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BEAM POLARIZATION IN e^+e^- ANNIHILATION

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Abstract

Transverse and longitudinal beam polarization greatly enhance the high precision experiments that probe the standard model of electroweak interactions and its radiative corrections. So far, polarization has contributed to the high precision measurement of the Z boson mass and is soon to provide substantial improvements in the measurements of the Z width and of the electroweak mixing angle, by accurate measurement of the left-right polarization asymmetry. This article reviews the main experimental techniques, the means to obtain highly polarized beams at LEP and SLC, and describes ongoing and future experiments.

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1 Introduction

This article will address two types of beam polarization experiments: firstly, transverse beam polarization, which is the natural polarization that occurs in storage rings; secondly, longitudinal polarization, that can be obtained directly from a source of longitudinally polarized electrons in a linear accelerator, or by spin rotation of the transverse polarization in a storage ring.

The relevance of beam polarization for precision measurements comes from the possibility of changing one – important – property of the initial state, the spin, without modifying any other. The resulting improvement in systematic errors is often decisive when precision experiments are performed.

In the particular case of e^+e^- annihilation at the Z pole, polarization plays a principal role for several important observables:

- Transverse polarization is precious for the beam energy calibration by resonant depolarization, which contributes sizeably to the reduction of systematic errors on the Z mass M_Z and width Γ_Z . In the future, it will also be useful in the determination of the W mass.
- Longitudinal polarization allows a very precise determination of the effective weak mixing angle $\sin^2 \theta_w^{\text{eff}}$ via the measurement of the Left-Right Polarization asymmetry, and a precise determination of the weak couplings of quarks by the Forward-Backward polarized asymmetries.

The combination of these observables, in particular Γ_Z and $\sin^2 \theta_w^{\text{eff}}$, allows bounds to be placed on the top quark mass and on the Higgs boson mass – or on whatever is playing its role. It can be argued that the improvement in precision brought about by polarized beams is the best way to make this game meaningful.

I will first discuss how one measures beam polarization, as it is an essential ingredient in both types of experiments. Then I will describe the observation of transverse beam polarization in the Large Electron Positron collider (LEP) at CERN and other storage rings, and its application for beam energy calibration, with present (M_Z) and future (Γ_Z, M_W) results. The longitudinal polarization experiments will be described next, with the first important results coming from the Stanford Linear Collider (SLC). The possibility of longitudinal polarization experiments at LEP will also be addressed. I will conclude with remarks on the potential physics output from a successful programme.

2 Beam Polarization Measurement Techniques

Two main techniques are used to measure the polarization of electron beams. Møller scattering of the beam on atomic electrons, and Compton scattering on polarized photons, usually produced by a laser. In both cases the spin dependent cross-sections are isolated by varying the relevant polarization of the target.

2.1 Møller scattering

The Møller polarimeters make use of the polarization asymmetries of the electron-electron scattering cross-section. At tree level, and in the limit $m_e = 0$, the differential cross-section for this process is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s} \frac{(3 + \cos^2 \theta)^2}{\sin^4 \theta} [1 - P_z^1 P_z^2 A_z(\theta) - P_t^1 P_t^2 A_t(\theta) \cos(2\phi - \phi_1 - \phi_2)] \quad (1)$$

where s is the center-of-mass energy squared; θ and ϕ are the center-of-mass angles of the scattered electron ($\phi = 0$ is arbitrary); P_z^1, P_z^2 (resp. P_t^1, P_t^2) are the longitudinal (resp. transverse) polarizations of the incident beam (1) and of the target (2); ϕ_1, ϕ_2 are the azimuths of the transverse polarization vectors; the longitudinal and transverse asymmetry functions $A_z(\theta), A_t(\theta)$ determine the analyzing power of the polarimeter, and are given by:

$$A_z(\theta) = \frac{(7 + \cos^2 \theta) \sin^2 \theta}{(3 + \cos^2 \theta)^2} \quad (2)$$

$$A_t(\theta) = \frac{\sin^4 \theta}{(3 + \cos^2 \theta)^2} \quad (3)$$

Figure 1:
Unpolarized differential cross-sections
for Møller and Bhabha scattering
as a function of the cm scattering angle.
The longitudinal and transverse
asymmetries for both processes are
also shown. From [2].

The values of the cross-section and analyzing power as a function of θ are shown in figure 1. Both asymmetry functions are maximal for 90° scattering, and decrease rapidly for small angles.

In a practical design such as that of the SLC [1, 2], the target is a thin iron foil, placed in the beam line. The interesting scattering occurs on the two external electrons (out of 26) of the iron atoms, that can be fully polarized by submitting the foil to an external magnetic field (around 100 gauss) parallel to the foils axis. The maximum polarization of the iron electrons is thus 8%. The target polarization can

Figure 2:

Left: The Linac Møller polarimeter at SLC. The linac electron beam interacts with the target, a thin iron foil magnetized by the field generated in the Helmholtz coils. Momentum selection is provided by the PEP extraction beam line. After appropriate collimation, the intensity profile of the scattered beam of around 100 electrons per pulse is detected in an array of silicon strip detectors.

Right: Measurement of the beam polarization at the end of the SLC linac with the Møller polarimeter. a) the Møller elastic peak appears at the angle given by two-body kinematics. b) the asymmetry upon beam polarization reversal measures the polarization to be $P_z = 0.244 \pm 0.016 \pm 0.015$. From [3].

be measured with a relative precision of a few percent. This constitutes the present limitation of the method. The orientation of the foil with respect to the beam can be varied to provide transverse or longitudinal polarization.

The scattering angles θ and ϕ are defined by momentum selection on the scattered electron (there is a one-to-one correspondence $P' = P/2(1 + \cos \theta)$ between the scattered electron momentum P' and θ) followed by azimuth selection. The Møller scattering is identified from the background of elastic and quasi-elastic nuclear interactions by the two-body kinematics, that fixes the laboratory scattering angle once the momentum is fixed: $\theta_{lab} = 2m_e(1/P' - 1/P)$. The extraction line Møller polarimeter of the SLC is shown in figure 2. The extraction line of SLC to PEP is used as momentum spectrometer at 15 GeV, corresponding to a center-of-mass angle of 110 degrees, and the scattered electrons are detected in an array of silicon strip detectors. The Møller scattering signal and asymmetry upon reversal of the beam polarization are shown in figure 2. A few hundred scatters occur at each beam passage, allowing a determination of the beam polarization to a few % per minute.

The main advantage of Møller polarimeter is that, in principle, it allows full determination of the polarization vector with a unique set-up. Its limitation, however, lies in understanding the target polarization. Also, the statistics of scatters are limited by the maximum thickness of the target.

2.2 Compton scattering

The use of a polarized laser beam as polarized target offers several advantages, as powerful beams with light polarization of nearly 100% are readily available, with practical spin reversal.

The differential polarized compton scattering cross-section is given by [4, 5]:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left(r_e \frac{q}{q_0} \right)^2 (\Phi_0 + \Phi_1 + \Phi_2) \quad (4)$$

where q_0 is the incident photon energy in the *electron* center-of-mass system, q the outgoing photon energy and r_e the classical electron radius. One has

$$\Phi_0 = (1 + \cos^2 \theta) + (q_0 - q)(1 - \cos \theta) \quad (5)$$

$$\Phi_1 = (\xi_1 \cos 2\phi + \xi_2 \sin 2\phi) \sin^2 \theta \quad (6)$$

$$\Phi_2 = \xi_3(1 - \cos \theta) \vec{\zeta} [\vec{q}_0 \cos \theta + \vec{q}] \quad (7)$$

where q, θ, ϕ are the photon momentum and scattering angles (in the electron center-of-mass) ; $\vec{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ is the electron polarization vector and $\vec{\xi} = (\xi_1, \xi_2, \xi_3)$ is the photon polarization vector [6] represented on the Poincaré sphere, namely: $\xi_3 = \pm 1$ corresponds to right- and left-circular polarization; $\xi_1 = \pm 1$ corresponds to linear polarization along the x and y axes respectively; $\xi_2 = \pm 1$ corresponds to linear polarization at $\pm 45^\circ$ to the x axis in the x-y plane. The collision is assumed here to take place head-on, with the incident electron moving in the +z direction. The term Φ_0 corresponds to the total unpolarized cross-section. Φ_1 corresponds to effects dependent on the photon linear polarization and can be useful for calibration purposes – or be a nuisance for systematic errors. Finally Φ_2 is the term of interest for polarimetry, as it depends on the product of the electron and photon polarizations. One can see that only ξ_3 enters in Φ_2 , so that *circular* photon polarization is used in electron beam polarimetry.

The kinematics of Compton scattering of 45 GeV electrons off laser photons is rather unusual. Both scattering products are swept forward at very small angles to the incident electron beam direction. The relation between the center-of-mass scattering angle θ and the laboratory angle and energy of the recoil photon is shown in figure 3, for the particular parameters of the LEP polarimeter, with a green laser ($\lambda = 530$ nm). High energy photons from 0 to 30 GeV are produced at very small angle, $\theta_{lab} = 1/\gamma \simeq 10^{-5}$ radians for 90° scattering.

Figure 3: Compton kinematics:

- a) the relation between the electron center-of-mass scattering angle and the photon energy and angle in the lab. frame; (from [7])
b) Spin dependent Compton cross-section vs scattering angle for transversely polarized electron beam. Line a shows the unpolarized case, while lines b and c show the right- and left-handed circular light on fully polarized electrons. (From [17]).

2.2.1 Transverse polarization measurement

Originally suggested by Baier and Khoze [8], the laser polarimeter has become a standard equipment in e^+e^- storage rings. The first compton polarimeter worked at SPEAR [9] followed by similar devices at VEPP2 and VEPP4 in Novosibirsk (U.S.S.R.) [10, 11, 12], DORIS in Hamburg (Germany) [13], CESR in Cornell (U.S.A) [14], HERA [15] and TRISTAN [16]. The LEP polarimeter [17] is of similar design.

Transverse electron polarization corresponds to $\vec{\zeta} = (0, P_e, 0)$ illuminated with circular polarized light $\vec{\xi} = (0, 0, \pm\xi)$. The asymmetry upon reversal of the light circular polarization from ξ to $-\xi$ is given by:

$$A(P_e, \xi) = \frac{d\sigma/d\Omega_R - d\sigma/d\Omega_L}{d\sigma/d\Omega_R + d\sigma/d\Omega_L} \quad (8)$$

$$= \frac{\Phi_2}{\Phi_0} = P_e \xi \sin \phi F(\theta, q_0) \quad (9)$$

It materializes as a ϕ asymmetry, resulting in an up-down asymmetry. The spin-dependent cross section is shown on figure 3b. The analyzing power is maximum for $\cos \theta = 0$.

In a practical design, a powerful laser is shined at the electron beam at low angle. The wave length chosen is a compromise between practicality and the analyzing power which increases with the photon energy. A wave length of 532

Figure 4: Set-up of the LEP polarimeter in LEP straight section 1. The laser beam is guided to the Laser Interaction Region (LIR) where it interacts with an angle of 3 mrad. The backscattered photons are separated from the electron beam 254 meters downstream at the beginning of the arc, and detected in a silicon strip calorimeter. (From [17]).

nm is used at LEP, and obtained from a frequency doubled Yag-Nd pulsed laser. Because of the very small scattering angle, a very long lever arm is needed to make the effect measurable. The LEP polarimeter set-up is shown in figure 4.

Two modes of operation can be envisaged for a Compton polarimeter.

- The *single photon* mode is used in HERA, where a low power laser illuminates continuously the electron beam. The backscattered photons are detected one by one, and a full analysis of the photon angular distribution versus its energy can be performed [18]. In particular the up-down asymmetry can be measured as a function of the photon energy, figure 5. The difficulty of the single photon method is that the background from beam gas bremsstrahlung is far from negligible. It can

Figure 5:
Vertical asymmetry as a function of the photon recoil energy for the HERA polarimeter. The beam energy is 27 GeV, the maximum photon energy is 13.2 GeV. η is the asymmetry in the average vertical position of the backscattered photons. The beam polarization is 53%. (From [18]).

Figure 6:
Measured vertical profile in the photon detector of the LEP polarimeter. The profile is measured with a 2 mm pitch silicon strip detector after 2.5 radiation lengths of tungsten.

Figure 7: Simulated and measured asymmetries in the vertical profiles of backscattered photons.
Top left: simulated profile and asymmetry for linear light;
Bottom left: measured asymmetry for linear light;
Top right: simulated profile and asymmetry for circular light on 100% polarized electrons;
Bottom right: measured asymmetry for circular light. The beam polarization is 10%. From [19].

be tolerated in HERA with a short (29 m) specially evacuated straight section. It would become completely unpractical for the 250 m path at LEP.

– The *multi-photon* mode uses a high peak power laser beam working at a low repetition rate (30 to 100 Hz for the LEP polarimeter, while the beam passage rate is 10 kHz). In this case the background is essentially negligible with respect to the several 10^3 backscattered photons per laser pulse. The integrated vertical profile of the backscattered photon pulse is recorded in the photon detector, figure 6. The measurement of transverse polarization consists in detecting the center-of-gravity shift $\Delta(Y)$ of this profile when reversing the polarization of the circular laser light from $-\xi$ to $+\xi$:

$$\Delta(Y) = \kappa \xi P_e \quad (10)$$

where $\kappa = 500 \pm 30 \mu\text{m}$ is the analyzing power of the polarimeter. A 10% beam polarization yields a $\sim 50 \mu\text{m}$ mean-shift for 100% circular light polarization. This

small effect – compared with the 250 m lever arm – requires extreme care in the light polarization reversal mechanism.

In LEP, polarization reversal is provided by a half-wavelength plate rotating synchronously with the laser pulsing [19]. By changing the time delay between the plate axis and the laser pulse one can switch to linearly polarized light, for which no vertical shift is expected but a change of shape in the distribution, due to the term Φ_1 in equation 7. This provides useful calibration and cross-check against systematic errors arising from possible geometrical differences between the two helicities. The expected and observed effects are shown in figure 7.

Transverse polarization at LEP can be measured to a precision of 1% every minute, with systematic errors estimated to be $\Delta P_e = (0.5 + 12P_e)\%$.

2.2.2 Longitudinal polarization measurement

Figure 8:
a) The Compton differential cross-section as a function of the recoil photon energy in the laboratory system for longitudinally polarized electrons. The photon polarization is denoted P_γ .
b) The corresponding asymmetry.

For longitudinally polarized electrons illuminated by circularly polarized light, the equations become:

$$\Phi_1 = 0 \quad (11)$$

$$\Phi_2 = \xi P_e (1 - \cos \theta) [q_0 + q] \cos \theta \quad (12)$$

so that the polarization effects show up only in the energy distribution of the outgoing photon – or electron. The resulting distributions for fully polarized electrons are shown in figure 8a and the corresponding asymmetry in figure 8b. The asymmetry depends strongly on the momentum of the recoil photon, or equivalently of the recoil electron.

Figure 9: left: The SLC compton polarimeter layout. right: the measured asymmetry upon light helicity reversal as a function of the scattered electron momentum. Each point corresponds to a cell of the Čerenkov counter or a channel of the proportional tube array.

The SLC compton polarimeter, shown in figure 9, analyses magnetically the recoil electrons to measure their spectrum in a integrated mode, using Čerenkov detectors, backed up by proportional tubes. The laser polarization is reversed at will by a Pockels cell, which is well matched to the 120 Hz repetition rate of the SLC. The maximum asymmetry is for $\cos\theta = -1$, e.g. the maximum photon energy, or the minimum electron energy of 17 GeV. An example of the measurement is shown in figure 9.

The main systematic error in the measurement comes from the understanding of the light polarization and of the exact analyzing power of the detector, stemming mostly from the possible detector non-linearity. At present, the SLC polarimeter provides a precision of $\Delta P_e/P_e = 2.7\%$ [20]. It is hoped to reduce this uncertainty to $\pm 1\%$ eventually.

Various setups have been proposed [21, 7] to measure longitudinal polarization in LEP. The recoil photon spectrum has to be analyzed. This can be done either by using the single photon mode, or, better, by using a converter/sweeping magnet combination to measure the spectrum of electrons from pair conversions in a way similar to the SLC polarimeter.

3 Physics with Transverse Beam Polarization

3.1 Polarized beams in electron storage rings

The complex topic of how to generate, store and manipulate beam polarization in storage rings is masterfully described in [22]. Transverse polarization builds up in a e^+e^- storage ring by the Sokolov-Ternov effect[24]: synchrotron radiation emission has a small spin-flip probability, with a large asymmetry in favor of orienting the particles' magnetic moment along the guiding magnetic field. In a perfect accelerator a large asymptotic transverse polarization ($-8/5\sqrt{3} \simeq 92.4\%$) would build up with a rise time τ_p :

$$\tau_p = \left(\frac{5\sqrt{3} \hbar r_e E_{beam}^5}{8 m_e^6 \rho^3} \right)^{-1} \quad (13)$$

The rise time depends very strongly on the beam energy E_{beam} and on the radius ρ of the accelerator. In LEP at 46 GeV per beam, the polarization time is 310 minutes.

In a real accelerator, depolarizing resonances occur, reducing the asymptotic degree of polarization P_∞ and its effective rise time τ_p^{eff} in the same ratio. More specifically, a depolarizing time τ_d competes with the polarization time τ_p so that

$$P_\infty = 0.924 \times \frac{1}{1 + \frac{\tau_p}{\tau_d}} \quad (14)$$

$$\tau_p^{eff} = \tau_p \times \frac{1}{1 + \frac{\tau_p}{\tau_d}} \quad (15)$$

The calculation of depolarizing effects, and of the attainable polarization degree, is extremely difficult. The problem lies in the combination of several unfavorable factors:

- The extreme sensitivity of the spin vector to transverse magnetic fields: the precession of the polarization vector around a transverse field is amplified with respect to the rotation of the particle by a factor ν called spin-tune, directly related to the beam energy by the anomalous magnetic moment $a_e = \frac{g_e - 2}{2} = 1.1596521884(43)10^{-3}$ [23] and the mass $m_e c^2 = 0.51099906(15)$ MeV of the electron:

$$\nu = a_e \gamma = \frac{g_e - 2}{2} \frac{E_{Beam}}{m_e c^2} = \frac{E_{Beam}}{0.4406486(1)} \quad (16)$$

The spin tune is also equal to the number of precessions over one turn of the machine. A spin tune $\nu = 103.5$ corresponds to the Z pole beam energy of 45.5 GeV. Any imperfection in the planarity of the storage ring is amplified accordingly.

- Of course, most of these imperfections are by essence unknown. This renders the predictions statistical – and the expected results fluctuating.

- The very long "polarization damping time" is equal to the polarization time – hours. As a consequence, the effect of imperfections is memorized by the polarization vector over typically 10^8 turns of the machine.
- Spin resonances occur each time the spin tune is in phase with the basic motions of the beam particles: turn around the machine (integer resonances); betatron oscillations and synchrotron oscillations (side bands of the integer resonances). These resonances correspond to specific particles energies. The spacing between resonances is energy independent, whereas the beam energy spread is a rapidly increasing function of energy. For instance, the distance in energy between integer resonances is 440 MeV, not comfortably large with respect to the beam energy spread of 40 MeV in LEP at the Z.

The motion of the polarization vector in a storage ring is given by the Thomas-BMT equation [25].

$$\frac{d\vec{P}}{ds} = \frac{e}{m_e c \gamma} \vec{P} \times \left[(1 + a_e \gamma) \vec{B}_\perp + (1 + a_e) \vec{B}_\parallel \right] \quad (17)$$

Where s is the coordinate along the ring, and $B_\perp$ and $B_\parallel$ are the components of the magnetic field transverse and longitudinal to the orbit. Unless the spin tune is an integer, there exist a stable periodic solution to this equation for an electron in periodic motion in a storage ring, denoted $\hat{n}(s)$. In a perfect, flat, machine, with $\vec{B}$ uniform, $\hat{n}$ is parallel to $\vec{B}$. Because of stochastic synchrotron radiation, in particular, not all particles have the same trajectory, but in a perfect ring they all have the same $\hat{n}$ vector, and polarization can quietly build up.

In a realistic situation, however, vertical kicks do occur, due in particular to quadrupoles misalignments, and the $\hat{n}$ vector is no longer vertical. Furthermore, its orientation depends on the trajectory (*spin-orbit coupling*). The spin vector of any given particle will precess around the $\hat{n}$ vector corresponding to its present trajectory and spin diffusion will occur, resulting in depolarization. This phenomenon is described by the Derbenev-Kondratenko formula [26] which we give here in a simplified form:

$$P_\infty = -\frac{8}{5\sqrt{3}} \frac{\sum_j |B_j|^3 L_j}{\sum_j |B_j|^3 L_j (1 + \frac{11}{18} |\Gamma_j|^2)} \quad (18)$$

The sum runs over the magnets in the ring. Their strength is B_j and their length L_j . This equation can be identified with equation 15 with:

$$\frac{1}{\tau_p} \propto \sum_j |B_j|^3 L_j \quad (19)$$

$$\frac{1}{\tau_d} \propto \sum_j |B_j|^3 L_j \frac{11}{18} |\Gamma_j|^2 \quad (20)$$

Note that the synchrotron radiation driving term $|B_j|^3 L_j$ is responsible for both polarization and depolarization.

Figure 10: Polarization measurements from SPEAR. The possible location of several higher order resonances is indicated.

The vector Γ_j is the spin orbit coupling vector at the magnet j :

$$\frac{1}{\gamma}\Gamma = \frac{\partial \hat{n}}{\partial \gamma} \quad (21)$$

It can be decomposed along the normal modes of motion, horizontal (x, x') and vertical (y, y') betatron motion and synchrotron oscillations(s):

$$\frac{1}{\gamma}\Gamma = \frac{\partial \hat{n}}{\partial s} + \frac{\partial \hat{n}}{\partial x}\eta_x + \frac{\partial \hat{n}}{\partial x'}\eta'_x + \frac{\partial \hat{n}}{\partial y}\eta_y + \frac{\partial \hat{n}}{\partial y'}\eta'_y \quad (22)$$

$$= \Gamma_s + \Gamma_x + \Gamma_y \quad (23)$$

The sum over the ring of these components are called spin-orbit coupling integrals [27, 28]. They have a resonant structure, so that strong depolarization takes place for:

$$\nu = m_0 + m_s Q_s + m_x Q_x + m_y Q_y \quad (24)$$

where $m_{0,s,x,y}$ are integers and $Q_{s,x,y}$ are the synchrotron and horizontal and vertical betatron tunes. Several of these resonances were observed at SPEAR [29], figure 10.

The calculation of the vector Γ can be done rigorously to first order using the formalism introduced by Chao [30], and implemented in the computer programme SLIM and several adaptations of it for more realistic simulations of defects [31]. These only calculate resonances with $|m_s| + |m_x| + |m_y| = 1$. The calculation of higher order resonances calls for analytical methods [32, 33], or complete spin-tracking [34], which requires enormous computing power, however. It is generally accepted that the driving terms of the first order resonances are likely to determine the strength of the higher order ones. For this reason, correction algorithms are based on first order calculations.

The most salient feature of spin simulation results for LEP is the dominance of the synchrotron resonances over betatron resonances. The driving terms of these

Figure 11: Polarization signal in LEP showing three consecutive phases:

- 1) stable polarization $P \simeq 9.6\%$;
- 2) depolarization by a static harmonic bump $\nu = 106$;
- 3) polarization rise.

The solid curve shows a fit to the polarization rise time of 35 ± 10 minutes corresponding to $P_\infty = (10.7 \pm 3.1)\%$.

Figure 12: Expected polarization degree as a function of beam energy in the LEP optics used in the first polarization measurements. From [35]

synchrotron resonances are proportional to $|\delta\hat{n}|^2$ where $\delta\hat{n}$ is the tilt of the equilibrium spin vector with respect to the normal to the plane of the accelerator. The immediate consequence is that depolarization increases like the square of the beam energy, and like the square of the size of the defects. A general strategy to improve polarization is therefore to align the orbit to the best possible precision.

Spin simulations have been used for i) choice of the appropriate working point; ii) compensation of known defects; iii) improvement of polarization.

The first order predictions of the polarization behavior show the dominance of integer resonances, and among them, of 'systematic' integer resonances [36]:

$$\nu = 8k \pm \text{int}[Q_{x/y}]. \quad (25)$$

Consequently, a beam energy $E = 46.5 \text{ GeV}$ ($\nu = 105.55$) was chosen for polarization studies, and the closest betatron resonances were moved away by choosing smaller values of the fractional parts of the betatron tunes. This choice, and careful vertical orbit corrections down to 0.5 mm RMS, allowed observation of 10% polarization at LEP in 1990 [19] and 1991, as shown in figure 11. The first order prediction was 20%, as shown in figure 12, but higher order effects were expected to reduce it somewhat.

An example of correction of known defects is the compensation of the spin rotation induced by the experimental solenoids. This is of practical importance since the first polarization experiments in LEP required turning off the experimental solenoids, a lengthy and dangerous procedure. The scheme adopted in LEP [39] was first suggested at DESY [37, 38]. A rotation opposed to that created by the solenoid is generated in the arcs on each side of the experimental straight section, by vertical closed bumps, figure 13. As a result the spin vector is vertical everywhere else in the machine. Spin simulations show that, indeed, the integer resonances

Figure 13: Left: the arc bumps used to spin-compensate the experimental solenoids in LEP. Right: motion of the tip of the $\hat{n}$ vector in the bump. The solenoid then creates a rotation of n_x by $+66\text{mrad}$. A second bump, opposite in to the first one, situated on the other side of the straight section, brings the spin back to vertical ($n_x = n_z = 0$).

Figure 14: Experimental verification of solenoid spin compensation at LEP. The polarization level is shown as function of time, while the solenoid of the ALEPH experiment was ramping from 0 to full field. At first no compensation is excited and the polarization degree drops. At time 1655 the solenoid compensation is turned on and polarization rises back up to the initial level.

are compensated when the tilt of the $\hat{n}$ vector is compensated. This was verified experimentally in 1992 [40], figure 14, and used since.

A similar *spin-matching* exercise is necessary if one wants to implement the spin 90° spin rotation required for obtaining longitudinally polarized beam, (see section 4.5). This was also performed with first order spin theory.

Improvement of polarization is based on the so-called *harmonic spin-matching* procedure. Here again, the fact that integer resonances and their Q_s side bands dominate the spectrum leads to simplification. The improvement of polarization can be obtained by reducing the tilt of $\hat{n}$, by appropriate spin rotations. It can be shown that, if the defects of the machine are dominated by the misalignment of the quadrupoles and the corresponding orbit correctors, the tilt of the $\hat{n}$ vector can be decomposed in the Fourier components (a_k, b_k) of the orbit, calculated in the spin

Figure 15: Improvement of polarization with harmonic spin matching. Left: over 50% (from about 15%) polarization was achieved at HERA by empirical spin-matching. Figures a) b) c) d) show the scan of the harmonic correctors. The bottom right picture shows the polarization rising over 50%.

precession frame [41]:

$$|d\hat{n}|^2 \propto \sum_k \frac{1}{(\nu - k)^2} (a_k^2 + b_k^2) \quad (26)$$

It is possible to construct combinations of bumps – similar to those shown above for the solenoid compensation – which generate exactly the desired harmonics. One can see from equation 26 that the most enhanced harmonics are those close to the spin tune, $\text{Int}(\nu) = k, k + 1$. Correcting for the real and imaginary components of each of these close harmonics requires four bumps. One beauty of the technique is that the harmonics are orthogonal to each other: the formalism provides a set of independent spin correctors.

This technique was first applied at PETRA [42], with improvements in the polarization degree from 20% to over 70%. The correctors were varied in turn to find the optimum polarization. The same procedure was successfully performed on HERA [18] in 1992, with polarization reaching 56%, figure 15a.

For LEP, the polarization rise time is much too long for this procedure to be successful. A more deterministic approach is necessary. If the beam orbit monitors (BOM) were infinitely precise, one could measure directly the harmonic components

of the orbit. The precision available was not sufficient in 1990-1992. The beam monitors were recently upgraded, and the machine realigned, so that the deterministic harmonic spin-matching has become possible. A polarization degree of 30 to 40% (from 10%) has been reproducibly obtained in 1993 by this method.

Further improvement for LEP could come from the use of dedicated polarization wigglers [43] designed to reduce the polarization time from ≈ 300 minutes to 30 minutes and simplify the optimization procedures. It is feared, however, that the unavoidable increase in the beam energy spread will at least partially cancel the benefits [33].

So far, the only source of depolarization considered were the energy jumps due to synchrotron radiation in the arcs. When the beams enter in collision, beam-beam collisions provide another source of stochastic orbit jumps, in the form of vertical kicks at the IP's. It is fair to say that this aspect has not been as carefully studied than the previous one. A rudimentary model based on random orbit kicks was implemented by Chao [30] in SLIM. Spin-matching conditions were derived by Buon [44], for both transverse and longitudinal polarization. They establish relations between the betatron tunes and the energy, so that one expects perfect spin matching only at discrete energies. Polarization in collisions was observed at SPEAR, CESR, VEPP and PETRA, but the phenomenon was sometimes irreproducible. Observations at PETRA indicate that depolarization only sets in when one approaches the beam-beam limit where vertical emittance is substantially increased. Clearly, this issue requires substantially more work. Polarization has not yet been observed in collisions at LEP.

3.2 Transverse polarization asymmetry

Figure 16: Transverse polarization asymmetry at SPEAR.

a) Observed hadron yield for all particles with $x > 0.3$ at $\sqrt{s}=7.4$ GeV;

b) Same as a) at the spin-depolarization resonance $\sqrt{s}=6.2$ GeV;

c) Transverse polarization asymmetry as a function of particle momentum, showing leading particle effect.

Transverse polarization has been observed in every electron storage ring where it was searched for and a few beautiful experiments were done. At SPEAR at SLAC (U.S.A.), observation of the transverse polarization asymmetry [45] – an azimuthal modulation of particle production dependent on the spin of the particle – was a clear confirmation of the conjecture that jets of hadrons originate from the production of spin $\frac{1}{2}$ partons (Figure 16). This experiment was performed with over 70% beam polarization in collision mode.

3.3 Beam energy calibration and hadronic resonances masses

Extremely precise calibration of the beam energy, to one part in 10^5 , can be done by exciting an artificial depolarizing resonance, using the electron spin in nearly the same way as the nuclear spins in a Nuclear Magnetic Resonance probe.

Accurate measurements of the masses of the ω , ϕ , J/ψ , ψ' , Υ , Υ' , and Υ'' resonances have been performed using this technique at VEPP2 and VEPP4 in Novosibirsk (U.S.S.R.) [10, 11, 12], at DORIS in Hamburg (Germany) [13], and CESR in Cornell (U.S.A) [14]. Figure 17 shows the observation of depolarization in VEPP4 and DORIS. For all these experiments the measurement of energy was

Figure 17: Examples of depolarization curves: The beam polarization is measured as an up-down asymmetry of back scattered light on the polarized beam. A small tunable RF magnetic field is applied on the beam, and polarization measured as a function of its frequency. a) VEPP4 in Novosibirsk [11]. 1 and 2: back scattering of synchrotron radiation light from the e^+ beam on the e^- beam and vice versa. 3: back scattering of polarized laser light on the e^- beam. $\tau_p = 50$ minutes, $P_\infty = 80\%$. b) at DORIS in Hambourg [13] with back scattered laser light. $\tau_p = 4$ minutes, $P_\infty = 80\%$, luminosity up to $1.5 \cdot 10^{31} \text{ cm}^2/\text{s}$.

performed while the accelerator was in normal colliding conditions with detectors taking data. Polarization degrees of up to 80% and luminosities approaching half the peak luminosity were obtained. The increase in precision obtained by experiments at VEPP2 and VEPP4 are listed in table 1.

3.4 Beam energy calibration for LEP

This section will describe how this powerful method has been applied to improve the determination of the Z boson mass, and is hoped to improve also the Z width measurement in the future.

3.4.1 Effect of beam energy uncertainties on the Z line shape parameters

The Z mass and width are extracted from a fit to the measured cross-section in $e^+e^- \rightarrow \text{hadrons}$ as a function of center-of-mass energy, as shown in figure 18. The determination of the Z mass is sensitive to the knowledge of the absolute energy scale, while the Z width is affected by possible errors in the differences between the energies of the scan points. Also sensitive to the relative energies is the forward-backward asymmetry for $e^+e^- \rightarrow \ell^+\ell^-$, as shown in figure 19, from which one of the most precise ¹ measurement of the effective weak mixing angle $\sin^2 \theta_w^{\text{eff}}$ is obtained. The Z line shape was scanned in 1990 and 1991, but not in 1992.

Energy errors are classified in four categories.

¹In absence of longitudinally polarized beams.

Table 1: A list of particles where the application of the resonant depolarization method increased the precision of the mass determination [12].

Particle	World average value (MeV)	Experimental results (MeV)	Year of publication	Accuracy improvement
$K^\pm$	493.84 ± 0.13	493.670 ± 0.029	1979	5
K^0	497.67 ± 0.13	497.661 ± 0.033	1987	4
ω	782.40 ± 0.20	781.780 ± 0.10	1983	2
Φ	1019.7 ± 0.24	1019.52 ± 0.13	1975	2.5
J/Ψ	3097.1 ± 0.90	3096.93 ± 0.09	1981	10
Ψ'	3685.3 ± 1.20	3686.00 ± 0.10	1981	10
Υ	9456.2 ± 9.50	9460.59 ± 0.12	1986	80
Υ'	$10016.0 \pm 10.$	10023.6 ± 0.50	1984	20
Υ''	$10347.0 \pm 10.$	10355.3 ± 0.50	1984	20

- The absolute energy scale error, $(\frac{\Delta E}{E})_{abs}$.
- The non-linearity $\alpha \pm \Delta\alpha$ of the response of the magnets to the exciting current.
- Point-to-point errors that account for possible higher order effects in the relation between dipole current and beam energy, $(\frac{\Delta E}{E})_{p-t-p}^{setting}$.
- Non-reproducibility errors, $(\frac{\Delta E}{E})_{rep}$, coming from several possible sources of variability, such as temperature, tidal effects, corrector settings, unknown variations in the RF status. The error from these sources average out with N_{scan} , the number of times the Z line shape is scanned. (7 times in 1990, 5 times in 1991).

The systematic errors on $M_Z, \Gamma_Z, \sin^2 \theta_w^{eff}$ resulting from energy uncertainties are as follows.

$$\frac{\Delta M_Z}{M_Z} = (\frac{\Delta E}{E})_{abs} \oplus 0.5 \left[\left(\frac{\Delta E}{E} \right)_{p-t-p}^{setting} \oplus \frac{1}{\sqrt{N_{scan}}} \left(\frac{\Delta E}{E} \right)_{rep} \right] \quad (27)$$

$$\frac{\Delta \Gamma_Z}{\Gamma_Z} = (\frac{\Delta E}{E})_{abs} \oplus 0.5 \frac{M_Z}{\Gamma_Z} \left[\left(\frac{\Delta E}{E} \right)_{p-t-p}^{setting} \oplus \frac{1}{\sqrt{N_{scan}}} \left(\frac{\Delta E}{E} \right)_{rep} \right] \oplus \Delta\alpha \quad (28)$$

$$\Delta \sin^2 \theta_w^{eff} = 0.0004 \cdot \frac{M_Z}{10 \text{ MeV}} \left[\left(\frac{\Delta E}{E} \right)_{p-t-p}^{setting} \oplus \frac{1}{\sqrt{N_{scan}}} \left(\frac{\Delta E}{E} \right)_{rep} \right] \quad (29)$$

where $\oplus$ represents a quadratic sum: $a \oplus b = \sqrt{a^2 + b^2}$. The coefficient 0.5 in front of the non-reproducibility and point-to-point errors is a numerical factor which can vary – slightly – depending on the number of points in the scan and on their energies.

Figure 18: The Z line shape. A fit to a theoretical formula gives the parameters M_Z and Γ_Z . The standard model predictions for $N_\nu = 2, 3, 4$ are shown. The lower plot shows the agreement of the theoretical shape to the measured one. The experiments (here ALEPH) are responsible for the vertical error bars, LEP and the LEP energy working group for the horizontal ones.

Figure 19: The forward backward asymmetry for leptons (from L3). The energy dependence is rather steep. The most important physics information is contained in the offset of the curve at $E_{c.m.} = M_Z$ and is affected by uncertainties in the beam energy.

3.4.2 Magnetic measurements

The determination of energies before polarization was available is described in [46], and is based on magnetic measurements. The beam energy, which is proportional to the field integral around the LEP ring, is derived from the measurement of the magnetic field in a LEP reference dipole magnet. A flipping coil is used to measure the field in a pure iron magnet powered in series with the ring's dipoles. This measurement is continuously available and is labeled E_{FD} (for Field Display). However, the reference magnet is of different nature than the LEP concrete-iron dipoles and situated in different conditions of temperature and humidity. An absolute calibration by other methods is necessary.

A direct measurement of the field generated by the dipoles is provided by cycling the magnets and measuring the induced current in a closed electrical loop, imbedded in the LEP dipoles. This "flux-loop" method is in principle rather accurate ($\pm 10^{-4}$), but cannot be applied while beams circulate. Furthermore it only measures the field produced by the dipoles, and not the additional fields which influence the beam energy, such as the earth magnetic field, the possible permanent magnetic properties of the beam pipe, as well as the dipole components of the orbit correctors and quadrupoles. An in-situ calibration with circulating beams is thus required.

An elegant solution is provided by comparing, at injection energy, the rev-

olution frequencies of protons and positrons circulating on the same orbit. This provides a calibration with a precision of $\pm 3 \cdot 10^{-5}$ [47] at 20 GeV. However, the extrapolation of this calibration to 45 GeV is entailed by the uncertainty in the non-linear fields mentioned above. Also, this method requires interruption of the high energy physics running.

Combining the three methods described above, an average correction to be applied to E_{FD} is obtained, which, for the 1990 data, was:

$$E_{Beam} = E_{FD} \times [1 - (6.4 \pm 2.4)10^{-4}]. \quad (30)$$

This allows an estimate of the absolute energy scale with an error of $(\frac{\Delta E}{E})_{abs} = \pm 2.2 \cdot 10^{-4}$. However, the non-reproducibility and point-to-point errors were essentially unknown and estimated to be about 10 MeV each. Non-linearity was not considered.

Altogether, the precision on M_Z was limited to ± 21 MeV, and on Γ_Z to ± 5 MeV, while the statistical errors for the 1990 data set were [48] ± 5 MeV and ± 9 MeV, respectively. The use of resonant depolarization was necessary to improve this result.

3.4.3 Resonant depolarization

The spin tune ν , or number of spin precessions in one turn around the ring, is proportional to the beam energy, equation (16). It is determined by resonant depolarization [49].

Resonant depolarization is produced by exciting the beam with an oscillating magnetic field generated by a vertical kicker magnet. This exciting field is thus perpendicular to the beam axis and situated in the plane (e.g. horizontal) of the ring. If the frequency of the resulting spin kick is in phase with the spin precession, a resonance condition occurs. The electron spins are coherently swept away from the vertical direction, and polarization disappears.

Because the beam encounters the transverse exciting field only once per turn, the frequency at resonance is determined by the fractional part of the spin tune, δ_s . The standard magnetic measurements are precise enough to provide the integer part without ambiguity. Furthermore, because the exciting field is a transverse field perpendicular to the beam axis, and not a rotating field around the vertical axis, a resonant condition occurs also for the mirror frequency $1 - \delta_s$. The mirror ambiguity is resolved by a second measurement after inducing a small change of beam energy with a change of RF frequency.

The actual spin precession frequency is the product of the spin tune by the revolution frequency f_{rev} of the particles around the LEP ring. The revolution frequency is obtained by dividing the RF frequency f_{RF} , nominally set to $f_{RF} = 352,254,170$ Hz and measured to better than 1 Hz, by the RF harmonic number, 31324. This yields, in nominal conditions, $f_{rev} = 11,245.5041(1)$.

To find the resonance, the horizontal field of maximum integrated strength bl is excited at a slowly varying frequency, in such a way as to cross the resonance

Figure 20: Polarization signal on 2 October 1991, showing the localization of the depolarizing frequency within the sweep.

Top: display of data points, with the frequency sweep indicated with vertical dashed lines. The full line represents the result of a fit with starting polarization $(-4.9 \pm 1.)\%$, polarization rise-time (60 ± 13) minutes, asymptotic polarization $(18.4 \pm 4.1)\%$.

Bottom: expanded view of the sweep period, with the individual data sets displayed (there are 10 sets per point); The frequency sweep lasted 7 data sets. The corresponding beam energy is shown in the upper box. Spin flip occurred between the two vertical dash-dotted lines.

condition. The expected signal for resonance crossing is a steep variation of the polarization degree, in a way that depends on the rapidity with which the equivalent spin tune is swept, $\frac{\Delta\nu}{\Delta t}$ [50]:

$$\frac{P(final)}{P(initial)} = 2e^{-\chi} - 1 \quad (31)$$

$$\chi = \frac{(\pi\nu \cdot \frac{bL}{BL})^2 \times f_{rev}}{\frac{\Delta\nu}{\Delta t}}, \quad (32)$$

where BL is the integrated guide field of LEP.

Formula (31) is derived for proton accelerators. Its application for e^+e^- storage rings is not straightforward, because of the excitation of energy oscillations of particles due to emission of synchrotron radiation. This effect leads to decoherence of the polarization component in the plane of the ring (horizontal component), with a decay time constant which is not yet well known. If this decay time is much longer than the resonance crossing time, a value of $\frac{P(final)}{P(initial)}$ of -1 can be obtained, leading to polarization reversal or "Spin-Flip". On the other hand, a short decay time of the horizontal polarization component would limit $\frac{P(final)}{P(initial)}$ to 0, i.e. depolarization.

Figure 21: Graphical representation of the depolarizations on 16 September 1991. Full line: depolarization or spin flip; dashed line: No depolarization.

An example of resonant depolarization is shown in figure 20. The resulting energy measurement is $E_{beam} = 46,466.6 \pm 0.6$ MeV, e.g. precise to $\pm 1.5 \cdot 10^{-5}$. Partial spin-flip took place, indicating that the decay time of the horizontal polarization component is of the same order of magnitude as the resonance crossing time, a few seconds.

In practice, the resonance is searched by successive frequency sweeps of varying range, as shown in figure 21. In 1991 the resonance could be located within a sweep range of $\Delta\delta_s = 0.005$, leading to an energy error of ± 0.6 MeV. This was improved in 1992 to $\Delta\delta_s = 0.002$, or a precision of ± 0.25 MeV.

Several tests were made to ascertain that the observed resonance was not a spurious one. In similarity to the static resonances of equation 24, some depolarization was observed at frequencies corresponding to the synchrotron sidebands of the main resonance. This confusion could cause an error of $\Delta E_{beam} = Q_s \times E_{beam}/\nu = 27$ MeV. However these side-bands were readily discarded by changing the synchrotron tune.

Other systematic errors were considered, in particular the possible interference of the resonance with the static depolarizing resonances present in the machine [51]. This effect was estimated to be less than 0.5 MeV.

In conclusion, the resonant depolarization of the electron beam could be located unambiguously and the energy measured at a given instant with a precision of ± 0.6 MeV. We will see, however, that other uncertainties limit the usefulness of the result.

3.5 *Energy variations and the tidal effect*

The beam energy measured during the polarization runs has to be traced over several weeks to the period where the LEP experiments are taking data. This is done with the field display, which has a more limited resolution of ± 1 MeV.

Furthermore, as can already be seen by comparing measurements 11 and 15 in figure 21, the position of the resonance varies, on a rather short time scale. When compiling the results of the 1991 calibrations, a scatter of ± 3 MeV in beam energy was noticed.

Variation of the magnetic field in the LEP dipoles with respect to the field display can originate from temperature variations, and more generally from other environmental parameters. The temperature coefficient of the LEP magnets was measured and compared with flux-loop measurements [52]. Correcting the depolarization data for the measured temperatures increased the scatter to ± 3.7 MeV!

It was suggested by G. Fischer and A. Hofmann [53] that earth tides could be the cause of these variations. Earth tides, generated by the combined effect of the gravitational fields of the moon and the sun have been known since the middle of the 19th century and precisely measured for more than 30 years [54]. Contrary to oceanic tides, earth tides are not a resonant phenomenon, and can be calculated successfully as a global deformation of our planet. They result in a linear dilatation of dimensions, with a maximum swing of $\pm 3.10^{-8}$.

How do earth tides affect the beam energy? The "central" orbit of LEP – the orbit that passes through the center of the quadrupoles and sextupoles – is also expanded (or shrunk) in this proportion, corresponding to a total change of the 27 km circumference by 600 μm . However, the true orbit of superrelativistic electrons is constrained by the fixed RF frequency to a fixed length. As a result,

Figure 22: Correlation between the energy measured with resonant depolarization in 1991 and the earth tide amplitude. The numbering on the measurements represents their sequence in time, and the error bars give the range in beam energy within which resonant depolarization was observed.

Figure 23: Beam energy variations measured over 24 hours compared to the expectation from the tidal LEP deformation.

when the machine expands (shrinks), the beam passes through the quadrupoles off center towards the inside (outside) of the ring. Since the quadrupoles are focusing on the average, the beam sees less (more) integrated magnetic field. This results in an energy change of

$$\frac{\Delta E}{E} = -\frac{1}{\alpha} \frac{\Delta R_{LEP}}{R_{LEP}} \quad (33)$$

where α is the *momentum compaction factor*; α depends on the horizontal betatron tune, so that the energy swing is larger, ± 7 MeV, for the 90° lattice used since 1992, than for the 60° lattice used in 1991, where the energy swing was ± 3.5 MeV.

The beam energy measurements of 1991 are plotted in figure 22 against the tide amplitude. A correlation of $\simeq 2 \sigma$ significance is visible. The calibrations sampled the tide effect sufficiently well, nevertheless, for their average to be applicable to the physics data.

A dedicated experiment was performed in 1992 to ascertain the effect [55]. The results, shown in figure 23, agree with the tide prediction with a precision, and stability, of better than 0.5 MeV over 24 hours.

3.6 Determination of the Z mass and width

Resonant depolarization measures the sum of spin precessions along the trajectory, which is proportional to the average energy in the arcs (the energy is not constant due the energy loss by synchrotron radiation). One need to relate this to the energy at the interaction points during the physics runs. This implies corrections to the average physics conditions, in particular for temperature and tidal effect. In addition a correction has to be made for the (known) asymmetry in the accelerating

cavities, which shifts the center of mass energies in OPAL and L3 by 12.8 and 12.7 MeV respectively.

A full account of the steps leading to the energy calibration of LEP in 1991 is given in [52], and the determination of the Z mass in [57]. The following values are found for the error components entering equation 29.

- Absolute energy error: $(\frac{\Delta E}{E})_{abs} = \pm 5.7 \cdot 10^{-5}$
- Non-linearity : $\Delta\alpha = \pm 1.5 \text{ MeV/GeV}$
- Further point-to-point error: $(\frac{\Delta E}{E})_{p-t-p}^{setting} = \pm 3.10^{-5}$
- Non-reproducibility: $(\frac{\Delta E}{E})_{rep} = \pm 10 \cdot 10^{-5}$.

The Z mass and width are extracted by a fit of the experimental cross-sections [56] to the theoretical line-shape [58]. The average of the four LEP experiments is:

$$M_Z = 91.187 \pm 0.0035_{stat} \pm 0.0063_{LEP} \text{ GeV} \quad (34)$$

$$\Gamma_Z = 2.488 \pm 0.0054_{stat} \pm 0.0045_{LEP} \text{ GeV} \quad (35)$$

The systematic error of 6.3 MeV on the Z mass is dominated by the absolute energy scale error, while the error on the Z width is dominated by the poorly known non-linearity.

3.7 Prospects for improvement on the Z line shape

Even though it is measured to a relative precision of $6 \cdot 10^{-5}$, the Z mass is a fundamental parameter to be measured with the best possible precision. The Z width is even more interesting since it provides, by comparison with M_Z , a powerful test of the Standard Model, sensitive to electroweak radiative effects.

The error on the width is still statistics limited. Furthermore the systematic error is substantial solely because the energy calibration was available at one energy only.

The 1993 running of LEP is dedicated to a new scan of the Z resonance. The LEP chamonix workshop [59] discussed the strategy in detail. Several improvements to the machine setup were successful, in particular the solenoid spin-compensation, and the observation of polarization on the same optics used for physics data taking. Polarization was observed – and energy calibration performed – at the two most favorable points on the line shape, $E_{beam} = 89.42$ ("peak - 2"), and $E_{beam} = 93.0$ ("peak + 2"). Energy calibration can be performed at the end of physics fills, and lasts only 3-4 hours – instead of typically 24 hours previously. Two calibrations are performed every week.

For a total integrated luminosity of $40\text{-}50 \text{ pb}^{-1}$, a total error of $\pm 3 \text{ MeV}$ or less on both M_Z and Γ_Z seems achievable.

3.8 The W mass

The energy of LEP will be upgraded to 90 GeV per beam towards the end of 1994. One of the important goals is the determination of the W boson mass. Several methods have been suggested to measure it [60], all require knowledge of the beam energy at LEP II, with a precision better than the expected statistical precision of the measurement, 25 MeV. If shown practical and feasible with the advertised accuracy, the proposal for measuring the beam energy by using the kinematical end-point in Møller scattering [61] could be interesting. In view of the present improvement in the polarization with realignment of LEP and harmonic spin-matching it is not impossible that energy calibration by resonant depolarization will be achievable at LEP II energies, or at least at a close enough energy. In any case, the LEP I energy measurement system, with the present polarimeter and the flux loop, will need to be kept operational for the LEP II era.

4 Physics with longitudinally polarized beams

In the energy range of LEP weak interactions dominate. The Standard Electroweak Model $SU(2)_L \times U(1)$ being left-right asymmetric, helicity effects are expected to play an important role. This is in contrast with lower energy e^+e^- accelerators where the QED-dominated physics leaves limited interest for longitudinally polarized beams.

The SLC was designed from the start to be a polarized machine, building on the success of the deep-inelastic experiments with polarized electrons [62]. The early LEP studies were quite aware of the importance of beam polarization [63]. The exact benefits were not realized, however, and the difficulties in obtaining polarized beams seemed overwhelming. The subject of longitudinally polarized beams for LEP has been extensively studied in 1988 [64].

4.1 Helicity effects in e^+e^- annihilation

Longitudinal polarization effects manifest themselves in many reactions. Most of all, longitudinally polarized beams allow measurements of the weak couplings of fermions to the Z in a clean and precise way. Helicity effects in $e^+e^- \rightarrow Z \rightarrow f\bar{f}$ are sketched in figure 24.

The fact that the Neutral Current Couplings are different for left-handed and right-handed fermions leads to polarization and forward-backward asymmetries which can be expressed in terms of the coupling asymmetries:

$$A_f \equiv \frac{g_{L_f}^2 - g_{R_f}^2}{g_{L_f}^2 + g_{R_f}^2} = \frac{2g_V f g_{A_f}}{g_V^2 + g_A^2} \quad (36)$$

The couplings are related to the weak mixing angle $\sin^2 \theta_w^{\text{eff}}$ by the well known relations:

$$g_{L_f} = I_{3f} - Q_f \cdot \sin^2 \theta_w^{\text{eff}} \quad (37)$$

Initial state helicity	cross-section	Final state helicity
Left e^- , right e^+ , $\mathcal{P} = -1$ $\begin{array}{ccc} \leftarrow & & \leftarrow \\ e^- & Z & e^+ \\ \rightarrow & & \leftarrow \end{array} \quad \propto g_{Le}^2$		Forward: $\begin{array}{cc} \leftarrow & \leftarrow \\ \bar{f} & f \\ \leftarrow & \rightarrow \end{array} \propto g_{Lf}^2$ Backward: $\begin{array}{cc} \leftarrow & \leftarrow \\ f & \bar{f} \\ \leftarrow & \rightarrow \end{array} \propto g_{Rf}^2$
Right e^- , left e^+ , $\mathcal{P} = +1$ $\begin{array}{ccc} \Rightarrow & & \Rightarrow \\ e^- & Z & e^+ \\ \rightarrow & & \leftarrow \end{array} \quad \propto g_{Re}^2$		Forward: $\begin{array}{cc} \Rightarrow & \Rightarrow \\ \bar{f} & f \\ \leftarrow & \rightarrow \end{array} \propto g_{Rf}^2$ Backward: $\begin{array}{cc} \Rightarrow & \Rightarrow \\ f & \bar{f} \\ \leftarrow & \rightarrow \end{array} \propto g_{Lf}^2$
$\begin{array}{ccc} \leftarrow & & \Rightarrow \\ e^- & Z & e^+ \\ \rightarrow & & \leftarrow \end{array} \quad 0$		
$\begin{array}{ccc} \Rightarrow & & \leftarrow \\ e^- & Z & e^+ \\ \rightarrow & & \leftarrow \end{array} \quad 0$		

Figure 24: Helicity effects in $e^+e^- \rightarrow Z$ production. The arrows $\leftarrow$ and $\Rightarrow$ indicate the helicity of the particles.

$$g_{Rf} = -Q_f \cdot \sin^2 \theta_w^{\text{eff}} \quad (38)$$

$$g_{Af} = 2(g_{Lf} - g_{Rf}) \quad (39)$$

$$g_{Vf} = 2(g_{Lf} + g_{Rf}) \quad (40)$$

These relations are nowadays used as *definition* for $\sin^2 \theta_w^{\text{eff}}$. The values of Neutral Current Couplings and their sensitivity to $\sin^2 \theta_w^{\text{eff}}$ are given in table 2.

The observables obtainable with longitudinal polarization are:

- the left-right asymmetry of Z production [65, 66, 67, 68, 1]:

$$A_{LR} = -\frac{1}{\mathcal{P}} \frac{\sigma_{\mathcal{P}} - \sigma_{-\mathcal{P}}}{\sigma_{\mathcal{P}} + \sigma_{-\mathcal{P}}} \simeq \mathcal{A}_e; \quad (41)$$

- the forward-backward polarized asymmetries [69]:

$$A_{FB}^{\text{pol}(f)} = -\frac{1}{\mathcal{P}} \frac{(\sigma_{\mathcal{P},F} - \sigma_{-\mathcal{P},F}) - (\sigma_{\mathcal{P},B} - \sigma_{-\mathcal{P},B})}{(\sigma_{\mathcal{P},F} + \sigma_{-\mathcal{P},F}) + (\sigma_{\mathcal{P},B} + \sigma_{-\mathcal{P},B})} \simeq \frac{3}{4} \mathcal{A}_f; \quad (42)$$

Table 2: Numerical values of quantum numbers, Neutral Current Couplings, chiral coupling asymmetry A_f , and sensitivity of A_f for the four types of fermions. The value of $\sin^2 \theta_w^{\text{eff}}$ is 0.23.

Fermion type	I_{3f}	Q_f	g_{Af}	g_{Vf}	A_f	$\frac{\delta A_f}{\delta \sin^2 \theta_w^{\text{eff}}}$
ν	1/2	0	1	1	1	0
e^-	-1/2	-1	-1	-0.08	0.16	-7.9
u	1/2	2/3	1	0.39	0.69	-3.5
d	-1/2	-1/3	-1	-0.69	0.94	-0.6

where

$$\mathcal{P} = \frac{P_{e^-} - P_{e^+}}{1 - P_{e^-} - P_{e^+}} \quad (43)$$

is the polarization of the e^+e^- system. In the case where only the e^- are polarized $\mathcal{P} = P_e$. The cross-sections for beam polarization $\pm \mathcal{P}$ are denoted $\sigma_{\pm \mathcal{P}}$. The indices F, B indicate integration over the forward or backward hemisphere.

Without polarization the forward-backward asymmetry can be measured:

$$A_{FB}^{(f)} \simeq \frac{3}{4} A_e A_f. \quad (44)$$

In either case, for the tau lepton, the polarization of the final state fermion is measurable. The tau polarization is a function of polar angle,

$$\mathcal{P}_\tau(\cos\theta) \simeq -\frac{A_\tau - \frac{2\cos\theta}{1+\cos^2\theta} \mathcal{P}_Z}{1 - \frac{2\cos\theta}{1+\cos^2\theta} \mathcal{P}_Z A_\tau}, \quad (45)$$

from which one can derive the average, integrated over polar angle,

$$\langle \mathcal{P}_\tau \rangle \simeq -A_\tau, \quad (46)$$

or an other version of the forward-backward polarization asymmetry [70],

$$A_{FB}^{\text{pol}(\tau)} = \frac{(\sigma_{\mathcal{P}_\tau=1,F} - \sigma_{\mathcal{P}_\tau=-1,F}) - (\sigma_{\mathcal{P}_\tau=1,B} - \sigma_{\mathcal{P}_\tau=-1,B})}{(\sigma_{\mathcal{P}_\tau=1,F} + \sigma_{\mathcal{P}_\tau=-1,F}) + (\sigma_{\mathcal{P}_\tau=1,B} + \sigma_{\mathcal{P}_\tau=-1,B})} \simeq \frac{3}{4} \mathcal{P}_Z, \quad (47)$$

where $\mathcal{P}_\tau$ is now the polarization of the tau lepton, $\mathcal{P}_Z$ is the longitudinal polarization of the produced Z,

$$\mathcal{P}_Z = -\frac{A_e - \mathcal{P}}{1 - A_e \mathcal{P}} \quad (48)$$

and $\mathcal{P}$ is the polarization of the e^+e^- system, equation 43. If the beams are unpolarized, $\mathcal{P}_Z = -A_e^2$.

If longitudinal polarization is available, measurements of A_{LR} and $A_{FB}^{\text{pol}(f)}$ for the various types of fermions allow a complete determination of the fermion couplings [69, 72, 73].

²Many unnatural minus signs come in these equations because A_f are defined as Left-Right asymmetries – this is the natural convention for the $SU(2)_L \times U(1)$ standard model –, while polarizations are traditionally defined as +1 for right-handed particles.

The Left-right polarization asymmetry A_{LR} is a rare example of an observable that conciles nearly all advantages. A_{LR} is simply obtained by comparing the total cross-section for producing a Z from a left-handed (σ_L) or right-handed (σ_R) e^+e^- system. It is obvious (Figure 24) that A_{LR} at the Z pole will not depend on the way how the Z decays, but only on the electron Neutral Current Couplings [66, 68]. Detailed calculations show that this is affected only by minor and well understood corrections [68, 71].

The statistical precision on A_{LR} and $\sin^2 \theta_w^{\text{eff}}$ from an exposure with alternating polarization P_e collecting N Z decays is:

$$\Delta A_{LR} = \frac{1}{P_e \sqrt{N}} \quad (49)$$

$$\Delta \sin^2 \theta_w^{\text{eff}} = \frac{\Delta A_{LR}}{7.9}. \quad (50)$$

Given the large cross-section at the top of the resonance a remarkable statistical precision can be envisaged: $\Delta A_{LR} = 0.002$ (statistical) for 10^6 events and 50% polarization. This corresponds to $\Delta \sin^2 \theta_w^{\text{eff}} = 0.0003$! More realistically for SLC, 10^5 events and 60% polarization are sufficient to match *with one single measurement* the precision of $\Delta \sin^2 \theta_w^{\text{eff}} = 0.0007$ obtained by combining all the unpolarized forward-backward asymmetries and τ polarization data from the LEP data at the time of the 1993 winter conferences [74].

4.2 Polarized beams at SLC

4.2.1 General layout

The overall SLC layout with polarized beams is shown in figure 25. A more complete description can be found in [1, 2, 75]. The orientation of the electron spin is shown along its transport from the electron gun, where polarization is generated, to the interaction point.

Pulses of longitudinally polarized electrons are produced by photoemission from a polarized cathode [76], at a rate of 120 Hz. The positrons are created by interaction of a second ("scavenger") electron bunch on a radiator situated 2/3 down the linac, and cannot be polarized. In order to reduce the emittance, the beam is transported to damping rings. The polarization is preserved by combination of spin precession in the transfer line and spin rotation in a superconducting solenoid, so that it is aligned along the damping ring magnetic field. Until 1992, two more superconducting solenoids were used to tune the spin orientation in the transfer line from the damping ring to the linac, so that, taking into account the precession in the arc, the polarization is longitudinal at the interaction point. In 1993, a new optical configuration with flat – instead of round previously – beam profile has been commissioned. The two solenoids situated after the damping ring not only act on the spin, but would generate betatron coupling which is detrimental to flat beam operation. They were replaced by a spin-rotation system based on bumps in the

Figure 25:

SLC arcs [78], in a way similar to the bumps described for solenoid compensation in LEP (section 3.1, figure 13).

The direction of the polarization vector is monitored at the end of the linac by a Møller polarimeter, and after the IP by a Compton polarimeter, as already described in section 2. The performance of SLC has been improving regularly in the last two years, both in luminosity and polarization: the 1992 analysis [20] is based on 10224 hadronic Z decays, with 22% typical polarization. As of July 1993, more than 40000 hadronic Z decays have been recorded with up to 64% longitudinal polarization.

4.2.2 The SLC polarized source

The first successful acceleration of polarized electrons to high energy was demonstrated at SLAC in 1974 using an atomic beam source [77]. In 1976, Pierce and Meier [79] observed polarized photoemission from negative affinity Gallium Arsenide. Most polarized electron sources that have been used in accelerators are based on this technique [80]. Such a photocathode polarized electron source was successfully employed in the atomic parity violation experiment [62]. The SLC polarized source is an improved version of it.

Figure 26: Left: the band structure of GaAs near gap minimum. Right: the corresponding energy levels of the $S_{1/2}$, $P_{3/2}$ and $P_{1/2}$ states. The allowed transitions for right-handed (left-handed) photons are marked by full (dashed) lines. The angular momentum state m_j indicates the projection on the incident photon direction. The relative transition rates are indicated by the circled numbers.

Figure 27: Measured Dependence of the polarization at the output of CsF coated GaAs photocathodes on the incident laser wavelength.

Gallium Arsenide is a semi-conductor with the band structure sketched on figure 26. The absorption of a photon with pumping of an electron from the valence band into the conduction band obeys the selection rule $\Delta m_j = \pm 1$ for right-/left handed photons. If a right-handed photon has an energy that allows the transition ($P_{3/2}$ to $S_{1/2}$), but not ($P_{1/2}$ to $S_{1/2}$), which has an energy gap larger by 0.34 eV, the electron will end up in the $m_j = -\frac{1}{2}$ state with a probability of 3 and in the $m_j = \frac{1}{2}$ state with a probability of 1. m_j is the projection of the angular momentum on the incident photon direction. In the S state, the electron spin carries the total angular momentum, so that $m_j = -\frac{1}{2}$ corresponds to a spin opposite to the photon direction. In a practical set-up where the laser hits a GaAs photocathode at normal incidence, the electron will be accelerated opposite to the incident photon direction. It will therefore be right-handed, with a polarization:

$$P_e = \frac{3-1}{3+1} = 0.5. \quad (51)$$

In order to make this a source one must allow the electron to come out of the material. This is not normally possible with pure GaAs, and a coating is necessary to reduce the free electron energy to below the conduction band ("negative affinity"). Best coatings are Cs_2O , and CsF. Keeping good quantum efficiency (1-5%) requires regular cesiation [76].

The actual polarization of the electron beam depends strongly on the energy of the incident photon, as shown in figure 27. It is difficult to obtain a sufficiently powerful laser at the low wavelength required by high polarization. In 1992 the laser was a Dye laser at 715 nm. The actual polarization of generated electrons was not 50% but rather lower, slightly below 30%.

It is possible to improve the polarization of the electron beam from GaAs if

Figure 28: Sketch of a strained GaAs photocathode. The laser beam hit the upper part, coated with Cesium/NF₃. An epitaxial layer of GaAs is grown on a support of GaAs_{1-x}P_x (x=0.24). The slight mismatch in the lattice structure results in a mechanical strain.

Figure 29: Measured Dependence of the polarization at the output of strained GaAs photocathodes on the incident laser wavelength.

one is able to split the energies of the $P_{\frac{3}{2},\frac{1}{2}}$ and $P_{\frac{3}{2},\frac{3}{2}}$ states so that the $P_{\frac{3}{2},\frac{3}{2}}$ state have higher energy –e.g. the gap is smaller. Then, provided the laser is tuned to a low enough energy, the $m=\frac{3}{2}$ state can be selected and the polarization can reach a theoretical maximum of 100%. This breakthrough was achieved [81] with strained GaAs photocathodes.

It is a well known fact³ that the band structure of solids is altered by mechanical strain. The band splitting can actually go one way or the other depending whether the material is under compression or tension. Strain photocathodes with high electron polarization were first obtained in a thin epitaxial layer of In_{1-y}Ga_yAs grown on a GaAs substrate, and later from epitaxial GaAs grown on a thick GaAsP buffer layer. The strain is due to a small lattice mismatch of the epitaxial layer with respect to the substrate. The strain can be varied by changing the concentration y or the thickness of the layer. Thinner layers lead to optimal polarization at a lower wavelength. High degrees of electron polarization have been obtained with an epitaxial layer of 0.15 microns of GaAs on a 2.5 microns layer of GaAs_{1-x}P_x (x=0.24) shown in figure 28. The wavelength dependence of the polarization is shown in figure 29. Polarizations of up to 90% were obtained, for a laser wavelength of 850 microns.

This technique is extremely demanding on the laser. In absence of commercial lasers meeting the specifications, a new Ti:Sapphire, optically pumped by two Nd:YAG lasers, was specially developed [82]. It is presently able to run at high power at 850 nm wavelength. Both the strained photocathodes and the laser came

³In solid state physics!

Figure 30: Layout of the SLC polarized electron source. Detail of the circular polarizer for fast helicity reversal is shown on the lower left corner.

into operation in 1993, yielding useable polarization of up to 70%.

The sign of the polarization of the beam is controlled by the helicity of the laser. This is controlled by Pockels cells and can be reversed on a pulse basis. The overall layout of the polarized electron source and the detail of the laser helicity reversal system are shown in figure 30. The SLC polarized source was able, in 1992, to deliver 4.10^{10} electrons per bunch at a repetition rate of 120 Hz with a polarization of 29%. In 1993 these numbers have been considerably improved to 7.10^{10} and 70%. The beam transport of the polarized beam to the IP results in some further loss, to 22% and 64% respectively.

4.3 Measurement of A_{LR} with SLD

The first measurement of A_{LR} [20] was performed at SLC with the SLD detector [83]. The measurement is based on 10244 Z decays recorded in 1992 with an average beam polarization of $(22.4 \pm 0.6)\%$. The center of mass energy was 91.55 ± 0.04 GeV, measured with magnetic beam spectrometers [84].

The left-right asymmetry is measured as:

$$A_{LR} = \frac{1}{|P_e|} \frac{N_L - N_R}{N_L + N_R} \quad (52)$$

where P_e is the average beam polarization of the e^- beam, and N_L, N_R are the

numbers of Z decays selected for left or right beam polarization.

The helicity of the beam was varied randomly at the source, and recorded by SLD with each event, and by the polarimeter. Hadronic Z decays, with a small content of τ pairs (1.5%) were selected with a calorimeter-based selection. The beam related backgrounds, as well as the physics backgrounds, "two-photon" interactions and e^+e^- final states, were efficiently reduced to a level of ($<0.7\%$, 0.1% , 0.7%) respectively. Backgrounds dilute the asymmetry and must be corrected for. Because of its large t-channel content, the e^+e^- final state has a different asymmetry than the other Z decays.

The helicity reversal can potentially change some of the beam parameters, namely the energy and, more critically, the luminosity and polarization. The energy difference might come from slightly different intensities resulting in different beam loading in the accelerating cavities. It matters here because of the steep energy dependence of the cross-section on energy when running off-peak. By a detailed study of possible asymmetry in the intensity, the effect was calculated and found to be negligible.

The luminosity integrated for each polarization state was measured directly with the SLD luminosity detectors, which count small angle Bhabha scattering events. Low angle Bhabha scattering has a very small left-right asymmetry, $3 \cdot 10^{-4}$ for the angular range of the SLD luminosity detectors, due to the small interference of $Z \rightarrow e^+e^-$ decays. The measured luminosity asymmetry was $(1.9 \pm 6.2) \cdot 10^{-3}$. This uncertainty would add significantly to the experimental error on A_{LR} . A better estimate of the luminosity asymmetry could, however, be inferred from a detailed study of possible asymmetries in the relevant beam parameters, intensity, beam position and beam size, to be $(1.8 \pm 4.2) \cdot 10^{-4}$.

The value of A_{LR} is inferred from the measured asymmetry $A_m = (N_L - N_R)/(N_L + N_R) = (2.23 \pm 0.99)\%$ by the formula

$$A_{LR} = \frac{A_m}{P_e} + \frac{1}{P_e} \left[A_m f_b + A_m^2 A_{P_e} - \frac{E}{\sigma} \frac{d\sigma}{dE} A_E - A_\epsilon - A_\mathcal{L} \right] \quad (53)$$

where f_b is the background fraction, A_{P_e} is the asymmetry in the beam polarization, and $A_E, A_\epsilon, A_\mathcal{L}$ are the asymmetries in the beam energy, detector acceptance and luminosity. The result is:

$$A_{LR} = 0.100 \pm 0.044_{\text{stat.}} \pm 0.004_{\text{syst.}} \quad (54)$$

The systematic error is a quadratic sum of uncertainties stemming from the beam polarization measurement (2.7%, see section 2.2.2) the luminosity asymmetry (1.9%) and the background contamination (1.4%). These errors are given in relative fraction of the measured A_{LR} .

From A_{LR} , after a very small (0.0003) correction for QED effects for the beam energy being shifted from the Z pole, one can derive the value of $\sin^2 \theta_w^{\text{eff}}$:

$$\sin^2 \theta_w^{\text{eff}} = 0.2378 \pm 0.0056_{\text{stat.}} \pm 0.0005_{\text{syst.}} \quad (55)$$

4.4 Prospects for improvements in the measurement of A_{LR} with SLD

From the smallness of i) the corrections, and ii) the systematic error on the result, one can deduce that the measurement of $\sin^2 \theta_w^{\text{eff}}$ from A_{LR} should soon be considerably improved. As mentioned in earlier sections, major improvements in SLC have taken place in all aspects of the project: integrated luminosity, beam polarization degree and polarimetry. The precision on A_{LR} is given by examination of formula 53 as a quadratic sum ($\oplus$) of the following terms:

$$\Delta A_{LR} = \frac{1}{P_e} \frac{1}{\sqrt{N}} \oplus A_{LR} \frac{\Delta P_e}{P_e} \oplus \frac{1}{P_e} \Delta A_c \oplus A_{LR} \Delta f_b. \quad (56)$$

It is clear that increasing the degree of polarization brings direct improvement of the measurement. The SLD detector [83] has recorded so far in 1993 more than 40000 Z decays, with an average polarization of 64%. The measurement should still be statistics dominated and reach a precision of $\Delta \sin^2 \theta_w^{\text{eff}} = \pm 0.0010$. This is already better than the measurement at LEP from the best channel, the b forward-backward asymmetry, where the average from the four LEP experiments (roughly $4 \cdot 10^6$ Z decays) yields an error of $\Delta \sin^2 \theta_w^{\text{eff}} = \pm 0.0011$, with a larger component of experimental and theoretical systematic errors, moreover. Eventually, it is hoped that SLD will record 750000 Z decays, with a polarization of 75%, yielding

$$\Delta \sin^2 \theta_w^{\text{eff}} = \pm 0.00025_{\text{stat.}} \pm 0.00012_{\text{syst.}} \quad (57)$$

4.5 Longitudinal polarization at LEP

In view of the great statistical power and systematic quality of the measurement of A_{LR} , and of the fundamental importance of a high precision measurement of $\sin^2 \theta_w^{\text{eff}}$, the feasibility of a longitudinal polarization program at LEP has been studied. The physics potential was reviewed in [64], and the feasibility of spin rotators in [85]. It is not expected that longitudinal polarization experiments take place before the high energy of LEP, which pushes them to after 1997. By then, high luminosity should be available in LEP by multibunching [86], so that exposures of $5 \cdot 10^6$ Z decays per experiment per year can be envisaged.

Running longitudinal polarization experiments at LEP requires:

- 1. the ability to build-up a large degree of polarization with colliding beams;
- 2. spin rotation;
- 3. control of the polarization of each bunch and polarization measurement;
- 4. measurement of A_{LR} in the experiments.

The polarization build-up has been abundantly discussed in section 3.1. At

present 30-40% polarization is obtained by harmonic spin-matching, in orbits that are fully compatible with high luminosity running. To go further would require correction for the only parameter that remains unknown in the orbit measurement, the offset between the position of the beam pick-ups and the center of the quadrupoles. This offset can be measured by survey, which would be very lengthy and not infinitely valid in time, or with a very elegant method that was developed at CESR [87], and recently tested at LEP [88].

The absolute center of a quadrupole is determined by exciting it with a slow frequency (50 Hz) AC current while scanning the beam position with an orbit bump. The response of the beam to the excitation, monitored with a pick-up, shows a steep minimum when the beam goes through the center of the quadrupole, where the magnetic field vanishes. This method allows absolute calibration of the offset with a precision of better than 50 microns. The practical implementation is still to be worked out. Given sufficient stability and reliability of the pick-ups, this should allow transverse polarization in excess of 50% to be easily obtained.

In order to run physics experiments, the high polarization has to be kept with high luminosity. The full compatibility of Polarization with the Pretzel optics necessary for multi-bunch operation is expected and was demonstrated experimentally [40, 89]. The depolarization due to beam-beam interaction, however, has not yet been studied.

Until a high degree of polarization with colliding beams has been observed, it is not possible to assert that longitudinal polarization experiments will be feasible at LEP.

The natural polarization mechanism in the arcs and wigglers is transverse. Spin rotators are needed to make it longitudinal locally in the interaction regions, while keeping it transverse in the arcs. This last point is crucial, fault of which very strong depolarization would take place. The principle sketch of spin rotators is shown in figure 31. After consideration of various schemes [90, 91], the scheme retained for LEP is the Richter-Schwitters spin rotator [92]. The spin-matching conditions [93, 94] for this rotator, shown in figure 31, are rather simple and a practical implementation is feasible [95, 96, 85] without substantial loss of polarization. The complete spin-compensation in presence of solenoids and beam-beam kicks remains to be fully worked out, however, as well as a fully satisfactory scheme to shield the experiments from the strong synchrotron radiation in the last bend of the rotator. A test spin-rotator is proposed [85] to study these issues.

Once the spin is rotated, one finds oneself in a situation where the electrons and positrons have the same helicity, and the net e^+e^- polarization, equation 43, is zero. Furthermore the cross-section is reduced. However, as suggested by [91], the helicity of the e^+e^- system can be made either positive or negative by depolarizing one beam or the other, with, e.g. the kicker used for energy calibration by resonant depolarization. In fact, any one of the bunches in the machine can be depolarized at will to a negligible level [97] offering the following succession of e^+e^- helicity states [98]:

Figure 31: Top: principle of spin rotators for LEP. The spin orientation of electrons (P^-) and positrons (P^+) is indicated.

Bottom: the Richter-Schwitters spin-rotator, showing the direction of the positron spin and the location of the radio-frequency cavities.

The comparison of the four respective total cross-sections:

$$\sigma_1 = \sigma_u(1 + P_{e+}A_{LR}) \quad (58)$$

$$\sigma_2 = \sigma_u(1 - P_{e-}A_{LR}) \quad (59)$$

$$\sigma_3 = \sigma_u \quad (60)$$

$$\sigma_4 = \sigma_u(1 - P_{e+}P_{e-} + (P_{e+} - P_{e-})A_{LR}) \quad (61)$$

allows a measurement of A_{LR} but also of P_{e+} and P_{e-} from the data. The role of the polarimeter is to monitor the evolution of the polarization with time and the possible differences between one bunch and another, but its absolute calibration is obtained from the data.

This scheme can certainly help reduce the experimental errors on the measurement of A_{LR} . These were studied in great detail in [64]. From this study, and from the knowledge acquired on cross-section measurements at LEP, one can safely assert that, *given sufficient polarization*, a measurement of $\sin^2 \theta_w^{\text{eff}}$ to a precision of

$$\Delta \sin^2 \theta_w^{\text{eff}} = \pm 0.0001 \quad (62)$$

should be achievable.

5 Implications of high precision measurements of $M_Z, \Gamma_Z, \sin^2 \theta_w^{\text{eff}}, M_W$

5.1 Electroweak radiative corrections

We have seen in this article that beam polarization plays, and will play even more in the future, a central role in high precision tests of the standard model. Beam energy by resonant depolarization provides accurate measurements of M_Z, Γ_Z, M_W . Longitudinal polarization gives access to the most sensitive measurement of $\sin^2 \theta_w^{\text{eff}}, A_{LR}$. These quantities are not predicted by the standard model, but their relationships are, modulo electroweak radiative corrections. The relations between these high energy observables, the Fermi constant G_F and the QED running constant $\alpha(M_Z^2)^{-1} = 128.8 \pm 0.1$ [100] can be written in terms of universal electroweak corrections $\Delta\rho, \Delta_{3Q}, \Delta r^{\text{ew}}$ as [99]:

$$\Gamma_Z = \frac{G_F M_Z^3}{24\sqrt{2}\pi} [1 + \Delta\rho] \left[1 + \left(\frac{g_V^t}{g_A^t} \right)^2 \right] \left(1 + \frac{3}{4} \frac{\alpha}{\pi} \right); \quad (63)$$

$$\Gamma_Z = \frac{\alpha(M_Z^2) M_Z}{48 \sin^2 \theta_w^{\text{eff}} \cos^2 \theta_w^{\text{eff}} (1 + \Delta_{3Q})} \left[1 + \left(\frac{g_V^t}{g_A^t} \right)^2 \right] \left(1 + \frac{3}{4} \frac{\alpha}{\pi} \right); \quad (64)$$

$$M_Z^2 = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_F (1 + \Delta\rho) (1 + \Delta_{3Q}) \sin^2 \theta_w^{\text{eff}} \cos^2 \theta_w^{\text{eff}}}; \quad (65)$$

$$M_W^2 = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_F (1 - \Delta r^{\text{ew}}) \left(1 - \frac{M_W^2}{M_Z^2} \right)}; \quad (66)$$

$$\frac{\sin^2 \theta_w^{\text{eff}} \cos^2 \theta_w^{\text{eff}}}{\left(1 - \frac{M_W^2}{M_Z^2} \right) \left(\frac{M_W^2}{M_Z^2} \right)} = \frac{1 - \Delta r^{\text{ew}}}{1 + \Delta\rho + \Delta_{3Q}}. \quad (67)$$

5.2 On the importance of $\alpha(M_Z^2)$

It can be seen from these relations that the error on $\alpha(M_Z^2)$ does not affect *all* of them. In particular, the relations

$$M_Z \leftrightarrow \Gamma_Z \quad \text{and} \quad \frac{M_W}{M_Z} \leftrightarrow \sin^2 \theta_w^{\text{eff}}$$

are unaffected by $\alpha(M_Z^2)$. Therefore,

the present limitation on the error in $\alpha(M_Z^2)$ should not be considered a intrinsic limit to the measurement of electroweak radiative corrections.

This being said, it is clear that a better knowledge of $\alpha(M_Z^2)$ would bring substantial value. The error on $\alpha(M_Z^2)$ is presently given by the experimental uncertainties in the measurement of the e^+e^- hadronic cross-sections between 1 and

10 GeV [100]. This error could certainly be reduced in two steps: i) a new compilation of e^+e^- cross-sections using the additional data now available; ii) further cross-section measurements using more modern detectors should easily improve on the available data, which often date back more than 15 years. The measurement of these cross-sections will soon become as important for precision physics than the measurement of any of the aforementioned quantities.

5.3 Higgsometry

Figure 32: A possible scenario for the ultimate precision of Electroweak measurements at LEP. Minimal Standard Model predictions in the $(\Delta\rho, \Delta_{3Q})$ plane. MSM predictions: dash-dotted line: M_t free, $M_H = 50$ GeV; full line: M_t free, $M_H = 200$ GeV; dotted line: M_t free, $M_H = 1000$ GeV; Experimental constraints: vertical band: $\Delta\Gamma_e = \pm 0.07$ MeV. 25° band: $\Delta M_W = \pm 30$ MeV. 45° band: $\Delta\sin^2\theta_w^{\text{eff}} = \pm 0.0001$. The MSM prediction for $M_t = 165 \pm 5$ GeV as expected from direct measurement of the top mass is also indicated.

As an example of the impact of the precision measurements concerned by beam polarization, the constraints in the $\Delta\rho$ vs. Δ_{3Q} plane were derived in [101] for the following set of measurements: Γ_Z to ± 2 MeV; M_W to ± 30 MeV; $\sin^2\theta_w^{\text{eff}}$ to ± 0.0001 . The standard model relation between Δr^{ew} and $\Delta\rho, \Delta_{3Q}$ was assumed. As can be seen in figure 32, combined with a measurement of the top quark mass, these measurements would certainly place significant limits on the mass of the Higgs boson, or on whatever plays its role.

6 Conclusions

Beam polarization greatly enhances the precision experiments that probe the standard electroweak model and its radiative corrections. So far, it has contributed an extremely precise measurement of the Z boson mass, and is soon to provide substantial improvements to the Z width and $\sin^2 \theta_w^{\text{eff}}$ measurements. The W mass measurement will also benefit from it. One certainly hopes that, before the end of this century, and to a great part thanks to beam polarization experiments, these measurements will have reached a precision sufficient to significantly constrain the most nebulous sector of modern particle theory, the spontaneous symmetry breaking.

Meanwhile, polarization provides the experimentalist with exquisitely delicate technological challenges, from orbit corrections to 50 microns over 27 km, to the deposition of 0.15 micron epitaxial layers on GaAs substrates. Subtle effects by earth tides and other lunacies are encountered on the way to extreme precision, broadening the experience of particle physicists to sometimes exotic fields. This close collaboration between particle experimentalists, accelerator physicists, solid state experts and laser wizards might be a practical version of the unification of forces we all seek.

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