MASS RELATIONS IN THE TWO HIGGS DOUBLET MODEL FROM THE ABSENCE OF QUADRATIC DIVERGENCES

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ABSTRACT

By setting the quadratic divergences to zero, four mass relations are obtained for the Standard Model generalized to two Higgs doublets. These four mass relations are obtained most simply in terms of the original fields in the Lagrangian, before spontaneous $SU(2) \times U(1)$ symmetry breaking is applied. Unlike the case of the Standard Model, the Higgs tadpoles of the two Higgs doublet theory do not furnish a complete set of mass relations, giving only three of the four.

1. Introduction

In reference [1], two mass relations were derived for the Standard Model [2] by requiring the absence of certain divergences in the theory. The two mass relations obtained in this way were then used to solve for the two unknown masses, namely the top quark mass and the Higgs boson mass [3].

Two Higgs doublet models were first introduced by Lee [4], and numerous authors [5] since then have explored the implications. These include charged Higgs bosons and, depending on the model, CP-violation in the Higgs sector. As in the Standard Model, the masses of the Higgs bosons are typically undetermined.

In this paper, we consider the Standard model with two Higgs doublets. Of all the theories incorporating the features of the Standard Model, it is probably the simplest generalization. In addition to the fields familiar from the Standard Model, this Two Higgs doublet model contains five physical Higgs bosons, two charged and three neutral. As in reference [1], we seek to establish relations between the coupling constants of the theory (mass relations) by computing its divergences and setting them to zero. In comparison with the Standard Model, there is a larger number of unknown coupling constants, and therefore a larger number of mass relations is required. Four mass relations are presented here, derived from a consideration of the Higgs self-energy graphs. Additional mass relations will be presented elsewhere. In the Two Higgs doublet model presented here, the two Higgs doublets Φ_1 and Φ_2 couple to fermions and to each other in any way consistent with the $SU(2) \times U(1)$ gauge symmetry. No additional symmetries are imposed. In particular, we do *not* insist that one doublet give mass only to the up quark while the other doublet give mass only to the down quark, as in supersymmetry [6]. Some special cases of this general theory will be considered in the conclusion. The Yukawa potential for this theory is given as follows:

$$\begin{aligned} \mathcal{L}_{Yukawa} = & -[(\bar{\Psi}_{uL}\Phi_1)G_u^1\Psi_{uR} + \bar{\Psi}_{uR}G_u^{1\dagger}(\Phi_1^\dagger\Psi_{uL})] - [(\bar{\Psi}_{uL}\Phi_2)G_u^2\Psi_{uR} + \bar{\Psi}_{uR}G_u^{2\dagger}(\Phi_2^\dagger\Psi_{uL})] \\ & - [(\bar{\Psi}_{dL}\Phi_1)G_d^1\Psi_{dR} + \bar{\Psi}_{dR}G_d^{1\dagger}(\Phi_1^\dagger\Psi_{dL})] - [(\bar{\Psi}_{dL}\Phi_2)G_d^2\Psi_{dR} + \bar{\Psi}_{dR}G_d^{2\dagger}(\Phi_2^\dagger\Psi_{dL})] \\ & - [(\bar{\Psi}_{qL}\Phi_1)G_q^1\Psi_{qR} + \bar{\Psi}_{qR}G_q^{1\dagger}(\Phi_1^\dagger\Psi_{qL})] - [(\bar{\Psi}_{qL}\Phi_2)G_q^2\Psi_{qR} + \bar{\Psi}_{qR}G_q^{2\dagger}(\Phi_2^\dagger\Psi_{qL})] \end{aligned}$$

where Ψ_{eR} , Ψ_{uR} , and Ψ_{dR} are 3-vectors of $SU(2) \times U(1)$ singlets in the space of generations

$$\Psi_{eR} = \begin{pmatrix} e_R^1 \\ e_R^2 \\ e_R^3 \end{pmatrix}, \quad \Psi_{uR} = \begin{pmatrix} u_R^1 \\ u_R^2 \\ u_R^3 \end{pmatrix}, \quad \Psi_{dR} = \begin{pmatrix} d_R^1 \\ d_R^2 \\ d_R^3 \end{pmatrix}, \quad (1.2)$$

and Ψ_{uL} and Ψ_{qL} are 3-vectors of doublets:

$$\Psi_{uL} = \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \end{pmatrix}, \quad \Psi_{qL} = \begin{pmatrix} u_L^1 \\ d_L^1 \\ u_L^2 \\ d_L^2 \\ u_L^3 \\ d_L^3 \end{pmatrix}. \quad (1.3)$$

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The notation $(\bar{\Psi}_L \Phi_L)$ implies that Φ_L is contracted with each of the doublets in $\bar{\Psi}_L$, and $\bar{\Phi}_i$ is defined by $\bar{\Phi}_i = i\gamma_5 \Phi_i^\dagger$. The coupling constants are contained in the complex 3×3 matrices $\mathbf{G}_e^1, \mathbf{G}_u^2, \mathbf{G}_d^3$, and $\mathbf{G}_\nu^4$, and the primes on the fields serve as a reminder that in this basis the mass matrices are not diagonal, so that the primed fields are not the physical fields.

The Higgs potential for this theory is given by Lee [4]:

$$\begin{aligned} V(\Phi_1, \Phi_2) = & \mu_1(\Phi_1^\dagger \Phi_1) + \mu_2(\Phi_2^\dagger \Phi_2) \\ & + \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + 2\lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + (\lambda_5 + i\lambda_6)(\Phi_1^\dagger \Phi_2)^2 + (\lambda_5 - i\lambda_6)(\Phi_2^\dagger \Phi_1)^2 \\ & + 2(\lambda_7 + i\lambda_8)(\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + 2(\lambda_7 - i\lambda_8)(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\ & + 2(\lambda_9 + i\lambda_{10})(\Phi_2^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + 2(\lambda_9 - i\lambda_{10})(\Phi_2^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) \end{aligned} \quad (1.4)$$

where $\mu_i, i = 1, 2$ and $\lambda_j, j = 1, \dots, 10$ are real coupling constants. In the N -Higgs doublet model, there are N real coupling constants μ_i associated with the quadratic terms and $\frac{1}{2}N^2(N^2 + 1)$ real coupling constants λ_j associated with the quartic terms in the Higgs potential.

The Two Higgs doublet model described here has many more undetermined constants than the Standard Model. However, the cancellation of divergences in this Two Higgs model also introduces a larger number of constraints. To define the divergences, it is necessary to choose an appropriate regulator. In this paper, as in reference [1], we use the point-splitting regularization [7], which introduces exponential functions of the loop momentum into all Feynman integrals. The quadratic divergences then appear as terms proportional to $\frac{1}{\delta^2}$, where δ is the amount of point-splitting, derived solve

One mass relation cancels the quadratic divergences of the Standard Model. However, this mass relation may be computed in two different ways: either by summing the Higgs tadpoles, or by summing the non-tadpole contributions to the Higgs self-energy. In contrast, four mass relations are necessary to cancel the quadratic divergences of the Two Higgs Model, only three of which may be obtained from the three sets of Higgs tadpoles. The complete set of four relations is obtained by summing the Higgs self-energy diagrams. In an N -Higgs doublet model, N^2 relations are needed to cancel the quadratic divergences from the Higgs self-energy diagrams, while a subset of $(2N - 1)$ relations cancels the divergences in the $(2N - 1)$ sets of Higgs tadpoles.

Both in the Standard Model and in the Two Higgs model, the mass relations derived by eliminating the quadratic divergences have an important property: they are independent of the vacuum expectation values of the scalar doublets. As a result, it is possible to calculate the quadratic divergences in terms of the original fields, without expanding about the minimum of the potential and without breaking the original $SU(2) \times U(1)$ symmetry. This procedure simplifies the calculation of the quadratic divergences both in the Standard Model and in the Two Higgs model, but

for the Two Higgs model, with the Higgs potential given by (1.4), this simplification is especially important.

We begin in section 2 by deriving the four independent relations necessary to eliminate the quadratic divergences from the Two Higgs Model. From the point of view of the original fields, there are four Higgs self-energy diagrams, shown in Figure 1. The first two each give one real relation, while the last two give complex conjugate relations, for total of four real relations, given explicitly at the end of section 2. In section 3, we define the physical fields, and show the relation between the quadratic divergences of the three Higgs tadpoles and the quadratic divergences of the Higgs self-energy terms discussed in section 2. In section 4, the conclusion, we discuss some special cases in which the number of undetermined couplings is reduced.

2. Higgs Self-Energy in Terms of Φ_1 and Φ_2

In this section, we calculate the quadratic divergences of the Higgs self-energy in terms of the fields Φ_1 and Φ_2 , without spontaneous breaking of the $SU(2) \times U(1)$ gauge symmetry. The relevant two-point functions are illustrated in Figure 1. Setting the quadratically divergent part of the amplitudes to zero yields four mass relations, given by equations (2.18) to (2.21). With the $SU(2) \times U(1)$ symmetry intact, the calculation of the quadratic divergences is relatively simple. Once the Lagrangian has been rewritten in terms of the physical fields, the algebra necessary to calculate the quadratic divergences is greatly increased, due primarily to the increased complexity of the Higgs potential (1.4).

The quadratically divergent diagrams contributing to the two-point functions in Figure 1 are illustrated in Figures 2, 3, and 4. The first group of diagrams, shown in Figure 2, gives the set of all diagrams with a fermion loop. The second group of diagrams, shown in Figure 3, gives the set of all diagrams which depend on the gauge-fixing parameter ξ [8], and the third group of diagrams, shown in Figure 4, gives the set of all diagrams which depend on the self-couplings of the Higgs doublets. The notation $(\Phi_i)_p$ denotes the p th component of the i th doublet, while A and B denote the $SU(2) \times U(1)$ Yang-Mills fields [9].

In the following, the sum of integrals illustrated by Figure 2 will be denoted by I_{ij} ($i, j = 1, 2$), where i and j are the doublet numbers of the ingoing and outgoing external lines. The expressions for individual diagrams, with virtual intermediate states x , will be denoted by $I_{ij|x}$, and the notation $\Rightarrow$ will be used to indicate that an integral has been replaced by its quadratically divergent part. The Higgs self-energy terms with a fermion loop (Figure 2) are given below [1]:

$$\begin{aligned} I_{ij} \Big|_e &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T \gamma} \left[(-i \mathbf{G}_e^1 \frac{1 + \gamma_5}{2} \delta_{pr}) \frac{i}{\not{k} - \frac{1}{2} \not{p}} \frac{1 - \gamma_5}{2} \right. \\ &\quad \left. (-i \mathbf{G}_e^j \frac{1 - \gamma_5}{2} \delta_{sq}) \frac{i \delta^{rs}}{\not{k} + \frac{1}{2} \not{p}} \frac{1 + \gamma_5}{2} \right] \\ &\Rightarrow -2I \gamma (\mathbf{G}_e^1 \mathbf{G}_e^j) \delta_{pq} \Delta \end{aligned} \quad (2.1)$$

$$\begin{aligned}
I_{ij}|_{u'} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} T_r \left[\left(-i G_u^{\dagger l} \frac{1 + \gamma_5}{2} \delta_{pr} \right) \frac{i}{\not{k} - \frac{1}{2} \not{p}} \frac{1 - \gamma_5}{2} \right. \\
&\quad \left. \left(-i G_u^{\dagger l} \frac{1 - \gamma_5}{2} \delta_{sq} \right) \frac{i \delta^{rs}}{\not{k} + \frac{1}{2} \not{p}} \frac{1 + \gamma_5}{2} \right] \\
&\Rightarrow -2T_r [G_u^{\dagger l} G_u^{\dagger l}] \delta_{pq} \Delta
\end{aligned} \tag{2.2}$$

$$\begin{aligned}
I_{ij}|_d &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} T_r \left[\left(-i G_d^{\dagger l} \frac{1 + \gamma_5}{2} \delta_{pr} \right) \frac{i}{\not{k} - \frac{1}{2} \not{p}} \frac{1 - \gamma_5}{2} \right. \\
&\quad \left. \left(-i G_d^{\dagger l} \frac{1 - \gamma_5}{2} \delta_{sq} \right) \frac{i \delta^{rs}}{\not{k} + \frac{1}{2} \not{p}} \frac{1 + \gamma_5}{2} \right] \\
&\Rightarrow -2T_r [G_d^{\dagger l} G_d^{\dagger l}] \delta_{pq} \Delta
\end{aligned} \tag{2.3}$$

where

$$\Delta = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \frac{1}{k^2}. \tag{2.4}$$

Here, the exponentials $e^{ik \cdot \delta}$ arise because of the point-splitting regularization. The indices $p, q, r, s = 1, 2$ refer to the components of the doublets Φ_i , while the indices $i, j, k, l = 1, 2$ refer to the doublet number.

The integrals which depend on the gauge-fixing parameter ξ (Figure 3) are given below:

$$\begin{aligned}
I_{ij}|_{\Phi \bar{\lambda}} &= \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(\frac{1}{2} i g(\tau_a) \delta_{ik} \delta_{jr} \left(k + \frac{3}{2} p \right)_\mu \right) \frac{i \delta^{kl} \delta^{rs}}{(k + \frac{1}{2} p)^2} \left(\frac{1}{2} i g(\tau_b) \delta_{ij} \delta_{sq} \left(k + \frac{3}{2} p \right)_\nu \right) \\
&\quad \frac{i \delta^{ab}}{(k - \frac{1}{2} p)^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{(k - \frac{1}{2} p)^\mu (k - \frac{1}{2} p)^\nu}{(k - \frac{1}{2} p)^2} \right] \\
&\Rightarrow -\xi \frac{3}{4} g^2 \delta_{ij} \delta_{pq} \Delta
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
I_{ij}|_{\Phi_H} &= \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-\frac{1}{2} i g' \delta_{ik} \delta_{pr} \left(k + \frac{3}{2} p \right)_\mu \right) \frac{i \delta^{kl} \delta^{rs}}{(k + \frac{1}{2} p)^2} \left(-\frac{1}{2} i g' \delta_{ij} \delta_{sq} \left(k + \frac{3}{2} p \right)_\nu \right) \\
&\quad \frac{i}{(k - \frac{1}{2} p)^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{(k - \frac{1}{2} p)^\mu (k - \frac{1}{2} p)^\nu}{(k - \frac{1}{2} p)^2} \right] \\
&\Rightarrow -\xi \frac{1}{4} g'^2 \delta_{ij} \delta_{pq} \Delta
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
I_{ij}|_{\bar{\lambda}} &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(\frac{i g^2}{2} g_{\mu\nu} \delta_{ab} \delta_{ij} \delta_{pq} \right) \frac{i \delta^{ab}}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \\
&\Rightarrow (3 + \xi) \frac{3}{4} g^2 \delta_{ij} \delta_{pq} \Delta
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
I_{ij}|_B &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(\frac{i g^2}{2} g_{\mu\nu} \delta_{ij} \delta_{pq} \right) \frac{i}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \\
&\Rightarrow (3 + \xi) \frac{1}{4} g^2 \delta_{ij} \delta_{pq} \Delta
\end{aligned} \tag{2.8}$$

Here the indices $a, b = 1, 2, 3$ refer to the components of the vector field $\hat{A}$. Notice that the result is proportional to δ_{ij} , and that the quadratically divergent part of the sum of ξ -dependent integrals is independent of ξ :

$$I_{ij} \Big|_{\sum \xi\text{-dependent}} \Rightarrow \left(\frac{9}{4} g^2 + \frac{3}{4} g'^2 \right) \delta_{ij} \delta_{pq} \Delta \tag{2.9}$$

The integrals which depend on the self-coupling of the Higgs doublets (Figure 4), through the Higgs potential (1.4), take an especially simple form when calculated in terms of Φ_1 and Φ_2 . There are two contributions to the two-point function with Φ_1 on the external lines:

$$I_{11}|_{\Phi_1} = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-2i \lambda_1 (\delta_{pq} \delta_{rs} + \delta_{ps} \delta_{rq}) \right) \frac{i \delta^{sr}}{k^2} \Rightarrow 6 \lambda_1 \delta_{pq} \Delta \tag{2.10}$$

$$I_{11}|_{\Phi_2} = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-2i (\lambda_3 \delta_{pq} \delta_{rs} + \lambda_4 \delta_{ps} \delta_{rq}) \right) \frac{i \delta^{sr}}{k^2} \Rightarrow (4 \lambda_3 + 2 \lambda_4) \delta_{pq} \Delta \tag{2.11}$$

For the two-point function with Φ_2 on the external lines, there are also two contributions, identical to the above two contributions after the replacement of λ_1 by λ_2 .

Two diagrams contribute to the two-point function with Φ_1 on the incoming line and Φ_2 on the outgoing line.

$$I_{12}|_{\Phi_1} = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-2i (\lambda_7 - i \lambda_8) (\delta_{pq} \delta_{rs} + \delta_{ps} \delta_{rq}) \right) \frac{i \delta^{sr}}{k^2} \Rightarrow 6 (\lambda_7 - i \lambda_8) \delta_{pq} \Delta \tag{2.12}$$

$$I_{12}|_{\Phi_2} = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-2i (\lambda_9 + i \lambda_{10}) (\delta_{pq} \delta_{rs} + \delta_{ps} \delta_{rq}) \right) \frac{i \delta^{sr}}{k^2} \Rightarrow 6 (\lambda_9 + i \lambda_{10}) \delta_{pq} \Delta \tag{2.13}$$

The corresponding results for the two-point functions with Φ_2 incoming and Φ_1 outgoing are obtained by complex conjugation. In equations (2.10) through (2.13), we have written massless propagators for the Higgs bosons, despite the quadratic terms in the Higgs potential (1.4), because the coefficient of Δ is independent of the mass term in the Higgs boson propagator.

Since each of the integrals (2.1-3) and (2.5-13) is proportional to δ_{pq} , I_{11} may be defined alternatively as the quadratically divergent part of the two-point function with the upper component of Φ_1 on both external lines or as the quadratically divergent part of the two-point function with the lower component of Φ_1 on both external lines. Similarly, the two-point functions that define I_{22} , I_{12} , and I_{21} may

be redefined using only the upper components or only the lower components of Φ_1 and Φ_2 . It follows that we may write the result in a form which is independent of the components of Φ_i , as:

$$I_{11} \Rightarrow \left(-2\text{Tr}\mathbf{G}_e^1\mathbf{G}_e^{1\dagger} - 6\text{Tr}\mathbf{G}_u^1\mathbf{G}_u^1 - 6\text{Tr}\mathbf{G}_d^1\mathbf{G}_d^{1\dagger} + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 4\lambda_3 + 2\lambda_4 \right) \Delta \quad (2.14)$$

$$I_{22} \Rightarrow \left(-2\text{Tr}\mathbf{G}_e^2\mathbf{G}_e^{2\dagger} - 6\text{Tr}\mathbf{G}_u^2\mathbf{G}_u^2 - 6\text{Tr}\mathbf{G}_d^2\mathbf{G}_d^{2\dagger} + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 4\lambda_3 + 2\lambda_4 \right) \Delta \quad (2.15)$$

$$I_{12} \Rightarrow \left(-2\text{Tr}\mathbf{G}_e^1\mathbf{G}_e^{2\dagger} - 6\text{Tr}\mathbf{G}_u^1\mathbf{G}_u^2 - 6\text{Tr}\mathbf{G}_d^1\mathbf{G}_d^{2\dagger} + 6(\lambda_7 - i\lambda_8) + 6(\lambda_9 + i\lambda_{10}) \right) \Delta \quad (2.16)$$

$$I_{21} \Rightarrow \left(-2\text{Tr}\mathbf{G}_e^2\mathbf{G}_e^{1\dagger} - 6\text{Tr}\mathbf{G}_u^2\mathbf{G}_u^1 - 6\text{Tr}\mathbf{G}_d^2\mathbf{G}_d^{1\dagger} + 6(\lambda_7 + i\lambda_8) + 6(\lambda_9 - i\lambda_{10}) \right) \Delta. \quad (2.17)$$

In conclusion, the four mass relations are obtained by setting the coefficients of Δ to zero in each of the above four relations:

$$\begin{aligned} -2\text{Tr}\mathbf{G}_e^1\mathbf{G}_e^{1\dagger} - 6\text{Tr}\mathbf{G}_u^1\mathbf{G}_u^1 - 6\text{Tr}\mathbf{G}_d^1\mathbf{G}_d^{1\dagger} + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_1 + 4\lambda_3 + 2\lambda_4 &= 0 \quad (2.18) \\ -2\text{Tr}\mathbf{G}_e^2\mathbf{G}_e^{2\dagger} - 6\text{Tr}\mathbf{G}_u^2\mathbf{G}_u^2 - 6\text{Tr}\mathbf{G}_d^2\mathbf{G}_d^{2\dagger} + \frac{9}{4}g^2 + \frac{3}{4}g'^2 + 6\lambda_2 + 4\lambda_3 + 2\lambda_4 &= 0 \quad (2.19) \\ -2\text{Tr}\mathbf{G}_e^1\mathbf{G}_e^{2\dagger} - 6\text{Tr}\mathbf{G}_u^1\mathbf{G}_u^2 - 6\text{Tr}\mathbf{G}_d^1\mathbf{G}_d^{2\dagger} + 6(\lambda_7 - i\lambda_8) + 6(\lambda_9 + i\lambda_{10}) &= 0 \quad (2.20) \\ -2\text{Tr}\mathbf{G}_e^2\mathbf{G}_e^{1\dagger} - 6\text{Tr}\mathbf{G}_u^2\mathbf{G}_u^1 - 6\text{Tr}\mathbf{G}_d^2\mathbf{G}_d^{1\dagger} + 6(\lambda_7 + i\lambda_8) + 6(\lambda_9 - i\lambda_{10}) &= 0 \quad (2.21) \end{aligned}$$

Notice that equations (2.18) and (2.19) are real equations, while equations (2.20) and (2.21) are complex conjugate equations. If the quadratic divergences of the Higgs self-energy are calculated in terms of the physical fields H_1 , H_2 , H_3 , and H^+ , to be defined in section 3, the same four mass relations are obtained, but the algebra is much more lengthy.

3. Relation of the Higgs Tadpoles to the Higgs Self-Energy

In section 2, we have calculated the quadratic divergences of the Two Higgs Model in terms of the original fields Φ_1 and Φ_2 . Expanding the fields about the vacuum expectation values of Φ_1 and Φ_2 introduces a new class of quadratically divergent graphs, the Higgs tadpoles. If all the quadratic divergences are to be cancelled from the theory, the Higgs tadpoles must be treated separately from the other Higgs self-energy graphs, since the Higgs tadpoles are the only quadratically divergent diagrams appearing e.g. in the expression for the electron self-energy. As in section 2, it is possible to obtain mass relations by summing the Higgs tadpoles and setting the coefficients of the quadratically divergent parts to zero. But as we show in this section, the new mass relations are simply linear combinations of the four mass relations given by equations (2.18) through (2.21).

If the calculation of section 2 is repeated for the Standard Model with one Higgs doublet, a single mass relation is obtained, which is identical to the mass relation derived by summing the Higgs tadpoles. In the Two Higgs Model, by contrast, there are three neutral Higgs fields and three sets of tadpoles, yielding three mass relations. These three mass relations are not sufficient to cancel the quadratic divergences from the Two Higgs Model. In this section, we define the physical fields of the Two Higgs Model, and proceed to show the relationship between the Higgs tadpoles and the other Higgs self-energy graphs.

Suppose the vacuum expectation values of Φ_j , $j = 1, 2$ are given by

$$\langle \Phi_j \rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v_j e^{i\theta_j} \end{pmatrix}, \quad (3.1)$$

where $v_j \geq 0$ and θ_j are real parameters. To exhibit the decoupling of the physical Higgs fields from the unphysical Higgs ghosts, and to simplify the expressions which follow, it is convenient to define new fields ϕ_1 and ϕ_2 according to

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \Phi_1 e^{-i\theta_1} \\ \Phi_2 e^{i\theta_2} \end{pmatrix}, \quad (3.2)$$

where β is defined by $\tan \beta = v_2/v_1$. As in section 2, it is possible to calculate the quadratic divergences of the two-point functions illustrated in Figure 5 without breaking $SU(2) \times U(1)$ symmetry, and without expanding about the minimum of the potential. The result, given in terms of the I_{11} , I_{22} , I_{12} , and I_{21} defined by relations (2.14) through (2.17), is found to be

$$J_{11} \Rightarrow I_{11} \cos^2 \beta + I_{22} \sin^2 \beta + [I_{12} e^{i(\theta_1 - \theta_2)} + I_{21} e^{i(\theta_2 - \theta_1)}] \sin \beta \cos \beta \quad (3.3)$$

$$J_{22} \Rightarrow I_{11} \sin^2 \beta + I_{22} \cos^2 \beta - [I_{12} e^{i(\theta_1 - \theta_2)} + I_{21} e^{i(\theta_2 - \theta_1)}] \sin \beta \cos \beta \quad (3.4)$$

$$J_{12} \Rightarrow [-I_{11} + I_{22}] \sin \beta \cos \beta + I_{12} e^{i(\theta_1 - \theta_2)} \cos^2 \beta - I_{21} e^{i(\theta_2 - \theta_1)} \sin^2 \beta \quad (3.5)$$

$$J_{21} \Rightarrow [-I_{11} + I_{22}] \sin \beta \cos \beta - I_{12} e^{i(\theta_1 - \theta_2)} \sin^2 \beta + I_{21} e^{i(\theta_2 - \theta_1)} \cos^2 \beta \quad (3.6)$$

Notice that the mass relations derived by setting the coefficient of Δ to zero in equations (3.3) and (3.4) are real, while the mass relations derived by setting the coefficient of Δ to zero in equations (3.5) and (3.6) are complex conjugates. Since the four expressions J_{11} , J_{22} , J_{12} , and J_{21} are linearly independent combinations of I_{11} , I_{22} , I_{12} , and I_{21} , they form an alternative basis for describing the quadratic divergences of the Two-Higgs Model.

The immediate advantage of working with the fields ϕ_1 and ϕ_2 , rather than Φ_1 and Φ_2 , is that the Higgs ghosts G^+ , G^- , and G_0 decouple from the five physical Higgs fields H^+ , H^- , H_1 , H_2 , and H_3 :

$$\phi_1 = \left(\frac{1}{\sqrt{2}}(v + a + iG_0) \right) \quad \phi_2 = \left(\frac{H^+}{\sqrt{2}(b + ic)} \right). \quad (3.7)$$

In the above expressions, $v = \sqrt{v_1^2 + v_2^2}$, and the fields a , b , and c are related to the three neutral Higgs fields H_1 , H_2 , and H_3 by an orthogonal transformation $\mathbf{R}$ which diagonalizes the mass matrix:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} \quad \mathbf{R}^T \mathbf{R} = I \quad (3.8)$$

The physical fermion fields are obtained by diagonalizing the fermion mass matrix. The physical leptons e , μ and τ are defined by a pair of unitary transformations U_e and V_e that act independently on the left-handed and right-handed fields

$$\begin{pmatrix} e'_L \\ \mu'_L \\ \tau'_L \end{pmatrix} = U_e \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \quad \begin{pmatrix} e'_R \\ \mu'_R \\ \tau'_R \end{pmatrix} = V_e \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \quad (3.9)$$

and which are chosen so that

$$U_e^\dagger (G_e^1 e^{i\theta_1} \cos \beta + G_e^2 e^{i\theta_2} \sin \beta) V_e = \frac{\sqrt{2}}{v} \mathbf{M}_e, \quad (3.10)$$

where $\mathbf{M}_e$ is diagonal:

$$\mathbf{M}_e = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}. \quad (3.11)$$

Equation (3.10) gives the coupling of ϕ_1 to the physical lepton fields; the coupling of ϕ_2 to the physical lepton fields is

$$U_e^\dagger (-G_e^1 e^{i\theta_1} \sin \beta + G_e^2 e^{i\theta_2} \cos \beta) V_e = \frac{\sqrt{2}}{v} (-\mathbf{F}_e^1 \tan \beta + \mathbf{F}_e^2 \cot \beta), \quad (3.12)$$

where $\mathbf{F}_e^1$ and $\mathbf{F}_e^2$ are 3×3 complex matrices satisfying $\mathbf{F}_e^1 + \mathbf{F}_e^2 = \mathbf{M}_e$. The physical quarks are obtained by an essentially identical procedure. The u , c , and t quarks are defined by a pair of unitary transformations U_u and V_u

$$\begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix} = U_u \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \quad \begin{pmatrix} u'_R \\ c'_R \\ t'_R \end{pmatrix} = V_u \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} \quad (3.13)$$

satisfying

$$U_u^\dagger (G_u^1 e^{-i\theta_1} \cos \beta + G_u^2 e^{-i\theta_2} \sin \beta) V_u = \frac{\sqrt{2}}{v} \mathbf{M}_u \quad (3.14)$$

where $\mathbf{M}_u$ is diagonal:

$$\mathbf{M}_u = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}. \quad (3.15)$$

Equation (3.14) gives the coupling of the u , c , and t quarks to $\phi_1 = i\tau_2 \phi_1^*$; similarly, the coupling of the u , c , and t quarks to $\phi_2 = i\tau_2 \phi_2^*$ is given by

$$U_u^\dagger (-G_u^1 e^{-i\theta_1} \sin \beta + G_u^2 e^{-i\theta_2} \cos \beta) V_u = \frac{\sqrt{2}}{v} (-\mathbf{F}_u^1 \tan \beta + \mathbf{F}_u^2 \cot \beta), \quad (3.16)$$

where $\mathbf{F}_u^1$ and $\mathbf{F}_u^2$ are 3×3 complex matrices satisfying $\mathbf{F}_u^1 + \mathbf{F}_u^2 = \mathbf{M}_u$. Comparison of equations (3.10) and (3.12) with equations (3.14) and (3.16) reveals that θ_1 and θ_2 change sign, but that after a change of the indices e and u the equations are otherwise identical. The sign change of θ_1 and θ_2 occurs because the mass terms for the u , c , and t quarks enter the Yukawa potential (1.1) through $\hat{\Phi}_i$ instead of Φ_i . Since the masses of the leptons e , μ , and τ , the d , s , and b quarks are the same way as the masses of the leptons e , μ , and τ , the d , s , and b quarks are defined as in equations (3.9) through (3.12).

To simplify the discussion which follows, we write all Feynman diagrams in terms of the fields

$$\phi_1 = \left(\frac{\phi_{1a}}{\sqrt{2}} + \phi_{1b} \right) \quad \phi_2 = \begin{pmatrix} \phi_{2a} \\ \phi_{2b} \end{pmatrix}, \quad (3.17)$$

where by equation (3.3), $\phi_{1a} = G^+$, $\phi_{1b} = \frac{1}{\sqrt{2}}(a + iG_0)$, $\phi_{2a} = H^+$, and $\phi_{2b} = \frac{1}{\sqrt{2}}(b + ic)$. In rough form, Figure 6 shows how the Higgs tadpoles are related to the other Higgs self-energy graphs. There is a one-to-one correspondence between Higgs tadpole graphs with incoming line ϕ_{1b} , which are denoted by T_1 , and non-tadpole Higgs self-energy graphs denoted by J_{11} , with incoming line ϕ_{1b} and outgoing line ϕ_{1b} . The coefficients of the quadratically divergent parts are the same up to a multiplicative factor $\frac{v}{\sqrt{2}}$. Similarly, there is a one-to-one correspondence between Higgs tadpole graphs with incoming line ϕ_{2b} , which are denoted by T_2 , and non-tadpole Higgs self-energy graphs denoted by J_{21} , with incoming line ϕ_{2b} and outgoing line ϕ_{1b} . Because the sum of graphs J_{11} produces one real mass relation, and the sum of graphs J_{21} produces one complex mass relation, the Higgs tadpoles together produce three real mass relations. The existence of three mass relations may be seen more directly by writing all expressions in terms of the physical fields H_1 , H_2 , and H_3 , but in this case the relationship between the Higgs tadpoles and the other Higgs self-energy graphs is obscured.

In the following, the notation $T_1|_x$ will be used to denote a tadpole with ϕ_{1b} on the incoming line and a virtual intermediate state x . The coupling of ϕ_{1b} to the physical fermion fields e , u , and d is illustrated in Figure 7a, with integrals given as follows:

$$\begin{aligned} T_1|_l &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_l} \left(-i \frac{\sqrt{2} m_l (1 + \gamma_5)}{v} \right) \right] \\ &\Rightarrow -2m_l^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad l = e, \mu, \tau \end{aligned} \quad (3.18)$$

$$\begin{aligned} T_1|_q &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_q} \left(-i \frac{\sqrt{2} m_q (1 + \gamma_5)}{v} \right) \right] \\ &\Rightarrow -2m_q^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad q = u, c, t \end{aligned} \quad (3.19)$$

$$\begin{aligned} T_1|_q &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_q} \left(-i \frac{\sqrt{2} m_q (1 + \gamma_5)}{v} \right) \right] \\ &\Rightarrow -2m_q^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad q = d, s, b \end{aligned} \quad (3.20)$$

Notice in the above equations that the coupling of ϕ_{1b} to fermions f is proportional to the fermion mass m_f . The coefficient of Δ is proportional to m_f^2 because the trace of the fermion propagator introduces an additional factor of m_f . Figure 7b shows the corresponding Higgs self-energy graphs, with integrals given by:

$$J_{11}|_l = - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_l} \left(-i \frac{\sqrt{2} m_l (1 + \gamma_5)}{v} \right) \right]$$

$$\begin{aligned} &\frac{i}{\not{k} - \frac{1}{2}\not{p} - m_l} \left(-i \frac{\sqrt{2} m_l (1 - \gamma_5)}{v} \right) \\ &\Rightarrow -2m_l^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad l = e, \mu, \tau \end{aligned} \quad (3.21)$$

$$\begin{aligned} J_{11}|_{qq} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_q} \left(-i \frac{\sqrt{2} m_q (1 - \gamma_5)}{v} \right) \right. \\ &\quad \left. \frac{i}{\not{k} - \frac{1}{2}\not{p} - m_q} \left(-i \frac{\sqrt{2} m_q (1 + \gamma_5)}{v} \right) \right] \\ &\Rightarrow -2m_q^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad q = u, c, t \end{aligned} \quad (3.22)$$

$$\begin{aligned} J_{11}|_q &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_q} \left(-i \frac{\sqrt{2} m_q (1 + \gamma_5)}{v} \right) \right. \\ &\quad \left. \frac{i}{\not{k} - \frac{1}{2}\not{p} - m_q} \left(-i \frac{\sqrt{2} m_q (1 - \gamma_5)}{v} \right) \right] \\ &\Rightarrow -2m_q^2 \left(\frac{\sqrt{2}}{v} \right)^2 \Delta \quad q = d, s, b \end{aligned} \quad (3.23)$$

The essential point is that the coefficients of Δ in equations (3.18) through (3.20) are identical to the coefficients of Δ in equations (3.21) through (3.23), except for a multiplicative factor $\frac{v}{\sqrt{2}}$. The results for the Higgs self-energy J_{11} are proportional to the fermion mass squared because ϕ_{1b} couples to the fermion loop twice. The Higgs tadpoles T_1 coupled to a fermion loop are illustrated in Figure 8a, and the associated integrals are given below:

$$\begin{aligned} T_2|_l &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_l} \left(-i \frac{\sqrt{2}}{v} \left(-\mathbf{F}_e^1 \tan \beta + \mathbf{F}_e^2 \cot \beta \right) \frac{1 + \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_l \left(\frac{\sqrt{2}}{v} \right) \left(-\mathbf{F}_e^1 \tan \beta + \mathbf{F}_e^2 \cot \beta \right) \Delta \quad l = e, \mu, \tau \end{aligned} \quad (3.24)$$

$$\begin{aligned} T_2|_q &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_l} \left(-i \frac{\sqrt{2}}{v} \left(-\mathbf{F}_u^1 \tan \beta + \mathbf{F}_u^2 \cot \beta \right) \frac{1 - \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_q \left(\frac{\sqrt{2}}{v} \right) \left(-\mathbf{F}_u^1 \tan \beta + \mathbf{F}_u^2 \cot \beta \right) \Delta_{qq} \quad q = u, c, t \end{aligned} \quad (3.25)$$

$$\begin{aligned} T_2|_q &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta T r} \left[\frac{i}{\not{k} - m_l} \left(-i \frac{\sqrt{2}}{v} \left(-\mathbf{F}_d^1 \tan \beta + \mathbf{F}_d^2 \cot \beta \right) \frac{1 + \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_q \left(\frac{\sqrt{2}}{v} \right) \left(-\mathbf{F}_d^1 \tan \beta + \mathbf{F}_d^2 \cot \beta \right) \Delta_{qq} \quad q = d, s, b \end{aligned} \quad (3.26)$$

As before, the trace of the fermion propagator yields the fermion mass as a multiplicative factor in the coefficient of Δ . Such a factor may also be introduced by coupling ϕ_b to the fermion loop, since the coupling is proportional to the fermion mass. The corresponding Higgs self-energy diagrams are illustrated in Figure 8b, with integrals given as follows:

$$\begin{aligned} J_{21}|_u &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \text{Tr} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_l} \left(-i\frac{\sqrt{2}}{v} (-\mathbf{F}_e^1 \tan \beta + \mathbf{F}_e^2 \cot \beta) \right)_u \frac{1 + \gamma_5}{2} \right. \\ &\quad \left. - \frac{i}{\not{k} - \frac{1}{2}\not{p} - m_l} \left(-i\frac{\sqrt{2}}{v} (\mathbf{M}_e)_u \frac{1 - \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_l \left(\frac{\sqrt{2}}{v} \right)^2 (-\mathbf{F}_e^1 \tan \beta + \mathbf{F}_e^2 \cot \beta)_u \Delta \quad l = e, \mu, \tau \end{aligned} \quad (3.27)$$

$$\begin{aligned} J_{21}|_{qq} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \text{Tr} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_q} \left(-i\frac{\sqrt{2}}{v} (-\mathbf{F}_u^1 \tan \beta + \mathbf{F}_u^2 \cot \beta) \right)_{qq}^\dagger \frac{1 - \gamma_5}{2} \right. \\ &\quad \left. - \frac{i}{\not{k} - \frac{1}{2}\not{p} - m_q} \left(-i\frac{\sqrt{2}}{v} (\mathbf{M}_u)_{qq} \frac{1 + \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_q \left(\frac{\sqrt{2}}{v} \right)^2 (-\mathbf{F}_u^1 \tan \beta + \mathbf{F}_u^2 \cot \beta)_{qq}^\dagger \Delta \quad q = u, c, t \end{aligned} \quad (3.28)$$

$$\begin{aligned} J_{21}|_{qq} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \text{Tr} \left[\frac{i}{\not{k} + \frac{1}{2}\not{p} - m_q} \left(-i\frac{\sqrt{2}}{v} (-\mathbf{F}_d^1 \tan \beta + \mathbf{F}_d^2 \cot \beta) \right)_{qq} \frac{1 + \gamma_5}{2} \right. \\ &\quad \left. - \frac{i}{\not{k} - \frac{1}{2}\not{p} - m_q} \left(-i\frac{\sqrt{2}}{v} (\mathbf{M}_d)_{qq} \frac{1 - \gamma_5}{2} \right) \right] \\ &\Rightarrow -2m_q \left(\frac{\sqrt{2}}{v} \right)^2 (-\mathbf{F}_d^1 \tan \beta + \mathbf{F}_d^2 \cot \beta)_{qq} \Delta \quad q = d, s, b \end{aligned} \quad (3.29)$$

Comparison of equations (3.24) through (3.26) with equations (3.27) through (3.29) reveals that the coefficients of Δ are identical except for a multiplicative factor $\frac{v}{\sqrt{2}}$. In R_ξ -gauge [8], the scalar doublets couple to Fadeev-Popov ghosts, as illustrated in Figure 9a. The integrals associated with these diagrams are given below:

$$\begin{aligned} T_1|_{\eta^A} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(-i\xi \frac{g^2}{4} \frac{v}{\sqrt{2}} \right) \frac{i\delta_{ab}}{k^2} \\ &\Rightarrow -\xi \frac{3g^2}{4} \frac{v}{\sqrt{2}} \Delta, \end{aligned} \quad (3.30)$$

$$\begin{aligned} T_1|_{\eta^B} &= - \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(-i\xi \frac{g^2}{4} \frac{v}{\sqrt{2}} \right) \frac{i}{k^2} \\ &\Rightarrow -\xi \frac{g^2}{4} \frac{v}{\sqrt{2}} \Delta. \end{aligned} \quad (3.31)$$

Here the indices $a, b = 1, 2, 3$ refer to the components of the vector field $\hat{A}$. The sum of terms (3.30) and (3.31) is not gauge-invariant, but there are additional ξ -dependent terms because of the Higgs coupling to vector bosons, illustrated in Figure 10a:

$$\begin{aligned} T_1|_A &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(\frac{ig^2}{2} \frac{v}{\sqrt{2}} g_{\mu\nu} \delta_{ab} \right) \frac{i\delta_{ab}}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \\ &\Rightarrow (3 + \xi) \frac{3g^2}{4} \frac{v}{\sqrt{2}} \Delta \end{aligned} \quad (3.32)$$

$$\begin{aligned} T_1|_B &= \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(\frac{ig^2}{2} \frac{v}{\sqrt{2}} g_{\mu\nu} \right) \frac{i}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \\ &\Rightarrow (3 + \xi) \frac{g^2}{4} \frac{v}{\sqrt{2}} \Delta \end{aligned} \quad (3.33)$$

Summing equations (3.30) through (3.33) gives the gauge-invariant result:

$$T_1|_{\sum \xi\text{-dependent}} = \left(\frac{9}{4} g^2 + \frac{3}{4} g'^2 \right) \frac{v}{\sqrt{2}} \Delta \quad (3.34)$$

Strictly speaking, the propagators should include a mass term, but since we are only interested in the quadratically divergent part of the amplitudes, these masses have been set to zero. The only case in which the propagator mass is important is the case of the Higgs tadpoles coupled to a fermion loop, which was treated exactly. The Higgs self-energy diagrams associated with the Fadeev-Popov ghost loops and the vector boson loops are illustrated in Figure 9b and Figure 10b, respectively. Essentially the same diagrams were treated earlier, in equations (2.4) through (2.7). The transformation (3.2) does nothing to alter the Higgs coupling to vector bosons. Hence, the integrals are given by:

$$\begin{aligned} J_{11}|_{\phi_1 A} &= \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(\frac{1}{2} ig(\tau_a)_{1\mu} (k + \frac{3}{2} p)_\mu \right) \frac{i\delta^{\mu\nu}}{(k + \frac{1}{2} p)^2} \left(\frac{1}{2} ig(\tau_b)_{1\nu} (k + \frac{3}{2} p)_\nu \right) \\ &\quad - \frac{i\delta^{ab}}{(k - \frac{1}{2} p)^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{(k - \frac{1}{2} p)^\mu (k - \frac{1}{2} p)^\nu}{(k - \frac{1}{2} p)^2} \right] \\ &\Rightarrow -\xi \frac{3}{4} g'^2 \Delta \end{aligned} \quad (3.35)$$

$$\begin{aligned} J_{11}|_{\phi_1 B} &= \int \frac{d^4 k}{(2\pi)^4} e^{ik\delta} \left(\frac{1}{2} ig'(k + \frac{3}{2} p)_\mu \right) \frac{i}{(k + \frac{1}{2} p)^2} \left(-\frac{1}{2} ig'(k + \frac{3}{2} p)_\nu \right) \\ &\quad - \frac{i}{(k - \frac{1}{2} p)^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{(k - \frac{1}{2} p)^\mu (k - \frac{1}{2} p)^\nu}{(k - \frac{1}{2} p)^2} \right] \\ &\Rightarrow -\xi \frac{1}{4} g'^2 \Delta \end{aligned} \quad (3.36)$$

$$J_{11}|_A = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(\frac{ig^2}{2} g_{\mu\nu} \delta_{ab} \right) \frac{i\delta^{ab}}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \Rightarrow (3 + \xi) \frac{3}{4} g^2 \Delta \quad (3.37)$$

$$J_{11}|_B = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(\frac{ig^2}{2} g_{\mu\nu} \right) \frac{i}{k^2} \left[-g^{\mu\nu} + (1 - \xi) \frac{k^\mu k^\nu}{k^2} \right] \Rightarrow (3 + \xi) \frac{1}{4} g^2 \Delta \quad (3.38)$$

Comparison of equations (3.30) through (3.33) with equations (3.35) through (3.38) reveals that the coefficient of Δ is the same except for a multiplicative factor $\frac{v}{\sqrt{2}}$. The field ϕ_2 does not couple to Fadeev-Popov ghosts, nor is there trilinear coupling between ϕ_2 and the vector bosons. The reason is that the vacuum expectation value of ϕ_2 is zero. This implies, by Figure 6, that the diagrams of Figures 9b and 10b should have no analogue with ϕ_2 on the incoming line and ϕ_1 on the outgoing line. In fact, there are no such diagrams, since the coupling to the vector bosons cannot change the doublet number; notice that each of the integrals (2.5) through (2.8) is proportional to δ_{ij} .

Finally, we consider the case of the Higgs loops. Because $\frac{v}{\sqrt{2}}$ and ϕ_{1b} always appear together in the combination $\frac{v}{\sqrt{2}} + \phi_{1b}$, there is a Higgs self-energy diagram with outgoing line ϕ_{1b} for every tadpole with incoming line ϕ_{1b} or ϕ_{2b} . The complete set of interaction terms contributing to T_1 , for example, is given by:

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \left(\frac{v}{\sqrt{2}} + \phi_{1b} \right) (\phi_{1a}^* \phi_{1a}) = (\phi_{1b}^* \phi_{1b}) (\phi_{1a}^* \phi_{1a}) + \frac{v}{\sqrt{2}} \phi_{1b} (\phi_{1a}^* \phi_{1a}) + \dots \quad (3.39)$$

$$\left(\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \left(\frac{v}{\sqrt{2}} + \phi_{1b} \right) \right)^2 = (\phi_{1b}^* \phi_{1b})^2 + 2 \frac{v}{\sqrt{2}} \phi_{1b} (\phi_{1b}^* \phi_{1b}) + \dots \quad (3.40)$$

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \left(\frac{v}{\sqrt{2}} + \phi_{1b} \right) (\phi_{2a}^* \phi_{2a}) = (\phi_{1b}^* \phi_{1b}) (\phi_{2a}^* \phi_{2a}) + \frac{v}{\sqrt{2}} \phi_{1b} (\phi_{2a}^* \phi_{2a}) + \dots \quad (3.41)$$

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \left(\frac{v}{\sqrt{2}} + \phi_{1b} \right) (\phi_{2b}^* \phi_{2b}) = (\phi_{1b}^* \phi_{1b}) (\phi_{2b}^* \phi_{2b}) + \frac{v}{\sqrt{2}} \phi_{1b} (\phi_{2b}^* \phi_{2b}) + \dots \quad (3.42)$$

From equations (3.39) through (3.42), it may be seen that every trilinear coupling x yields a contribution $T_1|_x$ given by

$$T_1|_x = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-iS \frac{v}{\sqrt{2}} \right) \frac{i}{k^2} \Rightarrow S \frac{v}{\sqrt{2}} \Delta, \quad (3.43)$$

while the associated quartic coupling X yields a contribution to the Higgs self-energy $J_{11}|_X$ given by

$$J_{11}|_X = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-iS \right) \frac{i}{k^2} \Rightarrow S \Delta, \quad (3.44)$$

where S is the symmetry factor for the quartic coupling. The symmetry factor is given by $S = 4$ for the interaction term (3.40), while $S = 1$ for each of the other three interaction terms. Although the coupling constants are not given, equations (3.43) and (3.44) share the same coupling for each pair of interactions. Hence, the coefficients of Δ are the same for the tadpole and the Higgs self-energy graph, except for a multiplicative constant $\frac{v}{\sqrt{2}}$.

The complete set of interaction terms contributing to T_2 is given by

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \phi_{2b} (\phi_{1a}^* \phi_{1a}) = (\phi_{1b}^* \phi_{2b}) (\phi_{1a}^* \phi_{1a}) + \frac{v}{\sqrt{2}} \phi_{2b} (\phi_{1a}^* \phi_{1a}) + \dots \quad (3.45)$$

$$\left(\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \left(\frac{v}{\sqrt{2}} + \phi_{1b} \right) \right) \phi_{2b} = (\phi_{1b}^* \phi_{2b}) (\phi_{1b}^* \phi_{1b}) + 2 \frac{v}{\sqrt{2}} \phi_{2b} (\phi_{1b}^* \phi_{1b}) + \dots \quad (3.46)$$

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \phi_{2b} (\phi_{2a}^* \phi_{2a}) = (\phi_{1b}^* \phi_{2b}) (\phi_{2a}^* \phi_{2a}) + \frac{v}{\sqrt{2}} \phi_{2b} (\phi_{2a}^* \phi_{2a}) + \dots \quad (3.47)$$

$$\left(\frac{v}{\sqrt{2}} + \phi_{1b} \right)^* \phi_{2b} \phi_{2b}^2 = (\phi_{1b}^* \phi_{2b}) (\phi_{2b}^* \phi_{2b}) + \frac{v}{\sqrt{2}} \phi_{2b} (\phi_{2b}^* \phi_{2b}) + \dots \quad (3.48)$$

It may be seen from equations (3.45) through (3.48) that every trilinear coupling x yields a contribution $T_2|_x$ given by

$$T_2|_x = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-iS \frac{v}{\sqrt{2}} \right) \frac{i}{k^2} \Rightarrow S \frac{v}{\sqrt{2}} \Delta, \quad (3.49)$$

while the associated quartic coupling X yields a contribution to the Higgs self-energy $J_{21}|_X$ given by

$$J_{21}|_X = \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot \delta} \left(-iS \right) \frac{i}{k^2} \Rightarrow S \Delta, \quad (3.50)$$

where S is the symmetry factor for the quartic coupling. The symmetry factor is given by $S = 1$ for interaction terms (3.45) and (3.47), and it is given by $S = 2$ for interaction terms (3.46) and (3.48).

Because the set of all graphs contributing to the quadratic divergences of the Higgs tadpoles T_1 and T_2 have been evaluated, and found to be equivalent, term by term, to the set of all graphs contributing to the quadratic divergences of the Higgs self-energy J_{11} and J_{21} , the considerations of this section lead to the conclusion

$$\begin{aligned} T_1 &\leftrightarrow \frac{v}{\sqrt{2}} J_{11} \\ T_2 &\leftrightarrow \frac{v}{\sqrt{2}} J_{21}, \end{aligned} \quad (3.51)$$

as indicated by Figure 6. The notation $\Leftrightarrow$ implies that the coefficient of the quadratic divergences of the sum of terms is identical, up to a multiplicative constant $\frac{v}{\sqrt{2}}$. The relation of J_{11} and J_{21} to the mass relations derived in section 2 can be seen from equations (3.3) and (3.6).

4. Conclusion

The four mass relations derived by cancelling the quadratic divergences of the Two Higgs model are given by equations (2.18) through (2.21). In the discussion so far, it has been assumed that the theory is invariant under $SU(2) \times U(1)$ gauge transformations, with no additional symmetries applied. But from the general formulas (2.18) through (2.21) it is a simple matter to write down the mass relations for any special case that may be of interest for reasons of phenomenology. The simplest example is the one where Φ_1 couples only to up quarks, while Φ_2 couples only to down quarks. If in addition we assume that the electron couples only to Φ_2 , then the large number of undetermined couplings in the Yukawa sector are eliminated. It follows that

$$\mathbf{G}_{1e} = 0, \quad \mathbf{G}_{2u} = 0, \quad \mathbf{G}_{1d} = 0, \quad (4.1)$$

and

$$Tr[\mathbf{G}_{2e}^\dagger \mathbf{G}_{2e}] = \left(\frac{\sqrt{2}}{v}\right)^2 [m_e^2 + m_\mu^2 + m_\tau^2] \sin^2 \beta \quad (4.2)$$

$$Tr[\mathbf{G}_{1u}^\dagger \mathbf{G}_{1u}] = \left(\frac{\sqrt{2}}{v}\right)^2 [m_u^2 + m_c^2 + m_t^2] \cos^2 \beta \quad (4.3)$$

$$Tr[\mathbf{G}_{2d}^\dagger \mathbf{G}_{2d}] = \left(\frac{\sqrt{2}}{v}\right)^2 [m_d^2 + m_s^2 + m_b^2] \sin^2 \beta, \quad (4.4)$$

by equations (3.10) and (3.14). If no further assumptions are made, the mass relations (2.20) and (2.21) take on the trivial form

$$\lambda_7 = -\lambda_9 \quad \text{and} \quad \lambda_8 = \lambda_{10}, \quad (4.5)$$

while equations (2.18) and (2.19) take the form

$$\frac{3}{2}m_w^2 + \frac{3}{4}m_z^2 + \left(\frac{v}{2}\right)^2 (6\lambda_1 + 4\lambda_3 + 2\lambda_4) = [3(m_u^2 + m_c^2 + m_t^2)] \cos^2 \beta \quad (4.6)$$

and

$$\frac{3}{2}m_w^2 + \frac{3}{4}m_z^2 + \left(\frac{v}{2}\right)^2 (6\lambda_2 + 4\lambda_3 + 2\lambda_4) = [m_e^2 + m_\mu^2 + m_\tau^2 + 3(m_d^2 + m_s^2 + m_b^2)] \sin^2 \beta, \quad (4.7)$$

with the aid of the relations $m_w = gv/2$ and $g' = g \tan \theta$.

The number of couplings in the Higgs potential (1.4) may be reduced by imposing the pair of discrete symmetries

$$\begin{array}{ll} 1) & \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2 \\ 2) & \Phi_1 \rightarrow -\Phi_1, \quad \Phi_2 \rightarrow \Phi_2. \end{array}$$

In this case, $\lambda_7 = \lambda_8 = \lambda_9 = \lambda_{10} = 0$, and the mass relations (2.20) and (2.21) are reduced to relations between the Yukawa couplings:

$$-2Tr\mathbf{G}_e^1\mathbf{G}_e^{2\dagger} - 6Tr\mathbf{G}_u^1\mathbf{G}_u^{2\dagger} - 6Tr\mathbf{G}_d^1\mathbf{G}_d^{2\dagger} = 0, \quad (4.8)$$

while equations (2.18) and (2.19) are unaffected. If the discrete symmetries given above are applied in conjunction with conditions (4.1), then the four mass relations are reduced to two mass relations, given by equations (4.6) and (4.7).

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Figure 1. Higgs self-energy diagrams in terms of Φ_1 and Φ_2 .

Figure 2. Higgs self-energy diagrams with fermion loops.

Figure 3. ξ -dependent Higgs self-energy diagrams.

Figure 4. Higgs self-energy diagrams with Higgs boson loops.

Figure 5. Higgs self-energy diagrams in terms of Φ_1 and Φ_2 .

Figure 6. Relation of tadpoles to self-energy diagrams.

Figure 7. (a) Higgs tadpoles T_1 with a fermion loop.
(b) The corresponding Higgs self-energy diagrams.

Figure 8. (a) Higgs tadpoles T_2 with a fermion loop.

(b) The corresponding Higgs self-energy diagrams.

Figure 9. (a) Higgs tadpoles with a FP -ghost loop.

(b) The corresponding Higgs self-energy diagrams.

Figure 10. (a) Higgs tadpoles with a vector boson loop.
 (b) The corresponding Higgs self-energy diagrams.