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On the Comparison of Matrix Element Calculations of $O(\alpha\alpha_s)$ with the Measurement of Photon Emission in Hadronic Z^0 Decays

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Abstract

The uncertainties in interpreting photon bremsstrahlung in the process $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$ with matrix element calculations of $O(\alpha\alpha_s)$ are discussed. We address the stability of the calculations with respect to the emission of collinear photons and to higher-order QCD corrections and discuss the bias due to experimental photon isolation cuts. We analyze the resulting uncertainties for various procedures to define an event with a final state photon. Of particular interest are (i) a two-step procedure where first jets are reconstructed from hadrons alone and in a second step the photon is required to be isolated from these jets, and (ii) a 'democratic' procedure where the photon is included in the jet reconstruction but a certain maximum hadronic energy is allowed in the photon jet. In both cases we estimate that the uncertainties of the theoretical predictions, hadronization effects and the experimental photon isolation are of the order of 4%. To obtain this level of accuracy, however, the democratic procedure requires very hard cuts that reduce the event samples significantly.

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1 Introduction

The large yield of hadronic Z^0 decays at the e^+e^- collider, LEP, and the possibility of identifying photon emission from quarks with a high purity offer a wide range of physics studies. Photons serve as probes for the QCD quark evolution [1], they enrich up-type quarks in the event sample thus allowing a measurement of quark couplings [2], or they may signal new kinds of Z^0 decays and the need for an extension of the standard model [3].

The potential reward from analyzing final state photon production depends crucially on the precision of the theoretical prediction and the uniqueness of relating these predictions to the data. First analytical results of a calculation in $\mathcal{O}(\alpha\alpha_s)$ have been published by Kramer and Lampe [4]. Recently, matrix element calculations of $\mathcal{O}(\alpha\alpha_s)$ have been cast into a Monte Carlo form [5, 6, 7]. These Monte Carlo calculations allow a flexibility in the definition of an event with a final state photon and provide an unambiguous theoretical basis avoiding model assumptions used in the popular QCD shower Monte Carlos. However, this is at the expense of providing only the differential distribution of the photon and of up to three partons instead of a complete event description. This lack of completeness introduces uncertainties in their relation to actual measurements of the process $e^+e^- \rightarrow Z^0 \rightarrow \gamma + \text{hadrons}$. These uncertainties will be discussed in this paper.

We start our discussion with some general considerations for the definition of a photon event which can be adopted by both theory and data. For an event definition which has been applied previously in experimental analyses [8, 9] we then discuss in detail in sections 3 and 4 the problems and uncertainties in relating theory to data. In section 3 we describe the ingredients of the matrix element calculations. The use of QCD shower models to relate the hard parton dynamics to the fully hadronized events actually measured is addressed in section 4. In sections 5 to 7 we analyze the stability of the matrix element calculation and the correction procedure for alternative event definitions. In section 8 we compare the uncertainties of these approaches. Conclusions will be given in section 9. We include in appendix A.1 a discussion of the phase space regions for $q\bar{q}\gamma$ final states selected by the main event definitions and in appendix A.2 details of photon isolation cuts as implemented in the matrix element Monte Carlo GNJETS [5].

Throughout this discussion we will completely neglect any uncertainty of the analysis due to detector effects. Those are discussed in the experimental literature (see e.g. discussion in reference [10]).

2 What is a Photon Event?

Due to both experimental and theoretical limitations it is impossible to determine the *total* yield of photons radiated from quarks without imposing cuts on photon properties restricting the measurement to a certain phase space region. These cuts lead to ambiguities as to what constitutes an event with a final state photon, and require a proper definition of a photon event.

Criteria for a good event definition are (compare [11])

- (i) It has to be unambiguously defined at the hadron level¹. Furthermore, for practical

¹The term 'hadron level' is used if the properties of photon events are reconstructed from the hadrons as either observed in the data or as simulated with a QCD model. In contrast, the term 'parton level' is used if properties are derived from the partons of a QCD matrix element calculation or from the partons at the end of a QCD cascade in a shower model.

purposes it should allow for a reasonable event yield and a high purity of photons. It should be possible to understand both the selection efficiency and the purity.

- (ii) The event definition should also be unambiguously defined at the parton level of the matrix element calculation and should be safe against singularities of photon and gluon emission.
- (iii) The relation between the matrix element calculation with, for $\mathcal{O}(\alpha_s)$, up to three partons and the hadron level with typically 30 hadrons should imply small uncertainties.
- (iv) It should allow the physics issues, mentioned in the introduction, to be easily covered. For example, to determine electroweak quark couplings or in some searches for exotic sources of photon production one has to know the absolute cross section to a high precision only in some well defined phase space region. For QCD studies, where one compares properties of the inclusive multihadronic Z^0 decays with those of photonic decays, it is desirable to use identical variables for both processes like the scaled mass of two particles, $y_{ij} = M_{ij}^2/E_{cm}^2$ used as a jet-separation measure in inclusive distributions.

With respect to (i), all LEP experiments [8, 9, 12, 13] apply similar cuts to minimize background from hadronic sources and to retain a decent efficiency. In all experimental analyses a minimum photon energy E_γ^{min} is required, and an isolation cut is imposed demanding no or little energy within a cone of opening angle θ_γ^{iso} around the photon candidate. The cuts used up to now are E_γ^{min} between 5 and 10 GeV and θ_γ^{iso} between 15 and 20 degrees. Only very little experimentally observed energy is allowed in the isolation cone.

One of the main ambiguities in the interpretation of the measurements is related to the asymmetry between photons and partons. Whereas the photon is a fundamental particle in both theory and data, the partons of the theory correspond more closely to jets rather than individual particles in the data. This leads to various possible event definitions, some of which are summarized in Fig. 1. In addition, if combined with the experimental isolation criterium, the different treatment of hadrons and photons leads to some subtle problems with respect to the photon-parton singularities.

Some of the LEP experiments [8, 9, 13] adopt a two-step procedure for an event definition involving a jet algorithm (branch A in Fig. 1). Since the hadrons in the observed events can only originate from partons, it seems natural to reconstruct jets from the hadrons alone, i.e. *excluding* the photon. This is done with the JADE algorithm [14] which is based on an iterative combination of pairs of partons and hadrons into jets as long as they have a mass y_{ij} below some predefined cut y_{cut} . In a second step the photon is required to be isolated from the jet by either demanding a minimum $y_{\gamma,jet}$ (A1) or a minimum $\theta_{\gamma,jet}$ (A2).

Another approach is to view photon emission as part of the showering process. This suggests jets should be determined in a 'democratic' manner (B) from all final particles *including* the photon. In this case one has to introduce a parameter ϵ_0 for the maximum hadronic energy E_{had} allowed inside the jet containing the photon ('photon jet'). It is convenient to express this energy relative to the photon energy. An event is accepted if

$$\frac{E_{had}}{E_\gamma} = \epsilon_{had} < \epsilon_0 \quad (1)$$

Finally, instead of reconstructing jets, one can just apply an energy and isolation cut (C), allowing a certain amount of hadronic energy inside the isolation cone θ_C . Again, an additional cut on the fraction of hadronic energy inside the cone, ϵ_{had}^C , is required.

In all these cases the final step of the data analysis has to be the correction for hadronization. Of foremost importance is the correction for losses due to hadrons leaking into the isolation cone. Even if all jets or partons are far away from the photon, the hadrons produced in the QCD showering process may come close to the photon such that the event is experimentally rejected.

In the following we will discuss the main steps and uncertainties of the analysis using the event definition A1. We will see that the results obtained with definition A2 are very similar to those of event definition A1. In sections 6 and 7 we will address the particular aspects of definitions B and C.

For the rest of this paper we will assume as experimental cuts $E_\gamma^{min} = 7.5$ GeV and no additional energy within a cone defined by $\theta_\gamma^{iso} = 15$ degrees. For the jet finding we will group hadrons into jets using the JADE algorithm based on $y_{ij} = M_{ij}^2/E_{cm}^2$. For most of the paper we use the E0 scheme with $M_{ij}^2 = 2E_i E_j (1 - \cos \alpha)$ [14], E_i the energy of each particle, and α the opening angle between particles i and j . The DURHAM scheme with $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos \alpha)$ [15] will be addressed in sections 4.5 and 7.3. We use the GNJETS Monte Carlo [5] as the basis of our analysis with detailed comparisons of their results from the calculations of Glover and Stirling [6] (EEPRAD) and of Kunszt and Trocsanyi [7] (KT) in section 3.4.

3 Matrix element calculation

In a matrix element calculation up to $\mathcal{O}(\alpha_s)$, the cross section for *isolated* photon production in Z^0 decays is a combination of the partonic cross sections for

$$e^+e^- \rightarrow Z \rightarrow q\bar{q}\gamma, \quad (2)$$

and

$$e^+e^- \rightarrow Z \rightarrow q\bar{q}g\gamma. \quad (3)$$

In the following discussion any photons from the incoming electrons are neglected. As shown in [1, 2], on the Z^0 their direct contribution and their interference with final state photons is strongly suppressed by the standard experimental cuts on the photon energy and its angle with respect to the beam axis.

Since the partonic cross sections are singular for non-isolated photons, the contribution from these configurations is infinite in perturbation theory. A theoretically well-founded prescription to avoid these collinear divergences has been formulated in the context of direct photon production in hadron colliders [16, 17] and recently also for e^+e^- annihilation [7]. The approach involves subtracting the divergences in the perturbative calculation and replacing them by a photon fragmentation function. The latter describes effects which cannot be derived in the framework of perturbation theory (although one may estimate them with the help of resummation techniques) but have to be determined from experimental data.

Instead of assuming a model of the fragmentation function one may simply neglect it at the expense of introducing cuts in the phase space for photon emission which exclude the singular contribution.

The recently published calculations use two different kinds of cut-offs:

- a minimum photon energy E_γ^{min} and an isolation angle θ_γ^{min} of the partons with respect to the photon, or

- a minimum scaled mass $y_{\gamma q}$, $y_{\gamma \bar{q}}$ combining the energy and angle requirement in one variable, called y_γ in the following.

These prescriptions imply that the calculations become sensitive to cut-off parameters which cannot be imposed in the experimental analysis since they are defined at the parton level. These cut-off parameters play the role of a factorization scale defining the separation of the calculation into a perturbative and a non-perturbative part [7]. Thus, the variations of the theoretical predictions with these partonic isolation cuts can be understood as an indication for the size of non-perturbative contributions which would be described in a complete calculation with the help of a photon fragmentation function.

Ideally the cuts applied at the hadron level should exclude those phase space regions which are affected by the partonic cuts of the matrix element calculation and consequently reduce non-perturbative contributions to a negligible level. However, this is not necessarily true as can be seen by considering the two-step treatment in event definition A1. As an example we show in Fig. 2 a $q\bar{q}\gamma$ event where $y_{\gamma q} \ll y_{cut}$, i.e. where the photon is not isolated from the quark. Nevertheless, if $y_{q\bar{q}} < y_{cut}$, the quark will be recombined according to definition A with the anti-quark into one jet $j = (q\bar{q})$ and the resulting invariant mass $y_{j\gamma}$ is larger than y_{cut} . Thus the event is accepted as a $1jet + \gamma$ event. With an event definition that relies on the reconstruction of jets, there is no way to get rid of this type of events with the help of cuts at the hadron level. It follows that the theoretical prediction is sensitive to the partonic isolation cut-off and therefore one has to expect also a sensitivity to non-isolated photon emission.

In this section we will particularly address the choice of parameter ranges for isolation cut-offs at the parton level, i.e. the sensitivity to non-isolated photons. They are a major source of uncertainty in the comparison of matrix element calculations with data. We start by describing in sections 3.1 and 3.2 the general procedure of implementing photon and gluon radiation into a matrix element Monte Carlo. In sections 3.3 and 3.4 we will discuss basic results of the Monte Carlos.

3.1 Lowest Order Formulas

The $q\bar{q}\gamma$ cross section in $\mathcal{O}(\alpha)$ will be expressed as a function of the normalized invariant masses squared

$$y_{ij} = (p_i + p_j)^2/s, \quad (4)$$

where $\sqrt{s} = E_{cm}$ and $i, j = q, \bar{q}, \gamma$. When going to the next order, $y_{ijk} = (p_i + p_j + p_k)^2/s$ with $i, j, k = q, \bar{q}, g, \gamma$ is also needed. Quarks are taken massless. The lowest order differential cross section is then

$$\frac{d^2\sigma^{(0)}}{dy_{q\gamma}dy_{\bar{q}\gamma}} \sim \frac{2y_{q\bar{q}} + y_{q\gamma}^2 + y_{\bar{q}\gamma}^2}{y_{q\gamma}y_{\bar{q}\gamma}} = \frac{M_{q\gamma}^{(0)}}{y_{q\gamma}} + \frac{M_{\bar{q}\gamma}^{(0)}}{y_{\bar{q}\gamma}}. \quad (5)$$

As shown in the above formula, using partial fractioning the cross section can be separated into parts which peak if either $y_{q\gamma} = 0$ or $y_{\bar{q}\gamma} = 0$. Since the cross section is symmetric under the exchange $q \leftrightarrow \bar{q}$, one has $M_{q\gamma}^{(0)} = M_{\bar{q}\gamma}^{(0)}(q \leftrightarrow \bar{q})$.

The $q\bar{q}\gamma$ phase space is shown in Fig. 3 and discussed in appendix A.1. The total cross section contains a logarithmic singularity arising from the integration over the boundaries $y_{q\gamma} = 0$ or $y_{\bar{q}\gamma} = 0$ where the photon is collinear with either the quark or the anti-quark. A double-logarithmic singularity arises from the corner $y_{q\gamma} = y_{\bar{q}\gamma} = 0$ where the photon is soft, $E_\gamma = 0$. These singularities are removed by the cuts in the phase space of photon emission (appendix A.1).

3.2 $\mathcal{O}(\alpha_s)$ Contributions

In the next-to-leading order $\mathcal{O}(\alpha_s)$, one has to include both real gluon emission and virtual corrections due to loops with an additional gluon. The latter can be written as a correction factor δ_V to the lowest order $q\bar{q}\gamma$ cross section:

$$d^2\sigma_{virt}^{(1)}(q\bar{q}\gamma) = d^2\sigma^{(0)}(q\bar{q}\gamma)(1 + \delta_V(y_{q\gamma}, y_{\bar{q}\gamma})). \quad (6)$$

The real corrections arising from $e^+e^- \rightarrow Z \rightarrow q\bar{q}g\gamma$ lead to a contribution to the cross section which is written as

$$d\sigma^{(1)}(q\bar{q}g\gamma) = \left(\frac{M_{q\gamma}^{(1)}}{y_{q\gamma}} + \frac{M_{qg}^{(1)}}{y_{qg}} + (q \leftrightarrow \bar{q}) \right) dPS(4), \quad (7)$$

where $M_{ij}^{(1)}$ are functions of the invariants y_{ki} and $dPS(4)$ is the 4-particle phase space. In the following, all statements will be made for quarks only and symmetrization with respect to $q \leftrightarrow \bar{q}$ will not be stated explicitly.

In addition to the singularities due to soft or collinear photons already present at lowest order, the $q\bar{q}g\gamma$ cross section contains additional divergences related to configurations with a collinear quark-gluon pair or a soft gluon, $y_{qg} = 0$. In order to cancel these gluonic singularities one has to integrate the $q\bar{q}g\gamma$ cross section over the corresponding phase space regions and combine the result with the virtual corrections to the $q\bar{q}\gamma$ cross section. In the following we describe a method which is known from QED and is referred to as 'phase space slicing method' in QCD²:

The singular contribution arising from Eq. (7) is isolated by introducing a parameter y_0 and by integrating over a small phase space region $y_{qg} < y_0$ containing the singularity. For small values of y_0 , the gluon will not give rise to a separate jet, but will be hidden inside the quark-jet. Thus the $q\bar{q}g\gamma$ events with $y_{qg} < y_0$ can be interpreted as a contribution to the $q\bar{q}\gamma$ cross section³. Adding this to the virtual corrections cancels the quark-gluon singularity and leads to a finite, but y_0 dependent, corrected cross section

$$d\sigma^{(1)}(q\bar{q}\gamma) = d\sigma_{virt}^{(1)}(q\bar{q}\gamma) + \int dPS(4) d\sigma^{(1)}(q\bar{q}g\gamma) \Theta(y_0 - y_{qg}). \quad (8)$$

The y_0 dependence disappears after adding the contribution from $q\bar{q}g\gamma$ final states, Eq. (7), for $y_{qg} > y_0$: $d\sigma^{(1)}(q\bar{q}g\gamma) \Theta(y_{qg} - y_0)$. Here $\Theta(x)$ is 0 for $x < 0$ and 1 for $x \geq 0$. The parameter y_0 is thus only a technical parameter and the final results should not depend on its value.

After introducing a jet definition and specifying γ isolation cuts, like those discussed in section 2, the $q\bar{q}\gamma$ cross section $d^2\sigma^{(1)}(q\bar{q}\gamma)$, Eq. (8), will contribute to $2jet + \gamma$ and $1jet + \gamma$ configurations and the $q\bar{q}g\gamma$ cross section $d\sigma^{(1)}(q\bar{q}g\gamma) \Theta(y_{qg} - y_0)$ gives rise to $njet + \gamma$ events with $n \leq 3$.

Jet definition and γ isolation can be expressed in terms of combinations of Θ -functions: $\Theta_1(njet + \gamma)$ for the $q\bar{q}\gamma$ final state which is a function of 3-particle kinematic variables and

²This is the method as implemented in the Monte Carlo GNJETS [5]. The program EEPRAD [6] is also based on the phase space slicing method, but with a slightly different definition of the singular phase space region. Ref. [7] uses a different approach called 'subtraction method' which is independent of a phase space slicing cut.

³This requires a definition of 2-parton + photon kinematic variables in the 3-parton + photon phase space. In GNJETS, this is realized by identifying $y_{qg\gamma}$ with $y_{q\gamma}$ and $y_{\bar{q}\gamma}$ with $y_{\bar{q}g}$. Another possibility would be to keep $y_{qg\gamma}$ and $y_{q\gamma} - y_{qg}$ as 2-parton + γ kinematic variables. The difference between these two choices would vanish with $y_0 \rightarrow 0$.

$\Theta_2(n\text{ jet} + \gamma)$ for the $q\bar{q}g\gamma$ final state which is defined in the 4-particle phase space. To guarantee the cancellation of the y_0 dependence, these two Θ -functions have to match each other exactly in the limit $y_{qg} \rightarrow 0$ where the phase space for the $q\bar{q}g\gamma$ final state reduces to a 3-particle phase space. This requirement has to be met for both the separation of $2\text{jet} + \gamma$ and $1\text{jet} + \gamma$ events, and for the definition of γ -isolation criteria needed to exclude QED singularities. In particular, the Θ -function Θ_2 must not restrict the integration over the gluon kinematic variables for $y_{qg} = y_0$. If this would be the case, the y_0 -dependence of $\sigma(q\bar{q}g\gamma)\Theta(y_{qg} - y_0)$ would be damped, whereas in the part which was included in $d\sigma^{(1)}(q\bar{q}\gamma)$ it was not, and as a consequence the final result would depend on y_0 . Therefore, a perfect isolation of the photon with respect to the gluon cannot be imposed. It can be shown that a condition like $y_{g\gamma} > y_\gamma$ would restrict the phase space for gluon emission also for $y_{qg} \rightarrow 0$.

There is some freedom in the way the perfect γ isolation is violated, and numerical differences in the predictions of the various matrix element calculations have to be expected. In GNJETS, photon isolation is imposed with respect to the qg -pair, but not with respect to the gluon, thereby avoiding restrictions of the gluon phase space. Only in a second step of the event analysis, can the photon become isolated from the individual partons, if these form separate jets. Further details are given in appendix A.2. EEPRAD [6] allows soft gluons, characterized by a parameter y_{min} with $y_{qg} < y_{min}$ and $y_{\bar{q}g} < y_{min}$, to fall into the isolation cone. The program of Kunszt and Trocsanyi (KT) [7] avoids treating gluons and quarks differently. They allow a maximum partonic energy E_{cone} , which may originate from quarks and gluons, inside the isolation cone, $E_{cone} < \epsilon_c E_\gamma$. The latter two calculations depend on an additional parameter (y_{min} or ϵ_c) which is not allowed to become zero.

According to these prescriptions, a Monte Carlo implementation of a calculation of $n\text{ jet} + \gamma$ rates simulates the two contributions from *i*) $q\bar{q}\gamma$ events including virtual $\mathcal{O}(\alpha_s)$ corrections and real corrections with unresolved qg ($\bar{q}g$) pairs, and *ii*) $q\bar{q}g\gamma$ events from Eq. (7) with $y_{qg} > y_0$. The latter can be separated further into two channels describing the y_{qg} -pole $\propto M^{(1)}/y_{qg}$ and the $y_{q\gamma}$ -pole $\propto M^{(1)}/y_{q\gamma}$. Events are first produced in a phase space region defined by γ -isolation criteria on the parton level. In a second step, a jet finding algorithm may be applied to the generated partons and the isolation of the photon with respect to jets can be checked. Any jet-finding algorithm and recombination scheme may be used in this step as far as it respects the requirements explained above. Otherwise, the y_0 -independence of the final result would be spoiled.

As discussed in appendix A.2 and shown in Fig. 4, there is no significant dependence of the predicted cross section on the technical cut-off y_0 for $y_0 \leq \mathcal{O}(10^{-5})$ in GNJETS. A similar result holds for EEPRAD [6].

3.3 Studies with the Matrix Element Monte Carlo Programs

In this section we discuss the sensitivity of the cross sections given by GNJETS to photon isolation cuts and the role of the $\mathcal{O}(\alpha_s)$ corrections. We use the event definition A1 as described in section 2 with values of $y_{cut} = 0.02, 0.06, 0.10$ and 0.20 . The lowest value is chosen since for $y_{cut} = 0.02$ the $3\text{jet} + \gamma$ fraction is large, about 30%, and the $\mathcal{O}(\alpha_s)$ correction to the $2\text{jet} + \gamma$ rate reaches -50% . This suggests that higher orders in α_s are important for $y_{cut} < 0.02$. The largest value of $y_{cut} = 0.2$ is the maximum value used in the experiments. As will be discussed below, for $y_{cut} > 0.2$, effects from the photon quark singularity become important.

3.3.1 Variation with y_γ

Fig. 5 shows the dependence of the predicted cross section on the partonic photon isolation cut y_γ . As discussed above and in appendix A.2, this cut-off is used to avoid the quark photon singularity. In the calculation only photons are considered which obey $y_{q\gamma} = M_{q\gamma}^2/E_{cm}^2 > y_\gamma$ and $y_{\bar{q}\gamma} = M_{\bar{q}\gamma}^2/E_{cm}^2 > y_\gamma$. We assume a value of $\alpha_s = 0.15$ (see 3.3.2) and consider values between $y_\gamma = 10^{-5}$ and $5 \cdot 10^{-2}$ which correspond to quark-photon masses of 300 MeV to 20 GeV. This range covers safely the hadronic mass scales and also the cut-offs in the QCD shower models that will be discussed later.

The variation of the total cross section with y_γ is shown in Fig. 5a. At least for $0.02 < y_{cut} < 0.10$ we observe a rather flat dependence between 10^{-5} and 10^{-3} . An increase of y_γ beyond 10^{-3} leads to a significant decrease. This suggests that for these values of y_γ the parton level isolation overlaps with the jet separation. Since the procedure and cuts used for the Monte Carlo generation of partons are not identical to those used for the physics analysis, the generated parton configuration does not necessarily cover the whole phase space. This has an effect at the analysis level even for values of y_{cut} that are nominally much larger than the internal cut-off in the Monte Carlo. From these results we conclude that values of $y_\gamma > 10^{-3}$ should not be considered.

The apparent flatness of the total cross section for $y_\gamma < 10^{-3}$ arises from a compensation between those individual $n\text{ jet} + \gamma$ rates (Figs. 5b - e) that do depend on y_γ . The three-jet rate is independent of y_γ . In this case, each parton is counted as a separate jet and the requirement $y_{\gamma jet} > y_{cut}$ imposed in the final step of the event analysis conceals the cut $y_{\gamma q} > y_\gamma$ since y_γ was chosen smaller than y_{cut} . However, there appears to be a feed-over from the one-jet to the two-jet rate with decreasing y_γ .

In view of these uncertainties due to photon isolation cuts, it is important to specify sensible ranges for y_γ . A lower bound of y_γ is suggested by the size of the $\mathcal{O}(\alpha_s)$ corrections. Decreasing y_γ to values below 10^{-4} leads to increasing negative corrections to the $1\text{jet} + \gamma$ rate for all values of y_{cut} (see Fig. 5) thus indicating that higher orders are important. Therefore, for $y_\gamma < 10^{-4}$ and in those cases where the $1\text{jet} + \gamma$ events contribute a major part to the cross section, the $\mathcal{O}(\alpha_s)$ calculation cannot be trusted. Also, if $M_{q\gamma} = \sqrt{y_\gamma} E_{cm}$ is of the order of hadron masses, non-perturbative QCD effects become important which are not included in the $\mathcal{O}(\alpha_s)$ calculation. Values of $y_\gamma < 10^{-4}$ imply a quark-photon mass of the order of the hadronic mass scale of 1 GeV and should be avoided. We consider $y_\gamma > 5 \cdot 10^{-4}$ a rather safe lower value.

This dependence on the internal photon cut-off is not an artifact of the use of y_γ or GNJETS. As can be seen from Fig. 6, a similar dependence holds also if the cut-off θ_γ^{min} is used. This corresponds to event definition A2 if partons are identified with jets. Here we use the EEPRAD matrix element calculation [6] for various values of the internal angular cut θ_γ^{min} as a function of y_{cut} for $E_\gamma^{min} = 7.5$ GeV. We vary θ_γ^{min} from 5 to 25 degrees which corresponds to y_γ of about 0.0006 to 0.007. Within this range, the cross section changes by $\pm 5\%$ for small and moderate values of y_{cut} and less for high values of y_{cut} . This variation is comparable to the change with y_γ in Fig. 5a. Also in the case of EEPRAD we observe that the variation of the total cross section with y_γ is smaller than the variation of either the one- or two-jet rate.

3.3.2 Dependence on α_s

A feeling for the importance of QCD corrections can be obtained from the α_s dependence of the total cross section. One should note that since the present matrix element calculations are

only in first order, the scale dependence of the cross section only enters through a variation of the value of $\alpha_s^{(1)}$. In Fig. 7 we display the change of the total cross section relative to the one for $\alpha_s^{(1)} = 0.15$, the value which is suggested by data [8, 13] (see below). For all values of y_{cut} we observe an almost linear decrease of the cross section with α_s .

Assuming a well-behaved perturbative expansion of the cross section in α_s , the coefficient of α_s indicates the possible importance of higher-order corrections. We observe a large variation of $\sim 30\%$ for the low value $y_{cut} = 0.02$, suggesting indeed significant corrections from higher orders in α_s . The change with α_s is only 6% and 3% at the moderate values of $y_{cut} = 0.06$ and 0.10. For large values of $y_{cut} \sim 0.2$, the α_s dependence increases again to $\sim 20\%$. Here the one-jet rate contributes more than half of the cross section implying an increased sensitivity to the quark-photon singularity.

For the comparison of data and theory, the choice of $\alpha_s(M_{Z^0})$ leads to a potential uncertainty. If QCD is correct, all experiments should be described by the same value of $\alpha_s(M_Z^2)$. In reality the value of α_s is experimentally determined by comparing measurements to QCD calculations of fixed order. Since higher-order corrections are not the same for all experiments, their neglect may give rise to different experimental values for the strong coupling constant. A value of α_s , characteristic for the configuration of the events, can be obtained from the ratio [8]

$$R_g = \frac{\sigma(3 \text{ jets} + \gamma)}{\sigma(3 \text{ jets} + \gamma) + \sigma(2 \text{ jets} + \gamma)} \quad (9)$$

or from the photon energy distribution [13]. The OPAL and L3 collaborations find consistent values of $\alpha_s^{(1)} = 0.18 \pm 0.01$ and 0.17 ± 0.04 . Although there is no strict argument for using this α_s value also for the total cross section, it seems reasonable to assume that higher-order corrections should affect both observables similarly. It is interesting to note that the value of $\alpha_s^{(1)}$ is consistent with results obtained from the jet rates in inclusive multihadronic Z^0 decays if compared to a *first* order QCD calculation [8, 13]. For this measurement, an $\mathcal{O}(\alpha_s^2)$ calculation exists, leading to a value of $\alpha_s^{(2)}(M_{Z^0}) = 0.12$. We assume an uncertainty of the cross section as given by a variation of α_s in the range between 0.12 and 0.20 to be a conservative estimate.

3.4 Comparison with other calculations

The available Monte Carlo programs of matrix element calculations [5, 6, 7] are all based on the same physics input: they use a perturbative calculation of the cross sections for $e^+e^- \rightarrow q\bar{q}\gamma$ and $e^+e^- \rightarrow q\bar{q}g\gamma$. The main differences between the three programs are in the implementation of the photon isolation and the treatment of the gluon. As discussed in section 3.2 (see also appendix A.2 and [7]) there is a conflict between the two requirements that i) the integration over the gluon momentum must not be restricted for small y_{qg} in order to cancel singularities from the virtual corrections and ii) the photon should be isolated. Since the first requirement is indispensable, in all programs the perfect isolation of photons with respect to partons is allowed to be slightly violated. Some of the features of the different calculations are summarized in table 1.

As discussed in the introduction to section 3, non-perturbative effects can be incorporated in the calculation with a photon fragmentation function. The KT program allows such an option. Effects for the isolated photon plus n jet production are found to be small for $n > 1$. [18]. Note that at the moment only isolated photon production is addressed by experimental measurements.

	Kramer, Spiesberger	Glover, Stirling	Kunszt, Trocsanyi
	GNJETS	EEPRAD	KT
Photon Fragmentation			optional
γ isolation	y_γ or $E_\gamma^{min}, \theta_\gamma^{min}$	$E_\gamma^{min}, \theta_\gamma^{min}$	y_γ or $E_\gamma^{min}, \theta_\gamma^{min}$
gluon treatment	combined with quark $y_{(qg)\gamma} > y_\gamma$	γg not separated if $y_{qg} < y_{min}$ and $y_{\bar{q}g} < y_{min}$	soft partons allowed inside cone: $E_{cone} < \epsilon_c E_\gamma$ if $y_{q\gamma} < y_{cut}$

Table 1: Main features of matrix element Monte Carlos

We compare in Fig. 8 the results of the three matrix element calculations [5, 6, 7]⁴. For $0.01 \leq y_{cut} \leq 0.20$ the ratios of the predictions are shown of either the EEPRAD or the KT calculation over the result of GNJETS. The results of the KT calculation do not include effects from the photon fragmentation function. In all cases $\alpha_s = 0.18$ was assumed and photon isolation is imposed by energy and angle cuts with $E_\gamma^{min} = 7.5$ GeV and $\theta_\gamma^{min} = 15$ degrees. The other parameters y_0 and ϵ_c are at their default values. We show the ratios for the total cross section and the various jet rates.

There is good agreement between the GNJETS and KT calculations for all jet ratios and all values of y_{cut} . The two-jet rate of EEPRAD agrees within 5 % for the whole range of y_{cut} with the others, except at very small values below 0.02. The three-jet rate of EEPRAD is about 5 % larger than those of the other calculations. The differences between EEPRAD and the other two calculations are particularly large for the one-jet rate: it differs by about $40 \pm 15\%$ for small y_{cut} (where the one-jet rate is tiny) and by $15 \pm 7\%$ at $y_{cut} = 0.2$. Depending on the fraction of the one-jet rate, the total rate is more or less in agreement between the calculations: there is good agreement for $y_{cut} \leq 0.12$, but increasing disagreement between EEPRAD and the other two programs for larger y_{cut} .

The reason for these differences is not obvious. However, it is suggestive to assume that it is due to the isolation with respect to the gluon since this is the main conceptual difference between the calculations. There is a freedom in the treatment of the γ isolation, since in the presently available programs the collinear singularity is removed with the help of partonic cuts. In a complete calculation, the singularity would be absorbed into the photon fragmentation function. Thus the differences between the calculations indicate the theoretical uncertainty related to collinear photon emission and missing information on the photon fragmentation function. The results show that at least for the two- and three-jet rate these uncertainties are rather small.

⁴The results of KT were given to us as private communication by the authors.

4 From Partons to Hadrons

To relate the calculations of $\mathcal{O}(\alpha_s)$ with at most three partons to the data, the much larger multiplicity of the experimentally observed hadrons has to be taken into account. This is achieved by grouping the hadrons into jets which then are identified with partons. However, there are caveats. Firstly, the hadronization may imply distortions of the jet energies and directions relative to those of the partons. Secondly, the losses due to the experimental energy and isolation cuts on the photon have to be estimated.

We denote by β the requirements defined in either one of the event definitions A, B or C discussed in section 2. For example, assuming the two-step procedure A1 with a value for y_{cut} , β stands for the jet finding and that the minimum jet photon mass is smaller than $\sqrt{y_{cut}} \cdot E_{cm}$. To interpret a measurement based on ~ 30 produced hadrons in terms of the $\mathcal{O}(\alpha_s)$ matrix element calculation, one has to estimate possible distortions of imposing the requirement on the data relative to the parton level. For the measurement one has in addition to account for experimental cuts like the minimum photon energy E_γ^{min} and the photon isolation angle θ_γ^{iso} with respect to hadrons. In principle also the distortions from the detector have to be estimated, however, those are neglected in this paper. The relation between the results on the parton and on the hadron level can be expressed in terms of the efficiency $f(\beta)$, the ratio of the number of events obtained at the hadron level after imposing experimental cuts, $N_{hadron}(\beta, \text{experimental cut})$, over the number of events $N_{parton}(\beta)$ on the parton level

$$f(\beta) = \frac{N_{hadron}^{PS}(\beta, \text{experimental cut})}{N_{parton}^{PS}(\beta)} = c_{PS}(\beta). \quad (10)$$

As is common practice in QCD analyses of inclusive multihadronic Z^0 decays, parton shower models ('PS') [19, 20, 21] are invoked to estimate effects from the higher-order gluon radiation and hadronization. It has been shown that these models reproduce the details of the hadron spectrum well [22]. Therefore the efficiency $f(\beta)$ is determined from parton shower models and the inverse $f^{-1}(\beta)$ is applied as a correction factor to the data. For this correction to be meaningful, one further has to establish the agreement of the matrix element calculation and the shower models at the parton-level. This will be addressed in section 4.4.

In the following sections we will discuss problems involved in the correction procedures and estimate their uncertainties. To evaluate these uncertainties, we separate the result into a contribution from isolated photons and one from collinear photon emission. The theoretical prediction is then

$$\sigma^{theo}(y_{cut}) = \sigma^{isol}(y_{cut}) + \sigma^{coll}(y_{cut}). \quad (11)$$

As a result, the observed cross section will be

$$\sigma^{meas}(y_{cut}) = f_{isol}(y_{cut}, \text{exptl. cut}) \cdot \sigma^{isol}(y_{cut}) + f_{coll}(y_{cut}, \text{exptl. cut}) \cdot \sigma^{coll}(y_{cut}) \quad (12)$$

where we allow for different experimental efficiencies for the isolated and the collinear parts.

The (in)dependence of the theoretical cross section on the collinear contribution has been discussed in sections 3.2 and 3.4. Here we want to address their potential effects in the experimental data analysis. We will discuss the isolated and collinear contributions separately. We will argue that f_{isol} can be well controlled. The collinear contribution will be shown to be negligible in the analyses performed so far, i.e. $f_{coll} \sim 0$.

4.1 Sensitivity to Collinear Photon Emission

As discussed above, with a matrix element calculation of fixed order, only photons in a limited phase space region can reliably be predicted. On the other hand, the special treatment of photons in the two-step approach A1 implies that photons may be retained that originate from less reliable phase space regions. An example was considered in section 3 (Fig. 2). We now address the uncertainties in the experimental analysis due to this type of events. For simplicity we denote all these events as 'collinear'.

Collinear photon emission may contribute directly to the observed photon sample or may sneak in with the correction for experimental losses. As will be seen from the following qualitative arguments, for both of these cases accepted photons have to be very energetic. We only consider the one-jet events to which the photon fragmentation function contributes almost exclusively [7]. Taking into account the imperfect experimental photon isolation and assuming a maximum hadronic energy E_{cone} of up to ~ 1 GeV inside the isolation cone, the photon energy for the center of mass energy $E_{cm} = M_{Z^0}$ has to be

$$x_\gamma = 2E_\gamma/E_{cm} > 1 - 2E_{cone}/E_{cm} \sim 0.98. \quad (13)$$

This may change, however, once losses of photon events due to the experimental cuts on the isolation angle are corrected for. These corrections are based on QCD shower models which include collinear emission to the extent that it is part of the shower process. Therefore, some fraction of it may sneak into the result. In this case the maximum photon energy is given by

$$x_\gamma > 1 - y_{cut} \geq 0.8 \quad (14)$$

where we assumed a maximum value of $y_{cut} = 0.2$ as used by present experiments. With or without these corrections photons are very energetic, they should therefore be rare and consequently hardly affect the measured total cross section.

This argument can be made more quantitative. Since the QCD shower models are based on a resummation of the leading-logarithmic corrections originating from collinear configurations, they should be better suited than matrix element calculations for describing the collinear photon emission. Although they are based on purely perturbative calculations, they may be viewed as a model for the photon fragmentation. One can investigate whether the event sample selected after kinematical cuts is sensitive to the contribution of collinear photon emission by varying the corresponding model parameters. If the results are independent of these model parameters, it suggests that the experimental cross section is insensitive to the non-perturbative photon fragmentation function.

The collinearity of photon emission can be steered with the cut-off Q_γ , a parameter used in QCD shower models to terminate photon emission. Its precise definition depends on the model.

- JETSET: $Q_\gamma^{JETSET} = m_q$, the minimum virtual mass of the radiating quark.
- HERWIG: $Q_\gamma^{HERWIG} \sim E_q \sqrt{1 - \cos \theta_{ij}}$, where θ_{ij} is the angle of the photon with respect to the direction of the emitting quark. For almost collinear emission $Q_\gamma^{HERWIG} \sim E_q \theta_{ij}$.
- ARIADNE: $Q_\gamma^{ARIADNE} = p_T$, the transverse momentum p_T with respect to an emitting dipole.

The default values for these cut-offs are $Q_\gamma \sim 1$ GeV. Equivalent cut-offs Q_g are used to terminate gluon emission.

We vary the Q_γ values in the models between 10 MeV and 10 GeV. This range covers safely the mass scales of hadrons, which is a natural choice for a cut-off since photons cannot be emitted from quarks after hadrons are formed. It also covers the cut-off y_γ used in the matrix element calculation which corresponds to quark photon masses of ~ 3 GeV. We leave the cut-offs for gluon emission on their default values.

In Fig. 9a we show the number of photon events, normalized to 1000 multihadronic Z^0 decays for different values of y_{cut} as a function of Q_γ . Here the HERWIG model was used. In Fig. 9b we display the efficiencies $f(\beta)$ for different y_{cut} as a function of Q_γ ⁵. We find that the total number of generated photonic events, i.e. before performing the jet analysis and imposing the cut on the jet-jet and jet-photon invariant masses, increases strongly with decreasing Q_γ . However, this increase does not affect the results once the y_{cut} requirement is applied. For values of Q_γ between 10 MeV and several GeV the event rates and efficiencies are constant. Only if $Q_\gamma > 1/3\sqrt{y_{cut}}E_{cm}$, i.e. the photon cut-off is close to the corresponding y_{cut} value, we observe a reduced accepted yield. We assign this decrease to hadronization effects. The general behaviour is the same for all models.

We conclude from these studies that for the considered values of y_{cut} , the measured photon yield and the correction procedure are insensitive to collinear photon emission.

4.2 Uncertainties Due to Higher-Order Gluon Emission

We now compare $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$ predictions to obtain an estimate of the influence of higher-order corrections. To this purpose the results of the matrix element Monte Carlo GNJETS are analyzed assuming α_s values of 0, 0.1 and 0.2.

4.2.1 Uncertainties from the Photon Energy Cut

The energy spectrum of photons is affected by higher-order QCD corrections at least in two ways: (i) higher-order gluon emission will increase the mass of the hadronic system and according to

$$x_\gamma = 1 - \frac{M_{had}^2}{E_{cm}^2},$$

will reduce the photon energy. This is counteracted by (ii) the increased probability of gluon emission. It implies that the photon is more often merged with a parton into one jet, and the event is rejected. Since $y_{\gamma,jet} \propto E_\gamma$, this particularly affects events with low photon energy. As a result, the energies of the retained photons are higher.

The change of the energy spectrum with α_s is qualitatively different for small, intermediate, and large values of y_{cut} . For $y_{cut} = 0.02$ (Fig. 10) it appears that events with low photon energy are 'cut away' by the gluon emission when increasing α_s from 0 to 0.2. The spectrum of photon energies of more than 20 GeV is hardly affected. For $y_{cut} = 0.06$ the photon spectrum is somewhat flatter with increasing α_s , but otherwise hardly affected by QCD corrections. For $y_{cut} = 0.2$ the sharp enhancement at $E_\gamma \sim E_{beam}$ is flattened out, accompanied by an increase of the yield of photons between 20 and 30 GeV.

⁵We are grateful to J. Stamm for this figure.

y_{cut}	E_{γ}^{min}	f PS	f ME	δf ME
0.02	5 GeV	0.888	0.887	+5.7%
	7.5 GeV	0.773	0.775	+9.3%
	10 GeV	0.661	0.684	+14.0%
0.06	5 GeV	1.000	1.000	
	7.5 GeV	0.970	0.964	-0.1%
	10 GeV	0.880	0.887	-1.7%

Table 2: Fraction of accepted events for various y_{cut} values and minimum photon energies. Values are given both for JETSET parton showers (column 2) and for the matrix element calculation with $\alpha_s = 0.2$ (column 3). The fourth column represents the change of the efficiency in the matrix element calculation by including first order QCD corrections with $\alpha_s = 0.2$ compared to an $\mathcal{O}(\alpha)$ calculation.

The sensitivity of the photon energy spectrum to higher-order gluon emission implies also a potential uncertainty in the experimental determination of the cross section for photon production. Since experiments require a minimum photon energy, the fraction of photons lost due to this cut may depend on which order of QCD corrections are implemented in the model to estimate the losses. In table 2 we list the fraction f of events with E_{γ} above some minimum value E_{γ}^{min} ⁶ as given by the $\mathcal{O}(\alpha\alpha_s)$ calculation for $\alpha_s = 0.2$. A significant fraction of events is lost for $y_{cut} = 0.02$, even for $E_{\gamma}^{min} = 5$ GeV. For $y_{cut} = 0.06$ the losses occur only for $E_{\gamma}^{min} \leq 7.5$ GeV. No events are lost for $y_{cut} = 0.2$, even for E_{γ}^{min} as high as 15 GeV. A feeling for the importance of higher-order effects is obtained by considering δf , the *change* of f , for an increase of α_s from 0 to 0.2 in the matrix element calculation (column 5). They are of $\mathcal{O}(10\%)$ for $y_{cut} = 0.02$, but very small at $y_{cut} = 0.06$. We also list the expectation from QCD shower models. For all values of E_{γ}^{min} and all values of y_{cut} the parton shower model and the matrix element calculation agree well with each other.

These results suggest that, at least for the energy requirement imposed in the experiments, the efficiency of retaining a photon is almost completely unaffected by higher-order gluon emission for $y_{cut} \geq 0.06$.

4.2.2 Uncertainties from the Photon Isolation Cut

Except for small values of y_{cut} the more important losses are due to the isolation cone around the photon. To estimate the related uncertainties, it is useful to distinguish two potential problems:

- (i) How often are jets, i.e. clusters of partons or hadrons, predicted to be inside the isolation cone? One has to test whether the isolation requirement cuts into the 'hard' parton structure.

⁶At this stage we consider all photons generated by the QCD shower models. We do not require the photon isolation θ_{γ}^{min} used in the experimental analysis.

- (ii) What is the probability that particles around these jets are scattered into the isolation cone? The corresponding corrections have to be estimated with QCD shower models.

To study (i) we show in Fig. 10 the angle α between the photon and the closest jet as obtained for the various values of y_{cut} and α_s . Since $y \propto 1 - \cos \alpha$, the angle α increases with y_{cut} . In all cases, the angles are significantly larger than those used as isolation cuts in experiments. By varying α_s between 0 and 0.2 we can estimate the effect from higher-order QCD corrections. For $y_{cut} = 0.02$ the distribution is significantly smeared out due to gluon emission. For $y_{cut} = 0.06$ the distribution hardly changes with α_s , for $y_{cut} = 0.2$ the peak around 180 degrees is reduced and the yield around $\alpha = 100$ degrees becomes more prominent. This suggests that higher-order QCD corrections will *not* lead to jet configurations that are affected by the isolation cut.

However, significant losses can arise from soft partons and hadrons inside the isolation cone. Firstly one should note that the colour flow in the hadronic system of mass M_{had} recoiling against the photon is unaffected by the photon emission. In fact, if the photon is emitted 'first', the hadron distribution in the hadronic center-of-mass system is identical⁷ to the one for hadrons produced in e^+e^- collisions at $E_{cm} = M_{had}$. The good description of the particle and energy flow for center-of-mass energies between 30 and 100 GeV by the QCD shower models suggests that these flows can be estimated reliably with these models. According to these models, assuming the currently applied isolation angles of 15 to 20 degrees, experiments loose about 50% of the events at low y_{cut} , but only about 10% at $y_{cut} > 0.06$ (cf. Fig. 9b). Depending on y_{cut} , the models agree in the estimated efficiency to within 3 – 5%. Given the quality of the shower models in reproducing the particle and energy flow, it seems appropriate to count a fraction of these losses as a systematic uncertainty.

Although a direct measurement of the efficiency is not possible, the quality of the simulation of the photon and energy flow can be cross-checked as summarized in [10]. One may hope that with a further understanding of the particle and energy flow and better means for identifying the photon, looser cuts can be applied and the corrections will become smaller.

4.3 Hadronization Corrections

Changes of the number of jets, their directions, and their energies at the hadron level relative to those at the parton level have to be taken into account. As is common practice in the study of inclusive multihadronic Z^0 decays, these hadronization corrections are estimated with the help of QCD shower models. To this end the relevant distributions are calculated with QCD shower models both for the generated partons and for the generated hadrons. Here we apply only the cut in $y_{ij} = M_{ij}^2/E_{cm}^2$ and partons and hadrons inside the experimental isolation cone are included in the calculation. The ratio

$$c_{had} = \frac{N_{parton}^{PS}(y_{cut})}{N_{hadron}^{PS}(y_{cut})} \quad (15)$$

is taken as a measure of hadronization effects. For the E0-JADE recombination scheme, the corrections of the total cross section are found to be 0.97 for $y_{cut} \leq 0.1$ and 0.95 at $y_{cut} = 0.2$.

Our studies show that the hadronization distortions can be separated into three groups. For the large majority of the events the number of jets is equal on the parton and hadron level, supporting the idea that the hadron state resembles the parton structure of the event. The

⁷Here we neglect the different flavour compositions at different values of E_{cm} .

second class of events has low energy parton jets which are absorbed in larger hadron jets after hadronization. The third and smallest group consists of events in which the number of jets is higher on the hadron level, due to large angle scattering of the particles during hadronization. We find for example at $y_{cut} = 0.06$, that 91% of the events accepted on the hadron level are accepted also on the parton level. All of these events have the same number of jets before and after hadronization. The remaining 9% are accepted only after hadronization, of which about half have undergone hadronization without change of the number of jets. This effect is compensated by events lost during hadronization; they amount to 6%, half of which again maintain their number of jets.

The hadronization corrections given by the various QCD shower models are in good agreement with each other. It is interesting to note that the corrections for the $2jet + \gamma$ configuration are similar to those estimated for the three-jet configuration of the inclusive multihadronic sample [23]. This suggests that for QCD studies they tend to cancel in the ratio of these two event types.

4.4 Jets in the Matrix Element Calculation and Shower Models

In calculating the corrections with QCD shower models to compare data with matrix element calculations, we finally have to confirm that the models do not significantly diverge from the calculations. It should be noted that, given a certain parton configuration, QCD shower models are believed to correctly evolve this system to the hadron level, at least on the average. This means that for a given parton configuration the losses can be estimated with the models. An inappropriate correction of the total cross section, however, can occur if the models and calculations would give significantly different distributions *and* the necessary corrections addressed in the previous sections would strongly depend on these distributions.

Results of the matrix element calculation and of ARIADNE [20] for $\alpha_s = 0.2$ are compared in Fig. 11. We show the normalized distributions of the angle of the photon with respect to the axis of the corresponding jet, and of the photon energy. The comparison is made at the parton level, i.e. without hadronization in the shower model.

- For $y_{cut} = 0.02$ the photon energy tends to be smaller in the QCD shower models. Whereas the shape of the spectra in the two calculations agree for $E_\gamma > 5$ GeV, the yield of photons at smaller energies is larger in the shower model than in the matrix element calculation. Since $y_{cut} \propto E_\gamma(1 - \cos \alpha)$ this implies larger angles in the shower models as observed in Fig. 11a. These differences suggest that for this value of y_{cut} higher-order corrections as approximately implemented in QCD shower models are important.
- For $y_{cut} = 0.06$ the agreement between parton shower models and the matrix element calculations is almost perfect.
- For $y_{cut} = 0.2$ there is general agreement. The parton shower models exhibit somewhat broader structures in regions where the one-jet contribution is relevant, i.e. for angles $\alpha \sim 180$ degrees, $M_{j\gamma} \sim E_{cm}$ and $E_\gamma \sim E_{beam}$. This broadening is probably due to higher-order gluon emission.

We therefore conclude that in general the parton shower models reproduce the matrix element calculations well. We also verified that the correction c_{PS} of Eq. (10) is not a strong function of the event properties. Therefore the parton shower models should allow to reliably correct the observed distributions for experimental cuts and hadronization.

4.5 The Use of the DURHAM Recombination Scheme

There is no unambiguous prescription to identify jets as reconstructed from hadrons in the experiment with partons of the theory. Instead several ways to combine hadrons into jets exist. Up to now we applied the JADE-E0 scheme [14]; we will now summarize which uncertainties appear if the DURHAM scheme [15] is used. In this case two particles are combined that have the smallest

$$y_{ij} = 2 \cdot \min(E_i^2, E_j^2)(1 - \cos \alpha_{ij}).$$

The DURHAM scheme is particularly interesting since it combines partons to jets on the basis of the infrared and collinear singularity structure of the matrix elements and therefore shows a good convergence property in the QCD perturbative expansion; it is particularly suited to resum non-leading corrections and leads to an assignment of particles to jets that is more intuitive.

Much of the discussion on the uncertainties in the E0 scheme can be directly applied to the results of the DURHAM scheme. The y_γ dependence, the influence of collinear photon emission, and the potential change of the experimental efficiency to select photons with an increase of α_s from 0 to 0.2 are unchanged. We find also that the α_s dependence of the predicted cross section is similar to what has been observed in the E0 scheme. A variation of α_s between 0.12 and 0.2 leads to a change of the cross section by less than 3 % for the considered values of y_{cut} . Also the hadronization correction is about the same as for the E0 scheme.

As a result, with respect to the correction procedure, there is no reason to give preference to either of the recombination schemes for the two-step procedure.

5 Isolation Using the Jet-Photon Angle

We now turn to the discussion of event definition A2. As in the previous discussion, in a first step jets are formed from hadrons alone *excluding* the photon. In the next step, the $y_{\gamma,jet}$ requirement of the previous discussion is replaced by demanding a minimum angle $\theta_{\gamma,jet}$ of the photon with respect to the *jets*. This procedure has been adopted by some LEP experiments [9, 13] with $\theta_{\gamma,jet} = 20$ degrees and $E_\gamma > 5$ GeV. At first sight this requirement appears to be identical to the experimental isolation cut, θ_γ^{iso} and to the theoretical cut-off θ_γ^{min} of the matrix element Monte Carlos. However, both are quite different. Firstly, the theoretical cut-off and the experimental isolation cut θ_γ^{iso} is with respect to *individual* partons or hadrons, respectively, whereas in the event definition A2 partons or hadrons are grouped into *jets* before the cut on the angle is applied. Secondly, as mentioned several times before, the cut in the matrix element calculation is with respect to at most three partons, whereas the experimental cut refers to typically 30 *hadrons*. These two remarks imply that also for the event definition A2 the theoretical prediction is sensitive to the theoretical cut-off θ_γ^{min} , and the experimental correction has to account for losses due to the experimental isolation. At least qualitatively, this event definition does not avoid the problems of A1.

Also quantitatively, there is no significant difference. As discussed in section 3.3 and Fig. 6, the dependence of the theoretical prediction on the cut-off θ_γ^{min} is quite similar to the one for y_γ . In addition, we showed in section 4.2.2 that for $y_{cut} \geq 0.02$ the minimum angle between photon and jets is anyway much larger than 20 degrees.

We conclude that the uncertainties of the two event definitions A1 and A2 based on the two-step procedure are similar. The requirement of a minimum angle between photon and jet has no advantage over a cut in $y_{\gamma,jet}$.

An analysis of the uncertainties of the matrix element predictions for the event definition similar to A2 has also been performed in [24]. There, jets are reconstructed in the E0 scheme with $y_{cut} = 0.05$. In a second step the photon is required to have at least $E_{\gamma}^{min} = 5$ GeV and an angle with respect to the closest jet of more than 20 degrees. Note that these requirements allow rather low values of $y_{\gamma,jet} \sim 0.003$ for which $\mathcal{O}(\alpha_s)$ corrections are large. Therefore, in the analysis of [24], the dependence on α_s and θ_{γ}^{min} is rather strong.

6 Democratic Event Definition

As discussed in section 3.3, the two-step procedures A1 and A2 are sensitive to partonic photon isolation cuts. As exemplified in Fig. 2, due to the selection procedure of first forming jets from hadrons only, contributions from the region close to the photon singularity may sneak into the accepted configurations. As discussed in section 3.3 there are qualitative arguments that the effect of the photon cut-off is important only in a rather small phase space region. However, to be on safer grounds it is better to have an event definition that largely avoids this dependence.

Analyzing the origin of the y_{γ} dependence for events like those of Fig. 2 suggests an alternative, 'democratic' event definition where the photon is included in the jet finding algorithm. In such a procedure, a jet may consist of hadrons and the photon. In particular, a collinear photon will be combined with the emitting quark. To obtain an unambiguous event definition one has to introduce an additional parameter ϵ_0 , the hadronic energy allowed inside the jet containing the photon (see Eq. (1)). Events are only accepted if $\epsilon_{had} < \epsilon_0$. In Fig. 3b we show the resulting phase space regions for $q\bar{q}\gamma$ final states. The full lines in this figure limit the regions with $1jet + \gamma$ and $2jet + \gamma$ configurations when *no* hadronic energy is allowed to accompany the photon, i.e. for $\epsilon_0 = 0$. Since the singularities at $y_{q\gamma} = 0$ and $y_{\bar{q}\gamma} = 0$ are excluded, the resulting cross sections are finite and in this democratic approach there is no need for a photon isolation criterium in lowest order.

For the $\mathcal{O}(\alpha_s)$ correction it is, however, not possible to completely dispose of photon isolation cut-offs. In order to cancel singularities between virtual and real contributions, the event definition is not allowed to restrict the phase space for soft gluons. Therefore, the choice $\epsilon_0 = 0$ is not possible. Allowing a non-zero value for ϵ_0 (see the dashed-dotted lines in Fig. 3b) reintroduces a sensitivity to collinear photon emission and photon isolation cuts have to be imposed (the dashed lines in Fig. 3b). However, as discussed below, the dependence on y_{γ} can be attributed almost exclusively to the one-jet plus photon configuration that can easily be cut out in the experimental analysis.

We now study the sensitivity of theoretical predictions to the required cut-off parameters y_{γ} and ϵ_0 and to the correction procedure for this specific event definition. In particular we will address the dependence on the new parameter ϵ_0 . In the experiment a precise measure of ϵ_0 requires also a good and well understood sensitivity to very low, i.e. $\mathcal{O}(100$ MeV), charged and neutral particles. For this phase space region QCD shower models have only a limited reliability. In addition there are potential experimental difficulties in reconstructing these low energy particles. In the following we assume the hadronic energy in the photon jet can be determined with some accuracy δE_{had} independent of the value of ϵ_0 . We use $\delta E_{had} = 200$ MeV which corresponds to $\delta\epsilon_0 \sim 0.01$.

6.1 Variations of Parameters in the ME Calculation

The dependence of the theoretical cross section for $2jet + \gamma$ events and values of $y_{cut} = 0.06$ and 0.2 is shown in Fig. 12. For example, relative to the value $\epsilon_0 = 0.01$, a choice of $\epsilon_0 = 0.1$ increases the theoretical cross section by $\sim 200\%$ for $y_{cut} = 0.06$, and $\sim 20\%$ for $y_{cut} = 0.20$. Taking into account the uncertainties in δE_{had} the steep dependence on ϵ_0 at low ϵ_0 suggests that one should assume larger values of ϵ_0 . On the other hand, the hadronic energy fraction should not be too large, otherwise non-perturbative effects or higher-order QCD corrections will become important. Therefore we assume in the following a value of $\epsilon_0 = 0.1$. For this value the variation of the one- and two-jet rates for values of y_γ between 10^{-5} and 10^{-2} is shown in Fig. 13 for various values of y_{cut} . The three-jet rate (not shown) is independent of y_γ and identical to the one of definition A. Also, for $y_{cut} \geq 0.04$, the two-jet rate becomes almost independent of y_γ . Only the one-jet rate changes significantly with y_γ . These studies suggest that, as in the two-step procedure, by restricting the analysis to $2jet + \gamma$ events, and by choosing a reasonably large value of y_{cut} , the dependence on y_γ can be made negligible and therefore the theoretical prediction much safer. In some sense, in the democratic approach, all the dependence on the cut-off is shifted to the one-jet rate. Thus, by restricting the analysis to the two-jet production theoretical uncertainties seem to be minimal.

This stability with respect to y_γ , however, is at the expense of an increased sensitivity of the two jet plus photon rate to the value of α_s . A change from $\alpha_s = 0$ to the preferred value of $\alpha_s = 0.15$ leads to a decrease in the predicted cross section by 35% even for a value of y_{cut} as high as 0.06 . This suggests considerable higher-order corrections for this event definition at low to moderate values of y_{cut} . Between $\alpha_s = 0.12$ and $\alpha_s = 0.20$, variations of less than 5% occur only for $y_{cut} > 0.1$. For the high value of $y_{cut} = 0.2$, the variation with α_s is 2% for $\epsilon_0 = 0.01$ and 0.5% for $\epsilon_0 = 0.1$. Thus, to avoid uncertainties due to higher-order corrections, large values of y_{cut} are recommended. This, however, increases the statistical uncertainty significantly. We confirmed that this α_s dependence is not due to the use of y_γ as the theoretical cut-off. Such a large dependence of the cross section on α_s is also observed if y_γ is replaced by $E_\gamma^{min} = 7.5$ GeV and $\theta_\gamma^{min} = 15$ degrees.

6.2 Uncertainties in Relating Partons and Jets

Given the negligible role of the collinear photon emission in the data analysis (cf. section 4.1), we address here only the relation between partonic and hadronic energy inside the photon jet and the variation of the cross section with ϵ_0 using the QCD shower models JETSET and ARIADNE.

In a first step we study the hadronization correction c_{had} (Eq. (15)). For the one-jet plus photon configuration we observe corrections of $\mathcal{O}(20\%)$. This again suggests that it will be difficult to use this configuration for a precision measurement. The corrections for the two-jet plus photon configuration are smaller. They become more prominent with increasing y_{cut} . For $y_{cut} = 0.2$ we find $\sim 8\%$ for JETSET and $\sim 2.5\%$ for ARIADNE. The correction is larger than for the two-step procedure. Moreover, the model dependence is stronger. One should be aware of the following difference between the two-step procedure and the democratic approach. In event definition A1 the partons and hadrons are first combined into a jet and thus the spread of the energy within these jets is of minor importance. In contrast, the spread affects the results of the democratic approach. Here the energy within the photon jet is summed up. This energy is correlated with the number of partons or hadrons which are considered in the analysis. The average number of partons at the end of a QCD cascade depends on the model assumed and

varies between 4.5 (ARIADNE) and 9 (JETSET). To avoid this model dependence and allow for a smoother transition to the matrix element calculation with at most three partons, one may choose the internal model parameters such that at most three partons are generated. This requires readjustment of other model parameters such that the model predictions for the hadron and energy flow agree with the data. An alternative would be to combine the partons generated in a matrix element calculation directly with a QCD shower algorithm. Such an analysis has not yet been performed.

The dependence of the cross section as a function of ϵ_0 as predicted by the ARIADNE and JETSET models is compared to the matrix element calculation in Fig. 13 for $y_{cut} = 0.06$ and 0.2. We show the change in the cross section $\sigma_{2jet+\gamma}(\epsilon_0)$ normalized to the cross section for $\epsilon_0 = 0.1$. For $y_{cut} = 0.06$ we find a significantly steeper ϵ_0 dependence in the matrix element calculation than for the QCD shower models. For $y_{cut} = 0.2$ the agreement is fairly good. Note that the ϵ_0 dependence at the hadron level can be measured outside the experimental isolation cone. Therefore, at least for large values of y_{cut} , the energy flow expected from the matrix element calculation can be experimentally checked.

6.3 Durham Recombination Scheme in the Democratic Approach

Using the democratic approach together with the Durham scheme, large hadronization corrections are observed if the energy in the photon jet is small; e.g. for a y_{cut} value of 0.006 and $\epsilon_0 = 0.01$ we find $c_{had} = 1.44$. Only by increasing ϵ_0 the corrections can be made small. For this value of y_{cut} the hadronic corrections are minimal, $c_{had} \sim 1$, for $\epsilon_0 = 0.1$. For other values of y_{cut} small hadronization corrections require different values of ϵ_0 . Thus, minimizing the corrections requires a fine tuning of ϵ_0 , i.e. experimentally a very precise control of the energy flow around the photon. Also for the Durham scheme we observe a model dependence of the hadronization correction. In contrast to the two-step approach, hadronization corrections in the democratic approach are significantly larger in the Durham scheme than in the E0 scheme.

7 No Jets at all

In principle one can make one further step to define a photon event by dropping the jet reconstruction completely (definition C). Here one requires just a minimum photon energy and a maximum hadronic energy ϵ_{had}^C in a cone of half opening angle θ_{cone} around the photon. Such a definition avoids all ambiguities in the jet reconstruction. However, its limitations are obvious already from the preceding discussion.

Such an event definition includes all jet configurations, and thus the badly behaved one-jet plus photon cross section fully affects the theoretical prediction. As discussed in the previous section it depends strongly on the internal cut-off parameters, and the value of α_s . These effects can be reduced by requiring large photon energies and cone angles, corresponding to a large y_{cut} between photon and hadrons. However, in this case the hadronization corrections become very large. For example, assuming an isolation cone of 15 degrees, they are $\sim 40\%$ and for 40 degrees they rise to even more than 100%. As a result it is difficult to see any advantage of this event definition over those considered in the previous sections.

Event definition		A1	B	
			$\epsilon_0 = 0.01$	$\epsilon_0 = 0.1$
Uncertainties related to matrix element				
Parameter	Range			
y_γ	$0.5 \cdot 10^{-3} - 1 \cdot 10^{-3}$	1.4%	4%	3%
α_s	0.12 - 0.20	1.5%	50%	10%
ϵ	$\delta\epsilon = 0.01$	-	100%	10%
Uncertainties related to QCD shower models				
Hadron. Corr.		3%	2 - 4%	4 - 6%
Statistical uncertainty				
10^6 hadronic Z^0 decays		3%	7%	5%
10^7 hadronic Z^0 decays		1%	2%	1.5%

Table 3a: Uncertainties due to various sources in the matrix element calculation and the QCD shower models for $y_{cut} = 0.06$. Listed are those for the two-step procedure (A1) and the democratic event definition (B). Also listed is the expected statistical uncertainty in an experiment collecting 10^6 or 10^7 hadronic Z^0 decays.

8 Comparison of the Various Event Definitions

To summarize the uncertainties involved in the various event definitions that have been considered in this article, we will restrict the comparison to A1 and B, since, according to the discussions in sections 4 and 5, the definitions A1 and A2 are almost identical, and C has large uncertainties. We consider both $y_{cut} = 0.06$ and 0.2, the values for which the uncertainties are smallest for the two event definitions. The most important contributions to the uncertainties are summarized in tables 3a and 3b.

Neglecting purely experimental uncertainties, the two-step procedure leads to a final uncertainty of $\sim 3.5\%$ for the moderate value of $y_{cut} = 0.06$ (table 3a). Assuming 3 million hadronic Z^0 decays, a yield that should be in the reach at the first phase of LEP operation, the additional statistical error will be $\sim 2\%$ suggesting that the purely experimental uncertainties are not dominating. The major uncertainty is due to the hadronization correction. It is unclear if this can be reduced with alternative jet definitions. Also the uncertainties in the theoretical prediction due to the dependence on α_s and y_γ are difficult to improve without a calculation including higher orders in QCD. The democratic approach for $y_{cut} = 0.06$ leads to reasonably small uncertainties only if a hadronic energy of up to 10% of the photon is allowed inside the isolation cone, but even then the uncertainty is of the order of 20%.

On the other hand, considering large values of y_{cut} , the democratic approach leads to rather stable results (see table 3b). The uncertainties due to the matrix element internal cut-off parameters and the α_s value turn out to be very small. A larger uncertainty results from the hadronization correction. If one gets a handle on this, an uncertainty in the matrix element prediction of $\sim 4\%$ seems to be feasible. The corresponding statistical error for three million Z^0 decays, however, will be about 5%. Thus, only for a substantial increase in statistics, the total error from the democratic approach may be pushed below that of the two-step procedure.

Event definition		A1	B	
			$\epsilon_0 = 0.01$	$\epsilon_0 = 0.1$
Uncertainties related to matrix element				
Parameter	Range			
y_γ	$0.5 \cdot 10^{-3} - 1 \cdot 10^{-3}$	4.2%	0.2%	0.5%
α_s	0.12 - 0.20	4.7%	4%	0.5%
ϵ	$\delta\epsilon = 0.01$	-	4%	2%
Uncertainties related to QCD shower models				
Hadron. Corr.		6%	7%	2 - 8%
Statistical uncertainty				
10^6 hadronic Z^0 decays		6%	10%	9%
10^7 hadronic Z^0 decays		2%	3%	2.8%

Table 3b: Uncertainties due to various sources in the matrix element calculation and the QCD shower models for $y_{cut} = 0.20$. Listed are those for the two-step procedure (A1) and the democratic event definition (B). Also listed is the expected statistical uncertainty in an experiment of 10^6 and 10^7 hadronic Z^0 decays.

The uncertainties of the two methods are to some extent complementary and therefore provide an interesting cross-check. However, it may be difficult to develop a reasonable procedure for averaging the two results.

Whereas the measurement of the absolute cross section is important for the determination of the electroweak quark couplings and the search for non-standard photon sources, the event configuration is of particular interest in view of QCD studies. A lot of QCD studies with inclusive multihadronic events have been performed studying the configurations and rates of n jet production in terms of the variable y_{cut} . It would be advantageous to express the $(n-1)$ jet plus photon configuration in a similar way. Both the two-step and the democratic approach try to achieve this. However, some caveats have to be kept in mind. Assuming the democratic approach, hadronic energy is allowed inside the photon jet, which obviously did *not* originate from the photon itself. Thus its energy and direction are distorted with respect to the original photon properties. In some sense such a flow of energy should also occur in the case of gluon instead of photon bremsstrahlung. But for gluons there is also a flow from the gluon to the quark. If there would be an exact balance in the exchange of energies from quarks and gluons, the two-step procedure would be more adequate for comparing the photon jet in an $(n-1)jet+\gamma$ event with a gluon jet in an $(n)jet$ event. Without further studies it is not obvious which procedure is more appropriate.

9 Conclusions

In this paper we discussed several aspects of the comparison of matrix element calculations of $\mathcal{O}(\alpha\alpha_s)$ with the measurement of isolated photon emission from quarks. By now, three matrix element Monte Carlo calculations have been published. Although they differ in details, their

results are consistent at least for the major part of the experimentally available phase space.

A problem of the matrix element calculation is the unsatisfactory description of collinear photon emission and missing information on the photon fragmentation function. This leads to a sensitivity of the theoretical prediction on cut-offs (y_γ or θ_γ^{min}) in the calculation. From physics arguments we suggest a reasonable range of these parameters such that their effect can be kept under control. The sensitivity depends on the way a photon event is defined and can be considerably reduced by, for example, restricting the analysis to $2jet + \gamma$ events in the democratic approach. In the two-step procedure, the total cross section turned out to be less sensitive to the photon isolation cut-offs than the $2jet + \gamma$ rate. We believe that the dependence on the parton level isolation cuts are an indication for the sensitivity to non-perturbative collinear photon emission. This, however, remains to be confirmed by a complete calculation including these effects with the help of a photon fragmentation function.

Higher-order corrections in α_s are expected to be rather small for large values of the photon-jet separation cut y_{cut} . Values of $y_{cut} \sim 0.06$ are appropriate for the two-step approach, whereas they have to be significantly higher (~ 0.16) for the democratic approach.

Another source of problems is the connection of partons in the matrix element calculation to the hadrons observed in the experiments. QCD shower models are invoked to obtain the relation between partons and hadrons. We find that, with the currently used definitions of photon events, the results do not depend on collinear photon emission. This suggests that *experimentally* there is also no sensitivity to the photon fragmentation function. Furthermore, the experimental requirement of photon isolation with respect to hadrons can be considered as an acceptance problem whose impact can be estimated with QCD shower models. We also observe that the configuration of events as predicted from the matrix element calculation is consistent with the predictions of parton shower models. This makes the use of shower models for the evaluation of effects from the hadronization and experimental cuts meaningful. Comparing the different sources of uncertainties we find the hadronization correction, i.e. the distortions of jet directions and energies at the hadron level relative to the ones on the parton level, will eventually be the major source of uncertainty.

We discussed the use of various procedures to define a photon event. It seems that the uncertainties are smallest in the two-step approach. For very high statistics also the democratic approach allows for a rather precise measurement of the cross section.

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A Appendix

A.1 Isolation cuts for the $q\bar{q}\gamma$ final state

In the matrix element calculations [5, 6, 7] two different sets of photon isolation cut-offs are used. One prescription uses a minimum photon angle with respect to partons θ_γ^{min} and a minimum photon energy E_γ^{min} to separate the singular phase space regions. The other definition combines these variables into $y_{\gamma i} = M_{\gamma i}^2/E_{cm}^2 = E_i E_\gamma (1 - \cos \theta_{\gamma i})$ with $i = q, \bar{q}, \text{ or } g$. In lowest order, i.e. for $q\bar{q}\gamma$ final states, the photon isolation cut-offs as used in the three matrix element Monte

Carlos agree with each other. For the $q\bar{q}g\gamma$ final state, however, different prescriptions are used. Details of the definitions used in the Monte Carlo GNJETS of Ref. [5] are given in appendix A.2.

The basic kinematic limits for the $q\bar{q}\gamma$ final state are:

$$0 \leq y_{q\gamma} \leq 1, \quad (16)$$

$$0 \leq y_{\bar{q}\gamma} \leq 1 - y_{q\gamma}. \quad (17)$$

γ -isolation using invariant mass cuts is expressed as

$$y_{q\gamma} \geq y_\gamma, \quad y_{\bar{q}\gamma} \geq y_\gamma. \quad (18)$$

In the case of cone-energy isolation, the photon is required to be separated by a minimum angle $\theta_\gamma^{\min}$ from the quarks (or jets) and to have a minimum energy $E_\gamma^{\min}$:

$$\theta_{q\gamma} \geq \theta_\gamma^{\min}, \quad \theta_{\bar{q}\gamma} \geq \theta_\gamma^{\min}. \quad (19)$$

$$E_\gamma > E_\gamma^{\min}. \quad (20)$$

The restriction to large angles between γ and quark or anti-quark can be expressed in terms of y_{ij} :

$$y_{q\gamma} \geq \frac{y_{\bar{q}\gamma}(1 - y_{\bar{q}\gamma})(1 - c)}{1 + c + y_{\bar{q}\gamma}(1 - c)}, \quad (21)$$

(the same for $q \leftrightarrow \bar{q}$) with

$$c = \cos \theta_\gamma^{\min}. \quad (22)$$

The cut on the photon energy $E_\gamma \geq E_\gamma^{\min}$ translates into

$$y_{q\gamma} + y_{\bar{q}\gamma} > \epsilon, \quad (23)$$

with

$$\epsilon = \frac{E_\gamma^{\min}}{\sqrt{s}}. \quad (24)$$

The relevant regions in the $q\bar{q}\gamma$ phase space are shown in Fig. 3a.

A.2 Isolation cuts in GNJETS

As explained in section 3.2, a perfect isolation of photons with respect to partons is not possible when for the inclusion of $\mathcal{O}(\alpha_s)$ corrections one has to consider the emission of an additional gluon. Photon isolation with respect to the gluon is treated in different ways in the available matrix element Monte Carlos. Here we give details of the prescriptions as realized in GNEJTS.

The gluonic singularity at $y_{qg} = 0$ is separated by the cut

$$y_{qg} > y_0. \quad (25)$$

Invariant mass isolation cuts are:

- for the $y_{q\gamma}$ -pole:

$$y_{q\gamma} > y_\gamma \quad (26)$$

- for the y_{qg} -pole:

$$y_{\bar{q}\gamma} > y_\gamma, \quad y_{qg\gamma} > y_\gamma \quad (27)$$

The cone-energy isolation cuts are:

- for the $y_{q\gamma}$ -pole:

$$\theta_{q\gamma} > \theta_{\min}, \quad E_\gamma > E_\gamma^{\min}. \quad (28)$$

- for the y_{qg} -pole:

$$\theta_{\bar{q}\gamma} > \theta_{\min}, \quad \theta_{(q+g),\gamma} > \theta_{\min}, \quad (29)$$

$$E_\gamma > E_\gamma^{\min}. \quad (30)$$

From Eqs. (27, 29) one observes that the photon isolation is not perfect: the isolation cut is not applied to $y_{g\gamma}$. A cut on $y_{g\gamma}$ would damp the y_0 -dependence in $\int \sigma(q\bar{q}g\gamma)\Theta(y_{qg} - y_0)$ and the final result would not come out independent of y_0 . The unsymmetric treatment of quarks and gluons implied by these prescriptions is, of course, unphysical. However, symmetry will be restored in the second step of the event selection algorithm where $y_{jet,\gamma} > y_{cut}$ is applied. Since there is no singularity at $y_{g\gamma} = 0$, a cut on $y_{g\gamma}$ is indeed not needed and would have only a non-leading effect.

Also, q and $\bar{q}$ are treated unsymmetrically in the y_{qg} -pole part. Not $y_{\bar{q}\gamma} > y_\gamma$ and $y_{q\gamma} > y_\gamma$, but $y_{\bar{q}\gamma} > y_\gamma$ and $y_{qg\gamma} > y_\gamma$ is used. Only with this choice, the requirement is fulfilled that the γ -isolation criteria applied to the $q\bar{q}g\gamma$ final state should match that applied to the $q\bar{q}\gamma$ cross section. It is also obvious that the cut $y_{qg\gamma} > y_\gamma$ does not damp the y_0 dependence since $y_{qg\gamma}$ was kept as a fixed variable in the calculation of $d^2\sigma^{(1)}(q\bar{q}\gamma)$. Since $y_{qg\gamma} > y_{q\gamma}$, the cut actually applied is weaker than $y_{q\gamma} > y_\gamma$. This increase of the phase space is corrected for in the second step of the Monte Carlo simulation where the experimental event selection algorithm is applied and γ -isolation is checked with respect to jets.

As is seen from these definitions, the $y_{q\gamma}$ - and y_{qg} -pole parts of the cross section Eq. (7) are treated differently: the cut on $y_{\bar{q}\gamma}$ (or $\theta_{\bar{q}\gamma}$) is not applied in the integration of $M_{q\gamma}^{(1)}/y_{q\gamma}$. This choice is made since there is no singularity at $y_{\bar{q}\gamma} = 0$ to be separated. The apparent unsymmetric treatment of quarks and anti-quarks will be restored in a second step of event generation where the events generated according to the above partonic γ -isolation criteria will be analyzed according to the experimental event selection criteria. It was checked numerically that applying also the cut $y_{\bar{q}\gamma} > y_\gamma$ in the $y_{q\gamma}$ -pole part already at the parton level, the normalized $n\text{ jet} + \gamma$ partial widths would be affected by less than $3 \cdot 10^{-6}$ throughout the range of y_{cut} values considered in this paper.

It was checked numerically for both the two-step procedure and the democratic approach of event definitions that the prescriptions discussed above indeed lead to y_0 -independent $n\text{ jet} + \gamma$ rates. For $y_\gamma = 10^{-3}$ and $\alpha_s = 0.18$ results are shown in Fig. 4 for the one-, two- and three-jet rates with event definition A1 for the four values of $y_{cut} = 0.02, 0.06, 0.10$, and 0.20 . y_0 is varied between 10^{-8} and 10^{-4} . At small values of y_0 , numerical instabilities lead to large statistical fluctuations. Values of y_0 larger than 10^{-5} should not be used for two reasons: i) if y_{cut} is small, the phase space cuts defined with the help of y_0 may start to overlap with those characterized by y_{cut} and ii) in the matrix element calculation implemented in GNJETS, terms that vanish with $y_0 \rightarrow 0$ have been neglected. The latter effect is clearly seen in Fig. 3: the one-jet rate at $y_0 = 10^{-4}$ relative to the one at lower y_0 is reduced by $24 \pm 4\%$ at $y_{cut} = 0.02$ and $5 \pm 0.7\%$ at $y_{cut} = 0.20$. In all other cases, no significant variation with y_0 is observed.

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Figure Caption

- Fig. 1 Schematic view of various event definitions.
- Fig. 2 Example of an event with non-isolated hard collinear photon radiation giving rise to a $1jet + \gamma$ event in event definition A1.
- Fig. 3 Phase space for $q\bar{q}\gamma$ final states in the $(x_q, x_{\bar{q}})$ plane, $x_q = 1 - y_{q\gamma}$, $x_{\bar{q}} = 1 - y_{\bar{q}\gamma}$. $1jet + \gamma$ and $2jet + \gamma$ configurations are separated by the full lines for event definition A1 in Fig. 3a and for definition B in Fig. 3b. The dashed lines show the effect of a cut on $y_{q\gamma}$ and $y_{\bar{q}\gamma}$, the dashed-dotted line in Fig. 3a represents the cut on $\theta_{q\gamma}$ and $\theta_{\bar{q}\gamma}$. The dotted line in Fig. 3a represents the cut $E_\gamma > E_\gamma^{min}$. In Fig. 3b, the dashed-dotted line shows the restriction coming from a cut on the hadronic energy allowed in the photon jet. In the resulting small cross-hatched triangular-shaped areas, events contain a jet consisting of a photon and a quark (anti-quark) with energy $E_q < \epsilon E_\gamma$ ($E_{\bar{q}} < \epsilon E_\gamma$).
- Fig. 4 Dependence of the $njet + \gamma$ rates on y_0 as obtained from GNJETS for $y_{cut} = 0.02, 0.06, 0.10$, and 0.20 . Here, $\alpha_s = 0.18$ and $y_\gamma = 10^{-3}$ is assumed. The results are given relative to the cross section for $y_0 = 10^{-6}$.
- Fig. 5 Dependence of the cross section on y_γ . GNJETS is used with $\alpha_s = 0.18$ and $y_0 = 10^{-6}$. The results are given relative to the cross section at $y_\gamma = 10^{-3}$. Total cross section for $y_{cut} = 0.02, 0.06, 0.10$, and 0.20 in a. In b - e, the $njet + \gamma$ rates are shown for the various values of y_{cut} .
- Fig. 6 Variation of the $njet + \gamma$ rates as a function of y_{cut} for various partonic isolation angles θ_γ^{min} using EEPRAD. Here $E_\gamma^{min} = 7.5$ GeV and $\alpha_s = 0.18$ is assumed. Shown is the result relative to the cross section for $\theta_\gamma^{min} = 15$ degrees.
- Fig. 7 α_s dependence of the total photonic cross section for $y_{cut} = 0.02, 0.06, 0.10$, and 0.20 in GNJETS.
- Fig. 8 Ratio of predictions from the various matrix element Monte Carlos: EEPRAD / GNJETS (full line), KT / GNJETS (dashed line). In all cases we assume $\alpha_s = 0.18$, $E_\gamma > 7.5$ GeV and $\theta_\gamma > 15$ degrees.
- Fig. 9 Rate of final state photon events from HERWIG 5.4 as a function of the cut-off Q_γ . Shown is the total photon rate and those for $y_{cut} = 0.02, 0.06, 0.10, 0.20$ (a). In (b): efficiency $f(y_{cut})$ for $y_{cut} = 0.02, 0.06$ and 0.20 as a function of Q_γ .
- Fig. 10 Event distributions for three values of $\alpha_s = 0.1$ (dashed line), 0.2 (dotted line) and 0 (full line). Shown is the angle between the photon and the closest jet (upper part), and the photon energy (lower part) for $y_{cut} = 0.02$ (a), 0.06 (b), and 0.20 (c). Results are from GNJETS. The wiggles are due to statistical fluctuations.
- Fig. 11 Comparison of event properties as given by GNJETS (histogram) with $\alpha_s = 0.2$ and the QCD shower model ARIADNE (stars). Shown are the angle between the photon and the closest jet (upper part), and the photon energy (lower part) for $y_{cut} = 0.02$ (a), 0.06 (b), and 0.20 (c).

Fig. 12 Dependence of the $2jet + \gamma$ cross section in the democratic approach on ϵ_0 , the hadronic energy fraction in the cone. Shown are the expectations from GNJETS assuming $\alpha_s = 0.18$ and those from JETSET and ARIADNE at the parton and hadron level. The results are normalized to the cross section with $\epsilon_0 = 0.1$ for each of the calculations.

Fig. 13 Dependence of $1jet + \gamma$ and $2jet + \gamma$ rates on y_γ in the democratic approach for $y_{cut} = 0.02, 0.06, 0.10$, and 0.20 . Results are from GNJETS for $\alpha_s = 0.18$ and $\epsilon_0 = 0.1$ and are given relative to the cross sections for $y_\gamma = 10^{-3}$.

Figure 1

Figure 2

Figure 3a

Figure 3b

Variation of the Jet Cross Section with y_0

Figure 4

Variation of the total X-section with y_γ

Figure 5a

Variation of the Jet Cross Section with y_γ

Figure 5 b-e

Figure 6

Variation of the Cross Section with α_s

Figure 7

Figure 8

Photon Rate in QCD Shower Model

Figure 9

Variation of Photon Distributions with α_s

Figure 10a

Figure 10b

Figure 10c

Figure 11a

Photon Distributions: Matrix element and QCD Shower Models

Figure 11b

Photon Distributions: Matrix element and QCD Shower Models

Figure 11c

Variation with ϵ_0 , Democratic Approach

Figure 12

Variation with y_γ Democratic Approach

Figure 13