DD

✓ CERN - ECP 93-6

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH fix 9401

CERN/ECP 93-6
(CERN/LAA-SF 92-24)
7 July 1993

9

CM-P00060665

MONTE CARLO SIMULATION AND PERFORMANCE EVALUATION FOR THE SCINTILLATING FIBRES TRACKING DETECTOR^{*)}

C. D'Ambrosio and S. Qian^{**)}

CERN, Geneva, Switzerland

Abstract

A Monte Carlo simulation has been performed to study the basic features of the Scintillating Fibres Tracking Detector (SFTD) to be used as the central tracker in a supercollider experiment. Charged particles with momenta up to 10 TeV/c have been tracked inside the detector at different polar angles in a 3.3 T solenoidal magnetic field, and then reconstructed using the Kalman filtering method. The detector performances obtained from this simulation agree fairly well with previous estimates based on analytic calculations at higher momenta p , but yield better results at low p where multiple scattering dominates, thus confirming that the SFTD is a very promising detector for high-precision measurements.

^{*)} The work reported here is part of the LAA Project.

^{**)} From Worldlab, Lausanne, Switzerland.

1 INTRODUCTION

This work is a supplement to a previous one [1], where the Scintillating Fibres Tracking Detector (SFTD) was proposed for experiments at future multi-TeV hadron colliders. In Ref. [1] momentum and vertex resolutions were calculated using analytic formulae. To check these results, a Monte Carlo simulation was performed which enables us to evaluate the SFTD resolutions more realistically, in particular for the non-negligible contribution from multiple scattering.

We have simulated the SFTD by using the GEANT Monte Carlo program [2]. The hits measured are simulated by digitizing the generated tracks which pass through the SFTD, and then by smearing these digitized points according to the SFTD spatial resolution. To reconstruct the tracks from the smeared hits we have used the Kalman filtering (KF) algorithm [3], [4]. By comparing the reconstructed tracks with the generated ones, some detector performances can be evaluated.

The SFTD has some distinguishing features [1] when compared with semiconducting (e.g. silicon or GaAs) and microstrip gas chambers: more hits per unit of radiation length, lower occupancy, less multiple scattering per hit, smaller cell-busy time, and better 2-track resolution. Furthermore, the number of readout channels is much smaller, with almost no heat production. However, some of the features (i.e. many hits per unit of radiation length), and the special geometrical set-up of the SFTD are a challenge for the KF method for track reconstruction. In LEP and HERA experiments, the KF method has proved able to cope properly with some typical problems (e.g. multiple scattering and non-uniform magnetic fields) in track reconstruction, as well as to identify and remove wrongly-assigned points. Our study demonstrates that the KF algorithm can also deal with the special geometrical configuration of the proposed SFTD, where multiple scattering and ionization energy loss of charged particles dominate the errors at momenta below 10 GeV/c.

In this paper, Sect. 2 describes the Monte Carlo simulation for the SFTD, including the hit digitization and smearing procedure. Section 3 briefly explains the KF method and the difficulties encountered in track reconstruction. Section 4 illustrates the results, and the conclusions are drawn in Sect. 5.

2 MONTE CARLO SIMULATION FOR THE SFTD

The SFTD configuration has been simulated using the Full Monte Carlo Chain (FMCC) [5] software package which provides a standard procedure to fully simulate physics events from pp interactions, and to track them through a detector. It makes use of the GEANT [2] program interfaced with two independent systems [6] (i.e. MEGA: Monte Carlo Event Generator Adaptor, and SUD: Set-Up Descriptor) to describe the event kinematics and detector parameters.

The simulated SFTD (Fig. 1) has three cylindrical shells composed of 1.6×10^8 scintillating fibres of 60 μm diameter, whose axes coincide with the collider beam line (i.e. the z axis). The three shells lie inside a uniform solenoidal magnetic field (whose z axis is also along the beam line) of 3.3 T. The space between the shells is filled with air. The colliding beams are running in a thin vacuum pipe.

Each fibre shell has four layers: two stereo (U and V) layers are sandwiched between two axial ones. The stereo angle is 4° in this study. This substructure (Fig. 1) in each shell is

simulated at the digitization level (i.e. no specified GEANT volume is assigned to the substructure); when digitizing the track, the hits in the axial layers are only smeared in the r - ϕ direction, whereas the hits in the stereo layers are smeared both in the r - ϕ and the z direction. The smearing is Gaussian with standard deviations $\sigma_{r-\phi} = 30 \mu\text{m}$ and $\sigma_z = \sigma_{r-\phi}/\sin \alpha = 430 \mu\text{m}$, where α is the stereo angle.

One of the major differences between the SFTD and the semiconductor trackers is that in the SFTD we obtain several signal hits per layer, which are randomly distributed along the particle path, and the number of hits follows a Poisson distribution, i.e. the probability to have n hits within the path length t is $P_n = (ht)^n/n! e^{-ht}$, with a mean value of $h = 3$ hits/mm thickness of scintillating fibres. In the proposed SFTD, the total thickness of the scintillating fibres is 40 mm for the three shells altogether, so the total number of hits along a track is ~ 120 , varying from 70 to 170 (depending on the distance from the photocathode readout). Therefore, for identical tracks, not only the total number of hits is different from one track to another, but also the radial positions of hits are hardly the same from track to track since any of the thin ($60 \mu\text{m}$) fibres can be stimulated to produce a signal hit. In contrast, in microstrip gas chambers or in silicon detectors, the number of hits is usually one per layer, and thus simply depends on the number of layers; the radial positions of the hits are also fixed by the layer radius (see Ref. [1] for more details about the SFTD arrangement).

When generated tracks pass through the detector, the multiple scattering and the ionization energy loss are simulated by GEANT. Namely, the generated trajectories are distorted wherever there exists a scatter, even when passing through a region of air. The digitized and smeared hits along the distorted trajectories are the input of the next step, i.e. track reconstruction.

A typical minimum bias pp event at 16 TeV centre-of-mass energy is shown inside the SFTD in Fig. 2. In our present study of the SFTD resolutions, single-track events are adequate. In a future stage, multi-track events will be generated, and a more precise digitization procedure will be investigated for pattern-recognition purposes.

3 TRACK RECONSTRUCTION

Normally, track reconstruction from digitized hits includes two major tasks: track finding and track fitting. Among various algorithms, the KF method offers an attractive feature to accomplish the two tasks simultaneously [3], [4]. It is also able to treat multiple scattering and non-uniform magnetic fields properly. The KF algorithm has already been adopted by several high-energy physics experiments (for example DELPHI [3] and ALEPH at LEP and ZEUS [4] at HERA) to reconstruct tracks in gaseous wire chambers.

However, the special features and the geometrical set-up of the SFTD introduce some difficulties with which the KF algorithm has not been confronted previously when applied to wire chambers. We have solved these difficulties and are confident that the KF method can also be applied (as proposed in Ref. [7]) for the SFTD in future hadron collider experiments.

Since the basic procedure of the KF method has been described in detail in Refs. [3], [4], we give here only a brief summary:

a) *Starting tracks*

A track segment is started by fitting three points (or by following a fitted track from a neighbouring detector component). The initial track parameters $\mathbf{p} = (\Phi, z, \theta, \phi, 1/R_{tr})$ at the fixed radial position r (Fig. 3), as well as their associated weight matrix W (which is the inverse of the covariance error matrix COV), are evaluated.

b) *Extrapolating tracks*

The parameters $\mathbf{p}$ and their weight matrix W (or COV) of the track segment are propagated from one r - ϕ surface to the next one, and the extrapolated parameters $\mathbf{p}^e$ and matrix W^e are obtained. Multiple scattering (see the Appendix for details) and energy loss by ionization between measured points can be taken into account at this stage.

c) *Adding points to the tracks*

The information of a measured point $\mathbf{m} = (\Phi_m, z_m, \text{any}, \text{any}, \text{any})$ with its weight matrix $U = \text{diag}(1/\sigma_\phi^2, 1/\sigma_z^2, 0, 0, 0)$ at the layer is added to the track by solving a linear system of equations $(W^e + U)(\mathbf{p}' - \mathbf{p}^e) = U(\mathbf{m} - \mathbf{p}^e)$, where Φ_m and z_m are the coordinates of a measured point; 'any' means arbitrary; σ_ϕ and σ_z are measurement errors; $\mathbf{p}'$ are the updated track parameters, and their weight matrix is $W' = W^e + U$. The quality of updating can be monitored by examining the χ^2 increment and can help to decide whether a point should be added to the track.

Steps (b) and (c) are repeated until the inner-most measured point. The difficulties encountered during this exercise are:

3.1 Track starting

Since each fibre shell is thin compared with the total track length [Fig. 1(a)], the momentum of an initial track segment fitted by three points within one shell is usually poorly determined. If the track segment is then extrapolated from the outermost to the second shell through 30 cm of space, it is possible that the track is bent so much that it can never intercept the second shell (thus terminating the reconstruction process), or that the intercepting position is too far away from the point measured in the second shell (thus giving a large χ^2 increment, and deteriorating the track fitting quality). To cope with this problem we have constructed the initial track from three special points, namely one in each of the three shells. Since the three shells span 60 cm radius, the track length obtained in this way provides a much more accurate initial momentum estimate. The parameters $\mathbf{p}$ and matrix W of the initial track are derived at the location of the innermost one of these three points, and are propagated to the outermost position of the track by a trivial helix extrapolation. Then the normal KF procedures (b) and (c) start.

3.2 The ionization energy loss

The three fibre shells of total 40 mm radial thickness cause at least 8 MeV energy loss which corresponds to ~ 0.1 – 1% of the energy for low p tracks and is comparable (i.e. in the same order of magnitude) with the SFTD momentum resolution. Therefore, in this study, the track momentum (which is related to one of the five track parameters, i.e. $1/R_{tr}$, since R_{tr} is proportional to p_T) has been adjusted in every tracking step according to the energy loss, so that the overall track fitting quality is secured.

3.3 The multiple scattering within the short path inside the fibre shells and over the long path in the air between the shells

A total fibre thickness of 40 mm is equivalent to 9.4% of radiation length which causes significant multiple scattering angles for low p_T tracks. Also, the distances are 25–30 cm between shells, 65 cm between the innermost shell and the beam pipe; even though these distances are only equivalent to ~ 0.1 – 0.2% of radiation length in air, the multiple scattering over the long path can produce a lateral displacement of the order of $(0.01$ – $0.02)/p_T$ cm (i.e. 40–80 μm for 5 GeV tracks with $\theta = 30^\circ$, where p_T is in GeV/c) which is comparable with the detector r - ϕ resolution for low p_T tracks. Therefore, the multiple scattering terms have to be added into the covariance matrix (COV) for every point along the reconstructed trajectory. The detailed formulae for the multiple scattering contribution to the COV are given in the Appendix.

Since only single-track events are used in this study, 100% efficiency for track finding is assumed. As already mentioned, a more dedicated program for pattern recognition in multi-track events will be investigated in the future.

4 RESULT OF THE MONTE CARLO STUDY AND DETECTOR PERFORMANCES

The single-track events were generated at fixed momentum p and polar angle θ for each simulation run, 240 events per run. Since future hadron colliders can be designed with a very narrow beam size (in r - ϕ) at the interaction point (e.g. 21 μm for the LHC, 10 μm for the SSC, and 3 μm for Eloisatron), the beam constraint can considerably improve the performance of the detector. So we have examined both cases, namely with and without the constraint $(\sigma_{r-\phi})_{\text{beam}} = 21 \mu\text{m}$, for comparison.

The track fitting quality is monitored by the pull [i.e. $(\mathbf{p}_{\text{fitted}} - \mathbf{p}_{\text{generated}})/\sigma_p$, where σ_p is the square root of the corresponding diagonal element in COV] plots for all the parameters of the track, and by the χ^2 probability plot. The three plots on the left side of Fig. 4 show the pull plots for a typical run ($p = 100$ GeV/c and polar angle $\theta = 90^\circ$) with 21 μm beam constraint. Figure 5 shows the same plots, but without beam constraint. Figure 6 displays the χ^2 distribution, and the corresponding χ^2 probability distribution for this run. The mean values of about zero, and the r.m.s. values of about unity in the pull plots, as well as the flatness of the χ^2 probability distribution, indicate that the track fitting is reliable.

From the corresponding deviation (i.e. $\mathbf{p}_{\text{fitted}} - \mathbf{p}_{\text{generated}}$) plots in Figs. 4 and 5, various resolutions of the detector can be observed. Figure 7 shows the momentum resolution dp/p plotted against p at $\theta = 90^\circ$, 60° , and 30° . The points with error bars are obtained by this Monte Carlo simulation, and the curves using Eq. (1) as quoted below from Ref. [1].

The momentum resolution (dp/p) consists of two components, the transverse one (index t), relative to the track curvature in the $(r$ - ϕ) plane [Fig. 1(a)], and the longitudinal one (index ℓ) with polar angle θ , indicating the slope in the $(r$ - z) plane [Fig. 1(b)], where the tracks are straight lines for particle momenta ≥ 10 GeV/c at $B = 3.3$ T

$$\left(\frac{dp}{p}\right)^2 = \left\{ \left[\left(\frac{G_c}{N}\right)^{1/2} \times \left(\frac{\sigma}{L^2}\right) \times \left(\frac{p \sin \vartheta}{0.3B}\right) \right]_{\text{curv.}}^2 + \left[t^{1/2} \times \left(\frac{0.016 \sin \vartheta}{0.3BL}\right) \right]_{\text{mult.sc.}}^2 \right\}_t + \left\{ \left[\left(\frac{G_{sl}}{N}\right)^{1/2} \times \left(\frac{\sigma \cos \vartheta}{L \sin \alpha}\right) \right]_{\text{slope}}^2 + \left[\left(\frac{t}{\sin \vartheta}\right)^{1/2} \times \left(\frac{0.014}{p \tan \vartheta}\right) \right]_{\text{mult.sc.}}^2 \right\}_\ell. \quad (1)$$

In Eq. (1), σ (in m) means the spatial precision, which is related to the fibre diameter; B (in Tesla) the magnetic flux density; L (in m) the distance in the $(r-\phi)$ plane between the beam axis and the outermost measured point (i.e. the radius of the outer shell); t the material layer in units of radiation lengths; α the stereo angle between axial and oblique fibre layers; N the number of hits along the track; G_c a number, which varies between 256 for optimum hit spacing, and 720 for a continuous hit distribution along the track [8] [in this latter case, (G_c/N) should read $(720/N+4)$]; and G_{sl} a number which varies in the same way between 2 and 10. The z coordinate emerges from the stereo angle α [Fig. 1(a)].

It can be seen that for tracks at polar angle $\theta = 90^\circ$, Monte Carlo and analytic results agree fairly well; at smaller polar angles, the discrepancy becomes more pronounced, in particular at low momenta where the Monte Carlo results are more realistic. The difference may be reduced if a second-order consideration for the multiple scattering is taken into account in Eq. (1).

Finally, Figs. 8 and 9 show the D_0 and Z_0 resolutions plotted against p .

5 CONCLUSIONS

With a Monte Carlo simulation based on GEANT, and with the track reconstruction KF method, we have evaluated the basic performances of the SFTD. These are found to be in good agreement with previous analytic calculations, thus confirming the expected qualities of the SFTD as central trackers in a forthcoming super-collider experiment. In addition, this study shows that the KF method is able to treat a complex detector configuration like the SFTD. A more detailed full-event simulation and reconstruction should be foreseen in order to investigate further SFTD's capabilities in terms of double-track resolution, pattern recognition efficiency, and vertex finding efficiency, etc.

Acknowledgements

We wish to thank L. Cifarelli and H. Leutz for continuous interest, encouragement, and help with our work. We are also grateful to G. La Commare and M. Marino for their assistance in using the FMC package. The work was supported by the LAA Group at CERN, led by Prof. A. Zichichi, and we would like to express our sincere gratitude to him.

APPENDIX

MULTIPLE SCATTERING CONTRIBUTION TO THE ERROR MATRIX

The multiple scattering angle is calculated from the Molière formula

$$\Theta = \frac{0.0136}{p} \sqrt{L} [1 + 0.038 \ln(L)] , \quad (A1)$$

where L is the thickness of the scattering material in units of radiation length, and p is the particle momentum in GeV/c. With y being the small lateral displacement from the track path due to the multiple scattering, the 2×2 covariance matrix Q for y and Θ along the track path [9] is

$$Q(y, \Theta) = \begin{pmatrix} L^2/3 & L/2 \\ L/2 & 1 \end{pmatrix} L \frac{d\Theta^2}{dL} . \quad (A2)$$

Its impact on the covariance matrix COV of the five general track parameters [i.e. $\mathbf{p} = (\Phi, z, \theta, \phi, 1/R_{tr})$ at the fixed radial position r (Fig. 3)] is simply the projections. The elements of the symmetric matrix COV($\mathbf{p}$) are numbered as:

$$\begin{pmatrix} 1 \\ 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{pmatrix} .$$

If $d\Theta^2/dL$ is constant, then the relations $Q_{\Theta\Theta} = \Theta^2$, $Q_{y\Theta} = L/2 \Theta^2$ and $Q_{yy} = L^2/3 \Theta^2$ hold.

Denoting $\beta \equiv \phi - \Phi$, the deviations of COV($\mathbf{p}$) are

$$\Delta\text{COV}(1) = \Delta\text{COV}_{\Phi\Phi} = Q_{yy} \frac{1}{r^2 \cos^2 \beta} . \quad (A3)$$

$$\Delta\text{COV}(2) = \Delta\text{COV}_{\Phi z} = Q_{yy} \frac{\text{ctg } \theta \sin \beta}{r \cos^2 \beta} . \quad (A4)$$

$$\Delta\text{COV}(3) = \Delta\text{COV}_{zz} = Q_{yy} (1 + \cos^2 \theta \text{tg}^2 \beta) / \sin^2 \theta . \quad (A5)$$

$$\Delta\text{COV}(4) = \Delta\text{COV}_{\Phi\theta} = 0 . \quad (A6)$$

$$\Delta\text{COV}(5) = \Delta\text{COV}_{z\theta} = Q_{y\Theta} / \sin \theta . \quad (A7)$$

$$\Delta\text{COV}(6) = \Delta\text{COV}_{\theta\theta} = Q_{\Theta\Theta} . \quad (A8)$$

$$\Delta\text{COV}(7) = \Delta\text{COV}_{\phi\phi} = (Q_{y\Theta} \frac{1}{\sin \theta} + Q_{yy} \frac{1}{R_{tr}} \text{tg } \beta) \frac{1}{r \cos \beta} . \quad (A9)$$

$$\Delta\text{COV}(8) = \Delta\text{COV}_{z\phi} = (Q_{y\theta} \frac{\text{tg } \beta}{\sin \theta} + Q_{yy} \frac{1}{R_{\text{tr}}} \frac{1}{\cos^2 \beta}) \text{ctg } \theta. \quad (\text{A10})$$

$$\Delta\text{COV}(9) = \Delta\text{COV}_{\theta\phi} = Q_{y\theta} \cos \theta \frac{1}{R_{\text{tr}}}. \quad (\text{A11})$$

$$\Delta\text{COV}(10) = \Delta\text{COV}_{\phi\phi} = \frac{Q_{\theta\theta}}{\sin^2 \theta} + Q_{yy} \frac{1}{R_{\text{tr}}} (\cos^2 \theta + \text{tg}^2 \beta) + Q_{y\theta} \frac{2}{R_{\text{tr}}} \frac{\text{tg } \beta}{\sin \theta}. \quad (\text{A12})$$

$$\Delta\text{COV}(11) = \Delta\text{COV}_{\phi R} = 0. \quad (\text{A13})$$

$$\Delta\text{COV}(12) = \Delta\text{COV}_{zR} = \text{ctg } \theta \frac{1}{R_{\text{tr}}} \Delta\text{COV}_{z\theta}. \quad (\text{A14})$$

$$\Delta\text{COV}(13) = \Delta\text{COV}_{\theta R} = \text{ctg } \theta \frac{1}{R_{\text{tr}}} Q_{\theta\theta}. \quad (\text{A15})$$

$$\Delta\text{COV}(14) = \Delta\text{COV}_{\phi R} = \text{ctg } \theta \frac{1}{R_{\text{tr}}} \Delta\text{COV}_{\phi\theta}. \quad (\text{A16})$$

$$\Delta\text{COV}(15) = \Delta\text{COV}_{RR} = (\text{ctg } \theta \frac{1}{R_{\text{tr}}})^2 Q_{\theta\theta}. \quad (\text{A17})$$

References

- [1] C. D'Ambrosio et al., Nucl. Instrum. Methods **A322** (1992) 20.
- [2] R. Brun et al., CERN/DD/EE 84-1 (1987).
- [3] P. Billoir, Comput. Phys. Commun. **57** (1989) 390.
- [4] P. Billoir and S. Qian, Nucl. Instrum. Methods **A294** (1990) 219;
P. Billoir and S. Qian, Nucl. Instrum. Methods **A295** (1990) 492.
- [5] J. Alberty et al., CERN/LAA-MSL 90-06 (1990).
- [6] F. Anselmo et al., MEGA: Monte Carlo event generator adaptor, Particle World, Vol. 2, No. 4 (1991) 108;
A. Bassi et al., Nuovo Cimento **105A** (1992) 1207.
- [7] P. Billoir and S. Qian, Proc. of the Symp. on Detector R&D for the SSC, Fort Worth, Texas, USA, Oct. 1990, World Scientific (1991) p. 618.
- [8] R.L. Gluckstern, Nucl. Instrum. Methods **24** (1963) 381.
- [9] R.K. Bock et al., Data Analysis Techniques for HEP Experiments, Cambridge Univ. Press (1990) p. 305.

30 to 60 points per track
 r, ϕ, z - coordinates are measured
 ~160 millions of fibres are readout by ~400 tubes
 which also provide ~1 μs delay for the first level trigger

Fig. 1 An arrangement of the SFTD: (a) r - ϕ plane seen at $z = 0$, with the magnified parts showing the substructure of axial ϕ - and stereo U- (V-) layers; (b) r - z plane showing the electromagnetically-focused delay tubes for the readout.

(a)

(b)

Fig. 2 A minimum bias event from pp collisions at $\sqrt{s} = 16$ TeV traced in the SFTD: (a) r-f plane; (b) r-z plane.

Fig. 3 A parametrization of the 2-D circular track at the reference point P

Fig. 4 Pull plots and deviation plots for the track parameters D_0 (i.e. the closest approach in the r - ϕ plane between the track and the beam), Z_0 (i.e. the z coordinate of the track at D_0), and $1/p$, when $p = 100$ GeV/c, $\theta = 90^\circ$ and $(\sigma_{r-\phi})_{\text{beam}} = 21 \mu\text{m}$ (i.e. with beam constraint).

Fig. 5 Same as Fig. 4, but without beam constraint.

Fig. 6 The χ^2 distribution and the χ^2 probability distribution corresponding to Fig. 4

Fig. 7 Momentum resolution dp/p plotted against p for tracks produced at different q angles. The points are results from Monte Carlo simulation, the curves are obtained from Eq. (1). Open points and solid curves are results with beam constraint; solid points and dashed curves are those without beam constraint.

Fig. 8 The D_0 resolution plotted against p for different θ

Fig. 9 The Z_0 resolution plotted against p for different θ