Natural Suppression of Higgsino-Mediated Proton Decay in Supersymmetric SO(10)*

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BA-93-26, May 1993

Abstract

In supersymmetric Grand Unified Theories, proton decay mediated by the color–triplet higgsino is generally problematic and requires some fine–tuning of parameters. We present a mechanism which naturally suppresses such dimension 5 operators in the context of SUSY SO(10). The mechanism, which implements natural doublet–triplet splitting using the adjoint higgs, converts these dimension 5 operators effectively into dimension 6. By explicitly computing the higgs spectrum and the resulting threshold uncertainties we show that the successful prediction of $\sin^2 \theta_W$ is maintained as a prediction in this scheme. It is argued that only a weak suppression of the higgsino mediated proton decay is achievable within SUSY SU(5) without fine–tuning, in contrast to a strong suppression in SUSY SO(10).

*Supported in part by Department of Energy Grant #DE-FG02-91ER406267
1. Introduction

The dramatically precise unification of couplings\(^1\) that occurs in the minimal supersymmetric \(SU(5)\) model has been much touted, and indeed is striking. A fit\(^2\) to all \(W,Z\), and neutral current data (using \(m_t = 138\text{GeV}\) and \(m_H = M_Z\)) gives \(\sin^2 \theta_W(M_Z) = 0.2324 \pm 0.0003\), while in SUSY \(SU(5)\) one has\(^2\) \(\sin^2 \theta_W(M_Z) = 0.2334 \pm 0.0036\) (we have combined the uncertainties due to \(\alpha_s(M_Z), \alpha(M_Z),\) sparticle thresholds, \(m_t, m_{h_0},\) high-scale thresholds and non-renormalizable operators). As a measure of how significant this agreement is consider that the addition of just one extra pair of light higgs doublet supermultiplets would increase the SUSY GUT prediction of \(\sin^2 \theta_W(M_Z)\) to about 0.256, leading to a discrepancy of over six standard deviations.

At the same time SUSY \(SU(5)\) suffers from a problem\(^3\) with proton decay arising from dimension-5 operators caused by the exchange of color-triplet higgsinos in the same SUSY-\(SU(5)\) multiplet with the light higgs doublets, \(H\) and \(H'\). For ‘central values’ of the model parameters the predicted proton life-time mediated by the dimension-5 operators\(^4,5\) is shorter than the current experimental limits.\(^6\) A certain amount of adjustment is then required for consistency, which pushes parameters to the corner of their allowed region. For short we will henceforth refer to this as the ‘higgsino-mediated proton decay (HMPD) problem’.

It would seem that one cannot take seriously the unification of couplings of SUSY-GUTs, however impressive, in the absence of satisfactory mechanism that suppresses higgsino-mediated proton decay. Two requirements for a ‘satisfactory’ mechanism that we will impose are that it involves no ‘fine-
tuning’ or artificial adjustments of parameters, and that the unification of
couplings is maintained as a prediction. (We emphasize that word because a
discrepancy in $\sin^2 \theta_W$ can often be remedied by introducing ad hoc new par-
ticles, threshold effects, etc; but we would not regard the resulting agreement
as being in any sense a prediction.)

The higgsino-mediated proton decay problem is easily described. In
SUSY $SU(5)$ models there exists a pair of higgs super-multiplets, that we
will denote $5_H+\bar{5}'_H$. Under the standard model group, $G_S = SU(3) \times SU(2) \times
U(1)$, these decompose as $5_H = \{(1,2,\frac{1}{2}) + (3,1,-\frac{1}{3})\} \equiv \{2_H + 3_H\}$, and
$\bar{5}'_H = \{(1,2,-\frac{1}{2}) + (\bar{3},1,\frac{1}{3})\} \equiv \{\bar{2}'_H + \bar{3}'_H\}$. The $2_H$ and $\bar{2}'_H$ are just the
familiar $H$ and $H'$ of the supersymmetric standard model, and their cou-
plings to the light quarks and leptons are therefore fixed in terms of their
vacuum expectation values and the light fermion masses. Then by $SU(5)$ the
Yukawa couplings of the $3_H$ and $\bar{3}'_H$ are fixed as well. If there is a Dirac mass
term connecting the higgsinos in $3_H$ and $\bar{3}'_H$ to each other, i.e., a term of
the form $M(3_H\bar{3}'_H)$, then the diagram shown in figure 1 exists, which depicts
a baryon-number violating process mediated by colored higgsino exchange.
The higgsino, of course, converts the quarks and leptons to their scalar part-
ers, so figure 1 needs to be ‘dressed’ for it to correspond to proton decay.
Dressing by $W$-ino exchange is by far the most dominant, the resulting life-
time for $p \to K^+\bar{\nu}_\mu$ for example (the anti-symmetry of the relevant operator
causes the change in flavor) has been estimated to be
\[ \tau(p \to K^+\bar{\nu}_\mu) = 6.9 \times 10^{28} \text{yr} \times \left( \frac{0.01 \text{GeV}^{-3}}{\beta} \right) \left( \frac{0.67}{A_s} \right) \left( \frac{\sin 2\beta_{\mu}}{1+y^{tk}} \right) \left( \frac{M_{Hc}}{10^{16} \text{GeV}} \right) \frac{T_{eV^{-1}}}{f(u,d)^{-1}} \left| f(u,e) \right|^2. \] (1)

Here \( \beta \) is the relevant nuclear matrix element which lies in the range \( \beta = (0.003 - 0.03) \text{GeV}^3 \). \( A_s \) is the short-distance renormalization factor (\( A_s \simeq 0.6 \)), \( \tan \beta_H = \langle H \rangle / \langle H' \rangle \), and \( y^{tk} \) parametrizes the contribution of the top family relative to the first two (\( 0.1 < |y^{tk}| < 1.3 \) for \( m_t = 100 \text{GeV} \)). The functions \( f \) arise from the one-loop integrals and \( f(u,d) \simeq m_{\tilde{W}_\pm}/m_{\tilde{Q}}^2 \) for \( m_{\tilde{W}_\pm} \ll m_{\tilde{Q}} \).

The prediction (1) is to be compared with the present experimental lower limit \( \tau(p \to K^+\bar{\nu}_\mu) > 1 \times 10^{32} \text{yr} \). There is already, as can be seen, somewhat of a difficulty reconciling these numbers. At least several of the following conditions should be fulfilled: (a) The nuclear matrix element \( \beta \) is near the lower end, \( \beta \simeq 0.003 \text{GeV}^3 \), (b) \( \tan \beta_H \) is not too large, (c) either the \( \tilde{W}_\pm \) is significantly lighter than the squark \( \tilde{Q} \) or vice versa, (d) the colored higgsino mass should exceed the GUT scale significantly (\( M_{Hc} \simeq 10^{17} \text{GeV} \)), (e) there is some cancellation in \( (1 + y^{tk}) \) between the third family and the first two family contributions. Obviously this pushes almost all parameters to their corners. Although not excluded, the problem begs for a more elegant explanation.

The necessity of the Dirac mass term \( M(3_H \bar{3}_H') \) for obtaining the \( D = 5 \) baryon-violating operator is a crucial point. A quartic term in the chiral superfields will be suppressed by \( \frac{1}{M_{\text{GUT}}} \) or \( \frac{1}{M_{\text{GUT}}^2} \) depending on whether it is an \( F \)-term or a \( D \)-term. To get an \( F \)-term all the left-handed chiral superfields
must be coming into the diagram (or out of it). This requires the chiralityflipping mass insertion coming from the $M(3_H\bar{3}'_H)$ term as shown in fig. 1. If the mass $M$ vanished, one could only write superfield diagrams like fig. 2, which clearly give $D$-terms and therefore are suppressed by $\frac{1}{M_{GUT}}$. (The $F$-terms correspond to higgsino exchange, the $D$-terms to higgs exchange.)

The foregoing considerations have suggested to several authors\textsuperscript{7,8} an approach to suppressing higgsino-mediated proton decay by imposing a (Peccei-Quinn type) symmetry that suppresses the Dirac mass between the $3_H$ and $\bar{3}'_H$. However, this approach leads to a dilemma. In the simplest SUSY $SU(5)$ model it is precisely the $M(3_H\bar{3}'_H)$ Dirac term that gives to the $3_H$ and $\bar{3}'_H$ the superlarge mass that they must have (else even the $D = 6$ operators would cause a disaster). How, then, can the $3_H$ and $\bar{3}'_H$ be made superheavy and yet not have a large Dirac mass connecting them to each other? To resolve this, one ends up introducing new color-triplet higgs superfields, a $\bar{3}_H$ to mate with the $3_H$ and a $\bar{3}'_H$ to mate with the $3'_H$, so that all color triplets become heavy while still leaving the unprimed and primed sectors disconnected. That means having four instead of the minimal two 5-plets of higgs. The situation can be represented diagrammatically as follows:

\[
\begin{array}{c}
\begin{pmatrix} 3 \\ 2 \end{pmatrix} & \begin{pmatrix} \bar{3} \\ 2 \end{pmatrix} & \begin{pmatrix} 3' \\ 2' \end{pmatrix} & \begin{pmatrix} \bar{3}' \\ 2' \end{pmatrix} \\
\parallel & \parallel & \parallel & \parallel \\
5_H & \bar{5}_H & 5'_H & \bar{5}'_H
\end{array}
\] (2)

where the solid horizontal lines representing superlarge Dirac masses. The 2 and $\bar{2}'$ are the usual light $H$ and $H'$ of the MSSM. But now it is to be noticed
that there are two additional light doublets, \( \bar{2} \) and \( 2' \). As noted above, this is unacceptable if the dramatic unification of gauge couplings is to be preserved as a prediction.

One could remove the extra pair of Higgs(ino) doublets to superlarge scales by introducing a mass term \( M(\bar{5}_H 5'_H) \), indicated by the dashed line in eq. (2). This, of course, would reintroduce the higgsino-mediated proton decay as shown in fig. 3. The parameter \( M \) would then control both the higgsino-mediated proton decay and the mass of the pair of extra doublets.

As we shall see in the later sections, this situation is typical of mechanisms that naturally suppress higgsino-mediated proton decay: (a) there is a doubling of the higgs sector, (b) there is the consequent danger to the unification of couplings of extra light fields in incomplete multiplets, and (c) there is a parameter \( M \) which controls both proton decay and the mass of these extra fields.

Two approaches, therefore, appear to be possible. (1) Weak suppression of higgsino-mediated proton decay, resulting from \( M \) being of order – but slightly less than – \( M_{GUT} \). For example, with \( M = \frac{1}{10} M_{GUT} \) higgsino-mediated proton decay is suppressed by a factor of \( 10^{-2} \) while at the same time those extra fields whose mass is given by \( M \) will only lead to small threshold corrections to \( \sin^2 \theta_W \). In this case the suppression of higgsino-mediated proton decay is just numerical; there is no symmetry or other qualitative explanation of it. One would have no a priori reason therefore to expect the suppression to be particularly large. A hope would therefore exist that \( p \rightarrow K^+ \bar{\nu}_\mu, n \rightarrow K^0 \bar{\nu} \), etc. might be seen at super-Kamiokande. (2) Strong suppression of higgsino-mediated proton decay would result if (due to
some approximate symmetry perhaps) $M$ were much less than $M_{\text{GUT}}$, say, $O(M_W)$. In that case it is imperative that there be no ‘extra’ fields (i.e., beyond the minimal supersymmetric standard model) in incomplete $SU(5)$ multiplets whose mass is proportional to $M$, or else the unification of couplings would be destroyed. To achieve this without fine-tuning turns out to be a non-trivial problem. One of the main conclusions of this paper is that such a strong suppression of proton decay can only be achieved in a satisfactory way in $SO(10)$ (or larger groups).

The whole problem of higgsino-mediated proton decay is of course intimately connected to the well known question of ‘doublet-triplet’ splitting.\textsuperscript{9–13} We will show that the most satisfactory treatments of this problem make use of an old but somewhat neglected idea for doublet-triplet splitting in $SO(10)$ using the adjoint higgs due to Dimopoulos and Wilczek.\textsuperscript{13}

Our paper is organized as follows. In section II we review the Dimopoulos-Wilczek idea for doublet-triplet splitting and show how both weak suppression and strong suppression of HMPD can be achieved naturally in SUSY $SO(10)$. In section III we consider SUSY $SU(5)$ and show that only weak suppression of HMPD can be achieved without fine-tuning parameters. In section IV we discuss flipped $SU(5)$. In section V a closer examination of the Dimopoulos-Wilczek mechanism in $SO(10)$ is undertaken, and we show that it can be made viable and consistent. Our conclusions are summarized in section VI. In appendix A we give the details of the minimization of a realistic $SO(10)$ superpotential. There we show that the gauge symmetry breaking can be achieved consistent with supersymmetry without generating pseudo goldstone bosons. Appendix B deals with the threshold corrections
to $\sin^2 \theta_W$.

2. Suppressing Proton Decay in SUSY SO(10)

The problem with doublet-triplet splitting arises in $SU(5)$ because of the tracelessness of its irreducible representations. Suppose the superpotential contains the term $\lambda_1 \bar{5}_H^\prime 24_H 5_H$ and $\langle 24_H \rangle = \text{diag}(x, x, x, y, y)$, then the $3_H$ and $\bar{3}_H^\prime$ higgsinos get a Dirac mass of $\lambda_1 x$ and the doublets $2_H$ and $\bar{2}_H^\prime$ get a mass $\lambda_1 y$. We need $\lambda_1 x \sim M_{\text{GUT}}$ and $\lambda_1 y \sim M_W$ but this is impossible since, by the tracelessness of $24$, $y = -\frac{3}{2} x$. This can be remedied by introducing a singlet superfield, $1_H$, with the coupling $\lambda_2 \bar{5}_H^\prime 1_H 5_H$ and $\langle 1_H \rangle \equiv z$ (or equivalently by a bare mass term), but only by tuning the parameters so that $(-\frac{3}{2} \lambda_1 x + \lambda_2 z) \sim 10^{-14}(\lambda_1 x + \lambda_2 z)$.

In $SO(10)$ such fine adjustment of parameters can be avoided because the analogue of the tracelessness condition does not exist. The 24, which is the adjoint of $SU(5)$, is contained in the 45 which is the adjoint of $SO(10)$. 45 is a rank-2 antisymmetric tensor and the VEV of $45_H$ can be brought to the canonical form

$$\langle 45_H \rangle = \eta \otimes \text{diag}(x_1, x_2, x_3, x_4, x_5),$$

$$\eta \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

which corresponds to the $U(5)$ matrix $\text{diag}(x_1, x_2, x_3, x_4, x_5)$. Because this is a $U(5)$ rather than an $SU(5)$ matrix its trace need not vanish. One can therefore have the VEV of $\langle 45_H \rangle$ take the form

\[\text{Equation (3)}\]
$$\langle 45_H \rangle = \eta \otimes \text{diag}(a, a, a, 0, 0).$$  \hspace{1cm} (4)

This is just what is needed to give mass to the $SU(3)_C$-triplet higgs(inos) and not the $SU(2)_L$-doublet ones. This is what we call the Dimopoulos-Wilczek mechanism.

There is another group-theoretical explanation for the doublet-triplet splitting in $SO(10)$. Under its maximal subgroup $SU(2)_L \times SU(2)_R \times SU(4)_C$, the standard model singlets of 45 which could acquire GUT-scale VEVs are contained in the $(1, 1, 15)$ and $(1, 3, 1)$ multiplets. The 10 of Higgs decomposes as $(2, 2, 1) + (1, 1, 6)$. If only the $(1, 1, 15)$ of 45 acquires a VEV, it gives the color triplets of $(1, 1, 6)$ a mass and not the doublets of $(2, 2, 1)$. If the $(1, 3, 1)$ acquires a VEV, it will supply a super-large mass to the doublets - and not to the triplets. (We shall shortly make use of the second property to suppress proton decay and at the same time preserve $\sin^2 \theta_W$ as a prediction.) Such options are not available in $SU(5)$, since $SU(5)$ has no intermediate symmetries, even for the sake of classification.

Consider the following coupling in the superpotential of a SUSY-$SO(10)$ model:

$$W \supset \lambda 10_{1H} 45_H 10_{2H}. \hspace{1cm} (5)$$

One must introduce two 10’s of Higgs(ino) fields because with just one the term $10_H 45_H 10_H$ would vanish by the antisymmetry of the 45. As we noted in the introduction, and shall see more clearly later, such a doubling is actually a useful thing from the point of view of suppressing higgsino-mediated proton decay.
decay. This is another appealing feature of $SO(10)$.

When the $45_H$ gets the VEV shown in eq (4) all of the triplet Higgs(ino) fields in $10_{1H}$ and $10_{2H}$ get superlarge masses. The situation can be represented schematically as follows

\[
\begin{array}{cccc}
(3_1 \ 2_1) & (3_2 \ 2_2) & (3_2 \ 2_2) & (3_1 \ 2_1) \\
\| & \| & \| & \|
\end{array}
\]

\[
\begin{array}{cccc}
5_{1H} & \bar{5}_{2H} & 5_{2H} & \bar{5}_{1H} \\
\end{array}
\]

where under $SO(10) \rightarrow SU(5), 10_{1H} = \bar{5}_{1H} + 5_{1H}$, and $10_{2H} = \bar{5}_{2H} + 5_{2H}$. By comparison with the scheme shown in eq. (2) one sees that the ‘unprimed sector’ consists of $5_{1H}$ and $\bar{5}_{2H}$, while the ‘primed sector’ consists of $5_{2H}$ and $\bar{5}_{1H}$. One can then identify $H \equiv 2_{1H}$ and $H' = \bar{2}_{1H}$.

Now we face the problem of generating mass for the ‘extra’ doublets which reside in $5_{2H} + \bar{5}_{2H} = 10_{2H}$. The simplest possibility is just to introduce into the superpotential, $W$, the term $M(10_{2H}10_{2H})$, with $M/M_{GUT}$ being less than - but not much - smaller than - one. The resulting threshold correction to $\sin^2 \theta_W$ is $+\frac{3\alpha(M_Z)}{16\pi} \ell n(M_{GUT}/M) \approx 10^{-3}$. For proton decay to be suppressed it is also necessary that only $10_{1H}$ but not $10_{2H}$ couple to light quarks and leptons with usual strength. All of this (including the absence of a $(10_{1H})^2$ term in $W$) can be enforced by a global symmetry. For example, one could have a symmetry under which $10_{2H} \rightarrow +10_{2H}, 10_{1H} \rightarrow -10_{1H}, 45_H \rightarrow -45_H,$ and $16_j \rightarrow i16_j$ (the light families, with $j = 1, 2, 3$).

The above appears to us to be the simplest way of achieving weak suppression of proton decay. (For comparison with $SU(5)$ see the next section.)
However, in $SO(10)$, but not in $SU(5)$, it is actually possible to achieve a strong suppression in a satisfactory way. To do this we need to give $2_2H$ and $\bar{2}_2H$ a superheavy Dirac mass (so as to not mess up $\sin^2 \theta_W$) without having a superheavy Dirac mass connecting $3_2H$ and $\bar{3}_2H$ (which could produce excessive proton decay). But this is just a doublet-triplet splitting problem – but upside down to the familiar one! Here the doublets but not the triplets need a mass term. This will prove to be not doable in $SU(5)$ without tuning, but in $SO(10)$ it can be done. What is required is another $45_H$ with a VEV

$$
\langle 45'_H \rangle = \eta \otimes \text{diag}(0, 0, a', a').
$$

As already noted, this is just as achievable as the VEV given in eq. (4). (See section V for the demonstration.)

There is a slight hitch in that we cannot, because of the antisymmetry of $45'_H$, simply write down $10_2H45'_H10_2H$ to give mass to $2_2H$ and $\bar{2}_2H$. However, this can be overcome by introducing an additional 10 of Higgs(ino) fields. Consider a superpotential containing

$$
W \supset \lambda 10_{1H}45_H10_{2H} + \lambda' 10_{2H}45'_H10_{3H}
+ M10_{3H}10_{3H}
+ \sum_{i,j=1}^3 f_{ij} 16_i 16_j 10_{1H}
$$

with $\langle 45_H \rangle$ and $\langle 45'_H \rangle$ being given by eqs. (4) and (7). Then the superheavy mass matrices of the color-triplet and weak-doublet higgs(inos) are
\[
(\bar{2}_1, \bar{2}_2, \bar{2}_3) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \lambda a' \\ 0 & -\lambda' a' & M \end{pmatrix} \begin{pmatrix} 2_1 \\ 2_2 \\ 2_3 \end{pmatrix}
\] (9)

and

\[
(\bar{3}_1, \bar{3}_2, \bar{3}_3) \begin{pmatrix} 0 & \lambda a & 0 \\ -\lambda a & 0 & 0 \\ 0 & 0 & M \end{pmatrix} \begin{pmatrix} 3_1 \\ 3_2 \\ 3_3 \end{pmatrix}
\] (10)

The doublet matrix is rank-two leaving a single pair of light doublets \( H \equiv 2_{1H} \) and \( H' \equiv 2_{1H} \). All triplets get superheavy mass; however, there is no mixing between \( 3_{1H} \) and \( \bar{3}_{1H} \) that would permit the diagrams shown in figures 1 or 3.

There are several questions to be answered concerning the \( SO(10) \) approaches to the proton decay problem. (1) Can the VEVs in eqs (4) and (7) arise from an actual (super) potential?\(^{14}\) (2) Can \( SO(10) \) be broken all the way to \( SU(3) \times SU(2) \times U(1) \) without destabilizing these VEVs? and (3), are the threshold corrections in such an \( SO(10) \) model likely to be small enough not to vitiate the successful prediction of \( \sin^2 \theta_W \)? We will show in section V that the answer to all these questions is ‘yes’. But first we will examine the possibilities that exist in \( SU(5) \) and flipped \( SU(5) \).
3. Suppressing Proton Decay in SUSY SU(5)

The only viable method of doublet-triplet splitting in SUSY SU(5) that does not involve fine tuning of parameters is the ‘missing partner mechanism’.\(^9\)\(^{,10}\) The so-called sliding-singlet mechanism\(^11\) has the problem in SU(5) that radiative corrections destroy the gauge hierarchy.\(^12\) For the missing partner mechanism in SU(5) one requires (at least) the set \(5 + \bar{5} + 50 + \bar{50} + 75\) of Higgs supermultiplets. In the \(50(\bar{50})\) there is a color \(3(\bar{3})\) but no weak \(2(\bar{2})\). Thus the couplings \(\lambda_{5H\bar{5}0H} \langle 75_H \rangle + \lambda'_{5H50H} \langle 75_H \rangle\) give mass to the triplets in \(5_H + \bar{5}_H\) but not to the doublets. Schematically,

\[
\begin{pmatrix}
3 \\
2
\end{pmatrix}
\begin{pmatrix}
\bar{3} \\
\text{other}
\end{pmatrix}
\begin{pmatrix}
3 \\
2
\end{pmatrix}
\begin{pmatrix}
\bar{3} \\
\text{other}
\end{pmatrix}
\begin{pmatrix}
5_H \\
\bar{50}_H \\
50_H \\
\bar{5}_H
\end{pmatrix}
\]

(11)

The horizontal solid lines represent superheavy triplet-higgs(ino) masses arising from the \(\langle 75_H \rangle\). As in the cases considered in previous sections, there is the question of how to make the ‘other’ fields in the \(50 + \bar{50}\) superheavy. (They contribute to the RGE at one loop the same as a pair of weak doublets.) If one wanted a ‘strong suppression’ of proton decay, it would require giving superlarge masses to all the ‘other’ fields in \(50 + \bar{50}\) but having the \(\bar{3}(\bar{50}_H)\) and \(3(50_H)\) not be connected by a large Dirac mass term. There is no analogue of the missing partner mechanism that would accomplish this in a natural way. It could only be done by fine-tuning. For example, two different representations \((1_H\) and \(24_H,\) or \(24_H\) and \(75_H)\) could couple \(50\) to
50 and be relatively adjusted to give light mass only to the $3(50_H) + \bar{3}(50_H)$. However, we have foresworn fine-tuning.

It is possible to achieve a natural weak suppression of higgsino-mediated proton decay in $SU(5)$ by introducing an explicit mass term $M(\overline{50}_H 50_H)$ into the superpotential and having $M/M_G$ be of order but somewhat smaller than unity. This works as well as the weak suppression mechanism in $SO(10)$ discussed in section II. However, in $SU(5)$ there is the necessity of introducing the somewhat exotic high rank representations 50, $\overline{50}$ and 75, whereas in $SO(10)$ only the low rank representations 10, 45, and 54 are required. If one were willing to live with multiple fine-tunings of parameters one could do with just doubling the higgs sector in $SU(5)$ to $5 + \bar{5} + 5' + \bar{5}'$ as discussed briefly in the introduction. With two fine-tunings one could make all the triplets heavy, and achieve weak suppression of proton decay. With a third fine-tuning one could give mass to the extra pair of doublets and yet achieve strong suppression of proton decay. There are papers in the literature that take this approach.$^7,^8$

4. Suppressing Proton Decay in Flipped $SU(5)$

As is well-known, the missing partner mechanism works much more economically in flipped $SU(5)$\(^{15}\) than in ordinary $SU(5)$\(^{16}\). One requires for the mechanism the $SU(5) \times U(1)$ representations $5_{H}^{-2} + \bar{5}_{H}^{2} + 10_{H}^{1} + \overline{10}_{H}^{-1}$, and the superpotential couplings $\lambda 5_{H}^{-2} 10_{H}^{1} 10_{H}^{1} + \lambda'5_{H}^{2} \overline{10}_{H}^{-1} \overline{10}_{H}^{-1}$. The $10_{H}(\overline{10}_{H})$ contains a color $\overline{3}(3)$ but no color-singlet, weak-doublet components. The $10_{H}$ and $\overline{10}_{H}$ get VEVs that break $SU(5) \times U(1)$ down to $G_S$ and also mate
the triplet higgsinos) in the $5_H + \bar{5}_H$ with these in the $10_H + \bar{10}_H$, leaving the doublets in $5_H + \bar{5}_H$ light. Schematically,

$$
\begin{pmatrix}
3 \\
2
\end{pmatrix}
\begin{pmatrix}
\bar{3} \\
\text{other}
\end{pmatrix}
\begin{pmatrix}
3 \\
\text{other}
\end{pmatrix}
\begin{pmatrix}
\bar{3} \\
2
\end{pmatrix}
\parallel
\parallel
\parallel
\parallel
5_H 
10_H 
\bar{10}_H 
\bar{5}_H
\end{array} \right) (12)
$$

Another beautiful feature of flipped $SU(5)$ is that there is no necessity to do anything else to give mass to the ‘other’ fields in the $10_H + \bar{10}_H$: they are all disposed of by the (super) Higgs mechanism! They are either eaten or become superheavy with the gauge/gaugino particles. Thus, in flipped $SU(5)$ one can strongly suppress higgsino-mediated proton decay without any fine-tuning and without leaving any ‘extra’ split multiplets lighter than $M_{GUT}$. We found this to be impossible in ordinary $SU(5)$. However, there is one major drawback: the group of flipped $SU(5)$ is really $SU(5) \times U(1)$ and so real unification of gauge couplings is not achieved. One has therefore lost, or rather never had, the unification of gauge couplings as a prediction.

5. A More Detailed Examination of SO(10)

In section II certain ideas were discussed for solving the doublet-triplet splitting problem and for suppressing higgsino-mediated proton decay in $SO(10)$ that made essential use of specific patterns of VEVs, in particular those shown in eqs. (4) and (7). The question arises whether such VEVs are natural. In ref. 14 Srednicki wrote down a superpotential for a 45 and a 54 of Higgs that has both of these forms as possible supersymmetric minima. Let
us denote the 45 and 54 by $A$ and $S$ respectively. Then the most general $SO(10)$-invariant superpotential involving just these fields has the form

$$W(A, S) = m_1 A^2 + m_2 S^2 + \lambda_1 S^3 + \lambda_2 A^2 S.$$  \hspace{1cm} (13)

The equations for a supersymmetric minimum are

$$0 = F_A = 2(m_1 + \lambda_2 S) A$$

$$0 = F_S = (2m_2 S + \lambda_2 A^2 + 3\lambda_1 S^2).$$  \hspace{1cm} (14)

Suppose we choose $\langle S \rangle = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \otimes \text{diag}(s, s, s, -\frac{3}{2}s, -\frac{3}{2}s)$ ($S$ is a traceless rank-two symmetric tensor of $SO(10)$) and $\langle A \rangle = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \text{diag}(a, a, a, b, b)$. Then equation (14) gives two equations:

$$(m_1 - \frac{3}{2}\lambda_2 s)b = 0,$$

$$(m_1 + \lambda_2 s)a = 0.$$  \hspace{1cm} (15)

Either $a$ or $b$ or both must therefore vanish (if $s \neq 0$). There are therefore three possible solutions. (1) $b = 0, s = -m_1/\lambda_2$; (2) $a = 0, s = \frac{2}{3}m_1/\lambda_2$; (3) $a = b = 0$.

For doublet-triplet splitting and weak suppression of proton decay we need only the solution (1). For strong suppression we need (at least) two adjoints, one with VEV corresponding to solution (1) and the other to solution (2). (See eqs. (4) and (7)). We will examine this latter more complicated case in greater detail. If the required pattern of VEVs can be achieved in a realistic model for that case, then a fortiori the simpler requirements for weak
suppression can be achieved also. The main issues are whether the VEVs in eqs. (4) and (7) can be achieved, the group $SO(10)$ broken completely to $SU(3) \times SU(2) \times U(1)$, and goldstone particles avoided. (The issue of threshold corrections to $\sin^2 \theta_W$ is dealt with in Appendix B.)

To begin with we double the superpotential shown in equation (13). That is, we have two 45’s, denoted $A$ and $A'$, and two 54’s, denoted $S$ and $S'$, with superpotential

$$W(A, S; A', S') = m_1 A^2 + m_2 S^2 + \lambda_1 S^3 + \lambda_2 A^2 S$$

$$+ m_1' A'^2 + m_2' S'^2 + \lambda_1' S'^2 + \lambda_2' A^2 S'.$$

We aim to have the $(A, S)$ sector have VEVs in solution (1), and the $(A', S')$ sector have VEVs in solution (2).

The superpotential in (16) is certainly not enough because, for one thing, nothing determines the relative alignment of the VEVs of the two sectors, and so there are goldstone modes corresponding to a continuous degeneracy whereby the sectors are rotated in $SO(10)$-space with respect to each other.

A second problem with eq. (16) is that $\langle A \rangle$ and $\langle S \rangle$ break $SO(10) \rightarrow [SU(3) \times U(1)] \times SO(4)$, while $\langle A' \rangle$ and $\langle S' \rangle$ break $SO(10) \rightarrow SO(6) \times [SU(2) \times U(1) \times U(1)]$. Altogether, then, the unbroken group is $SU(3) \times SU(2) \times U(1) \times U(1) = \text{rank 5}$.

To break all the way to the standard model further Higgs fields are needed. (They are needed for right-handed neutrino masses in any case.) The simplest choices are $16 + \bar{16}$ or $126 + \bar{126}$. Let us call these $C + \bar{C}$ where $C = 16$ or 126. One can write a superpotential that gives $C + \bar{C}$ VEVs which break $SU(10) \rightarrow SU(5)$. Together with $A, S, A'$, and $S'$ this will complete the
breaking of $SO(10)$ to $G_S$ and allow $\nu_R$ masses.

At this point a further somewhat subtle technical problem ar ises. There are certain generators of $SO(10)$ that are broken both by $C + \bar{C}$ and by the adjoints $A$ and $A'$, but not by the symmetric tensors $S + S'$, specifically the generators in $SO(6)/SU(3) \times U(1)$ and $SO(4)/SU(2) \times U(1)$ (where $SO(10) \supset SO(6) \times SO(4)$). Thus to avoid residual goldstone bosons there must be coupling between the $C + \bar{C}$ sector and the adjoints $A$ and $A'$. The technical problem is that the direct couplings $\bar{C}AC$ and $\bar{C}A'C$ would destabilize the desired VEVs of $A$ and $A'$. In particular, all the ‘diagonal’ components of $\langle A \rangle$ and $\langle A' \rangle$ (written as $U(5)$ matrices, that is) become non-vanishing. [This is because the VEVs of $C + \bar{C}$ that break $SO(10) \rightarrow SU(5)$ couple in $\bar{C}AC$ to the $SU(5)$-singlet combination $(3a + 2b)$, where $\langle A \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \times \text{diag}(a, a, a, b, b)$. This leads to a mass term proportional to $(3a + 2b)^2$ in the ordinary scalar potential, which in turn leads to cross-terms of the form $ab$, destabilizing the solution $a \neq 0$ and $b = 0$. The same thing would happen to $A'$.]

There are various solutions to this technical difficulty. The one we shall study here involves the introduction of a third adjoint, which we will denote $A''$, that serves as an intermediary between the $C + \bar{C}$ sector and $A$ and $A'$.

The part of the superpotential that does the complete breaking to the standard model and obviates all difficulties is given in full by
\[ W = m_1 A^2 + m_2 S^2 + \lambda_1 S^3 + \lambda_2 A^2 S + m'_1 A'^2 + m'_2 S'^2 + \lambda'_1 S'^3 + \lambda'_2 A'^2 S' + m''_1 A''^2 + m''_2 \bar{C} C + \lambda''_2 C A'' C + \lambda AA' A'' \]  

There are three sectors, \((A, S), (A', S'),\) and \((A'', \bar{C} + C),\) that are coupled together only by the last term \(\lambda AA' A''.\)

The term \(\lambda''_2 \bar{C} A'' C\) does serve to give \(A''\) a VEV in the \(SU(5)\)-singlet direction:

\[
\langle A'' \rangle = \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \otimes \text{diag}(a'', a'', a'', a'', a'')
\]

But this does not destabilize the VEVs of \(A\) and \(A'\) which are assumed to be of the forms given in eqs. (4) and (7). This is easily seen by examining the \(\lambda AA' A''\) term, which is the only thing linking \(A''\) to \(A\) and \(A'.\) Consider the \(F_A = 0\) equation. \((F_A)^{[ab]}\) is an antisymmetric tensor to which \(\lambda AA' A''\) contributes \(\lambda (A'A'')^{[ab]}\), which vanishes when the values of \(\langle A' \rangle\) and \(\langle A'' \rangle\) given in eqs (7) and (18) are substituted. Similarly, at the desired minimum, \(\lambda AA' A''\) gives no contribution to the \(F_A'\) and \(F_A''\) equations. In other words it can be neglected in doing the minimization! However, it does contribute to the Higgs(ino) masses, and, indeed, removes all of the possible goldstone modes discussed above.

In Appendix A we present details of the minimization of the superpotential, eq. (17), assuming \(C = 16.\) There we show explicitly that \(SO(10)\) may be completely broken to the standard model (without breaking SUSY),
uneaten goldstone bosons avoided, and VEVs of the desired form achieved. The masses of the various Higgs (super) multiplets enumerated in Appendix A will be used to estimate threshold corrections in the model.

It is conceivable that the superpotential of eq. (17) is the most general one compatible with some discrete symmetries, although due to the supersymmetric non-renormalization theorem one is not obliged to write all possible terms.

We have found other superpotentials and sets of Higgs fields that allow us to achieve the desired VEVs in a consistent and realistic way. We have presented eq. (17) as being algebraically simple to analyze. It should also be noted that the implementation of weak suppression of HMPD, where only a single $45_H$ is needed with VEV of the form in eq. (4), is a simpler task and fewer fields are required. We have not tried to find the absolutely minimal scheme.

At this point we wish to make an aside. If one is willing to give up $\sin^2 \theta_W$ as a prediction, there is a much simpler way to simultaneously achieve doublet-triplet splitting without fine-tuning and a strong suppression of higgsino mediated proton decay. All we need is then one $45_H$ of Higgs superfield with its VEV as given in eq. (4). Suppose the relevant superpotential term for doublet-triplet splitting is just $\lambda 10_H 45_H 10_{2H}$ as in eq. (5). This will make the color triplets heavy, but one is left with 2 pairs of light doublets. Now, if the $45_H$ does not couple to the sector that breaks $SO(10) \to SU(5)$ (via $C + \bar{C}$ superpotential), then in addition to the extra pair of doublets, one will have a \{$(3, 1; \frac{2}{3}) + H.C.$\} goldstone super-multiplet which remains light. (These are the goldstones corresponding to $SO(6)/SU(3) \times U(1)$ mentioned
earlier.) The combined effect of having an extra pair of light higgs doublets and the charge \( \frac{2}{3} \) higgs (super) fields is to alter \( \sin^2 \theta_W \) prediction to \( \approx 0.215 \) at one loop. The unification scale also comes down by an order of magnitude or so. If one ‘fixes’ these features by relying on particle thresholds, such a scenario may not be inconsistent. This scenario can be tested by directly searching for the \((3, 1, \frac{2}{3})\) - higgs and higgsino particles. This situation is somewhat analogous to the case studied in ref. 17. We do not advocate this scenario here, since our aim is to preserve the successful unification of couplings as a prediction.

Returning to the superpotential in eq. (17), it might be imagined that with \( 3(45) + 2(54) + \overline{16} + 16 \) the threshold corrections might be fairly large; large enough, perhaps to vitiate the successful ‘prediction’ of \( \sin^2 \theta_W \). Actually, this is not the case, especially if one assumes \( SO(10) \) breaks in two stages to the standard model: \( SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \). \( SO(10) \) is broken to \( SU(5) \) by the VEVs of \( C, \overline{C} \), and \( A'' \) at a scale \( M_{10} \), while \( SU(5) \) is broken down to \( SU(3) \times SU(2) \times U(1) \) by the VEVs of \( A, S, A' \), and \( S' \) at a scale \( M_5 \). The masses of particles will be of the form \( \alpha M_{10} + \beta M_5 \). In the limit that \( \beta M_5/\alpha M_{10} \rightarrow 0 \) for a given multiplet its one-loop threshold corrections to \( \sin^2 \theta_W \) will vanish since it will become a complete and degenerate \( SU(5) \) multiplet. Thus threshold corrections of complete \( SU(5) \)-multiplets go as \( \ell n(\alpha M_{10} + \beta M_5)/(\alpha M_{10}) \cong \beta M_5/\alpha M_{10} \) for \( M_5 \ll M_{10} \). Thus if \( M_{10} \) is assumed to be somewhat larger than \( M_5 \), the GUT-scale threshold corrections to \( \sin^2 \theta_W \) are substantially reduced. These will be discussed explicitly in Appendix B, where it is found that the uncertainties in \( \sin^2 \theta_W \) due to superheavy thresholds is typically in the range of \( 3 \times 10^{-3} \) to \( 10^{-2} \).
6. Conclusions

If one seeks a supersymmetric grand unified model in which proton decay mediated by color-triplet higgsino is strongly suppressed through a mechanism based on symmetry, in which there is no fine-tuning of parameters, and in which the remarkable prediction of $\sin^2 \theta_W$ is maintained as a prediction, then it seems that one must turn to $SO(10)$. On the other hand, a weak suppression due not to symmetry but to the smallness of a parameter is achievable in both $SU(5)$ and $SO(10)$ without either fine-tuning or sacrificing the $\sin^2 \theta_W$ prediction, though we believe $SO(10)$ allows the more economical solution. The $SU(5)$ solution, being based on the ‘missing-partner mechanism’, requires the introduction of Higgs(ino) multiplets in $50 + \overline{50} + 75$ (which are four and five index tensors), whereas the $SO(10)$ solution requires only the usual (rank one and two) tensors 45, and 54, and the spinors $16 + \overline{16}$.

The advantage of $SO(10)$ is due to the possibility of exploiting the elegant Dimopoulos-Wilczek mechanism of doublet-triplet splitting. We have studied that mechanism and found that it can be implemented in a fully realistic way.

In our view these results constitute yet another argument in favor of $SO(10)$. It is already well known that $SO(10)$ has the advantage over $SU(5)$ of allowing $R$-parity to be a gauge symmetry (that is because Higgs fields are in tensor representations and matter fields are in spinor representations). And, of course, $SO(10)$ achieves greater unification of quarks and leptons and requires the existence of right-handed neutrinos.

In any event, we have shown that higgsino-mediated proton decay is not a serious difficulty of supersymmetric grand unification as there are quite
simple and natural means to suppress it without undercutting the main success of those models. If the suppression is of the ‘weak’ type then there are grounds to hope to see proton decay in super Kamiokande.
Appendix A

In this Appendix, we give details of the minimization of the superpotential of eq. (17). We shall see explicitly that (a) $SO(10)$ breaks completely to the standard model in the supersymmetric limit, (b) the desired forms of the VEVs of $A$ and $A'$ are achieved, and (c) there are no unwanted pseudo goldstone modes which could potentially ruin the successful $\sin^2 \theta_W$ prediction.

$A^2$ in eq. (17) denotes $Tr(A^2)$, $AA'A''$ denotes $Tr(AA'A'')$ etc. We shall confine ourselves to the case where $C + \bar{C} \equiv 16 + \bar{16}$. The term $CA''C$ is explicitly written down as $\bar{C}\sigma_{\alpha\beta}A''_{\alpha\beta}C/4$, where $\sigma_{\alpha\beta}$ are the generators of $SO(10)$ algebra.\(^{18}\)

The VEVs for the fields are chosen as

\[
\begin{align*}
\langle A \rangle &= \eta \otimes \text{diag}(a, a, a, 0, 0) \\
\langle A' \rangle &= \eta \otimes \text{diag}(0, 0, 0, a', a') \\
\langle A'' \rangle &= \eta \otimes (a'', a'', a'', a'', a'') \\
\langle S \rangle &= I \otimes \text{diag}(s, s, s, -\frac{3}{2}s, -\frac{3}{2}s) \\
\langle S' \rangle &= I \otimes \text{diag}(s', s', s', -\frac{3}{2}s', -\frac{3}{2}s') \\
\langle C \rangle &= \langle \bar{C} \rangle = c
\end{align*}
\]

where $\eta \equiv \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. The equality $\langle C \rangle = \langle \bar{C} \rangle$ follows from the requirement of D-flatness. The vanishing of the F–terms lead to the following conditions, corresponding to $A, S, A', S', A'', C$ fields respectively:
\begin{align*}
0 &= m_1 + \lambda_2 s \\
0 &= m_2 s - \frac{3}{4} \lambda_1 s^2 - \frac{1}{5} \lambda_2 a^2 \\
0 &= m'_1 - \frac{3}{2} \lambda'_2 s' \\
0 &= m'_2 s' - \frac{3}{4} \lambda'_1 s'^2 + \frac{1}{5} \lambda'_2 a'^2 \\
0 &= m''_1 a'' + \frac{\lambda''_1}{8} c^2 \\
0 &= m''_2 - \frac{5}{2} \lambda''_2 a''.
\end{align*}

(A.2)

Since SUSY is unbroken, it is sufficient to investigate the Higgsino mass spectrum. The multiplets which transform as \((3, 1, 2/3) + H.C.\) under \(SU(3)_C \times SU(2)_L \times U(1)_Y\) have the following mass matrix:

\[
\mathcal{M}_1 = \begin{pmatrix}
0 & 2\lambda a'' & 0 & 0 \\
2\lambda a'' & -10\lambda'_2 b' & -2\lambda a & 0 \\
0 & -2\lambda a & -4m''_1 & -\lambda''_2 c \\
0 & 0 & -\lambda''_2 c & 2\lambda''_2 a''
\end{pmatrix}.
\]

(A.3)

This matrix has one zero eigenvalue by virtue of eq. (A.2). All the other 3 states become massive.

The mass matrix corresponding to \((1, 1, +1) + H.C.\) is
\[ M_2 = \begin{pmatrix} 10\lambda_2 b & 2\lambda a'' & -2\lambda a' & 0 \\ 2\lambda a'' & 0 & 0 & 0 \\ -2\lambda a' & 0 & -4m''_1 & -\lambda_2 c \\ 0 & 0 & -\lambda_2 c & 2\lambda''_2 a'' \end{pmatrix} \]  \hspace{1cm} \text{(A.4)}

Again, \( M_2 \) has one zero eigenvalue (using (A.2)) and three non-zero eigenvalues.

The mass matrix for \( \{(3, 2, -\frac{5}{6}) + H.C.\} \) is given by

\[ M_3 = \begin{pmatrix} 5\lambda_2 s & \sqrt{2}\lambda_2 a & 0 & 0 & -\lambda a' \\ \sqrt{2}\lambda_2 a & 2m_2 - \frac{3}{2}\lambda_1 s & 0 & 0 & 0 \\ 0 & 0 & -5\lambda'_2 s' & \sqrt{2}\lambda'_2 a' & -\lambda a \\ 0 & 0 & \sqrt{2}\lambda'_2 a' & 2m'_2 - \frac{3}{2}\lambda'_1 s' & 0 \\ -\lambda a' & 0 & -\lambda a & 0 & -4m''_1 \end{pmatrix} \]  \hspace{1cm} \text{(A.5)}

Using (A.2) one sees that \( M_3 \) has one of its eigenvalues equal to zero, while the rest are all nonzero.

The corresponding matrix for \( \{(3, 2, \frac{1}{6}) + H.C.\} \) is given by
This has one zero and five nonzero eigenvalues.

The goldstone modes in \( \mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3, \mathcal{M}_4 \) when combined with the zero mass mode corresponding to the phase of \((C + \bar{C})\)-singlet add up to the 33 mass–less modes needed for the symmetry breaking \( SO(10) \rightarrow SU(3) \times SU(2) \times U(1) \). All the remaining fields become massive. Their spectrum looks as follows. From the \((A, S)\) sector, we have

\[
\left\{(6, 1, -\frac{2}{3}) + H.C.\right\} = 2m_2 + 6\lambda_1s
\]

\[
\left\{(1, 3, \pm 1); (1, 3, 0)\right\} = 2m_2 - 9\lambda_1s
\]

\[
\left\{(1, 3, 0) + (1, 1, 0)\right\} = 10\lambda_2s
\]

\[
\left\{(8, 1, 0)\right\} = \begin{pmatrix}
0 & 2\sqrt{2}\lambda_2 a \\
2\sqrt{2}\lambda_2 a & 2m_2 + 6\lambda_1s
\end{pmatrix}
\]
\[(1, 1, 0) = \begin{pmatrix} 0 & -\frac{4}{\sqrt{5}}\lambda_2 a \\ -\frac{4}{\sqrt{5}}\lambda_2 a & 2m_2 - 3\lambda_1 s \end{pmatrix} \).

(A.7)

From the \((A', S')\) sector, one finds

\[
\{(6, 1, -\frac{2}{3}) + H.C.; (8, 1, 0)\} = 2m'_2 + 6\lambda_1 s'
\]

\[
\{(8, 1, 0); (1, 1, 0)\} = -4\lambda'_2 s'
\]

\[
\{(1, 3, \pm1)\} = 2m'_2 - 9\lambda'_1 s'
\]

\[
\{(1, 3, 0)\} = \begin{pmatrix} 0 & -2\sqrt{2}\lambda'_2 a' \\ -2\sqrt{2}\lambda'_2 a' & 2m'_2 - 9\lambda'_1 s' \end{pmatrix}
\]

\[
\{(1, 1, 0)\} = \begin{pmatrix} 0 & \sqrt{\frac{24}{5}}\lambda'_2 a' \\ \sqrt{\frac{24}{5}}\lambda'_2 a' & 2m'_2 - 3\lambda'_1 s' \end{pmatrix}.
\]

(A.8)

Finally, from the \((A'', C + \bar{C})\) sector, one finds

\[
\{(8, 1, 0) + (1, 3, 0) + (1, 1, 0)\} = -4m''_1
\]

\[
\{(3, 1, -\frac{1}{3}) + H.C.; (1, 2, \frac{1}{2}) + H.C.\} = 2m''_2 + 3\lambda''_2 a''
\]

\[
\{(1, 1, 0)\} = \begin{pmatrix} -4m''_1 & -\sqrt{\frac{3}{2}}\lambda''_2 c \\ -\sqrt{\frac{3}{2}}\lambda''_2 c & m''_2 - \frac{5}{2}\lambda''_2 a'' \end{pmatrix}
\]

(A.9)

Now to reduce threshold corrections somewhat (and simplify calculations)
we assume the scale of \(SO(10)\) breaking, \(M_{10}\), is somewhat greater than
the scale of \(SU(5)\) breaking, \(M_5\). (This means we assume \(m''_1, m''_2, c, a'' \gg\)
$m_1, m_2, m_1', m_2', a, a', s, s'$.) As explained in the text, the multiplets which have mass $O(M_{10})$ will give contributions to the threshold corrections suppressed by $M_5/M_{10}$. Thus in Appendix B we will only need the masses of particles which are $O(M_5)$. For sets of particles with the same $G_S$ quantum numbers we only will need to know the products of their masses. These are listed below.

\[
\begin{align*}
\{(3, 2, \frac{1}{6}) + h.c.\} &= (\frac{4}{25})\lambda_2\lambda'_2 a^2 a'^2 / ss' \quad (2\text{ states}) \\
\{(3, 2, -\frac{5}{6}) + h.c.\} &= (\frac{4}{25})(\lambda^2\lambda_2\lambda'_2 / \lambda''(a^2 a'' / ss' c^2) \\
&[2aa'(a + a') + 25(a's^2 + as'^2)] \quad (3\text{ states}) \\
\{(6, 1, -\frac{2}{3}) + h.c.\} &= (\frac{15}{2})\lambda_1 s + (\frac{7}{5})\lambda_2 a^2 / s \quad (1\text{ state}) \\
\{(1, 3, \pm1); (1, 3, 0)\} &= - (\frac{15}{2})\lambda_1 s + (\frac{7}{5})\lambda_2 a^2 / s \quad (1\text{ state}) \\
\{(1, 3, 0)\} &= 10\lambda_2 s \quad (1\text{ state}) \\
\{(8, 1, 0)\} &= 8\lambda_2^2 a^2 \quad (2\text{ states}) \\
\{(6, 1, -\frac{2}{3}) + h.c.; (8, 1, 0)\} &= (\frac{15}{2})\lambda'_1 s' - (\frac{2}{5})\lambda'_2 a'^2 / s' \quad (1\text{ state}) \\
\{(8, 1, 0)\} &= - 4\lambda'_2 s' \quad (1\text{ state}) \\
\{(1, 3, \pm1)\} &= - (\frac{15}{2})\lambda'_1 s' - (\frac{2}{5})\lambda'_2 a'^2 / s' \quad (1\text{ state}) \\
\{(1, 3, 0)\} &= 8\lambda'_2^2 a'^2 \quad (2\text{ states}) \\
\end{align*}
\]

We have expressed these in terms of the VEVs and Yukawa couplings and eliminated the mass parameters, $m_1, m_2$, etc., using eqs. (A.2).
Appendix B

Here we use the results of Appendix A to compute the threshold corrections to $\sin^2 \theta_W$ coming from superheavy fields. We look at the full model with three 45’s ($A, A', A''$) and two 54’s ($S, S'$), and we assume $SO(10)$ breaks to $SU(5)$ at a scale ($M_{10}$) which is higher than the scale at which $SU(5)$ breaks to the standard model ($M_5$).

It will prove convenient to define the parameters $M \equiv \lambda_2 s$, $x \equiv a/s$, $x' \equiv a'/s'$, $y \equiv \lambda_1/\lambda_2$, $y' \equiv \lambda'_1/\lambda'_2$, and $z \equiv \lambda'_2 s'/\lambda_2 s$. The correction to $\sin^2 \theta_W$ is given at one loop by

$$\Delta \sin^2 \theta_W(M_Z) = \frac{\alpha(M_Z)}{30\pi} \sum_j \left[5b'_1 - 12b'_2 + 7b'_3\right] \ln M_j$$

where the sum is taken over multiplets of $SU(3) \times SU(2) \times U(1)$. From eqs. (A.10) one obtains

$$\Delta \sin^2 \theta_W(M_Z) = \frac{\alpha(M_Z)}{30\pi^2} \left\{-21 \ln(\frac{4}{25} x^2 x'^2 z M^2) + 3 \ln(t M^3) + 51 \ln((\frac{15}{2} y + \frac{2}{5} x^2) M) + (-30 - 24) \ln((-\frac{15}{2} y + \frac{2}{5} x^2) M) - 24 \ln(10 M) + 21 \ln(8 x^2 M^2) + (51 + 21) \ln((\frac{15}{2} y' - \frac{2}{5} x'^2) z M) + 21 \ln(4 z M) - 30 \ln((-\frac{15}{2} y' - \frac{2}{5} x'^2) z M) - 24 \ln(8 x'^2 z^2 M^2)\right\}.$$  

(Here $t M^3$ is defined to be equal to the complicated expression on the right-hand side of the second equation of (A.10). $t$ is of order $M_5/M_{10}$. However,
the coefficient of this term is mercifully small, so that its effect is negligible. We have not displayed the threshold effects due to the doublet and triplet fields of eqs. (9)-(10) as they are negligible.) All logarithms are understood to have an absolute value in their arguments. We have used \( b_3(3) = \frac{1}{2}, \ b_3(6) = \frac{5}{2}, \ b_3(8) = 3, \ b_2(2) = \frac{1}{2}, \ b_2(3) = 2, \ b_1(\frac{y}{2}) = \frac{3}{2}(y/2)^2. \)

Collecting terms,

\[
\Delta \sin^2 \theta_W(M_Z) \cong \frac{\alpha(M_Z)}{30\pi} \left\{ (18 \ln 5 - 33 \ln 2) + 3 \ln t - 6 \ln z - 45 \ln(x^2) 
+ 51 \ln\left(\frac{15}{2}y + \frac{2}{5}x^2\right) - 54 \ln\left(\frac{15}{2}y - \frac{2}{5}x^2\right)
+ 72 \ln\left(\frac{15}{2}y' - \frac{2}{5}x'^2\right) - 30 \ln\left(\frac{15}{2}y' + \frac{2}{5}x'^2\right)
- 3 \ln M \right\}.
\]

The term \(-3 \ln M\) is due to the \((3, 2, -\frac{2}{3}) + h.c.\) that get eaten when \(SU(5)\) breaks and is present also in the minimal \(SU(5)\) model. If it were not for this eating, all the superheavy multiplets would be complete \(SU(5)\) multiplets and the dependence on \(M\) would disappear. (It is only splitting within multiplets that contribute at one loop to \(\sin^2 \theta_W\). Since all masses are scaled by \(M\), \(M\) drops out in the ratios.) In the first term, \(18 \ln 5 - 33 \ln 2 \cong 6.1\). All of the logarithms can have either sign. There are five potentially large terms with coefficients averaging about 50. If we assume the logarithms are of order one with arbitrary signs then the typical threshold correction to \(\sin^2 \theta_W\) would be expected to be about \(\pm(\alpha/30\pi)(10^2) \sim \pm10^{-2}\). This is to be contrasted with the effect of a pair of extra light Higgs doublets of \(\Delta \sin^2 \theta_W \cong 2.5 \times 10^{-2}\). It should also be compared to the theoretical uncertainties in \(\sin^2 \theta_W\) in minimal
SUSY $SU(5)$, referred to in the opening paragraph of this paper, of about $0.36 \times 10^{-2}$.

The expression in (B.2) simplifies considerably if we assume $y < \frac{4}{75} x^2$, $y' < \frac{4}{75} x'^2$. Then

$$
\Delta \sin^2 \theta_W \simeq \left( \frac{\alpha}{30\pi} \right) \{ 3 \ln t - 3 \ln(x^2) - 3 \ln(x'^2) \\
- 6 \ln z + (6 \ln 2 - 21 \ln 5) \}.
$$

(E.4)

Since $6 \ln 2 - 21 \ln 5 = -29.4$ one expects the threshold correction to be negative and about $-0.3 \times 10^{-2}$ in this limit. We mention this limit since it is a special solution of the superpotential of eq. (17), corresponding to setting the parameters $\lambda_1$, $\lambda'_1$ to zero. We note that this limit can be reached naturally without giving rise to any pseudo goldstones (see Appendix A).

We conclude that the threshold correction uncertainties to $\sin^2 \theta_W$ in the kind of $SO(10)$ model we are discussing are likely to be a few times larger than the total theoretical uncertainty in $\sin^2 \theta_W$ in minimal SUSY $SU(5)$, but several times smaller than the effect on $\sin^2 \theta_W$ of an extra pair of light Higgs doublets. We should emphasize that if one is satisfied only to suppress Higgsino mediated proton decay weakly, a much smaller Higgs sector may be adequate, with correspondingly smaller threshold corrections.
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Figure Captions

1. A diagram that gives a dimension 5 baryon-number-violating operator. The arrows indicate the direction a left-handed chiral superfield is flowing. Chirality shows this to be an $F$ term and hence suppressed by $(\text{Mass})^{-1}$. Chirality also implies that there must be a mass insertion (denoted by $M$) on the colored higgsino line. The suppression is thus $M/M_{\text{GUT}}^2$ which is naturally of order $M_{\text{GUT}}^{-1}$ in most models.

2. A diagram without the chirality-flipping mass insertion of Fig. 1 and thus representing a $D$ term. Such a term is effectively of dimension 6 and suppressed by $M_{\text{GUT}}^{-2}$. It corresponds to colored higgs boson exchange.

3. In the ‘weak suppression’ scheme discussed in section I (see eq. (2)) the primed and unprimed Higgs(ino) sectors are connected by a Dirac mass, $M$. This coupling of the two sectors allows a dimension-5, baryon-number-violating operator to arise as shown here.