Precision Electroweak Data, Supersymmetry and GUTs

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Abstract

Following an update of indirect constraints on m_t , M_H and $\alpha_s(M_Z)/\overline{MS}$ from precision electroweak measurements in the Standard Model, we analyze the data within the context of the Minimal Supersymmetric extension of the Standard Model (MSSM), including the constraint from supersymmetric grand unification. Within the Standard Model, we find somewhat larger values of m_t and M_H than previously, and a somewhat lower value of $\alpha_s(M_Z)/\overline{MS}$. The favoured value of m_t can be decreased (increased) by $\mathcal{O}(10 \text{ to } 20)$ GeV if the scalar particle masses (lightest chargino mass) are close to the direct experimental limits. We find for the first time a significant lower limit on supersymmetric Higgs masses. The supersymmetric GUT constraint gives a coupled upper limit on the scalar and gaugino masses that depends sensitively on the other MSSM parameters.

The Standard Model is being subjected to a barrage of increasingly precise tests using electroweak data from LEP and elsewhere. Thus far, the Standard Model is passing all these tests, and its unknown parameters, the top quark mass m_t and the Higgs boson mass M_H , are being predicted with increasing precision [1][2][3]. The hope is, of course, that the precision electroweak data will eventually reveal some discrepancy with the Standard Model that will show us how to go beyond it. In our view, one of the best-motivated scenarios for physics beyond the Standard Model is provided by supersymmetry, invoked on phenomenological grounds to alleviate the hierarchy and naturalness problems of Grand Unified Theories (GUTs).

LEP and other precision data have provided several circumstantial indications in favour of supersymmetry. One is the well-publicized striking agreement [4] of the LEP determination of $\sin^2\theta_w(M_Z)/\overline{MS}$ with the prediction of minimal supersymmetric GUTs with sparticle masses $\lesssim 1$ TeV, as suggested by the hierarchy and naturalness problems. Pre-LEP determinations of $\sin^2\theta_w(M_Z)/\overline{MS}$ already disfavoured [5] non-supersymmetric GUTs, but this discrepancy has been greatly sharpened by LEP. A second feature of the LEP data is that they favour a relatively low value of M_H , consistent with the prediction of the minimal supersymmetric extension of the Standard Model (MSSM) that $M_H \lesssim 130$ GeV [6]. In consistency with this indication, precision electroweak data disfavour alternative responses to the naturalness and hierarchy problems that invoke strongly-interacting composite Higgs bosons. One might also add that the precision electroweak data indicate a range for m_t that is consistent with ideas for dynamical symmetry breaking within the MSSM, and a superstring origin for the top quark Yukawa coupling.

In view of the strong motivations for supersymmetry that we perceive, it makes sense to analyze the precision electroweak data including radiative corrections calculated within the MSSM [7], as a supplement to less-focused model-independent analyses [8]. One can hope to refine predictions of m_t within this MSSM framework, and eventually

to constrain the parameters of the MSSM. In this paper, we report on an update on this programme [7], including various new theoretical effects, using the latest electroweak data made available in summer 1993 [9], and incorporating for the first time GUT constraints [10] directly in a fit to the precision electroweak data.

We first provide a brief update on fitting the precision electroweak data within the Standard Model, stressing that previous nagging discrepancies evidenced in [2] have now evaporated, presenting for the first time an analysis in the three-dimensional (α_s, m_t, M_H) parameter space, and emphasizing the prospect that LEP measurements of the Z decay rate and total cross-section may soon provide the most precise determination of $\alpha_s(M_Z)$. Next we recall the parametrization of the MSSM that we use [7], list the theoretical differences in this work from our own previous analyses, and introduce our parametrization of the supersymmetric GUT constraint [10]. Then we discuss m_t within the MSSM, including its sensitivity to the pseudoscalar supersymmetric Higgs mass m_A , the universal scalar mass m_0 , and the supersymmetric GUT constraint. This is followed by a discussion of precision electroweak constraints in the $(m_A, \tan \beta)$ plane, where we find for the first time a significant lower limit on m_A . We close with a discussion of the (m_0, m_β) plane, where precision electroweak data strengthen bounds from direct searches, whilst the supersymmetric GUT constraint can become significant for some specific choices of $\tan \beta$ and the Higgs superfield mixing parameter μ .

We preface our analysis of precision electroweak data within the MSSM by first updating our global fit within the Standard Model. The LEP data set we use is given in ref. [9], we use the SLD measurements of A_{LR} presented in ref. [11], and the high-energy data set is completed by the measurements of M_W in ref. [12]. In addition, we use the same low-energy data set as that compiled in ref. [2]. It should be emphasized that we fit these data directly in a model-independent parametrization in terms of the four squared chiral couplings, rather than treat them approximatively by the resulting effective value of some parameter as “ M_W ” or “ $\sin^2 \theta_w$ ”. The significance of the low-energy data is less than in previous global fits, because of the increase in precision of the high-energy (principally LEP) data, and their effect on the Standard Model parameters (m_t, M_H) is now marginal. However, they can play a useful rôle in constraining some

extensions of the Standard Model, particularly those with a threshold close to M_Z , as we discuss again later.

Previously, global fits to the precision electroweak data did not give constraints on $\alpha_s(M_Z)|_{\overline{MS}}$ that were as tight as other determinations, e.g. from $e^+e^- \rightarrow \text{jets}$ [13], and the latter were used as an external constraint on the fits. However, the precision of the LEP cross-section data in particular has now improved to a level where they give a determination of $\alpha_s(M_Z)|_{\overline{MS}}$ that is as precise as other measurements, and one has better control over theoretical and systematic errors. Therefore, we present for the first time the results of a Standard Model fit in a three-dimensional $(\alpha_s(M_Z)|_{\overline{MS}}, m_t, M_H)$ parameter space. The $\Delta\chi^2 = 1$ “potato” is shown in Fig. 1, together with its projections on the (m_t, M_H) , $(m_t, \alpha_s(M_Z)|_{\overline{MS}})$ and $(M_H, \alpha_s(M_Z)|_{\overline{MS}})$ planes. Fig. 1 exhibits a strong positive correlation between m_t and M_H , almost no correlation between m_t and $\alpha_s(M_Z)|_{\overline{MS}}$, and a correlation between M_H and $\alpha_s(M_Z)|_{\overline{MS}}$ that is stronger for larger M_H . We see that the direct search limits $m_t > 113$ GeV, $M_H > 63$ GeV are comfortably consistent with the global fit ($\Delta\chi^2 = 0.15$). This removes one nagging discrepancy quoted in ref. [2], where the fit was much less compatible with the direct search limits. The improvement is mainly due to an increase in the central value of M_H : we now find in the unconstrained fit

$$m_t = 140^{+21}_{-22} \text{ GeV}, \quad M_H = 35^{+205}_{-26} \text{ GeV}, \quad \alpha_s(M_Z)|_{\overline{MS}} = 0.116^{+0.007}_{-0.006}. \quad (1)$$

We note in passing that another previous nagging discrepancy between the different subsets of Z decay data associated with widths and asymmetries has also disappeared: they are now compatible at the 0.7σ level, compared with the 1.4σ level in our previous global fit. The range of M_H in eq. (1) is highly compatible with the range about M_Z predicted for the lighter scalar Higgs boson in the MSSM [6]. If we fix $M_H = M_Z$, we find $m_t = 146^{+16}_{-18}$ GeV. The precision electroweak data then determine

$$\alpha_s(M_Z)|_{\overline{MS}} = (0.117 \pm 0.006) \left| \begin{array}{l} M_H = M_Z \\ m_t = 146 \text{ GeV} \end{array} \right. \pm 0.001(m_t) \pm 0.002(M_H). \quad (2)$$

We note that the LEP data taken during 1993 are expected to lead to values of M_Z and Γ_Z that are more than twice as precise as those used here, as well as improvements in other LEP measurement errors. This should enable the error on $\alpha_s(M_Z)|_{\overline{MS}}$ in eq. (2) to be significantly improved. Therefore we expect that the determination of $\alpha_s(M_Z)|_{\overline{MS}}$ from precision electroweak measurements will soon be more precise than from, e.g., $e^+e^- \rightarrow$ jets. We note that, in contrast to all previous analyses, the jet value of $\alpha_s(M_Z)|_{\overline{MS}}$ is now *higher* than that from the total e^+e^- cross-section, i.e. precision electroweak measurements, leaving little room for a light gluino [14].

We are now ready to address the MSSM, for which we use the same theoretical description as in ref. [7]. The Lagrangian for the MSSM [15] contains a supersymmetric part that is determined by a superpotential W that includes a Higgs superfield mixing term $\mu H_1 H_2$, and soft supersymmetry breaking terms. The most essential of the latter are gaugino masses M_a , which we assume to be renormalized from a universal value $m_{\frac{1}{2}}$ at the GUT scale:

$$M_a = \frac{\alpha_a}{\alpha_{GUT}} m_{\frac{1}{2}} \quad (a = 1, 2, 3), \quad (3)$$

where we denote $M_3 = m_{\tilde{g}}$, and spin-0 masses $(m_0)_i$ that are also renormalized

$$m_{\tilde{f}_{L,R}}^2 = m_0^2 + c_{\tilde{f}_{L,R}} m_{\tilde{g}}^2 \pm M_Z^2 \cos 2\beta \left[T_{\tilde{f}_{L,R}}^3 - Q_{\tilde{f}_{L,R}} \sin^2 \theta_w \right], \quad (4)$$

where the renormalization coefficients $c_{\tilde{f}_{L,R}}$ are given in ref. [7]. We also include $\tilde{t}_{L,R}$ mixing as in ref. [7]. The supersymmetric Higgs sector is parametrized in the usual way by the neutral pseudoscalar Higgs mass m_A and the ratio $\tan \beta$ of the Higgs v.e.v.'s at the tree level, and we include the leading one-loop radiative correction [6]

$$\delta m^2 = \frac{3g^2 m_t^4}{8\pi^2 M_W^2 \sin^2 \beta} \ln \left(\frac{m_A^2}{m_t^2} \right), \quad (5)$$

that depends quartically on m_t . The MSSM is therefore parametrized well enough for

our purposes by $(\mu, m_{\tilde{g}}, m_0, m_A, \tan \beta)$ and m_t .

The main technical advance in our MSSM fit relative to ref. [7] is that we now include the m_t -dependent corrections to the $Z \rightarrow b\bar{b}$ vertex, due to loops involving $(t, H^\pm)$ and $(\tilde{t}, \tilde{W}^\pm/\tilde{H}^\pm)$ particles [16]. The most important MSSM radiative corrections that we do not include are one-loop vertex and box diagrams. Since these involve both stermions and charginos or neutralinos, decoupling theorems tell us that they are negligible for large m_0 or large $(\mu, m_{\tilde{g}})$. We therefore expect our approximate treatment to be accurate except possibly when both m_0 and $(\mu, m_{\tilde{g}})$ are small, a domain which is in any case largely excluded by direct searches.

If we compare our approach to those based on a model-independent parametrization, we note that the one-loop MSSM diagrams that we include are the same as those used, for example, in ref. [17], where the model-independent approach is adapted to analyze the MSSM. However, we analyze the data directly in the MSSM, whereas in the model-independent approach constraints on various lumped parameters ϵ_i are extracted from the data, and these are then compared with calculations of the ϵ_i in any model of interest. In the first version [18] of this model-independent approach, attention was focussed on a subset (Γ', A'_{FB}, M_W) of high-energy observables, which has now been extended to include hadronic observables at the Z^0 peak. Initially, three parameters $\epsilon_{1,2,3}$ characterizing oblique radiative corrections were introduced, but now a fourth parameter ϵ_b has been added to parametrize radiative corrections to the $Z \rightarrow b\bar{b}$ vertex, such as those in the MSSM. One difference between our approach and that of ref. [17] is in the use of low-energy data, which we treat exactly at the one-loop level, e.g. analyzing ν - q scattering in terms of the four chiral couplings $u_{L,R}, d_{L,R}$. Another difference is that we use the resummation procedure developed in the context of the Standard Model to treat the leading $(\ln m_t^2$ and $\ln(M_Z^2/m_t^2))$ higher-order radiative corrections [19]. As was emphasized in ref. [20], there is an ambiguity in the general formulation of the model-independent analysis, associated with the truncation of the perturbation series to the one-loop level. In general, one must make an Ansatz for the higher-order terms, e.g., equivalent to evaluating them all in the Standard Model with some fixed reference values of m_t and M_H . In the particular case of the MSSM, one knows that the leading

(in m_i^2 and $\ln(M_Z^2/m_i^2)$) higher-order radiative corrections are identical with those in the Standard Model, so it is correct to resum them in the same way. We also note that ref. [17] does not use all the GUT mass relations (see eqs. (3,4)) for sparticles, which have however recently been included in ref. [21].

Another new element in this MSSM analysis is the incorporation of the supersymmetric GUT constraint on the gauge couplings, which is often expressed as a prediction for $\sin^2\vartheta_w(M_Z)|_{\overline{MS}}$ in terms of the fine structure constant α_{em} and $\alpha_s(M_Z)|_{\overline{MS}}$. As discussed in ref. [10] the supersymmetric GUT prediction is

$$\sin^2\vartheta_w(M_Z)|_{\overline{MS}}^{\text{GUT}} = 0.2029 + \frac{7}{15} \frac{\alpha}{\alpha_s} + \delta_s(\text{light}) + \delta_s(\text{heavy}) \quad , \quad (6)$$

where the different light particles thresholds are included in $\delta_s(\text{light})$, and $\delta_s(\text{heavy})$ accounts for the thresholds for heavy particles close to the GUT mass. No useful information can be obtained if $\delta_s(\text{heavy})$ is large and unknown. In the case of the minimal supersymmetric SU(5) GUT, proton stability imposes constraints on the heavy particle spectrum that ensure $\delta_s(\text{heavy}) \geq 0$. We assume here that $\delta_s(\text{heavy})$ is negligible. The general form for the light threshold correction is

$$\delta_s(\text{light}) = \frac{\alpha}{20\pi} \sum_{\ell} C_s(R_{\ell}) \ln\left(\frac{t_{j\ell}}{M_Z}\right) \quad , \quad (7)$$

where the sum runs over light particles ℓ in representation R , with one-loop beta function coefficients

$$C_s(R) = \frac{10}{3} b_V(R) - 8 b_2(R) + \frac{14}{3} b_3(R) \quad (8)$$

and the threshold correction factors

$$t_j = 1, 1, e^{-1/21} \quad (9)$$

for particles of spin 0, $\frac{1}{2}$ and 1, respectively. In the limit $(|\mu|, m_1, m_0) \gg M_Z$ we find in the MSSM

$$\begin{aligned} \delta_s(\text{light}) \simeq \frac{\alpha}{20\pi} & \left[-3 \ln\left(\frac{m_t}{M_Z}\right) + \frac{28}{3} \ln\left(\frac{m_{\tilde{g}}}{M_Z}\right) - \frac{32}{3} \ln\left(\frac{\alpha_2 m_{\tilde{g}}}{\alpha_3 M_Z}\right) \right. \\ & - \ln\left(\frac{m_A}{M_Z}\right) - 4 \ln\left(\frac{|\mu|}{M_Z}\right) + \frac{5}{2} \ln\left(\frac{m_{\tilde{g}}}{M_Z}\right) \\ & \left. - 3 \ln\left(\frac{m_{\tilde{t}_R}}{M_Z}\right) + 2 \ln\left(\frac{m_{\tilde{t}_L}}{M_Z}\right) - \frac{3}{2} \ln\left(\frac{m_{\tilde{t}}}{M_Z}\right) \right] \quad , \end{aligned} \quad (10)$$

where we have approximated $m_{\tilde{g}, \tilde{t}_L, \tilde{t}_R}^2 \simeq m_0^2 + c_{\tilde{g}, \tilde{t}_L, \tilde{t}_R} m_{\tilde{g}}^2$, i.e., neglected the last $\mathcal{O}(M_Z)$ term in eq. (7), have also neglected terms of $\mathcal{O}(M_Z)$ in the neutralino, chargino and Higgs mass matrices, and set $m_i^2 \simeq m_{\tilde{g}}^2 + m_i^2$. The corresponding error in the estimate of $\sin^2\vartheta_w(M_Z)|_{\overline{MS}}^{\text{GUT}}$ is

$$\delta s_{\text{GUT}}^2 = \left\{ \left[\left(\frac{7}{15\alpha_s} + \frac{\delta_s(\text{light})}{\alpha} \right) \delta\alpha \right]^2 + \left[\frac{7\alpha}{15\alpha_s^2} \delta\alpha_s \right]^2 \right\}^{\frac{1}{2}} \quad , \quad (11)$$

where we use $\alpha = 1/(127.87 \pm 0.12)$ and take $\alpha_s(M_Z) = 0.123 \pm 0.006$ from ref. [13]. We incorporate the GUT constraint by comparing $\sin^2\vartheta_w(M_Z)|_{\overline{MS}}^{\text{GUT}}$ with the value of $\sin^2\vartheta_w(M_Z)|_{\overline{MS}}$ extracted from our global fit to the precision electroweak data, and adding

$$\chi_{\text{GUT}}^2 = \frac{\left(\sin^2\vartheta_w(M_Z)|_{\overline{MS}}^{\text{GUT}} - \sin^2\vartheta_w(M_Z)|_{\overline{MS}} \right)^2}{\delta s^2} \quad , \quad (12)$$

where δs^2 takes into account the correlated errors in the two terms in the numerator, to the χ^2 function extracted from the global fit.

We now present our results, discussing the dependences of the global fit on the major MSSM parameters $(\mu, m_{\tilde{g}}, m_0, m_A, \tan\beta)$ as well as m_t .

Figure 2a shows that the preferred value of m_t is quite insensitive to the value of m_A : as discussed below, values of m_H much lower than those shown are not only excluded by direct searches but also disfavoured by the precision electroweak data, and larger values of m_A give results very similar to that for $m_A = 300$ GeV, in agreement with decoupling. Figure 2b shows that the preferred value of m_t may vary significantly with m_0 . The main origin of this variation is the fact that if m_0 is small, the $(\tilde{t}, \tilde{b})$ doublet may contribute to the radiative corrections as well as the (t, b) doublet, decreasing the value of m_t required to fit the data [22]. As can be seen in Fig. 2b, this effect vanishes for large m_0 , as expected from decoupling. Figure 2c shows that the preferred value of m_t may be increased if there is a chargino $\chi^\pm$ with mass very close to $M_Z/2$ [23]: here this effect is exhibited by varying μ , and it can also be seen by varying $m_{\tilde{g}}$. Note, however, that this effect is significant only if $m_{\chi^\pm} \lesssim M_Z/2 + 10$ GeV. It is due to the $\chi^+\chi^-$ contribution to the Z_0 wave function renormalization, and enters our analysis only via Γ_Z , since it drops out of Z decay branching ratios and asymmetries and does not appear in low-energy data. This is why the effect is less pronounced than in some previous analyses: other data restrict the extent to which $m_{\chi^\pm}$ can be traded off against m_t in fitting Γ_Z .

Also seen in Fig. 2a are dashed curves including the contribution to the χ^2 function of the GUT constraint (12). We see that this does not alter significantly the preferred value of m_t for the stated choices of the MSSM parameters, which corresponds to the decoupling limit for μ , m_0 and $m_{\tilde{g}}$ in the global electroweak fit. This indicates good consistency between the value of $\sin^2\theta_w(M_Z)|_{\overline{MS}}$ extracted from data and that predicted (6) by supersymmetric GUTs [4]. We investigate later the extent to which this consistency can be used to constrain the MSSM parameters.

Figure 3 shows contours of the χ^2 function in the $(m_A, \tan\beta)$ plane for one fixed value of m_t . There is no significant variation in χ^2 for $m_A \geq 10^2$ GeV, even if the GUT constraint is included (not shown). On the other hand, we find for the first time

that low values of m_A are disfavoured by the precision electroweak data, at least for $\tan\beta \geq 5$. This constraint is not yet competitive with that from direct searches in Z^0 decay, but it may become so in the future. It remains to be seen whether the precision electroweak data will be able in the future to cast any light on the “difficult” region $m_A \sim (150 \text{ to } 200)$ GeV, $\tan\beta \sim 5$ that may not be explored directly by LEP II and the LHC [24].

Figure 4 shows how the Z^0 wave function renormalization due to $\chi^\pm$ [23] affects the χ^2 of the global fit as a function a) of μ , and b) of $m_{\tilde{g}}$. There is a local improvement in χ^2 , but the local minimum is not preferred significantly over the decoupling limit when μ and/or $m_{\tilde{g}}$ is large.

Finally, in Fig. 5 we show the precision electroweak constraint in the $(m_0, m_{\tilde{g}})$ plane, for selected values of the MSSM parameters $(\mu, m_A, \tan\beta, m_t)$, together with the the GUT constraint. As seen by comparing Figs. 5a,b,c this indirect constraint (shown by a dash-dotted line) depends strongly on the values of the other MSSM parameters. In our analysis the GUT constraint does not exclude “low” values of $(m_0, m_{\tilde{g}})$ close to the present direct and indirect electroweak constraints. In the case of Fig. 4a, the GUT constraint upper limit on $(m_0, m_{\tilde{g}})$ is so weak (above 10 TeV), that it cannot usefully be plotted on the same scale as the direct and indirect electroweak constraints. This is possible for the parameter choice of Fig. 4b, but only if all the constraints are plotted on a logarithmic scale. The tightest GUT constraint is for the MSSM parameter choice of Fig. 4c, in which case $(m_0, m_{\tilde{g}})$ are forced to lie close to the present experimental lower limits.

This analysis reinforces the message [10] [25] that although minimal supersymmetric GUTs are highly consistent with the precision electroweak data, all the MSSM parameters should be taken into account when analyzing quantitatively the GUT constraint. In view of the sensitivity to such unknown parameters as μ and $m_{\tilde{g}}$, the present consistency with supersymmetric GUTs cannot be used to derive experimentally relevant upper or lower limits on the sparticle masses [25].

In conclusion, we would like to mention a few possible improvements over this analysis. The anticipated higher precision of the LEP determinations of M_Z and particularly Γ_Z will be of great interest, especially for the value of $\alpha_s(M_Z)|_{\overline{MS}}$, as we have already mentioned. The expected SLD improvement in the measurement of A_{LR} will also be very relevant. It would still be nice to find the top quark some day, particularly in the context of the MSSM where knowing its mass could constrain scenarios with light sparticles. Technically, the MSSM fits could be improved by a more complete treatment of vertex and box corrections. The GUT constraint could be implemented more exactly, and it would be interesting to confront the precision electroweak data directly with supplementary model-building assumptions [26]. The full panoply of theoretical ideas and calculations has yet to be brought to bear on the precision electroweak data.

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Figure captions

Fig. 1 – $\Delta\chi^2 \leq 1$ preferred region for the global fit in the 3-parameter Standard Model space $(m_t, M_H, \alpha_s(M_Z)|_{\overline{MS}})$, together with the projections on the (m_t, M_H) , $(m_t, \alpha_s(M_Z)|_{\overline{MS}})$ and $(M_H, \alpha_s(M_Z)|_{\overline{MS}})$ planes. The vertical planes in the figure, whose projections are also shown, correspond to the direct experimental limit $M_H > 63$ GeV and the $e^+e^- \rightarrow$ jets value $\alpha_s(M_Z)|_{\overline{MS}} = 0.123 \pm 0.006$ [13].

Fig. 2 – The dependence of χ^2 in the MSSM as a function of m_t :

- (a) for different values of m_A with $\mu = 800$ GeV, $m_0 = 800$ GeV, $m_{\tilde{g}} = 800$ GeV, $\tan\beta = 2$ fixed;
- (b) for different values of m_0 with $\mu = -1$ TeV, $m_{\tilde{g}} = 250$ GeV, $m_A = 500$ GeV, $\tan\beta = 8$ fixed;
- (c) for different values of $m_{\tilde{\chi}^\pm}$, corresponding to $\mu = 230, 340, 600$ GeV, with $m_0 = 300$ GeV, $m_A = 300$ GeV, $m_{\tilde{g}} = 300$ GeV, $\tan\beta = 2$ fixed.

The addition to χ^2 from the GUT constraint is also shown in (a).

Fig. 3 – Contours of χ^2 in the $(m_A, \tan\beta)$ plane for $\mu = m_0 = m_{\tilde{g}} = 800$ GeV, $m_t = 140$ GeV fixed. Note the minimum at $m_A \simeq 80$ to 90 GeV almost independent of $\tan\beta$, and that there is no significant variation in χ^2 for larger m_A .

Fig. 4 – The dependence of χ^2 :

- (a) on μ for $m_A = m_{\tilde{g}} = m_0 = 300$ GeV, $\tan\beta = 2$, $m_t = 160$ GeV fixed;
- (b) on $m_{\tilde{g}} = M_{\tilde{g}} = (\alpha_3/\alpha_{GUT}) m_{\tilde{t}}$ for $m_A = m_0 = 300$ GeV, $\mu = -200$ GeV, $\tan\beta = 2$, $m_t = 160$ GeV fixed.

The lower panels show the corresponding spectra of lighter sparticles. Note that the dips in χ^2 at $\mu \simeq 260$ GeV and $m_{\tilde{g}} \simeq 150$ GeV are due to the Z wave function renormalization associated with virtual $\chi^\pm$.

Fig. 5 - $\Delta\chi^2 = 1, 2, 7$ contours in the $(m_0, m_{\tilde{g}})$ plane (solid lines), together with contours of m_ν (dotted lines), the region where $m_{\tilde{e}}^2 < 0$ (hatched), and the direct search limit on $\chi^\pm$ (dashed lines), for:

(a) $\mu = -500$ GeV, $m_A = 1500$ GeV, $m_t = 140$ GeV, $\tan\beta = 2$ fixed;

(b) $\mu = -850$ GeV, $m_A = 1500$ GeV, $m_t = 140$ GeV, $\tan\beta = 4$ fixed;

(c) $\mu = -1000$ GeV, $m_A = 1500$ GeV, $m_t = 140$ GeV, $\tan\beta = 8$ fixed.

Also shown in (b) and (c) are $\Delta\chi^2 = 2$ contours for the GUT constraint (dashed-dotted lines). Note that the position of this constraint depends strongly on the other parameters: it is at very large values of $(m_0, m_{\tilde{g}})$ in case (a), moderate values in case (b), and smaller values in case (c).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5