, $g = 2$). Notice that $L_{\text{free}}^R$ can be written as

$$L_{\text{free}}^R = \sum_{n=1}^{\infty} n(a_n^+ a_n + b_n^+ b_n + d_n^+ d_n + \tilde{d}_n^+ \tilde{d}_n) 
+ L_{\text{free}}^\perp \quad (11)$$

By substituting this expression in Eq. (10) one obtains a manifestly positive definite formula. The lowest mass states are those for which:
\[ a_n^+ a_n = 0, \quad b_n^+ b_n = 0, \quad \hat{d}_n^+ \hat{d}_n = 0, \quad a_n^+ a_n = 0 \quad (n \geq 2), \quad d_n^+ d_n = 0 \quad (n \geq 2). \]

In this case one has

\[ \alpha' E_R^2 = (1 - \epsilon)(a_1^+ a_1 + d_1^+ d_1). \quad (12) \]

In the limit \( \alpha' eH \to \infty \) (\( \epsilon \to 1 \)) all these states become massless, while the others remain strictly massive. The absence of tachyons in the Ramond sector can be understood by the fact that the massless spin \( 1/2 \) state cannot be tachyonic, analogous to the case of QED, while the positive square masses of the excited states is a pure stringy phenomenon due to the fact that \(|\epsilon| \leq 1\).

We now turn to the N-S sector. In this case the analog of Eq. (1) reads

\[ \alpha' E_{NS}^2 = \left( 2b_o^+ b_o + 1 \right) \frac{\epsilon}{2} - \epsilon \sum_{n=1}^{\infty} (a_n^+ a_n - b_n^+ b_n) \]

\[ - \epsilon \sum_{n=1/2}^{\infty} (d_n^+ d_n - \hat{d}_n^+ \hat{d}_n) + L_{frec}^{NS}. \quad (13) \]

Eq. (13) contains half-integral moded fermionic oscillators. The ground state energy is now \( \frac{\epsilon}{2} - \frac{1}{2} \). As usual the tachyon is eliminated by the GSO projection.

As for the bosonic string, the only states which can become tachyonic belong to the first Regge trajectory. They have the form

\[ (a_n)^{\tilde{n}} d_{1/2}^+ |0\rangle_{NS} \quad (14) \]

where \( |0\rangle_{NS} \) is the N-S vacuum. For these states Eq. (1) becomes

\[ \alpha' E_{NS}^2 = -\frac{\epsilon}{2} + (1 - \epsilon)\tilde{n}. \quad (15) \]

Eq. (15) gives tachyonic states for

\[ \tilde{n} < \frac{\epsilon}{2(1 - \epsilon)}. \quad (16) \]

Again, in the \( \alpha' eH \to \infty \) limit (\( \epsilon \to 1 \)) all (highest helicity component) states on the first Regge trajectory become tachyonic with square mass.
\[\alpha' m^2 = -\frac{1}{2}.\] (17)

The tachyonic mass indicates, as in the case of the electroweak theory, an instability of the superstring vacuum. For example, the first massive state becomes tachyonic for \(\epsilon = 2/3\), that is for \(\alpha' e H \pi = \sqrt{3} (e_1 = e_2 = e)\). This result gets modified when some of the transverse components are compactified. For instance, in the case of a torus compactification with radius \(R \gg (\alpha')^{1/2}\), the first excited state which becomes tachyonic is

\[d^+_1 | p = \frac{1}{R}.\]

Here \(p\) is the discrete compactified momentum, thus the state has mass \(\alpha' M^2 = \frac{\alpha'}{R^2} - \frac{1}{2} \epsilon\).
This mass becomes tachyonic at \(\epsilon = 2\alpha' / R^2\).

The magnetic field instability, in the electroweak theory, gives rise to a non-zero W and Z condensate and a new vacuum in which the electroweak symmetry may be restored. This phenomenon resembles high T phase transitions in point-field theory as discussed in Ref. 9. If a similar situation occurs in string theory, this may indicate that at high magnetic fields, the string undergoes a phase transition to a new vacuum with some huge unbroken symmetry, as suggested in Refs. 12,13 or may reach to a non-critical string vacuum as supported by the analysis of Ref. An effective Lagrangian formulation for a string in a constant magnetic field may unravel the nature of the phase transition and the structure of the new phase.

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