Natural Conservation of R Parity in Supersymmetry

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Abstract

Since baryon number B and lepton number L are no longer automatically conserved once the standard model is extended to include supersymmetry, the usually assumed conservation of \( R \equiv (-1)^{2} \cdot 4^{B} + 4^{L} \) is an imposed condition. For a more satisfactory realization of supersymmetry, we propose here a new model which conserves R automatically. It is unifiable under SO(14) and has exotic fermions at the 100 GeV scale. One important result is the enhancement of the Higgs-boson decay rates into two gluons and into two photons by factors of 25 and 15 respectively.
In discussing supersymmetry in particle physics, it is common practice to consider a quantity called R parity which is defined as

\[ R \equiv (-1)^{2j+3B+L}, \]

where \( j \) is the spin of the particle, \( B \) its baryon number and \( L \) its lepton number. If \( B \) and \( L \) are additively conserved, then it is obvious that \( R \) has to be multiplicatively conserved. However, in the supersymmetric standard model, this is not necessarily the case. Consider the quark and lepton superfields. In a notation where only the left chiral projections are counted, they transform under \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \) as follows: 

\[
\begin{align*}
Q &\equiv (u, d) \sim (3, 2, 1/6), \\
u^c &\sim (\bar{3}, 1, -2/3), \\
d^c &\sim (\bar{3}, 1, 1/3), \\
L &\equiv (\nu, e) \sim (1, 2, -1/2), \\
e^c &\sim (1, 1, 1),
\end{align*}
\]

where the family index has been suppressed. In addition, there must be two Higgs superfields \( \Phi_{1,2} \) transforming as \((1, 2, \mp 1/2)\) respectively. The desirable allowed terms in the superpotential are then \( \Phi_1 Qd^c, \Phi_2 Qu^c, \) and \( \Phi_1 Le^c \), which supply the quarks and leptons with masses as the neutral scalar components of \( \Phi_{1,2} \) acquire nonzero vacuum expectation values. However, the terms \( u^c d^c d^c, LQd^c, \) and \( LLe^c \) are also allowed \textit{a priori} and they violate the conservation of \( B \) and \( L \). Note also that \( \Phi_1 \) and \( L \) are indistinguishable by their transformations alone.

To obtain a realistic model, the usual procedure is to impose \( B \) and \( L \) conservation as an extra condition. In that case, \( R \) can be used to distinguish particles \((R = +1)\) from superparticles \((R = -1)\). As a result, the lightest supersymmetric particle (LSP) is stable and that is one of the essential features of supersymmetry upon which experimental search strategies are based. It seems to us that it would be far more satisfactory if \( B \) and \( L \) were automatically conserved as in the standard model without supersymmetry. Consider then the conventional left-right supersymmetric extension of the standard model. The gauge group is now \( \text{SU}(3) \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1) \). The quarks and leptons are

\[
\begin{align*}
Q &\equiv (u, d) \sim (3, 2, 1/6), \\
Q^c &\equiv (d^c, u^c) \sim (\bar{3}, 1, 2; -1/6), \\
L &\equiv (\nu, e) \sim (1, 2, 1; -1/2), \\
L^c &\equiv (e^c, \nu^c) \sim (1, 1, 2; 1/2).
\end{align*}
\]
Hence terms involving three such superfields are not possible in the superpotential because of gauge invariance, and the automatic conservation of B and L appears to have been achieved. However, if the Higgs sector consists of only triplets, bidoublets, and singlets, the scalar neutrinos must acquire nonzero vacuum expectation values, $\langle \tilde{\nu} \rangle \neq 0$, in order to break the left-right symmetry.[3] Hence B is conserved but not L. [There are other complications such as the inevitability of flavor-changing neutral currents (FCNC) because two bidoublets are needed for realistic quark mass matrices, as well as the fine tuning required to make $\langle \tilde{\nu} \rangle$ small and to keep the desirable equality $M_W = M_Z \cos \theta_W$ at tree level in the presence of the $SU(2)_L$ Higgs triplets.] A recent proposal by one of us[4] also conserves B but not L, unless it is imposed.

The problem of B and L conservation in supersymmetry may be considered also in the context of grand unification. If SU(5) is used, the problem persists because the $u^c d^c d^c$ term can still come from the invariant formed with a 10 and two 5's representations, and $\Phi_1$ and $L$ are still indistinguishable as 5's. If SO(10) is used, then the $u^c d^c d^c$ term is not allowed, and $\Phi_1$ and $\Phi_2$ belong in the 10 whereas $L$ and $L^c$ belong in the 16. This is a good solution for B conservation as long as the exotic SU(3) triplets in the 10 are made very heavy.[5] However, as SO(10) contains $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$, the details of the symmetry breaking still require nonzero vacuum expectation values for $SU(2)_R$ and $SU(2)_L$ doublets. Hence L conservation is broken either spontaneously[3] if the only such doublets are leptons, or explicitly[4] as well if there are additional Higgs doublet superfields and no extra discrete symmetry is assumed to distinguish them from the leptons.

Consider now the addition of another U(1) factor which contains electric charge but under which $Q, Q^c, L,$ and $L^c$ transform trivially. We also add the following superfields:

\begin{align*}
x \sim (3, 1, 1; -1/3, 1), & \quad x^c \sim (\bar{3}, 1, 1; 1/3, -1), \\
N \sim (1, 1, 1; -1, 1), & \quad N^c \sim (1, 1, 1; 1, -1),
\end{align*}

(4) (5)
\Phi_{12} \equiv \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ -\phi_1^- & \phi_2^- \end{pmatrix} \sim (1, 2, 2; 0, 0), \quad (6)

and

\Phi_3 \equiv (\phi_3^+, \phi_3^0) \sim (1, 2, 1; -1/2, 1), \quad \Phi_4 \equiv (\phi_4^0, -\phi_4^-) \sim (1, 1, 2; 1/2, -1). \quad (7)

Note that \Phi_3 and \Phi_4 transform differently from \(L\) and \(L^c\) under the extra U(1). Note also that the above particle content is not anomaly-free. In Ref. [4], two \(SU(2)_R\) doublets are used so that the structure is anomaly-free, but then since one of the new superfields transforms identically as \(L^c\), the conservation of lepton number cannot be maintained without being imposed. Here the allowed Yukawa terms are

\begin{align*}
\Phi_{12}Qq^c &= \phi_1^0 d^c + \phi_1^- u^c + \phi_2^0 uu^c - \phi_2^+ du^c, \\
\Phi_{12}LL^c &= \phi_1^0 ee^c + \phi_1^- \nu\nu^c + \phi_2^0 \nu\nu^c - \phi_2^+ \nu\nu^c, 
\end{align*}

and

\begin{align*}
\Phi_3Qx^c &= \phi_3^0 u^c x^c - \phi_3^+ dx^c, \quad \Phi_4Qq^c = \phi_4^0 x u^c + \phi_4^- x d^c, \\
\Phi_3LN^c &= \phi_3^0 \nu N^c - \phi_3^+ \nu e^c, \quad \Phi_4LN^c = \phi_4^0 \nu e^c + \phi_4^- \nu e^c. 
\end{align*}

Hence B and L are automatically conserved with B = 1/3 for Q and x, B = -1/3 for \(Q^c\) and \(x^c\), L = 1 for \(L\) and N, and L = -1 for \(L^c\) and \(N^c\), where the singlet quark \(x\) has charge 2/3 and the singlet lepton \(N\) is neutral. The Higgs superfields \(\Phi_{12}, \Phi_3,\) and \(\Phi_4\) have B = L = 0.

This assignment is automatic without the need of any extra imposed condition because they are in representations different from \(L, L^c, N,\) and \(N^c\). As a result, the spontaneous breaking of the gauge symmetry through the nonzero vacuum expectation values of \(\Phi_{12}, \Phi_3,\) and \(\Phi_4\) will not violate the conservation of B and L. However, \(\Phi_3\) and \(\Phi_4\) generate nonvanishing axial-vector anomalies as already mentioned and we should think about how they are to be canceled.

Consider the gauge group SO(10) \times U(1). It is obvious that \(Q, Q^c, L,\) and \(L^c\) are in the \((16,0)\) representation whereas \(\Phi_{12}\) is in the \((10,0)\). It is thus natural to assume that \(\Phi_3\) is
in the $\mathbf{(16, 1)}$ and $\Phi_4$ is in the $\mathbf{(16, -1)}$. The extra color triplets and singlets in the $\mathbf{(16, \pm 1)}$ will then render the theory anomaly-free. The fermions in these superfields will also have masses at the electroweak energy scale because they couple to $\Phi_{12}$ in analogy to the usual quarks and leptons. As for the singlet quarks $x$ and $x^c$ and the singlet leptons $N$ and $N^c$, although they do not generate anomalies at the $SU(3) \times SU(2) \times SU(2) \times U(1) \times U(1)$ level, they can also be considered as belonging to the $(\mathbf{10}, \pm 1)$ and $(\mathbf{120}, \pm 1)$ representations of $SO(10) \times U(1)$ respectively. Later on, we will show that it is natural to extend the gauge group further to $SO(14)$, but now let us return to the interaction structure of our model at low energies.

Since $x$ and $x^c$ are singlets, there is an allowed gauge-invariant mass term $xx^c$. Let $\langle \phi^0_{1,2,3,4} \rangle = v_{1,2,3,4}$, then the $6 \times 6$ mass matrix linking $(u, x)$ with $(u^c, x^c)$ is given by

$$M_{ux} = \begin{bmatrix} v_2 v_1^{-1} M_d & M_3 \\ M_4 & M_x \end{bmatrix},$$

where the $3 \times 3$ mass matrices $M_d$ and $M_x$ can be chosen to be diagonal, with $M_3$ and $M_4$ proportional to $v_3$ and $v_4$ respectively. The mixing of $u$ and $x$ is determined by the matrix

$$M_{ux}M_{ux}^\dagger = \begin{bmatrix} v_2^2 v_1^{-2} M_d M_u^\dagger + M_3 M_4^\dagger & v_2 v_1^{-1} M_d M_4^\dagger + M_3 M_5^\dagger \\ v_2 v_1^{-1} M_4 M_d^\dagger + M_3 M_3^\dagger & M_4 M_4^\dagger + M_5 M_5^\dagger \end{bmatrix},$$

and since $v_3$ breaks $SU(2)_L$ but $v_4$ breaks $SU(2)_R$, it is clear that $u - x$ mixing is very small and can be safely neglected. On the other hand, the mass matrix for the $u$ quarks is given by

$$M_u M_u^\dagger = v_2^2 v_1^{-2} M_d M_u^\dagger + M_3 M_4^\dagger - (v_2 v_1^{-1} M_d M_4^\dagger + M_3 M_5^\dagger) \times (M_4 M_4^\dagger + M_5 M_5^\dagger)^{-1}(v_2 v_1^{-1} M_4 M_4^\dagger + M_5 M_5^\dagger),$$

which is in general nondiagonal and can easily be phenomenologically correct even if $u - x$ mixing is very small. The mixing of $u^c$ and $x^c$ is determined by the matrix $M_{ux}^\dagger M_{ux}$ and can be quite large because $M_4$ and $M_5$ may be comparable in magnitude. However, because $u^c$
and $x^c$ transform identically under the standard $SU(2) \times U(1)$, their coupling to the $Z$ boson remains diagonal. Nevertheless, there are flavor-changing neutral currents in the $u$ sector due to the exchange of the $Z'$ boson from $SU(2)_R \times U(1)$ breaking and that of neutral Higgs bosons. One interesting implication[4] is the possibility of the decay $t \rightarrow c +$ Higgs boson, which may be observable once a large enough sample of $t$'s are available experimentally. We will come back to this point later.

In the leptonic sector, the analogous mass matrix linking $(\nu, N)$ with $(\nu^c, N^c)$ is given by

$$M_{\nu N} = \begin{pmatrix} v_2 v_1^{-1} M_\ell & M'_3 \\ M'_4 & M_N \end{pmatrix},$$

(15)

where $M_\ell$ is the $3 \times 3$ charged-lepton mass matrix. However, very delicate fine tuning would then be required to obtain the necessary small neutrino masses. A more natural solution is to allow Majorana masses for $N$ and $N^c$ which can come from the vacuum expectation values of superfields transforming as $(1,1,1;2,-2)$ and $(1,1,1;-2,2)$ respectively. [Actually their presence serves a dual purpose. Without them and with only $\Phi_1$, $\Phi_3$, and $\Phi_4$, a linear combination of the two $U(1)$ factors would stay unbroken in addition to the electromagnetic $U(1)$.] The induced Majorana mass matrix for $\nu^c$ is then given by $M'_4 M^{-1}_{eff} M_4$, where $M_{eff}$ is determined by the heavy $(N, N^c)$ mass submatrix, and can have large mass eigenvalues, say of order $10^2$ GeV. The Majorana neutrino mass matrix $M_\nu$ gets two see-saw contributions: one from the above-mentioned $\nu^c$ masses and is given by $v_2^2 v_1^{-2} M_\ell M^{-1}_{\nu} M'_4$, the other coming from $M'_3$ and a different $M_{eff}$. Both effects are highly suppressed, hence the observed neutrinos have naturally small Majorana masses in this model. Additive lepton number is now broken, but a multiplicative lepton number is still conserved: $L$, $L^c$, $N$, and $N^c$ are odd and all other superfields are even. Hence the automatic conservation of R parity remains valid.

The spontaneous breaking of the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ gauge symmetry is accomplished in this model first by the $(1,1,1;2,-2)$ and $(1,1,1;-2,2)$ singlets which reduce
the two U(1) factors into one. Then \( \Phi_4 \) breaks \( SU(2)_L \times SU(2)_R \times U(1) \) down to the standard \( SU(2)_L \times U(1) \) which is in turn broken down to the electromagnetic U(1) by \( \Phi_{12} \) and \( \Phi_3 \). The superpotential consisting of \( \Phi_{12}, \Phi_3, \) and \( \Phi_4 \) is given by

\[
W = m \text{det} \Phi_{12} + f \tilde{\Phi}_3^\dagger \Phi_{12} \tilde{\Phi}_4,
\]

where

\[
\tilde{\Phi}_3 = i \sigma_2 \Phi_3^* = \left( \begin{array}{c} \tilde{\phi}_3^0 \\ -\tilde{\phi}_3^0 \end{array} \right),
\]

and we have redefined \( \Phi_4 \) as \( \tilde{\Phi}_4 \) so that both \( \Phi_3 \) and \( \Phi_4 \) now have scalar components denoted by \( (\phi_3^+, \phi_3^0) \) and \( (\phi_4^+, \phi_4^0) \) respectively in accordance with the notation \( \Phi_1 = (\phi_1^+, \phi_1^0) \) and \( \Phi_2 = (\phi_2^+, \phi_2^0) \) for the scalar components of \( \Phi_{12} \). The soft terms of the Higgs potential, including those which break the supersymmetry, are given by

\[
V_{soft} = m^2 Tr(\Phi_{12}^\dagger \Phi_{12}) + mB(\text{det} \Phi_{12} + \text{det} \Phi_{12}^\dagger) + m_3^2 \Phi_3^\dagger \Phi_3 + m_4^2 \Phi_4^\dagger \Phi_4 + fA(\tilde{\Phi}_3^\dagger \Phi_{12} \tilde{\Phi}_4 + \tilde{\Phi}_4^\dagger \Phi_{12} \tilde{\Phi}_3).
\]

We look for a solution in which \( v_{1,2,3} \) are small compared to \( v_4 \). At the electroweak energy scale, the Higgs sector may reduce to three doublets, two doublets, or one doublet. The three-doublet case occurs only if \( f^2 = g^2 / 2 \) and both \( mB \) and \( fA v_4 \) are of order \((100 \text{ GeV})^2\). In the two-doublet case, consisting of \( \Phi_1 \) and a linear combination of \( \Phi_2 \) and \( \Phi_3 \), the minimal supersymmetric standard model (MSSM) is obtained in the limit \( f = 0 \). If \( f \neq 0 \), the two doublets will not have the couplings of the MSSM and a different mass spectrum will be found.[6] In the one-doublet case, we have of course only one physical Higgs boson as in the standard model. Specifically, it is given here by

\[
h = \sqrt{2}(v_1 \text{Re}\phi_3^0 + v_2 \text{Re}\phi_4^0 + v_3 \text{Re}\phi_3^0) / \sqrt{v_1^2 + v_2^2 + v_3^2}.
\]

Whereas \( h \) couples to \( dd^c \) according to \( \mathcal{M}_d \) as in the standard model, it couples to \( uu^c \) according to \( v_2 v_1^{-1} \mathcal{M}_d \) and \( uu^c \) according to \( \mathcal{M}_3 \), hence there are FCNC effects in the \( u \)
sector due to the exchange of $h$. [Recall there can be large mixing between $u^c$ and $x^c$ in this model.]

Consider now the decay $t \to c + h$. The coupling is equal to $(\xi g/2)(m_t/M_W)$, where $\xi$ is a suppression factor due to mixing. Hence

$$\frac{\Gamma(t \to c + h)}{\Gamma(t \to b + W)} = \xi^2 \left(1 - \frac{m_b^2}{m_t^2}\right)^2 \frac{M_W^2}{m_t^2} \left(1 + \frac{2M_W^2}{m_t^2}\right)^{-1}. \quad (20)$$

Since the value of $\xi$ is unconstrained by present experimental data, the above ratio may be substantial. Once produced, the Higgs boson $h$ will decay into $b\bar{b}$. The background to $t \to c + h$ is thus mainly $t \to b + W$, where the $W$ decays into $c\bar{b}$. However, the latter is suppressed by $|V_{cb}|^2 \sim 2 \times 10^{-3}$ and the $b\bar{b}$ invariant mass will not peak at $m_h$. In a hadron collider such as the Tevatron at Fermilab, a $t\bar{t}$ pair can be produced if kinematically allowed, then if $t \to b + W$, where the $W$ decays into an electron (or a muon) and its antineutrino, the decay $t \to c + h$, where $h \to b\bar{b}$, may have a chance of being observed through the use of vertex detectors.

Let us return now to the exotic fermions contained in the $(16,\pm 1)$ of $SO(10) \times U(1)$. They acquire masses through $\Phi_{12}$ and must therefore not be very heavy. They also interact with the singlet quarks $x$ and $x^c$ contained in the $(10,\pm 1)$ representations. Under $SO(10) \times U(1)$, there are in fact only four types of Yukawa terms: $(16,0)(16,0)(10,0)$, $(16,1)(16,-1)(10,0)$, and $(16,0)(16,\pm 1)(10,\mp 1)$. Assuming that at the $SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ level, the $(10,0)$ contains only $\Phi_{12}$ and the $(10,\pm 1)$ contain only $x$, $x^c$, and

$$y \sim (3, 1, 1; -1/3, -1), \quad y^c \sim (\overline{3}, 1, 1; 1/3, 1), \quad (21)$$

then every B and L assignment is uniquely determined. The complete list is given in Table 1. Note that because of the structure of this model, the new particles have unusual baryon and lepton numbers. Note also that the $(10,\pm 1)$ fermions have $R = +1$, whereas the $(16, \pm 1)$ fermions have $R = -1$. The decay products of the latter must then always include the
LSP, which we will choose for convenience to be the photino \( \tilde{\gamma} \).

Consider now the SU(3)-triplet fermion which has electric charge \( 5/3 \). It has a Yukawa coupling to \( L x^e \). Hence it will decay into \( e^+ u^c \) through \( u^c - x^e \) mixing and the exchange of the heavy squark \( \tilde{u}^c \). The absence of such a signal above background at the Tevatron so far suggests that it has a mass greater than 100 GeV. On the other hand, the magnitude of its Yukawa coupling is related to \( \mathcal{M}_3 \) of Eq. (12) in the SO(10) \( \times \) U(1) limit and is likely to be rather small for the physical \( u \) and \( c \) quarks. Therefore, we expect a decay rate orders of magnitude smaller than that of ordinary heavy quarks such as the \( t \) which can decay into a physical W boson + another quark. Consequently, a bound state of these exotic SU(3)-triplet fermions may exist up to a much higher mass than the usual quarkonia.[7] The scalar ground state will decay dominantly into two gluons, whereas the branching fraction into two photons is given by \( (5/3)^4(3/8)(\alpha/\alpha_s)^2 \) which is about 2%.

Another important consequence of the \((16, \pm 1)\) fermions is their contribution to the effective two-gluon and two-photon couplings of the Higgs boson \( h \). These couplings are absent at tree level but are nonzero to one loop where all particles which couple to \( h \) will contribute, depending of course on whether they also couple to gluons or to photons. In this model, \( \Gamma(h \rightarrow gg) \) is enhanced over that of the standard model by a factor of roughly \( (1 + 4)^2 = 25 \) because there are now four more heavy quarks. Assume for illustration that \( m_h = 90 \) GeV, then the cross section for \( p\bar{p} \rightarrow h + \text{anything} \) at a center-of-mass energy of 2 TeV is about 35 pb.[8] Similarly, \( \Gamma(h \rightarrow \gamma\gamma) \) is enhanced by a factor of roughly 15, and \( B(h \rightarrow \gamma\gamma) \) is about \( 9 \times 10^{-3} \).[9] These two large enhancements make it much easier for \( h \) to be discovered at the Tevatron with a signal of about 0.3 pb above a background[8] of about 0.1 pb for this value of \( m_h \). Note that the heavy-quark contribution to the background is negligible when the invariant mass of the photon pair is much less than that of the quark pair.[10] Details of this and other phenomenological implications will be given elsewhere.
If further unification is desired beyond $\text{SO}(10) \times \text{U}(1)$, the natural choice is $\text{SO}(14)$. Consider the latter’s $\text{SO}(10) \times \text{SO}(4)$ decomposition. It is clear that the spinorial 64 representation of $\text{SO}(14)$ splits up into four 16 representations of $\text{SO}(10)$, and each is a component of the 4 representation of $\text{SO}(4)$. Since the U(1) decomposition of the latter has charges 1, 0, 0, −1, it is also clear that our $(16,0)$ and $(16,\pm 1)$ representations can be accommodated. Similarly, the $(10,\pm 1)$ and $(120,\pm 1)$ representations are accommodated in the product 64 $\times$ 64 of $\text{SO}(14)$.

In conclusion, we have shown how R parity can be automatically conserved in a realistic model of supersymmetry. It is based on the gauge group $\text{SU}(3) \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1) \times \text{U}(1)$ and is unifiable under $\text{SO}(14)$. A necessary condition is to make sure that the Higgs superfields needed for the spontaneous breaking of the gauge symmetry are in representations different from those of the leptons. We want to use only Higgs superfields which are doublets or singlets under the standard $\text{SU}(2)_L \times \text{U}(1)$ so that the tree-level equality $M_W = M_Z \cos \theta_W$ can be maintained, hence the choice of $\Phi_{12}$, $\Phi_3$, and $\Phi_4$. Motivated by the necessity of anomaly cancellation and the possibility of grand unification, we put $\Phi_3$ and $\Phi_4$ in the $(16,\pm 1)$ representations of $\text{SO}(10) \times \text{U}(1)$. The fermions contained therein must not be very heavy because they get their masses through $\Phi_{12}$. They also interact with the singlet superfields $x$ and $x^c$ which are introduced to mix with $u$ and $u^c$ to obtain a realistic $\mathcal{M}_u$ despite having only one $\Phi_{12}$. Consequently, every B and L assignment is uniquely determined in this model, as shown in Table 1. Two particularly interesting phenomenological implications are the possibility of heavy bound states of exotic color-triplet fermions with significant branching fractions into two photons and that of greatly enhanced two-gluon and two-photon couplings of the Higgs boson $h$. 
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References


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<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$L_-$</td>
<td>$(1, 2, 1; -1/2, -1)$</td>
<td>$(-1, -2)$</td>
<td>-1</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$L^c_-$</td>
<td>$(1, 1, 2; 1/2, 1)$</td>
<td>$(2, 1)$</td>
<td>1</td>
<td>-1</td>
<td>-</td>
</tr>
<tr>
<td>$Q_+^c$</td>
<td>$(3, 2, 1; 1/6, 1)$</td>
<td>$(5/3, 2/3)$</td>
<td>1/3</td>
<td>-1</td>
<td>-</td>
</tr>
<tr>
<td>$Q_-^c$</td>
<td>$(3, 1, 2; -1/6, -1)$</td>
<td>$(-2/3, -5/3)$</td>
<td>-1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$Q_-$</td>
<td>$(3, 2, 1; 1/6, -1)$</td>
<td>$(-1/3, -4/3)$</td>
<td>-2/3</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$Q^c_+$</td>
<td>$(3, 1, 2; -1/6, 1)$</td>
<td>$(4/3, 1/3)$</td>
<td>2/3</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Particle content of this model under $SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ and the associated electric charge, baryon number, lepton number, and $R$ parity of the fermions.