A MATHEMATICAL MODEL OF A THREE-GAP THYRATRON SIMULATING TURN-ON

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Abstract

Kicker magnets are required for all ring-to-ring transfers in the 5 rings of the proposed KAON factory synchrotron. The kick must rise/fall from 1% to 99% of full strength during the time interval of gaps created in the beam (80 ns to 160 ns) so that the beam can be extracted with minimum losses. Approximately one-third of the injection and extraction kicker magnets will operate continuously at a rate of 50 pulses per second; the others operate at 10 pulses per second. The kicker magnet FFN voltages will be in the range 50kV to 80kV, hence multi-gap thyatrons will be used for the injection and extraction kicker systems. Displacement current arising from turn-on of a multi-gap thyatron flows in the external circuit and can thus increase the effective rise-time of the kick. A mathematical model of a three-gap thyatron, which includes the drift spaces, has been developed for simulating turn-on, and is described in this paper. The thyatron model has been used to investigate ways to suppress the effects of displacement current on the kick, and to reduce thyatron switching loss. A ferrite saturating inductor may be connected adjacent to each thyatron to reduce the switching loss, so that thyatron life can be extended and the kick rise-time improved. This inductor can also be used to reduce the effect of anode displacement current during turn-on of a multi-gap thyatron. The research has culminated in a predicted kick rise time (1% to 99%) of less than 50 ns for a TRIUMF 10 cell prototype kicker magnet. The proposed improvements are currently being implemented on our prototype kicker system.

Introduction

Many of the kicker magnets for the proposed KAON factory synchrotron require kick rise/fall times of less than 82µs[1]. In order to achieve the required kick rise/fall times in the available space pulse forming network (PFN) voltages of approximately 50kV are typically required[1], hence multi-gap thyatrons are to be used for the high-voltage switches.

The design of the pulse generator proposed for the injection and extraction kicker magnets will be based on that of the CERN PS division[2, 3]: three gap thyatrons will be used in the high voltage switching. The individual gaps in a three gap thyatron break down in sequence[4, 5]. Initially the gap closest to the cathode conducts and the full FFN voltage is shared between the centre and anode gaps. Approximately 50ns later the centre gap starts to conduct and the full FFN voltage builds up across the anode gap[4].

The voltage redistribution between the parasitic capacitance of each of the gaps is associated with a flow of displacement current[5]. The displacement current also flows in the external circuit[5], and hence through the kicker magnet, and can increase the effective rise-time of the kick.

A simulation[1] shows a typical voltage pulse from a pulse generator borrowed from CERN PS Division: cathode displacement current, up to 8% of the magnitude of the flat-top, occurs for about 100ns before the main pulse. Anode displacement current causes a slight reduction in the flat-top of the measured pulse approximately 50ns before the pulse ends[1].

One-third to one-third of the injection and extraction kicker magnets for the proposed KAON factory will operate continuously at a rate of 50 pulses per second: the others will operate at 10 pulses per second[1]. Switching loss in a thyatron operating at 50Hz is more important than in lower frequency applications[4]. Saturating ferrite cores connected adjacent to the anode of the thyatron may be used to reduce the effect of anode displacement current[6], reduce switching loss[4], and improve current rise-time[4, 6].

In order to be able to investigate ways to suppress the effect of anode and cathode displacement current upon kick, and to reduce thyatron switching loss, a representative mathematical model of a thyatron has been developed. The circuit analysis code PSpice[7] is utilized for all the mathematical simulations whose results are reported in this paper.

Mathematical Model of a Thyatron Simulating Turn-On

Single Gap Thyatron Model

A non-linear switching characteristic for an EEEV CX1168[9] thyatron simulating turn-on may be represented using the equivalent circuit shown in Fig. 1: this mathematical model is based on the CERN SPS Division's simulation of the CX1168[9]. Hsense and Hcontrol are both current-controlled voltage sources with unity gain, which are used for controlling voltage source Eproduct. The potential difference across the output terminals of Eproduct is given by the product of the current through Hsense and Hcontrol. ISA is a current source whose current decays exponentially, with time-constant τ1, from a high-current (e.g. 50kA) to a low current (e.g. 20mA).

The low current value of ISA can be chosen to give a representative conduction voltage drop at the required load current. This results in a simulated conduction voltage drop which is proportional to load current, which is not the situation in reality[4]. However, for the majority of simulations the resulting error is acceptable.

Lpara, in Fig. 1 represents the parasitic inductance of a gap and/or drift space of a thyatron. The inductance of the CX1171 thyatron, in its coaxial housing, has been deduced to be 80nH[4].

Fig. 5 of reference[4] shows a thyatron current rise-time of about 32ns (10%–90%), for a reservoir voltage of 5.85 V and a FFN pre-charge of 80kV. Since the EEEV CX1171 thyatron[8] used for the tests documented in reference[4] was manufactured in 1980, its rise time performance may be slower than that of currently produced tubes because of detail improvements[4]. A simulated value of 7ns for τ1 results in a predicted 10% to 90% current rise-time of approximately 32ns for the equivalent circuit of Fig. 1.

Three Gap Thyatron Model

Figs. 2 and 3 show equivalent circuits for the CX1171[8] three-gap thyatron for simulating turn-on. The equivalent circuit of Fig. 2 simulates non-linear switching characteristics for three-gaps and two drift spaces. The equivalent circuit of Fig. 3 lumps the non-linear switching characteristics for the two drift spaces with those of the cathode and central gaps.

The 14MΩ resistors in Figs. 2 and 3 are for d.c. voltage grading[4]: the 540Ω resistors close the thyatron drift spaces[4]: Cgap represents the inter-electrode capacitance of each of the three gaps. The data sheet value for Cgap is 15pF to 20pF[8], and 20pF has been used throughout the reported simulations.

**XTHYRATRON** in Figs. 2 and 3 represents the equivalent circuit shown in Fig. 1. Analysis of measured anode, cathode and gradient grid voltages during turn-on of a multi-gap thyatron, indicates that the time duration associated with the collapse of voltage across a gap is similar for all the gaps[10]. Hence it is permissible to utilize the same non-linear switching characteristic for each of the three gaps in the equivalent circuit of the CX1171 thyatron. Cdrift in Fig. 2 represents the drift space capacitance. Cdrift is in the range 25pF to 30pF for the CX1171 thyatron[10]. 30pF has been used for simulating the thyatron. Cgrid in Figs. 2 and 3 represents the parasitic capacitance from the grading ring to ground. The value
Figure 2: Three gap thyatron: drift-spaces simulated

of this capacitance affects the predicted cathode displacement current (see Fig. 4).

Scaling a typical measured pulse (Fig. 2 of reference [1]), from a pulse generator borrowed from CERN PS Division, to correspond with a PFN pre-charge of 80kV, gives 31A and 72A, respectively, for the first and second peak of cathode displacement current. Assuming a value of \( \tau \) of 7ns, which gives a predicted cathode current rise-time of about 32ns (as above):

- when Cgnd=10.6pF, the 3-gap thyatron model with drift spaces (Fig. 2) results in a similar magnitude of predicted cathode displacement current (Fig. 4) [7A and 72A for the first and second peaks, respectively], to the measured current;
- when Cgnd=8.4pF, the 3-gap thyatron model with drift spaces lumped with the cathode and central gaps (Fig. 3) results in an almost identical magnitude of predicted cathode displacement current [72A and 72A for the first and second peaks, respectively], to the measured current.

Where drift-spaces are simulated the delay between a gap turning-on and the associated drift-space ‘turning-on’ is assumed to be half of the delay between consecutive gaps turning-on; this assumption is consistent with measurements of anode delay times for two different grid connections [10].

Fig. 5 shows predicted anode and cathode displacement current when the equivalent circuit shown in Fig. 2 is utilised: the cathode displacement current is significantly greater in magnitude than the anode displacement current. \( \tau \) for the thyatron model is assumed to be 7ns, and in order to err on the pessimistic side a value of 12pF, rather than 10.6pF, is assumed for Cgnd. Unless stated otherwise \( \tau \approx 7ns \) and Cgnd=12pF are utilised for the remainder of the predictions reported in this paper. In addition, unless stated otherwise, the equivalent circuit shown in Fig. 2 is utilised for the thyatron.

When the simulated time-constant \( \tau \) is greater than 4ns the equivalent circuit of Fig. 3 results in similar predictions to those obtained when using the equivalent circuit of Fig. 2. However for a simulated time constant of less than 4ns (corresponds to a 10% to 90% current rise-time of about 17ns when magnetic assistance is not used) there is also a significant displacement current flow associated with the ‘turn-on’ of the drift-spaces, and therefore it is necessary to utilise the equivalent circuit shown in Fig. 2. Hence for \( \tau > 7ns \) the simplified circuit of Fig. 3 may be utilised for simulating a CX1171 thyatron, as there is a reduction in CPU time but little loss in accuracy.

Validation of Mathematical Model of Thyatron

An investigation of switching loss of a CX1171A thyatron [8] is reported in reference [4]. The thyatron under test, in reference [4], was installed in a system with a characteristic impedance \( Z_{0} \) of 150, with the thyatron cathode at negative high voltage. Parasitic capacitance of the isolating transformer and grid drive components of the cathode region resulted in a 40% initial overshoot of anode current [4]. Fig. 1 of reference [4] shows the test circuit used for investiga-

![Figure 3: Three gap thyatron: drift-spaces lumped with cathode and central gaps](image)

Figure 3: Three gap thyatron: drift-spaces lumped with cathode and central gaps

![Figure 4: Effect of the value of Cgnd upon predicted cathode displacement current. Z0=30Ω, V_{PFN}=80kV](image)

Figure 4: Effect of the value of Cgnd upon predicted cathode displacement current. \( Z_{0}=30\Omega \), \( V_{PFN}=80kV \) tigating switching loss; Fig. 6 of this paper shows the basic circuit simulated for comparing predictions with measurements reported in reference [4]. \( C_{L} \) in Fig. 6 represents the parasitic capacitance associated with the cathode region. A value of 500pF for \( C_{L} \) results in an overshoot of 40% in the predicted anode current, hence 500pF was used for the remainder of the investigations to validate the mathematical model of the thyatron.

As per the investigations reported in reference [4], thyatron loss is determined from \( \int (V_{a} - V_{c}) t \) dt, where (see Figs. 2 and 3):

- \( V_{a} \) is instantaneous anode voltage;
- \( V_{c} \) is instantaneous cathode voltage;
- \( I_{a} \) is instantaneous anode current.

The integration is carried out from pre-pulse current zero to post-pulse current zero, and conduction loss is subtracted from the resultant energy loss. Integrating over this period of time ‘neutralises’ the reactive energy which is stored in the parasitic inductance while the thyatron is carrying load current [4].

The predicted sum of switching and conduction losses, for a PFN pre-charge of 80kV, is 3.0J; subtracting a conduction loss of 0.2J results in a predicted switching loss of 2.8J, which is in excellent agreement with the 2.8J reported in reference [4]. The integral \( \int (V_{a} - V_{c}) t \) dt assumes that the measured anode current is simultaneously flowing through the three gaps and two drift spaces. A detailed analysis of the predicted power dissipation in each of the gaps and drift spaces, where the energy dissipation for each gap and drift space is calculated from the predicted instantaneous voltage drop associated with the gap or drift space and the instantaneous current through the same gap or drift space, indicates that the integral \( \int (V_{a} - V_{c}) t \) dt overestimates energy dissipation in the thyatron by approximately 0.3J per pulse. Approximately 86% of the predicted switching loss, where the total loss is calculated from \( \int (V_{a} - V_{c}) t \) dt, is associated with the anode gap: measurements indicated that about 70% of the total loss is associated with the anode gap [4]. The predictions show that the peak power dissipation in the anode gap (35MW) is almost a factor of 20 greater than the peak dissipation in any of the other gaps.

![Figure 5: Predicted anode and cathode displacement current. Z0=30Ω, V_{PFN}=80kV](image)
gaps or drift spaces.

Subsequently, a saturating inductor consisting of 72m2 of CMD5005 ferrite[1] was simulated as being connected adjacent to the anode of the thyratron. A linear inductor was modelled in series with the saturating inductor, to give a total saturated inductance of 37nH[4]. The predicted sum of switching and conduction losses, calculated from \( \int (V_i - V_a) \times I_{dc} \, dt \) for a PFN pre-charge of 80kV, is 0.74J; subtracting conduction losses of 0.21J results in a predicted switching loss of about 0.5J, which is in good agreement with the 0.4J reported in reference[4]. A detailed analysis of the predicted dissipation indicates that, for this case, the integral \( \int (V_i - V_a) \times I_{dc} \, dt \) overestimates energy dissipation in the thyatron by approximately 0.15J per pulse. Approximately 40% of the predicted switching loss, where the total loss is calculated from \( \int (V_i - V_a) \times I_{dc} \, dt \), is associated with the anode gap. The predictions show that the peak power dissipation in the anode gap (8MW) is almost a factor of 3 greater than the peak dissipation in any of the other gaps or drift spaces. Hence the saturating inductor has reduced the predicted energy loss during switching by a factor of approximately 5.5, and reduced the peak power dissipation by a factor of about 4.5.

Unless stated otherwise, the remainder of the investigations reported in this paper assume a system with a characteristic impedance of 30Ω (as per the prototype TRUMPF kicker magnet), with the thyatron anode connected to a positively charged PFN (see Fig. 7). In addition the PFN pre-charge is simulated as 80kV.

**Predicted Thyratron Dissipation**

Fig. 7 shows the circuit simulated to investigate the effect of saturating ferrites upon predicted switching losses for the TRUMPF 50Ω system. A switching loss reduction saturating inductor (S.L.S.I.) is connected to the anode of the thyatron, as this helps to reduce the effect of anode displacement current upon the field in the kimmer magnet (see below).

The S.L.S.I. is assumed to be manufactured from split toroidal cores of CMD5005 ferrite[11]. An air gap of 0.04mm is included to reduce the remanent flux density in the ferrite. The inner and outer diameters for the switching loss ferrite have been chosen to be 1 cm and 10 cm respectively[4]. The ferrite is assumed to be housed in a cylindrical aluminium housing with an inside diameter of 12cm. In order to calculate end-to-end capacitance, and capacitance to the aluminium housing, the relative permittivity of the ferrite is assumed to be 15. The magnetic cross-sectional area (CSA) of the ferrite is swept through a range of values.

Fig. 8 shows a plot of the predicted switching loss within the thyatron, for capacitances from thyatron cathode to ground of 20pF and 500pF. A S.L.S.I. with a magnetic CSA of 72cm² reduces switching loss by a factor of about 3, in comparison with no S.L.S.I.. As the magnetic CSA of the S.L.S.I. is increased beyond 80 cm² there is little further reduction in the thyatron switching loss.

**Predicted Thyratron Pre-Pulse Current**

No S.L.S.I.'s Simulated

Anode and cathode displacement current are fairly insensitive to the characteristic impedance of the system[12]. Similarly the magnitude of the anode and cathode displacement current are virtually independent of the magnitude of stray capacitance from thyatron cathode to ground[12]. However the energy dissipated within the thyatron is dependent upon both the stray capacitance from cathode to ground (Fig. 8) and the characteristic impedance of the system[12].

Figure 9 shows the effect of S.L.S.I. upon predicted pre-pulse cathode current.
Two mathematical models of a three gap thyatron have been developed to simulate turn-on. One of the mathematical models simulates non-linear switching characteristics for the three gaps and two drift spaces separately, whereas the other model lumps the non-linear switching characteristics for the drift spaces with those of the cathode and central gaps. In general, for 10% to 90% current rise-times of greater than about 10μs, the model which lumps the drift spaces with the cathode and central gaps is adequate. The mathematical model, which simulates the two drift spaces separately, has been validated by comparing predicted switching losses, with and without a saturating inductor present, with switching losses measured at CERN: the agreement is good.

The mathematical model of the thyatron has been used to assess the effect of a S.L.S.I. upon pre-pulse current, and energy dissipation per pulse within the thyatron. The optimum position for a S.L.S.I. is adjacent to the PPN side of the thyatron. A S.L.S.I. with a magnetic CSA of 70cm² significantly reduces switching losses in the thyatron. As the magnetic CSA is increased beyond 80cm² there is not any further significant reduction in switching loss. The S.L.S.I. can also be used to reduce kick rise time. A separate D.I.S.I., connected on the input to the kicker magnet, is effective at reducing the effect of pre-pulse cathode current upon the pre-pulse kick. These improvements are presently being carried out to the borrowed CERN pulser.

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References

[9] Private Communication with V. Sandberg, LAMPF, Los Alamos, New Mexico, USA.