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Tests of the Electroweak Theory at LEP

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Abstract

LEP offers a rich choice of tests of the electroweak theory such as the measurement of hadronic and leptonic cross sections, leptonic forward-backward asymmetries, τ polarization asymmetries, partial widths and forward-backward asymmetries of heavy quark flavours, of the inclusive $q\bar{q}$ charge asymmetry and of final state radiation in hadronic events. We discuss experimental aspects of these measurements and their theoretical parametrization and summarize the results available so far. We present several analyses which reveal specific aspects of the results, such as their constraints on Standard Model parameters and on new particles, the sensitivity to deviations from the Standard Model multiplet structure and an analysis in a framework which provides a model independent search for new physics.

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1 Introduction

The first experimental success of the Standard Model of electroweak interactions [1] was the discovery of the neutral current in neutrino scattering experiments [2]. Since then many experiments contributed to establish the theory in considerable detail, well before the start of Z^0 -pole physics at the e^+e^- colliders LEP and SLC [3]. The analysis of e^+e^- collisions at LEP and SLC offers a unique opportunity to extend these tests of the theory to a variety of aspects inherent to its quantum structure and to probe tiny modifications to the tree level processes. The electroweak precision tests in e^+e^- collisions at the Z^0 resonance can be compared with tests of QED like those of the $g - 2$ experiments. In contrast to pure QED, radiative corrections in the electroweak theory are also sensitive to particles with masses far beyond the range of direct production. Their precise measurement might therefore provide a window to new physics in an energy domain which is beyond the reach of present or even the next generation of accelerators. Within the framework of the Standard Model precision measurements can provide limits on the masses of the elusive top quark, M_t , and the Higgs boson, M_H . A consistent description of all our data with a unique value of M_t and M_H constitutes a stringent test of the theory.

This report reviews electroweak precision tests at the e^+e^- storage ring LEP. Section 2 summarizes the important features of the LEP e^+e^- collider and of the four major experiments installed in the interaction regions of LEP. We then give an overview of the theoretical parametrizations used for the analysis of LEP electroweak results and define the notation in Section 3. The aim of Section 4 is to provide the reader with the experimental analysis techniques and a basic set of results for the measurement of hadronic and leptonic cross sections, leptonic forward-backward asymmetries, τ polarization asymmetries, partial widths and forward-backward asymmetries for b and c quarks and the inclusive $q\bar{q}$ charge asymmetry. Most of the results quoted in this report are based on a summary of published and preliminary data prepared by the LEP collaborations and the LEP Electroweak Working Group for the Europhysics Conference on High Energy Physics, Marseille, July 22–28 1993 and the International Symposium on Lepton-Photon Interactions at High Energies, Cornell, August 10–15, 1993 [4].

Section 5 presents several analyses which discuss selected aspects of the LEP electroweak results. In Subsection 5.1 we show that all measurements are consistent with the Standard Model prediction and investigate the resulting parameter constraints within the Standard Model framework. Subsection 5.2 is devoted to constraints from the total and partial widths of the Z^0 for the production of new particles. In Subsection 5.3 we present an analysis of the effective electroweak couplings of leptons and quarks. Rather than constructing tests of the Standard Model with maximum sensitivity to new physics, the emphasis of this analysis is to set the scale at which we can probe effects related to specific fermions. In the Standard Model the effective couplings of fermions to the Z^0 have a common dependence on yet unknown parameters, like the masses of the top quark and the Higgs boson, which enter via radiative corrections. As new physics can be more easily disentangled if not masked by our ignorance of Standard Model parameters, several authors have proposed variables which are free from large M_t dependences and therefore particularly sensitive to new physics. In Subsection 5.4 we investigate one specific approach, proposed in [5], which tries to minimize the assumptions that need to be made about the contribution from possible new physics, which may enter either via radiative corrections or as a modification to the Born diagram.

The status of electroweak precision tests is summarized in Section 6 and an outlook to future measurements is given.

2 The LEP e^+e^- Collider and the Detectors

LEP is an e^+e^- storage ring with a circumference of about 27 km. By the end of 1992 the four LEP collaborations had recorded a total of $4.7 \cdot 10^6$ hadronic Z^0 decays. The number of e^+ and e^- bunches in the machine was initially four of each. After successful tests during 1992 this number has been doubled in 1993, where a luminosity of $1.5 \cdot 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ has been reached for many fills.

The physics goals of LEP are the test of the Standard Model of electroweak and strong interactions and the search for new particles and rare processes. These studies profit from the clean environment of an e^+e^- collider and the high statistics at the Z^0 resonance. An important part of the LEP programme is a precise determination of the mass of the Z^0 and its total decay width. For this purpose several energy scans with centre-of-mass energies within $\pm 3 \text{ GeV}$ of the Z^0 mass have been performed.

In circular e^+e^- -colliders under favorable conditions a natural transverse polarization may build up due to the interaction of the e^+ and e^- with the magnetic guide field, a phenomenon referred to as the Sokolov-Ternov effect [6]. Since the first observation of transverse polarization at LEP in August 1990 significant progress has been made in exploiting polarization as a tool to calibrate the LEP energy scale. This progress motivated a high statistics scan in 1993 with the aim of measuring M_Z and Γ_Z with a precision of 3 MeV or better. Besides the calibration of the LEP energy scale, polarization offers a wide field of precision tests of the electroweak theory. These studies need longitudinal polarization which can be achieved by means of spin-rotators, once transverse polarization is established. The use of longitudinal polarization has been studied in [7] as an option for future LEP running.

The present centre-of-mass energy of LEP is limited to 110 GeV. An energy upgrade is in progress in order to allow LEP to operate above the W^+W^- pair production threshold. The kernel of this upgrade is the replacement of the normal conducting copper cavities by superconducting cavities with higher accelerating gradients. At the time of writing the observation of the first W^+W^- pairs is expected in 1996 [8]. The energy upgrade will also give a new impact for particle searches, especially the search of the Higgs boson.

Four out of the eight interaction zones in LEP are capable of accomodating large experiments. They are occupied by the ALEPH [9], DELPHI [10], L3 [11] and OPAL [12] detectors. All the above are general purpose detectors with almost 4π solid angle coverage. As an example, the OPAL detector is shown in Figure 1.

In all experiments the interaction occurs in the centre of a high-resolution vertex chamber. Most of the experiments started with dedicated drift chambers, which have now been complemented by silicon microvertex detectors capable of reaching an impact parameter resolution of typically $10 \mu\text{m}$. The vertex chamber is surrounded by a large volume tracking chamber within a magnetic field of 0.4-1.5 Tesla oriented along the beam axis. The reconstruction of tracks within these chambers allows the determination of the momentum of charged particles with an accuracy of typically $\Delta p/p^2 \approx 10^{-3} \text{ GeV}^{-1}$. The specific energy loss of charged particles can be determined by multiple sampling with an accuracy of up to 3%, allowing particle identification also in the region of the relativistic rise of the energy loss-momentum relation. The design of the inner tracking chamber differs among the experiments: ALEPH and DELPHI choose a TPC, OPAL a Jet Chamber with a z-coordinate determination by charge division and by a layer of special 'Z'-chambers with orthogonal wires, L3 has a small inner tracking chamber which operates in the time expansion mode.

Between the central tracking detectors and the calorimetry some experiments have introduced a layer of time of flight hodoscopes. DELPHI also has a ring imaging Čerenkov system providing additional particle identification capabilities.

All experiments emphasize the detection of electromagnetic energy. The calorimeters surrounding the inner detectors are therefore subdivided into a high-resolution electromagnetic calorimeter and a hadronic calorimeter of several interaction lengths. There are various approaches to the detection of electromagnetic energy: sandwiched layers of lead and proportional wire planes (ALEPH), a high-density projection chamber, where the ionization produced between lead layers is drifted to the ends of 90 cm long modules (DELPHI), BGO crystals (L3) or a lead glass calorimeter (OPAL).

In most experiments the iron of the return yoke is interleaved with streamer tubes and serves as hadron calorimeter. The outer layer of each detector is again a system of tracking chambers for muon identification. L3 differs in design from the other experiments as the coil surrounds the entire detector to allow a precise momentum measurement in the muon chambers.

The luminosity is determined by measuring small-angle Bhabha scattering in a system of tracking chambers and calorimeters located in the inner part of the detector endcaps. The LEP experiments have supplemented their luminosity detectors with silicon devices to improve the precision of the measurement.

3 Observables and Theoretical Parametrizations

In this Section we first discuss the basic experimental observables for electroweak precision tests at LEP. We then give a few simple expressions for the lowest order cross sections of the process $e^+e^- \rightarrow f\bar{f}$. For $f \neq e$ we only have to consider the annihilation of the initial e^+e^- pair via a photon or a Z^0 into the final $f\bar{f}$ state (Figure 2 a)), whereas the treatment of the process $e^+e^- \rightarrow e^+e^-$ is complicated by the presence of t -channel diagrams (Figure 2 b)). We discuss how these expressions are modified by radiative corrections and describe the electroweak program libraries and parametrizations used by the LEP experiments.

3.1 Experimental Observables

The direct experimental observables for electroweak precision tests at LEP can be characterized as ratios of the observed numbers of events in various final states and the distribution of their production angles.

A special case of such ratios is the number of events for a specific final state divided by the number of small-angle Bhabha events. The cross section for small-angle Bhabha scattering is large as compared to e^+e^- annihilation processes and is well known as it is dominated by pure photon exchange with very small corrections from Z^0 exchange and the γZ^0 -interference. small-angle Bhabha scattering therefore serves as a reference process in e^+e^- collisions in deriving absolute cross sections. The ratio of hadronic events divided by small-angle Bhabha events as a function of the centre-of-mass energy allows the derivation of important parameters of the Z^0 like its mass, M_Z , and its total width, Γ_Z . The ratio of hadronic to leptonic events, when combined with the former determines the partial decay widths of the Z^0 into hadronic, leptonic and invisible channels.

The production angle, θ , of final state fermions is defined as the angle between the incoming e^+ -direction and the outgoing antifermion $\bar{f}$ direction (Figure 3). Theory predicts the angular distribution

for the process $e^+e^- \rightarrow f\bar{f}$ to be¹:

$$\frac{d\sigma}{d(\cos\theta)} \propto 1 + \cos^2\theta + B \cos\theta \quad (1)$$

for $f \neq e$. A more complicated expression results for $f = e$ due to the presence of the t -channel process (see e.g. [14] for a parametrization of the differential cross section). Having verified that the data follow the theoretical prediction, the information content of the angular distribution can be summarized in a single number, the ‘forward-backward asymmetry’, A_{FB} , given by:

$$A_{\text{FB}} = \frac{N_{\text{F}} - N_{\text{B}}}{N_{\text{F}} + N_{\text{B}}} = \frac{3}{8}B \quad (2)$$

Here the number of ‘forward’ events (N_{F}) is defined to be the number of events for which $\theta < \frac{\pi}{2}$. Similarly, N_{B} is the number of events for which $\frac{\pi}{2} < \theta < \pi$.

The study of event ratios and production angles can also be extended to initial or final states with a specific helicity (see Subsection 3.6).

3.2 Lowest Order Predictions

In the Minimal Standard Model with one iso-doublet of complex Higgs fields the lowest order predictions for the process $e^+e^- \rightarrow f\bar{f}$ can be described, neglecting fermion masses, with only 3 free parameters, which have to be determined from measurements. These 3 free parameters are usually expressed in terms quantities which are measured with the highest accuracy. At the Z^0 -pole the most natural choice is:

$$\alpha, G_{\text{F}} \text{ and } M_Z \quad (3)$$

where α is the electromagnetic coupling constant and G_{F} the Fermi constant.

For the process $e^+e^- \rightarrow f\bar{f}$ with $f \neq e$ the total cross section in the vicinity of $\sqrt{s} = M_Z$ is dominated by Z^0 exchange. At Born level the cross section can therefore be written as:

$$\sigma(s) = \sigma_{\text{ff}}^0 \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \gamma Z^0 + \gamma \quad (4)$$

where σ_{ff}^0 represents the cross section for the process $e^+e^- \rightarrow f\bar{f}$ at $\sqrt{s} = M_Z$, ‘ γ ’ and ‘ γZ^0 ’ represent small $\mathcal{O}(1\%)$ contributions from pure photon exchange and the γZ^0 -interference. The pole cross section, σ_{ff}^0 , can be written in terms of the Z^0 partial decay widths into e^+e^- and $f\bar{f}$ final states, Γ_{ee} and Γ_{ff} :

$$\sigma_{\text{ff}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{ff}}}{\Gamma_Z^2} \quad (5)$$

In the Standard Model the partial widths of the Z^0 are not free parameters but can be written in terms of vector and axial-vector coupling constants, g_v^f and g_a^f , of the Z^0 :

$$\Gamma_{\text{ff}} = \frac{G_{\text{F}}M_Z^3}{6\pi\sqrt{2}} [(g_a^f)^2 + (g_v^f)^2] \quad (6)$$

At the tree level the couplings can be expressed as

$$g_a^f = \sqrt{\rho} I_f^3 \quad (7)$$

$$g_v^f = \sqrt{\rho}(I_f^3 + 2Q_f \sin^2\theta_W) \quad (8)$$

¹Radiative corrections and fermion mass terms introduce corrections, which are, however, small compared to the present statistical accuracy.

with

$$\sin^2\theta_W \cos^2\theta_W = \frac{\pi\alpha}{G_F\sqrt{2}} \frac{1}{\rho M_Z^2} . \quad (9)$$

Here θ_W represents the weak mixing angle and I_f^3 the weak isospin of the fermion f . The value of the ρ parameter, which measures the relative strength of neutral and charged currents, is determined by the Higgs structure of the theory. In the Minimal Standard Model, which we assume when referring to the Standard Model in the following, $\rho = 1$ at the tree level².

3.3 Radiative Corrections

Radiative corrections modify the above relations. By convention they are separated into 3 classes, as indicated in Figure 4, which will be discussed below.

3.3.1 Photonic Corrections

The term photonic corrections refers to all diagrams with real or virtual photons added to the Born diagram. These corrections are large ($\mathcal{O}(30\%)$) and depend on experimental cuts. The dominant contribution arises from diagrams where a photon is radiated off the initial state, thus modifying the effective centre-of-mass energy, which has a substantial effect on cross sections close to a resonance. Photonic corrections are taken into account by convoluting the cross section for the hard scattering process by a radiator function, which can be calculated within the framework of QED. As the result of a substantial theoretical effort (see [15] and references therein) photonic corrections to the s -channel are by now well understood. The theoretical accuracy is estimated to be 0.1% or better, thus still³ matching the data statistics and the experimental systematics. For the process $e^+e^- \rightarrow e^+e^-$ the situation before the start of the Z^0 -pole experiments was considered as unsatisfactory [16]. A significant improvement for large-angle Bhabha scattering was achieved by the work of [17]. The authors quote an uncertainty of 0.5% for these calculations. Much work has also been invested into higher order corrections to small angle Bhabha scattering [18]. The theoretical accuracy for small-angle Bhabha scattering varies between 0.25% and 0.3% for the measurements quoted in this report. More work, however, is needed to fully exploit the anticipated experimental accuracy of forthcoming LEP data.

3.3.2 Non-Photonic Corrections

Non-photonic corrections denote the electroweak complement⁴ to photonic corrections. A familiar example of non-photonic diagrams is the vacuum-polarization of the photon, which leads to an s -dependent correction of the electromagnetic coupling constant:

$$\alpha \rightarrow \alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

The dominant uncertainty of $\alpha(M_Z^2) = 1/128.82$ is due to the contribution of light quarks to the vacuum-polarization of the photon and amounts to $\delta\Delta\alpha(M_Z^2) = 0.0009$ at present [19].

²The relation $\rho = 1$ at the tree level also holds for an arbitrary number of Higgs doublets but radiative corrections to the ρ parameter are different from the Minimal Standard Model.

³With 10^7 hadronic events and a reasonable extrapolation of experimental systematics, LEP will have several observables which have an experimental systematic error of $\mathcal{O}(0.1\%)$: the hadronic and leptonic pole cross section, the ratio of hadrons to leptons and the leptonic pole asymmetry.

⁴A separation of electroweak radiative corrections into photonic and non-photonic diagrams is rigorous only in $\mathcal{O}(\alpha)$; it can be justified, however, by the smallness of higher order corrections.

In the electroweak theory we have to take into account besides the photon vacuum polarization similar corrections related to Z^0 -exchange and additional diagrams involving heavy gauge bosons. In pure QED a precise measurement of radiative corrections would never give us any hint of particles which have a mass far above the energy scale of the process under consideration. This is a consequence of exact charge conservation, as the associated symmetry results in a suppression of heavy physics appearing in internal loops. The electroweak symmetry is broken, however, and therefore radiative corrections involving heavy particles may have observable consequences. This is one of the most interesting aspects of electroweak radiative corrections and electroweak precision tests: They potentially probe the complete particle spectrum and not only the part which is accessible at a given energy scale.

Non-photonic radiative corrections require modifications to the Born description of the hard scattering process which can be summarized to a very good approximation by the following [20]:

- An s -dependent photon vacuum polarization correction $\Delta\alpha(s)$.
- An s -dependent Z^0 total width which can be approximated by:

$$\Gamma_Z(s) = s/M_Z^2 \cdot \Gamma_Z(s = M_Z^2).$$

In the following we will use $\Gamma_Z \equiv \Gamma_Z(s = M_Z^2)$.

- Effective vector and axial-vector couplings denoted by $\hat{g}_v$ and $\hat{g}_a$. These effective couplings exhibit an s -dependence, which is negligible in the vicinity of the peak.

The above modifications essentially retain the Born structure of the hard scattering process and are therefore referred to as the ‘improved Born approximation’.

3.3.3 QCD Corrections

QCD corrections to the process $e^+e^- \rightarrow f\bar{f}$ account for gluon radiation off real and virtual quarks. They modify the $q\bar{q}$ final state, thus affecting the Z^0 partial widths for decays into $q\bar{q}$ -pairs, $\Gamma_{q\bar{q}}$, the forward-backward asymmetry, $A_{\text{FB}}^{q\bar{q}}$, of the reaction $e^+e^- \rightarrow q\bar{q}$ and also internal loops.

QCD corrections to $\Gamma_{q\bar{q}}$ can be written as an expansion of the strong coupling constant:

$$\Gamma_{q\bar{q}} = \Gamma_{q\bar{q}}(1 + \delta_{QCD}).$$

The QCD correction to the Z^0 hadronic partial width has recently been calculated to $\mathcal{O}(\alpha_s^3)$ [21] for massless quarks, superseding an earlier erroneous calculation with:

$$\delta_{QCD} = \frac{\alpha_s(M_Z^2)}{\pi} + 1.409 \left(\frac{\alpha_s(M_Z^2)}{\pi} \right)^2 - 12.805 \left(\frac{\alpha_s(M_Z^2)}{\pi} \right)^3 + \dots$$

The effects of quark masses on these corrections are of $\mathcal{O}(0.1\%)$ and included into the LEP parametrizations [22].

QCD corrections to $A_{\text{FB}}^{q\bar{q}}$ have been studied by [23, 24, 25]. The first order correction to $A_{\text{FB}}^{q\bar{q}}$ can be written as:

$$A_{\text{FB}}^{q\bar{q}} = A_{\text{FB}}^{q\bar{q}} \left(1 - k \frac{\alpha_s(M_Z^2)}{\pi} \right).$$

For massless quarks $k = 1$. Quark mass effects and the method by which the direction of the primary quark is determined experimentally modify the value of k . For b quarks it is customary to approximate the primary quark direction by the thrust axis of the event with appropriate sign conventions. The value of k is also sensitive to explicit and implicit experimental cuts which influence the relative acceptance for two and three jet events⁵.

Besides corrections to the $q\bar{q}$ final state, gluon radiation also induces corrections to internal loops, which have been calculated to $\mathcal{O}(\alpha\alpha_s)$.

3.4 Treatment of Radiative Corrections at LEP

The determination of the Z^0 resonance parameters at LEP is based on calculations which have been described in detail in [26, 27, 30]. Photonic corrections to the hard scattering process are calculated in complete $\mathcal{O}(\alpha)$, including leading $\mathcal{O}(\alpha^2)$ contributions and soft photon exponentiation. The t -channel and s - t -interference contributions in large-angle $e^+e^- \rightarrow e^+e^-$ scattering are treated as a correction to the s -channel based on [17]. For the s -channel hard scattering process without photonic corrections two approaches are followed frequently:

One approach is based on the precise calculation of radiative corrections within the Minimal Standard Model. The programs include a full one-loop calculation with leading $\mathcal{O}(\alpha^2 M_t^4)$ and $\mathcal{O}(\alpha\alpha_s)$ terms, using M_Z , M_t , M_H and α_s as input parameters. QCD corrections to the $q\bar{q}$ final states are calculated in $\mathcal{O}(\alpha_s^3)$, taking into account $b\bar{b}$ mass effects [22].

Substituting appropriate values for the effective couplings, $\hat{g}_v^f$ and $\hat{g}_a^f$, the differences between the improved Born approximation and the full Standard Model calculation are much smaller than the present experimental accuracy. This motivates the second approach which uses the improved Born approximation as a starting point for a more model independent test of radiative corrections.

3.5 The Differential Cross Section $e^+e^- \rightarrow f\bar{f}$

To facilitate the discussion of the results we give below the differential cross section for the hard scattering process $e^+e^- \rightarrow \ell^+\ell^-$ in the improved Born approximation neglecting small $\mathcal{O}(0.1\%)$ corrections due to fermion masses:

$$\begin{aligned} \frac{2s}{\pi\alpha^2} \frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow f\bar{f}) &= \left| \frac{1}{1 - \Delta\alpha} \right|^2 (1 + \cos^2\theta) \\ &+ 4\text{Re} \left\{ \frac{2}{1 - \Delta\alpha} \chi(s) [\hat{g}_v^e \hat{g}_v^\ell (1 + \cos^2\theta) + 2 \hat{g}_a^e \hat{g}_a^\ell \cos\theta] \right\} \\ &+ 16|\chi(s)|^2 \left[(\hat{g}_a^{e^2} + \hat{g}_v^{e^2})(\hat{g}_a^{\ell^2} + \hat{g}_v^{\ell^2})(1 + \cos^2\theta) + 8 \hat{g}_a^e \hat{g}_a^\ell \hat{g}_v^e \hat{g}_v^\ell \cos\theta \right], \end{aligned} \quad (10)$$

with

$$\chi(s) = \frac{G_F M_Z^2}{8\pi\alpha\sqrt{2}} \frac{s}{s - M_Z^2 + is\Gamma_Z/M_Z}. \quad (11)$$

The superscripts ‘e’ and ‘ ℓ ’ refer to initial state electron and the final state lepton couplings. To compare this expression to the experimental data it still has to be convoluted with a radiator function

⁵At the time of writing several aspects of this topic were still uncovered; we therefore use $k = 1$, if not explicitly stated differently.

which accounts for photonic corrections. Eqn. (10) also can be applied to $q\bar{q}$ final states, taking into account a colour factor of 3 and QCD corrections.

As stated above the information in the angular distribution is often expressed by a single number, the forward-backward asymmetry. If one considers only Z^0 exchange, the asymmetry at $s = M_Z^2$ without photonic corrections, denoted by $A_{\text{FB}}^{0,\text{f}}$ in the following, can be written as:

$$A_{\text{FB}}^{0,\text{f}} \equiv \frac{3}{4} \mathcal{A}_e \mathcal{A}_f \quad (12)$$

with

$$\mathcal{A}_f = \frac{2 \hat{g}_f^{\text{f}} \hat{g}_a^{\text{f}}}{\hat{g}_f^{\text{f}^2} + \hat{g}_a^{\text{f}^2}}. \quad (13)$$

3.6 Polarization Asymmetries

The Standard Model predicts parity violation not only for charged currents but also for neutral currents. Parity violation in neutral currents was first observed in the scattering of polarized electrons on deuterium at SLAC [31]. Parity violation in neutral currents also allows the verification of tiny effects of the γZ^0 -interference in atomic transitions. Today parity violation in neutral currents is well established. For the process $e^+e^- \rightarrow f\bar{f}$ it manifests itself by

- A final state fermion polarization.
- An asymmetry of the production cross section with respect to left-handed and right-handed polarization of the incoming electron beam.

The measurement of polarization asymmetries in e^+e^- collisions at the Z^0 -pole serves as a precision test of the lepton couplings.

For unpolarized e^+e^- beams the polarization $\mathcal{P}_f$ of the final state fermions is defined as:

$$\mathcal{P}_f = \frac{1}{\sigma_f^{\text{tot}}} (\sigma_f(h = +1) - \sigma_f(h = -1))$$

where $\sigma_f(h = +1)$ and $\sigma_f(h = -1)$ refer to the production cross section of the process $e^+e^- \rightarrow f\bar{f}$ for positive ($h = +1$) and negative ($h = -1$) helicity fermions, respectively, and σ_f^{tot} denotes the total fermion production cross section. A negative value for $\mathcal{P}_f$ means that fermions produced in neutral current reactions are preferentially left-handed, as observed in charged current reactions. From an analysis of the angular distribution of the final state fermion polarization a forward-backward asymmetry $A_{\text{FB}}^{\mathcal{P}_f}$ can be defined:

$$A_{\text{FB}}^{\mathcal{P}_f} = \langle \mathcal{P}_f \rangle_{\cos \theta > 0} - \langle \mathcal{P}_f \rangle_{\cos \theta < 0}.$$

Up to now the polarization of the final state fermions has only been measured for τ leptons.

For polarized e^+e^- beams the left-right asymmetry A_{LR} is defined as:

$$A_{\text{LR}} = \frac{1}{\sigma_f^{\text{tot}}} (\sigma_{\text{L}} - \sigma_{\text{R}})$$

where σ_L (σ_R) denotes the total production cross section $e^+e^- \rightarrow f\bar{f}$ for a left-handed (right-handed) polarization of the incoming electrons. From an analysis of the angular distribution of the final state fermions a polarized forward-backward asymmetry can be derived as:

$$A_{\text{FB}}^{\text{pol}} = \langle A_{\text{LR}} \rangle_{\cos\theta > 0} - \langle A_{\text{LR}} \rangle_{\cos\theta < 0} \quad .$$

The general formalism of how to include fermion helicities into the description of the differential cross section for the process $e^+e^- \rightarrow f\bar{f}$ can be found in [32]. The formulae simplify considerably if only Z^0 -exchange is considered. Neglecting photonic corrections the asymmetries defined above then have a simple relation to the vector and axial-vector couplings of the Z^0 for $s = M_Z^2$:

$$\mathcal{P}_f(s = M_Z^2) = -\mathcal{A}_f \quad (14)$$

$$A_{\text{FB}}^{\text{P}_f}(s = M_Z^2) = -\frac{3}{4}\mathcal{A}_e \quad (15)$$

$$A_{\text{LR}}(s = M_Z^2) = \mathcal{A}_e \quad (16)$$

$$A_{\text{FB}}^{\text{pol}}(s = M_Z^2) = \frac{3}{4}\mathcal{A}_f \quad (17)$$

The effects from the full $e^+e^- \rightarrow f\bar{f}$ amplitude and from photonic corrections are small, each increasing the absolute value of the predicted asymmetries at the Z^0 -pole by approximately 0.002.

3.7 Definition of an Effective Electroweak Mixing Angle

The introduction of effective couplings $\hat{g}_v^f$ and $\hat{g}_a^f$ also motivates the definition of an effective ρ -parameter and an effective mixing angle, the two sets of effective parameters being related via eqns. (7) and (8). The definition of these effective quantities in the literature differs slightly by the treatment of higher order corrections and the inclusion of vertex and box diagrams (see e.g. [29, 33, 34]). Frequently used for the effective mixing angle are the following notations:

$$\sin^2\theta_{eff}^f \equiv \kappa_f \sin^2\theta_W \quad (18)$$

with $\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$ and κ_f representing a flavour dependent form factor containing the non-photonic corrections. Also, a universal quantity $\sin^2\bar{\theta}_W$ can be defined which is related to $\sin^2\theta_{eff}^f$ via:

$$\sin^2\theta_{eff}^f = (1 + \Delta\kappa_{f,\text{vertex}}) \sin^2\bar{\theta}_W \quad (19)$$

Here $\Delta\kappa_{f,\text{vertex}}$ is the non-universal part of the form factor κ_f originating from vertex corrections. Within the Standard Model $\Delta\kappa_{f,\text{vertex}}$ can be calculated to a good approximation as a function of α , G_F , M_Z and known fermion masses, except for the b quark, for which vertex corrections are top mass dependent. For leptons we have $\sin^2\theta_{eff}^{\text{lept}} \approx \sin^2\bar{\theta}_W + 0.0007$.

A definition of the effective mixing angle which is related to a specific experimental measurement and used by the LEP experiments is the following:

$$\sin^2\theta_{eff}^{\text{lept}} \equiv \frac{1}{4}(1 - \hat{g}_v^\ell / \hat{g}_a^\ell) \quad (20)$$

where the ratio $\hat{g}_v^\ell / \hat{g}_a^\ell$ has been derived from the leptonic pole asymmetry $A_{\text{FB}}^{0,\ell}$ using eqn. (12). With this definition one obtains an effective mixing angle which corresponds to $\sin^2\theta_{eff}^f$ for leptons in eqn. (18). This determination of $\sin^2\theta_{eff}^{\text{lept}}$ does not require any knowledge of an effective ρ parameter. Similarly, the results of polarization asymmetries can be mapped directly onto $\sin^2\theta_{eff}^{\text{lept}}$ by using their

effective Born representation and eqn. (20). The resulting $\sin^2\theta_{eff}^{lept}$ is equivalent to the one determined from $A_{FB}^{0,\ell}$, as long as the improved Born approximation is a valid representation of the data. The determination of $\sin^2\theta_{eff}^{lept}$ from the $b\bar{b}$ or $c\bar{c}$ forward-backward asymmetry requires the knowledge of corrections particular to the hadronic vertex. These corrections, however, are small compared to the present experimental accuracy due to the reduced sensitivity of $A_{FB}^{q\bar{q}}$ to the hadronic couplings.

Determinations of $\sin^2\theta_{eff}^{lept}$ from partial widths of the Z^0 or from the inclusive $q\bar{q}$ charge asymmetry require a knowledge of effective ρ -parameters. There are ways to express the effective ρ -parameters in terms of the effective weak mixing angles (loosely speaking by using eqn. (9) with the substitution $\alpha \rightarrow \alpha(s)$). To a certain level of accuracy the effect of electroweak radiative corrections can be condensed into one single parameter, designating the effective mixing angle as a convenient quantity to compare the sensitivity of various measurements to non-photon corrections. The combined set of electroweak data, however, determines the effective mixing angle with an accuracy at which the difference between definitions is significant (c.f. Section 5.1).

4 Experimental Aspects and Results

4.1 The Z^0 Lineshape and Leptonic Forward-Backward Asymmetries

4.1.1 Statistics and Systematics

Cross sections are determined from experiments via the relation:

$$\sigma = \frac{N_{sel} - N_{bg}}{\epsilon \int \mathcal{L} dt}.$$

where N_{sel} and N_{bg} refer to the number of events passing the selection cuts and the number of background events in the selected sample. $\int \mathcal{L} dt$ denotes the integrated luminosity and the correction factor ϵ accounts for the trigger efficiency, the geometrical acceptance and the efficiency of the selection cuts.

The design of the LEP detectors allows a trigger on hadrons and leptons with high redundancy, accepting 100% of the events, with an uncertainty of less than 0.1%, within the solid angle considered in the analysis.

The selection of hadrons and lepton pairs at LEP is conceptually easy, as they can be discriminated by a few simple cuts like cluster or track multiplicities, deposited energy and energy balance, against backgrounds which are of $\mathcal{O}(0.1\%)$ for hadrons, e^+e^- and $\mu^+\mu^-$, and of $\mathcal{O}(1\%)$ for $\tau^+\tau^-$ pairs. The challenge of the analysis is motivated by the aim to match the systematic error of the efficiency and acceptance corrections and the statistical error. The tools used to reach this goal involve both elaborate detector simulations and cross-checks with the data themselves.

The absolute luminosity is obtained from the ratio of the number of events measured in small-angle Bhabha scattering and the theoretical prediction for the cross section of this process within the acceptance of the luminosity monitor. The acceptance in polar angle of the forward detectors typically ranges from 25–120 mrad. At small polar angles the cross section for Bhabha scattering is given by :

$$\frac{d\sigma}{d\theta} \approx \frac{32\pi\alpha^2}{s} \frac{1}{\theta^3},$$

resulting in an integrated cross section:

$$\sigma_{Bhabha} \approx \frac{16\pi\alpha^2}{s} \left(\frac{1}{\theta_{min}^2} - \frac{1}{\theta_{max}^2} \right),$$

where θ_{min} and θ_{max} define the limits of the acceptance. The most important task of the luminosity determination is therefore the precise monitoring of the edge of the acceptance at the inner radius⁶. The current theoretical uncertainties, mainly related to uncalculated amounts to about 0.3% [35], depending on the specific experimental cuts involved. This accuracy is still better than all published experimental accuracies related to the luminosity measurement. Theoretical progress, however, is eagerly expected as recently installed new forward detectors based on silicon technology are likely to decrease the experimental systematic error to 0.1% in the near future⁷. Tables 1 and 2 summarize the statistical and systematic errors related to the cross section measurement.

To determine the lepton forward-backward asymmetries, $A_{FB}^{\ell\ell}$, the same event selection as for the cross sections is applied. The measurement requires charge identification. It is, however, not sensitive to overall normalization factors. Furthermore, if A_{FB} is determined from a maximum likelihood fit of eqn. (1) to the experimental angular distribution, the result for A_{FB} is also insensitive to a symmetric θ dependence of the acceptance. The systematic errors for $A_{FB}^{\ell\ell}$ are due to charge misidentification, angular resolution and uncontrolled displacements of the event vertex. The experimental systematic errors quoted for the '92 data ($\Delta A_{FB} = 0.001 - 0.004$, uncorrelated among the three lepton species) are still smaller than the statistical errors. They are expected to further improve with statistics as there are no known limitations and larger data samples will allow more detailed systematic studies to be made.

4.1.2 The LEP Energy Calibration

Very important for the measurement of the Z^0 mass and width is the accuracy of the LEP centre-of-mass energy determination. The techniques first applied for the published results based on the 1990 data [36] were based on flip coil measurements in a reference magnet connected in series with the magnets in the LEP ring, on the calibration of the magnetic field by flux loops mounted on the pole faces of all dipole magnets and on the measurement of the revolution frequency of 20 GeV protons circulating on the same orbit as the positrons.

With the first observation of transverse polarization at LEP [37] at the end of the 1990 running period a new method started to be applied in 1991. It is based on the resonant depolarization of the beam particles by a weak oscillating magnetic field. Resonant depolarization occurs if the frequency of the magnetic field applied matches the precession frequency of the electron spins around the magnetic bending field. The feasibility of the method for LEP has been proven by several successful measurements [38, 39]. The intrinsic accuracy of this technique is of $\mathcal{O}(1 \text{ MeV})$.

An impressive demonstration of the sensitivity of the resonant depolarization method is the fact, that the result of the measurement can be related to the effect of the tides on the LEP ring. The terrestrial tides due to the moon and the sun move the earth surface up and down. This motion has also lateral forces resulting in a change of the LEP circumference. The length of the beam orbit being fixed by the constant RF-frequency, the change of the LEP circumference will force the beams to go

⁶As a rule of thumb, the precision of the inner edge affects the accuracy of the luminosity $\mathcal{L}$ like: $\Delta\mathcal{L}/\mathcal{L} \approx 2\Delta\theta_{min}/\theta_{min}$, i.e. $\Delta\mathcal{L}/\mathcal{L} \approx 2\% \cdot \Delta r_{min} [\text{mm}]$ for an inner radius of the luminosity monitor $r_{min} = 10 \text{ cm}$.

⁷Before LEP start-up an experimental accuracy of 2% for the luminosity determination was considered as a sonic barrier.

off-center through the quadrupole magnets which leads to an energy variation of ± 10 MeV over a period of 24 hours. In a dedicated experiment [40] frequent resonant depolarization measurements of the LEP energy have been performed over a period of 24 hours. The measurements perfectly confirm the change in energy expected from calculations of the effect of the tides on the length of the LEP circumference.

In 1991 only few calibrations based on this method were performed at a nominal centre-of-mass energy of 93 GeV. Additional uncertainties arise when applying this calibration to all energies of the scan and to the entire running period using magnetic measurements. The combination of these methods has led to a systematic uncertainty of 6 MeV on M_Z in 1991 [39]. The calibration of the LEP energy scale in 1992 [41] was performed using a similar procedure. In 1992, however, calibrations with resonant depolarization were successful only late in the year and showed a large spread resulting in an error of ± 18 MeV on the centre-of-mass energy.

For the present 1993 running period LEP succeeded in performing calibrations based on resonant depolarization under machine conditions which are fully identical to those of the data taking. This allows frequent calibrations at the end of the fills used for physics analysis. In contrast to the scan in 1991, all energy points of the scan have been calibrated using the resonant depolarization method. A significant reduction of the point-to-point energy uncertainty, which is the dominant systematic error for Γ_Z , is expected.

4.1.3 Results

Figures 5 and 6 give an example of measured cross sections and leptonic forward-backward asymmetries as a function of the centre-of-mass energy. The LEP experiments determine the Z^0 resonance parameters in a combined fit to their measured hadronic and leptonic cross sections and leptonic forward-backward asymmetries. The parameters are obtained using a χ^2 minimization procedure taking into account the full covariance matrix of the data. The covariance matrix also includes the uncertainties arising from the LEP energy calibration. Several parameter sets have been chosen to characterize the measurements. In order to facilitate the combination of the results from the four experiments, the LEP collaborations also present, in addition to their favorite choices, one parameter set:

$$M_Z, \Gamma_Z, \sigma_h^0, R_e, R_\mu, R_\tau \text{ and } A_{\text{FB}}^{0,e}, A_{\text{FB}}^{0,\mu}, A_{\text{FB}}^{0,\tau}.$$

In the above parameter set $R_e \equiv \Gamma_{\text{had}}/\Gamma_{ee}$, $R_\mu \equiv \Gamma_{\text{had}}/\Gamma_{\mu\mu}$ and $R_\tau \equiv \Gamma_{\text{had}}/\Gamma_{\tau\tau}$, where Γ_{ee} , $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ denote the partial widths of the Z^0 for the decay into electron, muon and tau pairs, respectively. The pole asymmetries, $A_{\text{FB}}^{0,e}$, $A_{\text{FB}}^{0,\mu}$ and $A_{\text{FB}}^{0,\tau}$ (see eqn. 12) refer to the leptonic forward-backward asymmetries at $\sqrt{s} = M_Z$ without photonic corrections and originating from Z^0 exchange only for electron, muon and tau pairs, respectively. This parametrization is closely related to the experimental measurements, and correlations between parameters are small. The results of the 4 LEP experiments are summarized in Table 3.

Methods to combine the results of the individual LEP experiments have been studied [4, 36, 42] by the LEP Electroweak Working Group with representatives from the four LEP Collaborations. The conclusions of their first report based on the 1990 LEP data [36] can be summarized as follows: The 4 LEP experiments use equivalent procedures to determine the Z^0 parameters. At the present level of accuracy, simple parameter averages taking into account common systematic errors are as good as complicated combined fits to data points. Different parameter sets are useful to emphasize particular aspects of the data. To characterize the data, however, any single parameter set is sufficient, provided, it has symmetric Gaussian errors and is accompanied by the correlation matrix. With the

statistics available for [36] the correlation matrices obtained by each individual experiment or from a combination of all data are sufficiently similar to be interchanged.

Meanwhile the accuracy of the combined LEP parameters has been improved by a factor two to three. The conclusions of [36] are still valid to a very good approximation. To allow for a more general treatment of common systematics⁸ the procedure has been slightly refined for the results presented in [4, 42]:

- Each experiment provides the set of nine parameters described above, accompanied by its correlation matrix.
- From the covariance matrices for the 9 parameters of the individual experiments a 36×36 covariance matrix $\mathcal{V}$ is constructed, which can be subdivided in sixteen 9×9 submatrices as shown in Table 4. The diagonal submatrices are the parameter covariance matrices provided by the individual experiments, which include all common systematic errors. The contribution of the LEP centre-of-mass energy uncertainty to the covariance matrix of the parameters for a single experiment can be obtained by repeating the fit to the data points without including these errors. This contribution has been found to be sufficiently similar among the experiments so that it can be approximated by a single 9×9 submatrix appearing in all off-diagonal subblocks. The common systematic uncertainty for the theoretical calculation of the small-angle Bhabha cross section mainly affects the diagonal covariance matrix element for σ_h^0 and has been added as such to the off-diagonal subblocks. For a detailed set of covariance matrices used for obtaining the combined results presented in this report we refer to [4].
- Then a combined parameter set is obtained by minimizing $\chi^2 = \Delta^T \mathcal{V}^{-1} \Delta$, where Δ denotes the vector of residuals of the combined parameter set to the results of the individual experiments.

The LEP combined result for the parameter set above and its correlation matrix are given in Table 5 and Table 6.

Lepton universality is an important assumption of the Standard Model and of several extensions. It states that the gauge couplings of the lepton doublets should be equal. This assumption can be used to reduce the set of 9 parameters given above to a set of 5 parameters. Due to mass corrections to $\Gamma_{\tau\tau}$ we expect, however, a small (0.2%) difference between the values for R_e and R_μ and the value for R_τ . The procedure as to how to take into account these mass terms, when specifying a single partial width, $\Gamma_{\ell\ell}$, for the decay of the Z^0 into leptons is not unambiguous and has to be defined. In this report we follow the convention of [4] and define $\Gamma_{\ell\ell}$ as the partial width of the Z^0 for the decay into a pair of a massless charged leptons.

Figure 7 shows, for each leptonic species, the one standard deviation contours of the combined result in the R_ℓ - $A_{\text{FB}}^{0,\ell}$ plane. Good agreement is observed with the hypothesis of lepton universality⁹. Repeating the combination of results from the 4 experiments imposing lepton universality leads to the parameters given in Table 7 with the correlation matrix given in Table 8. The one standard deviation contour imposing lepton universality and the comparison with the Standard Model prediction are also indicated in Figure 7.

⁸E.g. the treatment of common systematic errors which introduce correlations among parameters such as uncertainties in the LEP centre-of-mass energy or the treatment of theoretical errors which are highly correlated among the experiments but have a size which depends on the acceptance of the individual experiments

⁹At the present accuracy the effect of mass terms for R_τ can be neglected for a qualitative statement on lepton universality

The parameters M_Z , Γ_Z , σ_h^0 , R_ℓ and $A_{\text{FB}}^{0,\ell}$ are convenient for fitting and averaging since they have minimal correlations amongst them. In Section 5 we will present a number of other parameters which can be derived from the previous set by simple parameter transformations.

Finally we wish to give a warning: There may be more information in the Z^0 line shape and the lepton asymmetries than just the 9 parameters in Table 5 ! These parameters do not tell us anything about the γZ^0 -interference ! In quoting them, the contributions from the γZ^0 -interference have been assigned their Standard Model values¹⁰. Some experiments have performed a more general fit: L3 has analysed their data using an S -matrix approach [43, 44]; OPAL also reports fits where the interference term for hadrons is multiplied by a free parameter or an extended improved Born approach for leptonic cross sections and asymmetries [45]. These studies reveal the subtle interplay between the shape of the interference and the value of M_Z . They also explicitly test the energy dependence of the lepton asymmetries. At present none of these studies is in contradiction with the Standard Model. There is also no inconsistency between parameters derived in these more general frameworks and those of the standard 9 parameter fits.

The moderate interest in these studies is due to the lack of models which could introduce effects on the interference in the vicinity of the Z^0 peak comparable in size to the present parameter uncertainties, and simultaneously reproducing the striking agreement with the Standard Model for the rest of the experimental results. We should be aware, however, that these studies are very important not to end up in a ‘Standard Model bootstrap’.

4.2 The τ Polarization Asymmetries

The τ lepton plays an exceptional role in the investigation of final state fermion polarizations in e^+e^- -collisions because fermion and antifermion can easily be discriminated, the τ has a short lifetime and parity is violated in its weak decays. Assuming the $V - A$ structure of the weak charged current the decay products can therefore be used as spin analysers.

The classical example for the determination of the τ helicity from its decay products is the $\tau \rightarrow \pi(K)\nu$ decay (Figure 8). The π is spinless and therefore the ν_τ has to carry the spin of the τ . The $V - A$ structure of the weak charged current interactions requires the ν_τ to be left-handed and therefore the angular distribution in the τ -restframe depends on the polarization state of the τ lepton and is given by:

$$W(\cos \theta^*) = \frac{1}{N} \frac{dN}{d\cos \theta^*} = \frac{1}{2} (1 + \mathcal{P}_\tau \cos \theta^*) ,$$

where $\cos \theta^*$ refers to the decay angle of the π in the τ rest frame. The angular distribution of the decay products in the τ rest frame has a one-to-one correspondence to the energy spectra in the e^+e^- centre-of-mass frame. Neglecting mass terms this relation is for the $\tau \rightarrow \pi(K)\nu$ decay:

$$\cos \theta^* \approx 2 \frac{E_\pi}{E_\tau} - 1 .$$

Figure 9 shows the π energy spectrum measured in τ decays.

Most of the experiments determine the τ polarization by fitting a linear combination of the simulated $h = +1$ and $h = -1$ distributions to their data. The simulation includes an event generation taking into account radiative corrections, the detector response and the effect of event selection cuts.

¹⁰For hadrons this holds for all the LEP experiments. The treatment of the interference term for leptons differs slightly among the experiments, but these differences can be neglected at the present accuracy.

A complementary approach has been pursued by the OPAL collaboration. They first deconvolute the data for the effects of event selection, detector resolution and photonic corrections and then fit the expected distribution of kinematic variables directly to the data to obtain $\mathcal{P}_\tau$.

The branching fraction of the classical $\tau \rightarrow \pi(K)\nu$ channel being only 12%, the τ polarization measurement benefits significantly by also considering other decay channels. The purely leptonic decay modes $\tau \rightarrow e\nu\bar{\nu}$ and $\tau \rightarrow \mu\nu\bar{\nu}$ have a large branching fraction, but the polarization signal is diluted as the decay angles cannot be reconstructed because of the two neutrinos in these decays. Only the lepton energy can be measured. Assuming the V–A-structure for the charged current and neglecting terms of $\mathcal{O}(M_\tau/M_Z)$ and $\mathcal{O}(M_\ell/M_\tau)$, the expected distribution of the lepton energies is

$$W(x) = \frac{1}{N} \frac{dN}{dx} = \frac{1}{3} [4x^3 - 9x^2 + 5 + \mathcal{P}_\tau(8x^3 - 9x^2 + 1)]$$

with $x = E_\ell/E_\tau$. Among the semileptonic decay modes the $\tau \rightarrow \rho\nu$ and the $\tau \rightarrow a_1\nu$ decays have been analysed. The ρ and the a_1 meson, however, have spin 1 allowing for two possible spin configurations in τ decays (Figure 10). To fully exploit the information available, experiments therefore also take into account the angular distribution of the ρ and a_1 decay products in the ρ and a_1 restframe. Table 9 compares the statistical weights for the various τ decay channels. The LEP averages of the τ -polarization measurements for the individual τ decay channels are summarized in Table 10.

Table 11 displays the average values of $\mathcal{A}_\tau$ from each of the four LEP experiments. In this combination the average result for $\mathcal{P}_\tau$ as quoted by the individual experiments has been used to allow for correlations between the individual decay channels. To obtain $\mathcal{A}_\tau$ from the measured value of $\mathcal{P}_\tau$ small corrections have been applied (see also Subsection 3.6) to account for photon exchange and the γZ^0 -interference, initial state radiation and the energy dependence of $\mathcal{P}_\tau$ in the neighborhood of $s = M_Z^2$.

An analysis of the angular distribution of $\mathcal{P}_\tau$, which is shown in Figure 11, allows the determination of $\mathcal{A}_e$. Two experiments have performed such an analysis, the results are given in Table 12.

4.3 Electroweak Results with Quarks

The determination of the effective quark couplings requires event samples with different compositions of the primary quark flavours. Up to now an exclusive separation of primary quark flavours has only been achieved for b and c quarks. In the following we will present several methods to determine quark partial widths and forward-backward asymmetries.

4.3.1 The b Quark Sector

The ability to separate b quarks from other quark flavours has several attractive features:

- The b quark is the isospin partner of the yet unseen top quark. Already a rough determination of its neutral current couplings is sufficient to establish its Standard Model weak isospin assignments [56], and thereby giving experimental evidence for the existence of the top quark.
- Precision measurements of the b quark couplings give complementary information on the structure of radiative corrections.

- A study of particle-antiparticle oscillations in the $B^0\bar{B}^0$ system can give important information on the nature of CP violation.
- We know today that the Higgs - if it exists - is heavy. If its mass is within the energy range accessible in the near future, it will predominantly couple to b quarks. Especially at LEP200 the identification of b quarks will be vital to assess the question of the origin of particle masses.

Primary b quarks exhibit several distinct signatures: As b-flavoured hadrons are heavy, their decay products have a large p_T with respect to the primary B-hadron direction. Also the fragmentation of light quarks is much softer as they lose a larger fraction of their energy by gluon radiation, whereas the hadrons containing the primary b quark carry away on average about 70% of the beam energy. Furthermore B hadrons have long lifetimes, typically about 1.5 ps, resulting in decay vertices which are displaced from the main vertex by about 2 mm. These signatures lead to 3 b-tagging techniques, which will be described in the following: lepton tagging, event shape tagging and lifetime tagging.

Initially the most common method was to separate b quarks using their semileptonic decays $b \rightarrow \ell X$. Leptons from primary b-flavoured hadron decays are characterized by a high momentum p because of the hard b quark fragmentation. They also have a relatively high transverse momentum p_T with respect to the nearest jet because of the large b-flavoured hadron mass. The method therefore essentially consists in counting the number of high p and high p_T leptons.

There are several sources of inclusive leptons which are summarized in Figure 12. The inclusive leptons from the various sources differ in their p and p_T spectra, as shown in Figure 13. To separate these contributions the observed p versus p_T distribution of inclusive leptons is fitted to an expression of the form:

$$N(p, p_T) = 2N_{\text{had}} \sum_i C_i f_i(p, p_T) .$$

In this expression, N_{had} denotes the total number of hadronic events in the data, C_i the total fraction of events of class i in the lepton sample and $f_i(p, p_T)$ the probability densities for each contribution, as a function of p and p_T . For the contribution originating from primary b quarks:

$$C_i \equiv C_{b\bar{b}} = \frac{\Gamma_{b\bar{b}}}{\Gamma_{\text{had}}} Br(b \rightarrow \ell X) \eta_{b\bar{b}} .$$

Here $Br(b \rightarrow \ell X)$ refers to the semileptonic branching ratio for b-decays and $\eta_{b\bar{b}}$ to the efficiency for detecting the associated lepton. Measurements which only use single lepton tags determine the branching ratio product $Br(b \rightarrow \ell X) \Gamma_{b\bar{b}}/\Gamma_{\text{had}}$, i.e. for a determination of $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ the semileptonic branching ratio $Br(b \rightarrow \ell X)$ has to be imported from measurements at the $\Upsilon(4S)$ resonance or in the e^+e^- continuum at PEP/PETRA. The accuracy of $Br(b \rightarrow \ell X)$ represents the limiting factor for the precision of $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ determined from single lepton tags.

Another approach to separate b quarks is based on global event shape variables (see [57, 58, 59] for a definition of observables). As there is no single event shape observable which is good enough on its own and correlations among observables are important, these analyses use neural networks to discriminate $b\bar{b}$ and $c\bar{c}$ events from the hadronic sample. The limiting factor for these tagging methods is the Monte Carlo modelling of fragmentation and decay.

High-resolution silicon microvertex detectors have opened a new field of b quark separation, the lifetime tagging. Two observables which signal the presence of long lived B hadrons are used: The decay length, which is defined as the distance between the primary Z^0 decay vertex and a secondary vertex, and the impact parameter which is defined as the distance of closest approach of a track to the primary Z^0 decay vertex. For both observables a sign can be defined as shown in Figure 14. Decays of

b-flavoured hadrons are expected to have a positive decay length and on average several charged tracks with a large positive impact parameter. Figure 15 shows purity and efficiency for a lifetime tagging method which is based on the reconstruction of the 3-dimensional impact parameters of tracks.

An interesting trick is the double tagging method. The scope of this method applies to all tagging techniques discussed above. The fraction of single event hemispheres, T_s , passing the cuts of a specific method and the fraction of events, T_d , in which both hemispheres pass these cuts can be written as:

$$T_s = (1 - \lambda_c - \lambda_b)\epsilon_{uds}^s + \lambda_c\epsilon_c^s + \lambda_b\epsilon_b^s \quad (21)$$

$$T_d = (1 - \lambda_c - \lambda_b)\epsilon_{uds}^d + \lambda_c\epsilon_c^d + \lambda_b\epsilon_b^d \quad (22)$$

$$\text{with} \quad \lambda_{uds} + \lambda_c + \lambda_b = 1. \quad (23)$$

In these equations λ_{uds} , λ_c and λ_b refer to the fraction of hadronic events containing primary uds, c and b quarks, respectively. In the vicinity of the Z^0 peak the λ_i can be written to a good approximation as $\lambda_i = \Gamma_i/\Gamma_{\text{had}}$, where Γ_i refers to the partial width of the Z^0 for decays into a quark antiquark pair of species i. The parameters ϵ_i^s and ϵ_i^d denote the single and double tag efficiencies, including all effects, such as the branching ratios of hadrons containing the primary quarks, detector acceptance and selection efficiencies.

Eqns. (21)–(23) describe 2 measurements with 8 unknowns, i.e. 6 of the parameters have to be supplied from external sources. The parameter space can be reduced by assuming no correlations for the efficiency between the tags in the two event hemispheres, i.e. $\epsilon_i^d = (\epsilon_i^s)^2$. As these correlations are small they can be introduced as corrections, usually computed by Monte Carlo. The benefit of the double tag method can be illustrated with an oversimplification:

$$\begin{aligned} \epsilon_{uds}^{s,d} = \epsilon_c^{s,d} = 0 & \quad ; \quad \text{i.e. 100\% b purity} \\ \epsilon_b^d = (\epsilon_b^s)^2 = (\epsilon_b)^2 & \quad ; \quad \text{i.e. no hemisphere correlations.} \end{aligned}$$

Then the efficiency, ϵ_b , and $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ can be obtained directly from the measurement:

$$\epsilon_b = T_d/T_s \quad (24)$$

$$\Gamma_{b\bar{b}}/\Gamma_{\text{had}} = T_s^2/T_d. \quad (25)$$

An extension of the double tagging method are so called mixed tag methods. Here two or more tagging techniques are used simultaneously and are applied separately to each hemisphere of the event. The event is classified according to the different tagging patterns. The analysis of these event classes allows the simultaneous determination of a number of parameters, which otherwise have to be determined from external sources, and can lead to a reduction of the decay model dependence for the determination of $\Gamma_{b\bar{b}}$. Published analyses combine an event shape tag [57] or a lifetime tag [60] with a lepton tag.

Figure 16 shows the LEP results for $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ for the three different tagging methods. The averages for the individual tagging methods take into account common systematic errors. For the results based on lepton tagging, these are due to the semileptonic decay model, semileptonic branching ratios of b and c hadrons and the contributions from $c\bar{c}$ events. For the event shape method all errors arising from a variation of Monte Carlo parameters have been treated as common. The two main contributions to the common systematic errors for methods based on lifetime tagging are hemisphere correlations and the charm contribution.

In calculating the overall average of all methods, the results from the three categories were assumed to be entirely uncorrelated. This assumption has been imposed for simplicity: the different

measurements are sensitive to different aspects of the fragmentation process and the correlations are difficult to extract. As there are several sources of correlations, cross checks on the average have been performed assuming different models for these correlations. At the present accuracy, the variation of the average assuming different models is still small. Averaging procedures which are adequate for the anticipated accuracy of future results are under discussion among the LEP experiments.

For the measurement of quark forward-backward asymmetries the jet originating from the primary quark must be discriminated against the jet originating from the primary antiquark.

The inclusive lepton analysis can be naturally extended to determine the $b\bar{b}$ forward-backward asymmetry $A_{\text{FB}}^{b\bar{b}}$. For the total asymmetry the contributions in Figure 12 have to be considered separately. Furthermore it is very important to consider the $B^0\bar{B}^0$ mixing which contributes to wrong sign leptons (see Figure 17). The amount of mixing can be expressed by a parameter λ_B defined as:

$$\lambda_B = \frac{Br(b \rightarrow \bar{B}^0 \rightarrow B^0 \rightarrow \ell^+ X)}{Br(b \rightarrow \ell^\pm X)}.$$

$B^0\bar{B}^0$ mixing at LEP has been determined using the ratio of like-sign and unlike-sign inclusive dilepton events [61]. Alternatively, the charge of one b quark has been tagged by a high p , high p_T electron or muon and the charge of the other b quark has been measured from the momentum weighted average of charges of particles within the jet in the opposite hemisphere [62].

The contributions shown in Figures 12 and 17 lead to an experimentally observed asymmetry which is given by:

$$A_{\text{FB}}^{r_{aw}} = (f_{b-\ell} - f_{b-c-\ell} + f_{b-\bar{c}-\ell})(1 - 2\lambda_B)A_{\text{FB}}^{b\bar{b}} - f_{c-\ell}A_{\text{FB}}^{\bar{c}\bar{c}} + f_{bg}A_{\text{FB}}^{bg}$$

where f_i denotes the fraction of contribution i in the inclusive lepton sample.

Another approach to discriminate between quark and antiquark is jet charge measurement (see also Subsection 4.3.3) which can be combined with the lifetime tag for an efficient selection of events with primary b quarks. The dominant systematic error arises from uncertainties in the modelling of fragmentation and decay of b -flavoured hadrons. To reduce these uncertainties also the lepton tagged sample has been used to calibrate the jet charge measurement [4].

Figure 18 shows recent LEP measurements of the $b\bar{b}$ asymmetry. Correlated systematic errors arise from the imprecise understanding of the production and decay of hadrons containing b - and c quarks. The evaluation of common systematic errors is described in [4]. The quoted values of $A_{\text{FB}}^{b\bar{b}}$ based on a lepton tag and the ALEPH measurement using a vertex tag have all been corrected using the same LEP average value for the mixing parameter, $\lambda = 0.115 \pm 0.009 \pm 0.006$ [4]. The combined result is

$$A_{\text{FB}}^{b\bar{b}}(\sqrt{s} = 91.27 \text{ GeV}) = (9.4 \pm 0.5 \pm 0.2 \pm 0.3)\% \quad (\chi^2/(d.o.f.) = 3.8/5).$$

The three quoted errors are the statistical, that due to uncorrelated systematic errors and that due to common systematic uncertainties respectively. A LEP average of the $b\bar{b}$ asymmetry below and above the Z^0 peak is given in Table 13. In order to derive the pole asymmetry, $A_{\text{FB}}^{0,b}$ (defined in an analogous way to those for leptons in eqn. (12)), the corrections accounting for pure photon exchange and for the γZ^0 -interference, the s -dependence of the asymmetry in the vicinity of the peak and an unfolding of photonic and QCD corrections have to be applied. With these corrections one obtains $A_{\text{FB}}^{0,b} = 0.099 \pm 0.006$.

4.3.2 The c Quark Sector

Several methods to tag events originating from primary $c\bar{c}$ quarks have been used: One method is to extend the inclusive lepton analysis in the region of low p and p_T [63, 64, 65] where a sizable fraction of the events can be traced to primary c quarks. For the determination of $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$ the branching ratio $Br(c \rightarrow \ell X)$ is needed, which has been used from measurements at energies below the Z^0 -pole for the published results. ALEPH has also determined $\Gamma_{c\bar{c}}$ and $Br(c \rightarrow \ell X)$ simultaneously in a combined fit to inclusive single and dilepton distributions [4].

DELPHI has presented a neural network analysis based on event shape and lifetime variables to separate simultaneously $b\bar{b}$ and $c\bar{c}$ events in the hadronic sample [66].

Other methods are based on the reconstruction of $D^{*\pm}$ decays. The inclusive method exploits that the mass difference between the $D^{*\pm}$ and the D^0 is only a few MeV larger than the mass of the $\pi^\pm$ so that the decays $D^{*+} \rightarrow D^0\pi^+$ and $D^{*-} \rightarrow \bar{D}^0\pi^-$ result in an excess of tracks with very low transverse momentum with respect to the jet axis. DELPHI has analysed the p_T^2 distribution of charged tracks with momentum $1.5 < p [\text{GeV}] < 2.5$ [67]. Together with the probability for a c quark to fragment into a $D^{*\pm}$ from measurements at $\sqrt{s} = 10.55 \text{ GeV}$ and assuming this probability to be energy independent, the ratio $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$ has been derived. The dominant systematic error of this method arises from the modelling of the background from non $c\bar{c}$ reactions to the low p_T^2 region.

The reconstruction of exclusive decays of the $D^{*\pm}$ allows a determination of both, $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$ and the $c\bar{c}$ forward-backward asymmetry. The main decay channel investigated is [4, 68, 69, 70]:

$$D^{*+} \rightarrow \pi^+ D^0 \quad \text{with} \quad D^0 \rightarrow K^- \pi^+.$$

The high combinatorial background for the $D^0\pi$ mode can be largely suppressed by exploiting the low Q-value of the $D^* \rightarrow D^0\pi$ decay, yielding a prominent signal in an otherwise phase space suppressed region. The mass difference $\Delta M = M(K\pi\pi) - M(K\pi)$ for all events with a $K\pi$ combination in the region of the D^0 mass $1790 < M(K\pi) [\text{MeV}] < 1940$ is shown in Figure 19b). A significant enhancement is apparent at $\Delta M = 146 \text{ MeV}$. Cutting on the mass difference $143.5 < \Delta M [\text{MeV}] < 147.5$ one obtains the invariant mass spectrum for D^0 candidates shown in Figure 19a). A clear signal is visible around the nominal D^0 mass. The second peak around 1600 MeV is attributed to the decay $D^0 \rightarrow K^- \pi^+ \pi^0$ where the π^0 is not reconstructed. Using the product branching ratio that a D^* meson is produced from a primary c quark and then decays in the channel $D^* \rightarrow D^0\pi \rightarrow \pi(\pi K)$ from experiments below the Z^0 -pole the ratio $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$ can be determined.

Figure 20 summarizes the status of LEP measurements for $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$. As the two most precise measurements result from independent methods (inclusive leptons and the reconstruction of $D^* \rightarrow D^0\pi$) common systematic errors have been neglected in the average. Using the present LEP average, $\Gamma_{\text{had}} = 1740.3 \pm 5.9 \text{ MeV}$, we obtain:

$$\Gamma_{c\bar{c}} = 298 \pm 24 \text{ MeV}$$

in good agreement with the Standard Model prediction [71] $\Gamma_{c\bar{c}} = 297_{-3}^{+4} \text{ MeV}$. The inclusive lepton tag as well as the reconstruction of exclusive $D^{*\pm}$ decays have also been used to determine the forward-backward asymmetry of c quarks $A_{\text{FB}}^{c\bar{c}}$. Figure 21 shows the status of the $c\bar{c}$ forward-backward asymmetries. The evaluation of common systematic errors is described in [4]. The combined result is:

$$A_{\text{FB}}^{c\bar{c}}(\sqrt{s} = 91.25 \text{ GeV}) = (6.6 \pm 1.2 \pm 0.7 \pm 0.7)\% \quad (\chi^2/(d.o.f.) = 4.5/5).$$

The three quoted errors are the statistical, that due to uncorrelated systematic errors and that due to common systematic uncertainties, respectively. The $c\bar{c}$ asymmetry below and above the Z^0 peak has only been quoted by one experiment and is given in Table 13. Applying to $A_{\text{FB}}^{c\bar{c}}$ the equivalent corrections described for $A_{\text{FB}}^{b\bar{b}}$, one obtains for the pole asymmetry $A_{\text{FB}}^{0,c} = 0.075 \pm 0.015$.

4.3.3 The $q\bar{q}$ Charge Asymmetry

For any fermion f the average charge produced in the forward and backward hemispheres is given by $\langle Q_F^f \rangle = q_f A_{FB}^{ff}$ and $\langle Q_B^f \rangle = -q_f A_{FB}^{ff}$, leading to a charge flow $\langle Q_{FB}^f \rangle = 2q_f A_{FB}^{ff}$. Summed over 5 quark flavours the charge flow is:

$$\langle Q_{FB} \rangle = \sum_{f=(u,d,s,c,b)} 2q_f A_{FB}^{f\bar{f}} \frac{\Gamma_{f\bar{f}}}{\Gamma_{had}}$$

To measure a charge flow in hadronic events, the hadron charges within jets have to be related to the charge of the primary $q\bar{q}$ pair produced in the hard scattering process. The algorithms to recover these primary quark charges are based on the idea that the primary parton charge manifests itself in the leading hadrons. The interpretation of the measured distributions requires a detailed modelling of the fragmentation process, which introduces significant systematics into this measurement. For this observable not all experiments provide a value of the average charge asymmetry deconvoluted for detector and fragmentation effects, but rather quote an effective mixing angle, which has been derived from fitting the observed asymmetry to the Monte Carlo prediction. The LEP results are summarized in Table 14 in terms of $\sin^2 \theta_{eff}^{lept}$.

Using the Standard Model calculation with $M_Z = 91.187 \pm 0.007$ GeV and $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ the average value for $\sin^2 \theta_{eff}^{lept}$ corresponds, at parton level, to a hadronic charge asymmetry of:

$$\langle Q_{FB} \rangle = 0.0405 \pm 0.0032.$$

4.3.4 Inclusive Separation of Up and Down Type Quarks

Final state photons emitted in hadronic Z^0 decays are an interesting probe of strong and electroweak interactions. Their differential distributions have been used to study the interplay between quark and gluon radiation [75]–[83]. Three experiments have used the total production rate to study the Z^0 couplings of up and down type quarks [75, 78, 80, 83]. Defining $\mathcal{C}_q \equiv \hat{g}_a^{q^2} + \hat{g}_v^{q^2}$ and assuming universality among the couplings of up and down type quarks; i.e. $\mathcal{C}_u = \mathcal{C}_c$ and $\mathcal{C}_d = \mathcal{C}_s = \mathcal{C}_b$ ¹¹ the total hadronic width can be expressed in terms of $\mathcal{C}_q$:

$$\Gamma_{had} \propto \sum_q \mathcal{C}_q = 3\mathcal{C}_d + 2\mathcal{C}_u.$$

The final state photon yield $N_{q\bar{q}\gamma}$ is proportional to the square of quark charges e_q^2 of the produced quarks:

$$N_{q\bar{q}\gamma} \propto \sum_q e_q^2 \mathcal{C}_q = \frac{1}{9}(3\mathcal{C}_d + 8\mathcal{C}_u).$$

Combining both measurements allows the determination of $\mathcal{C}_u$ and $\mathcal{C}_d$ or, alternatively, $\Gamma_{u\bar{u}}$ and $\Gamma_{d\bar{d}}$ separately. The results derived from the final state photon analysis are summarized in Table 15.

Combining the LEP average for $3\mathcal{C}_d + 8\mathcal{C}_u$ obtained from final state photons with the LEP average $\Gamma_{had} = 1740.3 \pm 5.9$ MeV we obtain

$$\begin{aligned} \mathcal{C}_d &= 0.405 \pm 0.026 & \Gamma_{d\bar{d}} &= 419 \pm 27 \text{ MeV} \\ \mathcal{C}_u &= 0.236 \pm 0.038 & \Gamma_{u\bar{u}} &= 244 \pm 39 \text{ MeV} \end{aligned}$$

in agreement with the Standard Model prediction [71] for $\frac{1}{3}(\Gamma_{d\bar{d}} + \Gamma_{ss} + \Gamma_{b\bar{b}}) = 381 \pm 3$ MeV and $\frac{1}{2}(\Gamma_{u\bar{u}} + \Gamma_{c\bar{c}}) = 297_{-3}^{+4}$ MeV. Note that the results for $\Gamma_{u\bar{u}}$ and $\Gamma_{d\bar{d}}$ are fully anticorrelated.

¹¹In the Standard Model small deviations of 2-3% are expected for the couplings of the b quark.

5 Interpretation of Results

5.1 Standard Model Constraints

In this section we first want to show that all available data are consistent with a unique set of Standard Model parameters and then combine the data to constrain the model's unknown input parameters.

Table 16 summarizes the measurements included in this analysis. We have chosen only those measurements where the errors are comparable to the expected variation of the Standard Model prediction when varying the top mass in the range $90 \leq M_t$ [GeV] ≤ 200 or α_s in the range $0.11 < \alpha_s(M_Z^2) < 0.13$. The results from LEP data will then be compared to further electroweak precision tests from ν N-scattering and $p\bar{p}$ colliders, also listed in Table 16. The measurements in comparison to their Standard Model prediction as a function of M_t are shown in Figure 22. The bands in the Standard Model predictions reflect the expected variation of each quantity due to a variation of the strong coupling constant $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ [89] and M_H in the interval $60 \leq M_H$ [GeV] ≤ 1000 for $M_Z = 91.187$ GeV.

In the Standard Model framework any observable X can be parametrized as:

$$X = f(\alpha, G_F, M_Z, M_t, M_H, \alpha_s). \quad (26)$$

Before applying eqn. (26) blindly to the data we would like to map the data onto a common scale in order to demonstrate their internal consistency. There are several ways to achieve this.

The scale which is most commonly chosen is undoubtedly the effective mixing angle, $\sin^2\theta_{eff}^{lept}$, as defined in eqn. (20). For leptonic forward-backward asymmetries and τ polarization asymmetries $\sin^2\theta_{eff}^{lept}$ can be calculated from the results quoted without making any strong model specific assumptions, with the exception of lepton universality. This also holds for the quark forward-backward asymmetries $A_{FB}^{b\bar{b}}$ and $A_{FB}^{c\bar{c}}$ as quark asymmetries have a reduced sensitivity to the hadronic couplings and therefore to corrections particular to the hadronic vertex. The determination of $\sin^2\theta_{eff}^{lept}$ from the hadronic charge asymmetry, $\langle Q_{FB} \rangle$, involves additional assumptions as $\langle Q_{FB} \rangle$ depends on the fractional composition of the primary quark flavours in the hadronic sample. As mentioned in Subsection 3.7 partial widths could in principal also be mapped on the variable $\sin^2\theta_{eff}^{lept}$, but the use of a relation between the effective ρ parameter and an effective mixing angle introduces a strong model dependence. Figure 23 shows the result from various determinations of $\sin^2\theta_{eff}^{lept}$.

It should be stressed that for the purpose of mapping all measurements onto a single variable, comparing their accuracy and checking their consistency, $\sin^2\theta_{eff}^{lept}$ is just one possible choice. Inverting eqn. (26) and expressing each observable as a measurement of M_Z :

$$M_Z = g(\alpha, G_F, X, M_t, M_H, \alpha_s) \quad (27)$$

enables us to choose any other variable predicted by the Standard Model as common scale by repeated application of eqn. (26). Of course, it may be argued, that this scale is more model dependent than $\sin^2\theta_{eff}^{lept}$ but it also fulfills the purpose above. Among all possible scales we have chosen as an example M_Z as it is the most precise measurement, LEP can provide. Figure 24 confronts the direct measurement of M_Z with the prediction for M_Z from three groups of observables: i) hadronic and leptonic cross sections and leptonic forward-backward asymmetries; ii) polarization and quark forward-backward asymmetries; iii) the non-LEP precision measurements listed in Table 16. The resulting ± 1 standard deviation bands are shown as a function of M_t for M_H fixed to 300 GeV. The

value of the strong coupling constant has been constrained to $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ [89]. All bands overlap for a top mass ranging from 145–175 GeV.

The purpose of Figure 24 is to illustrate the consistency of the measurements, but it does not allow the derivation of confidence intervals for M_t . The one standard deviation confidence intervals for M_t , resulting from a fit to the LEP data only, and a fit to the full set of input data in Table 16, are given in Table 17. For the quoted results the two dominant theoretical uncertainties have been propagated in the fits. These are due to the uncertainties arising from the light quark contribution to the vacuum polarization and to the uncertainties from the choice of scale in the heavy quark loops. The magnitude of these two theoretical uncertainties has been taken from [29]. Taking these into account slightly affects the central value of M_t by about 2 GeV and increases its uncertainty by about 1 to 2 GeV. Therefore, at present their effect in the top mass determination is small, but will be significant for the anticipated accuracy of future LEP results.

Note, that for the fits in Table 17 no external constraint has been imposed on the value of $\alpha_s(M_Z^2)$. The result for $\alpha_s(M_Z^2)$ from electroweak precision tests is complementary in all aspects of systematic and theoretical uncertainties to the value determined in traditional QCD analyses based on event topologies and it is of similar accuracy.

Figure 25 shows, for the fit in Table 17 column 3, the χ^2 value as a function of M_t for the three values of M_H considered in Table 17. These curves demonstrate that the data impose stringent constraints on the mass of the yet elusive top quark.

With the direct determination of M_t and future improvements in the accuracy of LEP results we may also hope to obtain interesting constraints on M_H . At present, the increase in χ^2 when M_H is changed between 60 GeV and 1000 GeV is about 2.0, which does not allow the derivation of any meaningful constraints on M_H . In addition, most of the present increase of χ^2 when varying M_H from 60 GeV to 1000 GeV is likely to arise from statistical fluctuations of the data around the Standard Model prediction which are still large compared to the effects due to the variation of M_H (for a discussion of statistical biases in the determination of M_H from electroweak precision tests we refer to [90, 91] and the comments in [4]).

5.2 Constraints from the Z^0 Total and Partial Widths

One of the major goals of e^+e^- physics at the Z^0 -pole is the determination of the partial width of the Z^0 into invisible channels, Γ_{inv} . In the Standard Model the only channels contributing to Γ_{inv} are massless neutrinos. The number of generations and therefore the number of light neutrino species, N_ν , are not predicted by the Standard Model. Upper limits from experiments before the start of Z^0 -pole physics were ranging between 4–5 [92]. The first results of N_ν from LEP [93] were based on an integrated luminosity of $\approx 190 \text{ nb}^{-1}$, collected during the first 15 days of physics running. At that time N_ν was determined from the hadronic line shape alone, assuming the Standard Model prediction for all partial widths:

$$\sigma_h^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_{\ell\ell}^{SM} \Gamma_{\text{had}}^{SM}}{\Gamma_Z^2} \quad \text{with} \quad \Gamma_Z = \Gamma_{\text{had}}^{SM} + (3 + \delta_m) \Gamma_{\ell\ell}^{SM} + N_\nu \Gamma_{\nu\nu}^{SM},$$

where $\delta_m = -0.0023$ [20] represents a small correction for the mass of the tau lepton. With this method, the dominant sensitivity to N_ν was given by the absolute normalization, reflected in σ_h^0 , rather than by the measurement of the total width. With increased statistics the partial widths for hadrons and leptons could be determined separately, allowing a model independent determination of

Γ_{inv} :

$$\Gamma_{\text{inv}} = \Gamma_Z^{\text{meas}} - \Gamma_{\text{had}}^{\text{meas}} - (3 + \delta_m)\Gamma_{\ell\ell}^{\text{meas}}.$$

Table 18 summarizes the current status of the determination of partial widths at LEP.

Any partial width shows a non-negligible top mass dependence due to radiative corrections. As most of these corrections are universal, the bulk of this dependence cancels in the ratio of partial widths. In the Standard Model and in extensions which only introduce universal corrections to partial widths, it is therefore more advantageous to derive N_ν from the ratio $\Gamma_{\text{inv}}/\Gamma_{\ell\ell}$. Comparing the present LEP average:

$$\Gamma_{\text{inv}}/\Gamma_{\ell\ell} = 5.936 \pm 0.054$$

to the Standard Model prediction [71] $\Gamma_{\nu\nu}/\Gamma_{\ell\ell} = 1.992 \pm 0.003$ results in:

$$N_\nu = 2.980 \pm 0.027.$$

This result is supported by direct measurements of the Z^0 invisible width at LEP by counting the number of events with only a single photon but otherwise no further activity in the detector [94, 95, 96].

We do not necessarily expect an integer number for N_ν . A heavy neutrino with Standard Model couplings contributes to N_ν with:

$$\delta N_\nu = \left(1 - 4\frac{M_\nu^2}{s}\right)^{1/2} \left(1 - \frac{M_\nu^2}{s}\right).$$

With the present accuracy such a heavy neutrino can be excluded for $M_\nu < 45.5$ GeV at the 95% confidence level. The experimental determination of Γ_{inv} at LEP is a powerful constraint on various extensions of the Standard Model predicting new particles that couple to the Z^0 . The resulting limits on light and heavy neutrinos have an important impact on our understanding of cosmological and astrophysical models.

Below we generalize the study above, giving limits on new physics which may occur in the visible channels as well. We express the inclusive partial widths Γ_{had} , $\Gamma_{\ell\ell}$ and Γ_{inv} as the sum of their Standard Model contributions and some additional (not necessarily positive) contributions, $\Sigma_{\text{x, had}}$, $\Sigma_{\text{x, } \ell\ell}$ and $\Sigma_{\text{x, inv}}$, respectively:

$$\begin{aligned}\Gamma_{\text{had}} &= \Gamma_{\text{had}}^{\text{SM}} + \Sigma_{\text{x, had}} \\ \Gamma_{\ell\ell} &= \Gamma_{\ell\ell}^{\text{SM}} + \Sigma_{\text{x, } \ell\ell} \\ \Gamma_Z &= \Gamma_Z^{\text{SM}} + \Sigma_{\text{x}}\end{aligned}$$

with $\Sigma_{\text{x}} = \Sigma_{\text{x, had}} + 3\Sigma_{\text{x, } \ell\ell} + \Sigma_{\text{x, inv}}$. Here $\Sigma_{\text{x, } \ell\ell}$ has to be interpreted as the average contribution to the three known leptonic channels. We furthermore assume that Γ_Z , σ_h^0 , R_ℓ and $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ are the only measurements in Table 16 which can be modified with respect to their Standard Model prediction¹². The results of a fit where we treat, besides M_Z , M_t and α_s , also $\Sigma_{\text{x, had}}$, $\Sigma_{\text{x, } \ell\ell}$ and Σ_{x} as free parameters are listed in Table 19. Figure 26 shows χ^2 curves as a function of Σ_{x} . From these curves we derive a 95% confidence level upper bound of 8.5 MeV on Σ_{x} , allowing positive as well as negative contributions. If one interpretes Σ_{x} as partial width, Γ_{x} , of a new particle x, one has to impose the constraint, that all contributions have a positive sign, resulting in

$$\Gamma_{\text{x}} < 13.9 \text{ MeV} \quad 95\% \text{ CL}.$$

This result implies that the branching ratio of the Z^0 to any new particle is less than $5.6 \cdot 10^{-3}$ (95% CL), irrespective of its decays.

¹²This assumption is not unreasonable for positive additional contributions, which may be due to additional particles with very little impact on the couplings to known fermions. For negative additional contributions the assumption is of course very arbitrary, as these may originate from effects like fermion mixing or radiative corrections, which are likely to affect the fermion couplings in a collective way.

5.3 The Electroweak Couplings of Fermions

Below we want to establish the level of sensitivity for anomalies related to specific fermions. The Standard Model predicts relations among the couplings of fermions to the electroweak gauge bosons. Any violation of these relations indicates the presence of new physics. For example deviations from lepton universality may arise from the mixing of the conventional leptons with new lepton species [97]. Precision tests at the Z^0 -pole are a unique tool to test the hypothesis of lepton universality for the neutral current couplings of charged leptons. We will also summarize those tests of lepton universality in charged current reactions which receive significant contributions from LEP. LEP offers several methods to determine the neutral current couplings of quarks, an area which the very precise results in νN -scattering have dominated up to now. We will show that results from LEP are complementary and of similar accuracy.

5.3.1 Lepton Universality in Neutral Current Reactions

For the determination of the neutral current couplings of charged leptons we combine the Z^0 -pole averages for hadronic and leptonic cross sections and leptonic forward-backward asymmetries (Table 5) with $\mathcal{A}_\tau$ and $\mathcal{A}_e$ from the τ polarization asymmetries (Table 11 and 12). The results of a fit to the effective vector and axial-vector couplings of leptons with and without the assumption of universality are summarized in Table 20. The ratios of e , μ and τ couplings are:

$$\begin{aligned}\hat{g}_a^\mu / \hat{g}_a^e &= 1.0006 \pm 0.0026, \quad \hat{g}_a^\tau / \hat{g}_a^e = 0.9990 \pm 0.0029, \\ \hat{g}_v^\mu / \hat{g}_v^e &= 0.77 \pm 0.21, \quad \hat{g}_v^\tau / \hat{g}_v^e = 1.00 \pm 0.13.\end{aligned}$$

Figure 27 shows the one standard deviation contours in the $\hat{g}_a^\ell$ - $\hat{g}_v^\ell$ plane. The axial-vector couplings are essentially determined by the leptonic partial widths, whereas the asymmetry data measure $\hat{g}_v^\ell / \hat{g}_a^\ell$. The error on the electron axial-vector coupling is roughly $\sqrt{2}$ times smaller than the others as the electron couplings enter in both initial and final state for the process $e^+e^- \rightarrow e^+e^-$. The accuracy of the vector coupling for the τ lepton significantly profits from the measurement of $\mathcal{P}_\tau$. The results are in very good agreement with the hypothesis of lepton universality in the neutral current sector.

5.3.2 Lepton Universality in Charged Current Reactions

Universality in charged current reactions between electron and muon has been established with high accuracy in low energy experiments resulting in $(g_e/g_\mu) = 0.9989 \pm 0.0016$ [98]. High statistics and modern detector technology enable LEP to make significant contributions in exploring this question for the τ lepton. Lepton universality for the τ lepton can be tested by comparing the decay widths for muons and tau leptons into an electron and two neutrinos:

$$\begin{aligned}\Gamma(\mu \rightarrow e\nu\nu) &= \frac{1}{\tau_\mu} = \frac{G_\mu^2}{192} \frac{m_\mu^5}{\pi^3} (1 + \delta) \\ \Gamma(\tau \rightarrow e\nu\bar{\nu}) &= \frac{Br(\tau \rightarrow e\nu\bar{\nu})}{\tau_\tau} = \frac{G_\tau^2}{192} \frac{m_\tau^5}{\pi^3} (1 + \epsilon).\end{aligned}\tag{28}$$

Here G_μ represents the Fermi constant and G_τ the equivalent for τ decays; τ_μ (τ_τ) and m_μ (m_τ) the lifetime and the mass of the muon (tau); δ and ϵ are small quantities arising from phase space and

radiative corrections [99]. Neglecting phase space and radiative corrections eqns. (28) result in:

$$\left(\frac{G_\tau}{G_\mu}\right)^2 = \left(\frac{\tau_\mu}{\tau_\tau}\right) \left(\frac{m_\mu}{m_\tau}\right)^5 Br(\tau \rightarrow e\nu\bar{\nu}). \quad (29)$$

Going from the Fermi theory to the Glashow-Salam-Weinberg model, $G_\mu \propto g_\mu g_e$ and $G_\tau \propto g_\tau g_e$, resulting in $(G_\tau/G_\mu) = (g_\tau/g_\mu)$. Earlier data [100] give a ratio of $g_\tau/g_\mu = 0.972 \pm 0.012$ which differs from unity by 2.3 standard deviations. Since then the accuracy of m_τ has been improved substantially by a measurement of BES [49, 101] and new results on $Br(\tau \rightarrow e\nu\bar{\nu})$ became available from CLEO, ARGUS and LEP. Furthermore LEP contributed with a very accurate determination of the τ lifetime. Assuming e- μ universality by using for $Br(\tau \rightarrow e\nu\bar{\nu})$ a phase space corrected average of $Br(\tau \rightarrow e\nu\bar{\nu})$ and $Br(\tau \rightarrow \mu\nu\bar{\nu})$ the new measurements of the τ parameters result in [49]:

$$(g_\tau/g_\mu) = 0.996 \pm 0.006.$$

5.3.3 Combined Analysis of Quark and Lepton Neutral Current Couplings

The determination of the effective neutral current couplings of charged leptons has been presented above. In this subsection we want to extend the scope of the analysis and include the determination of neutrino and quark couplings.

In e^+e^- -collisions the only observable which is sensitive to neutrino couplings is the partial width of the Z^0 into invisible decay channels, Γ_{inv} . There are two complementary measurements of Γ_{inv} :

- The indirect method based on the measurement of the total width of the Z^0 in combination with the measurement of all partial widths into visible decay channels, resulting in $\Gamma_{\text{inv}} = 497.6 \pm 4.3$ MeV (see Section 5.2).
- The direct measurement of Γ_{inv} from an analysis of the yield of single photons which can be related to the reaction $Z^0 \rightarrow \nu\bar{\nu}\gamma$. The unweighted LEP average for this measurement is $\Gamma_{\text{inv}} = 491 \pm 30$ MeV [94, 95, 96].

To check the agreement between both methods is an important test of the Standard Model. For the determination of fermion couplings we will restrict ourselves, however, to the input data from the far more precise indirect method.

The measurements which carry information on quark couplings are: Γ_Z , σ_h^0 , R_ℓ , $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$, $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$, $A_{\text{FB}}^{b\bar{b}}$, $A_{\text{FB}}^{c\bar{c}}$, $\langle Q_{FB} \rangle$ and the final state photon yield, $N_{q\bar{q}\gamma}$. The determination of the effective neutral current couplings of quarks is, however, less straightforward than for leptons, because all the observables above also depend on QCD corrections. Furthermore, the forward-backward asymmetries of quarks are much more strongly dependent on the initial state electron couplings than on the quark couplings. The leptonic couplings also enter in Γ_Z and σ_h^0 . Lepton universality being established to a high level of accuracy, we will impose this assumption in the following analysis.

In the determination of effective vector and axial-vector couplings of quarks we also encounter a technical problem: Partial widths as well as the forward-backward asymmetry at the peak are symmetric with respect to an interchange of $\hat{g}_v^f$ and $\hat{g}_a^f$. For the charged lepton couplings this problem no longer manifests itself as the precision of the leptonic data is high enough to allow the fit to find a stable local minimum. The situation is different for quarks, especially for the b-quark, as $\hat{g}_v^b$ and

$\hat{g}_a^b$ are close in value to each other as compared to the present experimental accuracy. There are two solutions to this problem, described below.

One solution is a transformation to the effective chiral neutral current couplings:

$$\hat{g}_L^f \equiv (\hat{g}_v^f + \hat{g}_a^f)/2 \quad (30)$$

$$\hat{g}_R^f \equiv (\hat{g}_v^f - \hat{g}_a^f)/2 \quad (31)$$

Expressing partial widths and forward-backward asymmetries at the Z^0 -pole in terms of effective chiral couplings:

$$\Gamma_{f\bar{f}} \propto \hat{g}_L^{f^2} + \hat{g}_R^{f^2} \quad (32)$$

$$A_{\text{FB}}^{0,\ell}(\text{f}\bar{\text{f}}) \propto \frac{\hat{g}_L^{f^2} - \hat{g}_R^{f^2}}{\hat{g}_L^{f^2} + \hat{g}_R^{f^2}}. \quad (33)$$

The measurement of the forward-backward asymmetry of quarks therefore allows a discrimination between $\hat{g}_L^{f^2}$ and $\hat{g}_R^{f^2}$ as soon as the data are accurate enough to determine the sign of the forward-backward asymmetries.

The second solution is to include data of the quark forward-backward asymmetries, which are off-peak and therefore measure the contribution of the γZ^0 -interference to the forward-backward asymmetry. Looking at eqn. (10) we see that the energy dependence of the forward-backward asymmetry in the vicinity of the peak is proportional to $\hat{g}_a^f / (\hat{g}_a^{f^2} + \hat{g}_v^{f^2})$ i.e. no longer symmetric in $\hat{g}_a^f$ and $\hat{g}_v^f$.

We will adopt the latter solution by including $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$ and $A_{\text{FB}}^{\text{c}\bar{\text{c}}}$ by including not only $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$ and $A_{\text{FB}}^{\text{c}\bar{\text{c}}}$ at the Z^0 peak but also the off-peak values in Table 13. As an exclusive separation of primary quark flavours in e^+e^- reactions has been achieved so far only for b and c quarks and as we expect small deviations between the couplings of b quarks and other down type quarks even within the framework of the Standard Model due to radiative corrections, we want to perform an analysis which does not impose any assumptions on quark universality. We therefore only add to the measurements which were used previously to determine the leptonic couplings $\Gamma_{\text{b}\bar{\text{b}}}/\Gamma_{\text{had}}$, $\Gamma_{\text{c}\bar{\text{c}}}/\Gamma_{\text{had}}$, $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$ and $A_{\text{FB}}^{\text{c}\bar{\text{c}}}$ and we treat R_ℓ as free parameter of the fit¹³. To account for QCD corrections to $\Gamma_{\text{b}\bar{\text{b}}}$, $\Gamma_{\text{c}\bar{\text{c}}}$, $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$ and $A_{\text{FB}}^{\text{c}\bar{\text{c}}}$ we vary in addition the strong coupling constant in the range $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ [89]. The results of such a fit to the effective vector and axial-vector couplings of leptons and quarks are given in Table 21.

Figure 28 displays the one standard deviation contours for the effective couplings of b and c quarks. The size of Figure 27 for the leptonic couplings is indicated in Figure 28 by a small rectangle. The fit also determines $\hat{g}_v^{\nu^2} + \hat{g}_a^{\nu^2}$. Within the Standard Model context $\hat{g}_v^\nu = \hat{g}_a^\nu$ and the result can be interpreted as a precision determination of $2\hat{g}_a^{\nu^2}$. In the $\hat{g}_v^f - \hat{g}_a^f$ plane the result can be displayed as a circle with a width corresponding to the $\pm$ one standard deviation error of the measurement. Similarly the results for $\mathcal{C}_d$ and $\mathcal{C}_u$ from the analysis of final state radiation in hadronic events (see subsection 4.3.4) can be visualized as circles in this plane. The comparison with the contours for b and c quarks supports the hypothesis of universality among up type and among down type quarks.

We also give in Table 21 the transformation to the effective chiral couplings and the comparison with the determination of up and down quark neutral current couplings in νN -scattering. We see that LEP now provides complementary information for the couplings of b and c quarks with comparable accuracy. The comparison again supports the hypothesis of universality among up type and down type quarks. The experimental results are in good agreement with the Standard Model prediction [71], also given in Table 21. The comparison of the measurement errors for quark couplings with the variation of the Standard Model prediction with M_t and M_H shows, however, that the sensitivity of the measurement of quark couplings is not yet sufficient to probe effects of similar size.

¹³This effectively amounts to treating Γ_{had} as free parameter.

5.4 A Test for New Physics Based on ‘ ϵ ’ Parameters

In the Standard Model the effective couplings of fermions to the Z^0 share common contributions from radiative corrections which depend on yet unknown parameters, like the masses of the top quark and the Higgs boson. As new physics can be more easily disentangled if not masked by our ignorance of Standard Model parameters, several authors have proposed variables which are free from large M_t dependences and therefore particularly sensitive to new physics. In this section we investigate one specific approach, proposed in [5], which tries to minimize the assumptions that need to be made about the contribution from possible new physics, which may enter either via radiative corrections or as a modification to the Born diagram.

The analysis below is based on a definition of an effective mixing angle, s_0^2 , which contains only pure QED corrections:

$$s_0^2 c_0^2 \equiv \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F M_Z^2} \quad (34)$$

with $c_0^2 = 1 - s_0^2$. The fine structure constant at $s = M_Z^2$, $\alpha(M_Z) = \alpha/(1 - \Delta\alpha(M_Z))$, has been evaluated with $\Delta\alpha(M_Z = 91.187 \text{ GeV}) = 0.0599 \pm 0.0009$ resulting in $s_0^2 = 0.2313 \pm 0.0003$, where the quoted errors are due to the uncertainty in the contribution of light quarks to the photon vacuum polarization [29].

The authors of [5] have chosen as primary defining measurements of their parameters the leptonic partial width, $\Gamma_{\ell\ell}$:

$$\Gamma_{\ell\ell} = \frac{G_F M_Z^3}{6\pi\sqrt{2}} \left(\hat{g}_v^2 + \hat{g}_a^2 \right) R_{QED}^\ell, \quad (35)$$

where the factor R_{QED}^ℓ arises from final state photonic corrections with:

$$R_{QED}^\ell = 1 + \frac{3}{4} \frac{\alpha}{\pi} Q_f^2, \quad (36)$$

and the leptonic pole asymmetry, $A_{FB}^{0,\ell}$:

$$A_{FB}^{0,\ell} = 3 \frac{\hat{g}_v^2 \hat{g}_a^2}{\left(\hat{g}_v^2 + \hat{g}_a^2 \right)^2}. \quad (37)$$

To exhibit the dependence of $\hat{g}_a$ and $\hat{g}_v$ upon radiative corrections, these can be written in terms of two parameters, $\Delta\rho$ and $\Delta\kappa$, which both contain a strong dependence upon M_t , and which for large M_t become proportional to M_t^2 [20, 29]:

$$\hat{g}_a = I_f^3 \sqrt{1 + \Delta\rho} \quad (38)$$

$$\frac{\hat{g}_v}{\hat{g}_a} = [1 - 4(1 + \Delta\kappa)s_0^2]. \quad (39)$$

Here I_f^3 refers to the weak isospin of fermion species f . To separate the strong M_t dependence the authors of [5] perform a parameter transformation:

$$\epsilon_1 \equiv \Delta\rho \quad (40)$$

$$\epsilon_3 \equiv c_0^2 \Delta\rho + (c_0^2 - s_0^2) \Delta\kappa. \quad (41)$$

The strong quadratic M_t dependence of $\Delta\rho$ and $\Delta\kappa$ cancels in ϵ_3 ¹⁴.

¹⁴In [5] a parameter ϵ_2 , related to a precise measurement of M_W , is also defined. This parameter, however, does not enter into this analysis.

With the definition of $\hat{g}_i$ and $\hat{g}_a$ via eqns. (35) and (37) one can extend the analysis to further leptonic observables if one assumes the validity of the improved Born approximation with:

$$\hat{g}_i^\ell = \hat{g}_i \quad \text{and} \quad \hat{g}_a^\ell = \hat{g}_a . \quad (42)$$

The definition of $\Delta\rho$ and $\Delta\kappa$ already contains some small vertex corrections which are largely independent of M_t . If one wishes to apply the analysis also to observables which involve not only lepton couplings, then one must allow for small fermion dependent differences in these vertex corrections. This is achieved by writing the effective fermion couplings in the following form:

$$\hat{g}_a^f = I_f^3 \sqrt{1 + \epsilon_1} + \Delta_a^f \quad (43)$$

$$\hat{g}_i^f = I_f^3 \sqrt{1 + \epsilon_1} [1 - 4|Q_f|(1 + \Delta\kappa)s_0^2] + \Delta_i^f . \quad (44)$$

For all fermions $f \neq b$ the offsets accounting for the non-universal parts in the vertex corrections, Δ_a^f and Δ_i^f , are also largely independent of M_t . Also, to a very good approximation one has $\Delta_a^f = \Delta_i^f = \Delta^f$. In the analysis below we will use the following values of Δ^f , computed by the program ZFITTER [26]: $\Delta^f = 0.16 \cdot 10^{-3}$ for u and c quarks, $\Delta^f = -0.39 \cdot 10^{-3}$ for d and s quarks and $\Delta^f = 0.68 \cdot 10^{-3}$ for neutrinos.

As the $b\bar{b}$ vertex exhibits a strong top mass dependence it needs special consideration. This motivates the introduction of a further defining measurement, the $b\bar{b}$ partial decay width:

$$\Gamma_{b\bar{b}} = \sqrt{1 - 4\mu_b} \frac{G_F M_Z^3}{2\pi\sqrt{2}} \left((1 + 2\mu_b)(\hat{g}_a^{b^2} + \hat{g}_i^{b^2}) - 6\hat{g}_a^{b^2}\mu_b \right) R_{QCD}^b R_{QED}^b . \quad (45)$$

with $\mu_b = (m_b/M_Z)^2$, R_{QED}^b given in eqn. (36) and R_{QCD}^b denoting the QCD correction to $\Gamma_{b\bar{b}}$ [22]. To a good approximation $\hat{g}_a^{b^2}$ and $\hat{g}_i^{b^2}$ can be written in terms of a single additional free parameter ϵ_b , which is defined in [5] via the relations:

$$\hat{g}_a^{b^2} = I_f^3 \sqrt{1 + \epsilon_1} (1 + \epsilon_b) \quad (46)$$

$$\frac{\hat{g}_i^{b^2}}{\hat{g}_a^{b^2}} = \frac{[1 - 4|Q_b|(1 + \Delta\kappa)s_0^2] + \epsilon_b}{1 + \epsilon_b} . \quad (47)$$

The parameter ϵ_3 has a top mass dependence which is complementary to ϵ_1 .

Deriving ϵ_1 and ϵ_3 from the defining measurements, $\Gamma_{\ell\ell}$ and $A_{FB}^{0,\ell}(\ell^+\ell^-)$ is a pure parameter transformation. Extending the analysis to additional observables needs further assumptions to maintain a consistent parameter definition, such as the validity of the improved Born approximation, the validity of QCD corrections, the validity of the vertex corrections and the assumption, that the observed invisible width of the Z^0 is accounted for by three massless neutrino species.

We have followed an analysis strategy which allows one to keep track of the assumptions used. The results for the fits described below are summarized in Table 22:

- In ‘Fit 1’ ϵ_1 and ϵ_3 have been derived from leptonic data only. Input data to this fit are leptonic and hadronic cross sections, lepton forward-backward asymmetries and $\mathcal{A}_\tau$ and $\mathcal{A}_e$ from τ polarization asymmetries. The complications inherent to the hadronic sector are avoided by treating Γ_Z and σ_h^0 as free parameters constrained by their inputs.
- In ‘Fit 2’ we add to the input data above $\Gamma_{b\bar{b}}/\Gamma_{had}$. $\Gamma_{b\bar{b}}$ is parametrized based on eqns. (45–47). QCD corrections to $\Gamma_{b\bar{b}}$ are calculated with α_s being an additional parameter constrained by $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. The fit now determines ϵ_3 as well as ϵ_1 and ϵ_2 .

- In ‘Fit 3’ we parametrize Γ_{had} in terms of quark couplings using the quark vertex corrections given above and we assume $\Gamma_{\text{inv}} = 3\Gamma_{\nu\nu}(\epsilon_1, \epsilon_3, \Delta^\nu)$. This allows to parametrize σ_h^0 and Γ_Z in terms of 5 parameters: M_Z , ϵ_1 , ϵ_3 , ϵ_b and α_s .
- In ‘Fit 4’ we add to the input data of ‘Fit 3’ $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$, $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$, $A_{\text{FB}}^{\text{c}\bar{\text{c}}}$ and $\langle Q_{\text{FB}} \rangle$.
- ‘Fit 5’ is identical to ‘Fit 4’ but without the constraint $\alpha_s(M_Z^2) = 0.123 \pm 0.006$.

The different assumptions imposed lead to results which are compatible within the present accuracy of the data.

The ϵ parameters in Table 22 are all compatible with zero. This means that an improved Born approximation which takes into account only the running of the electromagnetic fine structure constant happens to be a good representation of our data. Within the Standard Model this observation points towards a particular range of top quark masses where certain electroweak corrections cancel.

The solid contours in Figure 29 represent the one standard deviation parameter contours for ‘Fit 5’ without α_s constraint. Also indicated is the Standard Model prediction [71] for the fitted parameters. The results are in good agreement with the Standard Model. The correlations among the parameters are small, except for a correlation of 0.89 between ϵ_1 and ϵ_3 and an anticorrelation of -0.84 between ϵ_b and α_s . This strong correlation between ϵ_b and α_s is also responsible for the increased error of $\alpha_s(M_Z^2)$ with respect to the Standard Model fit (see Table 17), where the $Z^0\text{b}\bar{\text{b}}$ vertex is fixed by the theory. The constraints of QCD corrections from future electroweak precision tests will be derived from $R_\ell = \Gamma_{\text{had}}/\Gamma_{\ell\ell}$, $\sigma_{\mu\mu}^{\text{pole}}$ and Γ_Z . All these variables also depend on the $\text{b}\bar{\text{b}}$ vertex. Therefore the interpretation of ϵ_b relies strongly on an independent determination of $\alpha_s(M_Z^2)$ in order to disentangle our understanding of QCD corrections to quark partial widths from electroweak effects particular to the $\text{b}\bar{\text{b}}$ vertex. To illustrate the effect of an independent determination of α_s we also display in Figure 29, as dashed contours, the result of ‘Fit 4’ with α_s constrained.

Assuming that new physics does enter only through vacuum polarization corrections, the parameters ϵ_1 , ϵ_2 and ϵ_3 can be related to the variables S , T , and U , proposed in [102] to investigate the possible origin of the symmetry breaking sector. Using the relations $\epsilon_1 = \epsilon_1^0 + \alpha T$ and $\epsilon_3 = \epsilon_3^0 + [\alpha/(4s_0^2)]S$, where $\epsilon_1^0 = 2.87 \cdot 10^{-3}$ and $\epsilon_3^0 = 6.10 \cdot 10^{-3}$ define a common origin for $M_Z = 91.187$ GeV, $M_t = 150$ GeV, $M_H = 300$ GeV and $\alpha_s(M_Z^2) = 0.123$, we obtain from ‘Fit 4’ in Table 22:

$$\begin{aligned} T &= -0.16 \pm 0.42 \\ S &= -0.33 \pm 0.37. \end{aligned}$$

The virtue of the ϵ or S , T , U variables is that they can easily be calculated in extensions of the Standard Model. For a recent comparison of electroweak precision tests with specific models we refer to [103].

6 Summary and Outlook

A wealth of electroweak precision tests has become available from the LEP experiments, some already reaching today an accuracy at the $\mathcal{O}(0.1\%)$ level. All measurements presented show good agreement among the experiments and with the Standard Model prediction.

In the first running phase of LEP the measurement of the invisible width of the Z^0 has determined unambiguously the number of light neutrino species. Today this measurement constitutes a precision

determination of the Z^0 coupling to neutrinos. An analysis of electroweak precision data at LEP limits the Z^0 partial width of any new particle to 13.9 MeV irrespective of its decay modes.

The data allow tests of lepton universality in neutral current reactions with an unprecedented precision. The measurement of production rates and forward-backward asymmetries of b and c quarks has reached a sensitivity which allows the separation of quark and lepton couplings. A comparison with results from νN scattering or with measurements of final state radiation in hadronic events provides an experimental verification of universality among up and among down type quarks. Contrary to lepton couplings, the accuracy of quark couplings is still low compared to the effects expected from radiative corrections in the Standard Model framework.

The combination of available measurements provides a significant constraint on the mass of the top quark. The best current estimate is:

$$M_t = 164_{-17}^{+16} {}_{-21}^{+18} \text{ GeV}.$$

where the central value and the first error refer to a mass of the Higgs boson of $M_H = 300$ GeV, fixed, and the second error refers to Higgs masses spanning the interval $60 \leq M_H [\text{GeV}] \leq 1000$. The same set of electroweak precision tests provides simultaneously a determination of $\alpha_s(M_Z^2)$ which is complementary in all aspects of experimental and theoretical uncertainties to the methods used in traditional QCD analyses and of similar accuracy:

$$\alpha_s(M_Z^2) = 0.120 \pm 0.006 \pm 0.002.$$

the second error reflecting again the ignorance of M_H .

In the year 1993 LEP has continued scanning with frequent energy calibrations based on the method of resonant depolarization. One is hoping to reach a precision of about 3 MeV for both M_Z and Γ_Z . Note, that the attempted relative precision on M_Z compares to that of the Fermi constant G_F . The experiments have upgraded their luminosity monitors and aim at an experimental precision $\Delta\mathcal{L}/\mathcal{L} = \mathcal{O}(0.1\%)$. Many heavy flavour results became available in 1992 which manifestly profit from the use of silicon microvertex chambers. These studies could profit substantially from collecting statistics at the peak in 1994 and 1995. During 1994 the total statistics at LEP is expected to exceed 10^7 hadronic events. After 1995 LEP plans to run above the W-pair production threshold. This will also be an interesting domain for direct particle searches, especially the search for the Higgs boson.

We have shown that up to now electroweak precision tests do not allow any significant indirect constraint to be placed on the mass of the Higgs boson. At the end of this report we wish to give a glimpse into the future (a detailed discussion has been presented in [90]). In Figure 30 we display the direct measurement of M_Z and the indirect determination of M_Z from all other electroweak precision tests available today, for fixed $M_H = 300$ GeV. This band as a function of M_t has been derived as discussed in subsection 5.1. A decrease of the width of this band by a factor of two within the next two years is probably a conservative estimate. It has been shown that a direct measurement of M_W at LEP200 could be performed with an accuracy of 37 MeV [104]. The indirect determination of M_Z resulting from such a measurement is also indicated in Figure 30 for $M_H = 60$ GeV and $M_H = 1000$ GeV. Finally, we expect a direct determination of the mass of the top quark with a precision of 5–10 GeV at the Tevatron or future hadron colliders, corresponding to the vertical band in Figure 30. At future e^+e^- colliders this precision might even be improved to $\mathcal{O}(0.5 \text{ GeV})$ [105].

The message of Figure 30 is twofold: Firstly, we will become sensitive to M_H and might produce interesting indirect constraints on M_H , provided a direct measurement of M_t becomes available. And last but not least: We have good chances that in the near future bands will no longer overlap pointing towards physics beyond the Standard Model.

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		ALEPH	DELPHI	L3	OPAL	LEP
$q\bar{q}$	'90-'91	451	365	423	454	1693
	'92	686	695	677	733	2791
	total	1137	1060	1100	1187	4484
$\ell^+\ell^-$	'90-'91	55	37	40	58	190
	'92	82	76	58	88	304
	total	137	113	98	146	494

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z^0 line shape and lepton forward-backward asymmetries.

	ALEPH		DELPHI		L3		OPAL	
	'91	'92 prel.	'91	'92 prel.	'91	'92 prel.	'91	'92
$\mathcal{L}^{\text{exp. (a)}}$	0.45%	0.35% ^(b) 0.15% ^(c)	0.5 %	0.6 %	0.5 %	0.5 %	0.60%	0.41%
σ_{had}	0.2 %	0.17%	0.2 %	0.28%	0.15%	0.14%	0.20%	0.20%
σ_e	0.4 %	0.4 %	0.5 %	0.6 %	0.4 %	0.4 %	0.45%	0.22%
σ_μ	0.5 %	0.5 %	0.5 %	0.5 %	0.5 %	0.5 %	0.25%	0.19%
σ_τ	0.6%	0.5%	0.75%	0.75%	0.7 %	0.7 %	0.76%	0.44%

Table 2: The experimental systematic errors for the analysis of the Z^0 line shape [4, 46, 47, 48, 45].

^(a)Only the experimental error including the statistics of small-angle Bhabha events is quoted. In addition, there is a theoretical error for the calculation of the small-angle Bhabha cross section.

^(b)Without the ALEPH silicon calorimeter.

^(c)With the ALEPH silicon calorimeter.

	ALEPH	DELPHI	L3	OPAL
$M_Z(\text{GeV})$	91.187 $\pm$ 0.009	91.187 $\pm$ 0.009	91.195 $\pm$ 0.009	91.182 $\pm$ 0.009
$\Gamma_Z(\text{GeV})$	2.501 $\pm$ 0.011	2.482 $\pm$ 0.012	2.493 $\pm$ 0.010	2.483 $\pm$ 0.012
$\sigma_h^0(\text{nb})$	41.61 $\pm$ 0.16	41.02 $\pm$ 0.27	41.33 $\pm$ 0.26	41.71 $\pm$ 0.23
R_e	20.58 $\pm$ 0.15	20.70 $\pm$ 0.18	20.90 $\pm$ 0.16	20.83 $\pm$ 0.16
R_μ	20.82 $\pm$ 0.15	20.48 $\pm$ 0.15	21.02 $\pm$ 0.16	20.78 $\pm$ 0.11
R_τ	20.62 $\pm$ 0.17	20.88 $\pm$ 0.20	20.78 $\pm$ 0.20	21.01 $\pm$ 0.15
$A_{\text{FB}}^{0,e}$	0.0185 $\pm$ 0.0059	0.0237 $\pm$ 0.0092	0.0135 $\pm$ 0.0078	0.0062 $\pm$ 0.0080
$A_{\text{FB}}^{0,\mu}$	0.0147 $\pm$ 0.0047	0.0143 $\pm$ 0.0050	0.0167 $\pm$ 0.0064	0.0099 $\pm$ 0.0042
$A_{\text{FB}}^{0,\tau}$	0.0182 $\pm$ 0.0053	0.0213 $\pm$ 0.0068	0.0257 $\pm$ 0.0089	0.0205 $\pm$ 0.0052

Table 3: Line shape and asymmetry parameters from 9-parameter fits to the data of the four LEP experiments.

$\mathcal{V}$:	ALEPH	DELPHI	L3	OPAL
ALEPH	A	$C(A, D)$	$C(A, L)$	$C(A, O)$
DELPHI	$C(A, D)$	D	$C(D, L)$	$C(D, O)$
L3	$C(A, L)$	$C(D, L)$	L	$C(L, O)$
OPAL	$C(A, O)$	$C(D, O)$	$C(L, O)$	O

Table 4: The covariance matrix $\mathcal{V}$ used for the LEP average of parameters. The 9×9 covariance matrices quoted by the individual experiments (A, D, L, O) include the effect of experiment specific and common uncertainties. The 9×9 submatrices C account for common systematic uncertainties due to the LEP energy calibration and the theoretical uncertainty in the the small-angle Bhabha cross section for the absolute normalization.

Parameter	Average Value
$M_Z(\text{GeV})$	91.187 ± 0.007
$\Gamma_Z(\text{GeV})$	2.489 ± 0.007
$\sigma_h^0(\text{nb})$	41.56 ± 0.14
R_e	20.743 ± 0.080
R_μ	20.764 ± 0.069
R_τ	20.832 ± 0.088
$A_{\text{FB}}^{0,e}$	0.0153 ± 0.0038
$A_{\text{FB}}^{0,\mu}$	0.0132 ± 0.0026
$A_{\text{FB}}^{0,\tau}$	0.0204 ± 0.0032

Table 5: Average line shape and asymmetry parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The $\chi^2/(d.o.f.)$ of the average is 27.1/27.

	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{\text{FB}}^{0,e}$	$A_{\text{FB}}^{0,\mu}$	$A_{\text{FB}}^{0,\tau}$
M_Z	1.000	-0.157	0.006	0.029	-0.002	-0.003	0.025	0.056	0.048
Γ_Z	-0.157	1.000	-0.070	0.001	0.006	-0.001	0.000	0.004	0.006
σ_h^0	0.006	-0.070	1.000	0.071	0.100	0.085	0.008	-0.002	-0.001
R_e	0.029	0.001	0.071	1.000	0.102	0.072	-0.023	0.021	0.018
R_μ	-0.002	0.006	0.100	0.102	1.000	0.076	-0.001	0.007	-0.002
R_τ	-0.003	-0.001	0.085	0.072	0.076	1.000	0.000	-0.002	0.006
$A_{\text{FB}}^{0,e}$	0.025	0.000	0.008	-0.023	-0.001	0.000	1.000	-0.062	-0.048
$A_{\text{FB}}^{0,\mu}$	0.056	0.004	-0.002	0.021	0.007	-0.002	-0.062	1.000	0.119
$A_{\text{FB}}^{0,\tau}$	0.048	0.006	-0.001	0.018	-0.002	0.006	-0.048	0.119	1.000

Table 6: The correlation matrix for the set of parameters given in Table 5.

Parameter	Average Value
$M_Z(\text{GeV})$	91.187 ± 0.007
$\Gamma_Z(\text{GeV})$	2.489 ± 0.007
$\sigma_h^0(\text{nb})$	41.56 ± 0.14
R_ℓ	20.763 ± 0.049
$A_{\text{FB}}^{0,\ell}$	0.0158 ± 0.0018

Table 7: Average line shape and asymmetry parameters from the results of the four LEP experiments given in Table 3, assuming lepton universality. The $\chi^2/(d.o.f.)$ of the average is 30.8/31.

	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{\text{FB}}^{0,\ell}$
M_Z	1.000	-0.157	0.007	0.012	0.075
Γ_Z	-0.157	1.000	-0.070	0.003	0.006
σ_h^0	0.007	-0.070	1.000	0.137	0.003
R_ℓ	0.012	0.003	0.137	1.000	0.008
$A_{\text{FB}}^{0,\ell}$	0.075	0.006	0.003	0.008	1.000

Table 8: The correlation matrix for the set of parameters given in Table 7.

τ decay mode	branching fraction [%] [49]	sensitivity $S = (\Delta\mathcal{P}_\tau\sqrt{N})^{-1}$	statistical weight $\propto Br \cdot S^2$
$\tau \rightarrow e\nu\bar{\nu}$	17.88 ± 0.15	0.22	0.07
$\tau \rightarrow \mu\nu\bar{\nu}$	17.42 ± 0.17	0.22	0.07
$\tau \rightarrow \pi(K)\nu$	12.04 ± 0.24	0.59	0.35
$\tau \rightarrow \rho\nu$	24.48 ± 0.31	0.48	0.47
$\tau \rightarrow a_1\nu$	$8.44 \pm 0.20^{(a)}$	0.22	0.03

Table 9: The τ decay channels used in the polarization analysis and their statistical weights. The sensitivity $S = (\Delta\mathcal{P}_\tau\sqrt{N})^{-1}$ has been derived from a fit to simulated data in the limit $N \rightarrow \infty$, assuming perfect acceptance [50].

^(a)The $\chi^2/(d.o.f.)$ for the $\tau \rightarrow a_1\nu$ channel is 4.8.

decay	$\mathcal{P}_\tau$
$\tau \rightarrow e\nu\bar{\nu}$	-0.109 ± 0.047
$\tau \rightarrow \mu\nu\bar{\nu}$	-0.125 ± 0.040
$\tau \rightarrow \pi(K)\nu$	-0.150 ± 0.023
$\tau \rightarrow \rho\nu$	-0.136 ± 0.021
$\tau \rightarrow a_1\nu$	-0.122 ± 0.054

Table 10: Summary of τ -polarization measurements for the individual decay channels [51].

ALEPH	('90 + '91) [52]	0.143 ± 0.023
DELPHI	('90 + '91 + '92), prel. [53, 4]	0.151 ± 0.029
L3	('90 + '91 + '92), prel. [54, 4]	0.133 ± 0.024
OPAL	('90 + '91), prel. [55, 4]	0.117 ± 0.046
LEP Average		0.139 ± 0.014

Table 11: LEP results for $\mathcal{A}_\tau$.

ALEPH	('90 + '91) [52]	0.120 ± 0.026
OPAL	('90 + '91), prel. [55, 4]	0.231 ± 0.083
LEP Average		0.130 ± 0.025

Table 12: LEP results for $\mathcal{A}_e$.

$A_{\text{FB}}^{\text{bb}}(\sqrt{s} = 89.6 \text{ GeV})\text{prel.}$	$0.037 \pm 0.027 \pm 0.005$
$A_{\text{FB}}^{\text{bb}}(\sqrt{s} = 92.8 \text{ GeV})\text{prel.}$	$0.096 \pm 0.024 \pm 0.007$
$A_{\text{FB}}^{\text{cc}}(\sqrt{s} = 89.35 \text{ GeV})$	$-0.14 \pm 0.14 \pm 0.03$
$A_{\text{FB}}^{\text{cc}}(\sqrt{s} = 92.85 \text{ GeV})$	$0.18 \pm 0.12 \pm 0.03$

Table 13: LEP averages for $A_{\text{FB}}^{\text{bb}}$ [4, 65] and $A_{\text{FB}}^{\text{cc}}$ [70] below and above the Z^0 peak.

Exp.	data	$\sin^2\theta_{\text{eff}}^{\text{lept}}$
ALEPH	89-92 prel.	$0.2317 \pm 0.0013 \pm 0.0011$
DELPHI	90-91	$0.2345 \pm 0.0030 \pm 0.0027$
OPAL	90-91	$0.2321 \pm 0.0017 \pm 0.0028$
Average		$0.2320 \pm 0.0011 \pm 0.0011$

Table 14: Results for the effective mixing angle from the measurement of the $q\bar{q}$ charge asymmetry [4, 72, 73, 74]. The first error quoted is statistical the second accounts for systematics. The fragmentation errors quoted have been treated as fully correlated among the experiments. The present average is not very sensitive to the treatment of common uncertainties.

experiment	$3\mathcal{C}_d + 8\mathcal{C}_u$
DELPHI	$4.15 \pm 0.62 \pm 0.21$
L3	$2.97 \pm 0.29 \pm 0.16$
OPAL	$3.09 \pm 0.20 \pm 0.17$
LEP	$3.10 \pm 0.16 \pm 0.17$

Table 15: LEP results for $3\mathcal{C}_d + 8\mathcal{C}_u$ determined from final state photon radiation in hadronic events [78, 80, 83]. The first error quoted includes statistics and uncorrelated experimental systematics. The second error accounts for theoretical uncertainties in the determination of the final state photon yield and is treated as fully correlated in the averaging procedure.

measurement	result
a) <u>LEP</u> line-shape and lepton asymmetries: M_Z Γ_Z σ_h^0 R_ℓ $A_{\text{FB}}^{0,\ell}$ + correlation matrix (Table 8) τ polarization asymmetries: $\mathcal{A}_\tau$ $\mathcal{A}_e$ b and c quark results: $A_{\text{FB}}^{0,b}$ $A_{\text{FB}}^{0,c}$ $\Gamma_{b\bar{c}}/\Gamma_{\text{had}}$ q $\bar{q}$ charge asymmetry: $\sin^2\theta_{eff}^{\text{lept}}$ from $\langle Q_{\text{FB}} \rangle$	 91.187 ± 0.007 GeV 2.489 ± 0.007 GeV 41.56 ± 0.14 nb 20.763 ± 0.049 0.0158 ± 0.0018 0.139 ± 0.014 0.130 ± 0.025 0.099 ± 0.006 0.075 ± 0.015 0.2200 ± 0.0027 0.2320 ± 0.0016
b) <u>p$\bar{p}$ and νN</u> M_W/M_Z (UA2, [84]) M_W (CDF [85]) $\sin^2\theta_W(\nu N)$ (CDHS [86], CHARM [87], CCFR [88])	 0.8813 ± 0.0041 79.91 ± 0.39 0.2256 ± 0.0047

Table 16: Summary of measurements included in the combined analysis of Standard Model parameters. Section a) summarizes LEP averages [4], section b) electroweak precision tests from hadron colliders and νN -scattering.

	LEP	LEP + Collider and ν data
M_t (GeV)	$166_{-19}^{+17} {}_{-22}^{+19}$	$164_{-17}^{+16} {}_{-21}^{+18}$
$\alpha_s(M_Z^2)$	$0.120 \pm 0.006 \pm 0.002$	$0.120 \pm 0.006 \pm 0.002$
$\chi^2/(d.o.f.)$	3.5/8	4.4/11
$\sin^2\theta_{eff}^{\text{lept}}$	$0.2324 \pm 0.0005 {}_{-0.0002}^{+0.0001}$	$0.2325 \pm 0.0005 {}_{-0.0002}^{+0.0001}$
$\sin^2\theta_W$	$0.2255 \pm 0.0019 {}_{-0.0003}^{+0.0005}$	$0.2257 \pm 0.0017 {}_{-0.0003}^{+0.0004}$
M_W (GeV)	$80.25 \pm 0.10 {}_{-0.03}^{+0.02}$	$80.24 \pm 0.09 {}_{-0.02}^{+0.01}$

Table 17: Results of fits to LEP and other data for M_t and $\alpha_s(M_Z^2)$ [4]. No external constraint on $\alpha_s(M_Z^2)$ has been imposed. In the third column also the data from the p $\bar{p}$ and from the neutrino scattering experiments (see Table 16 b) are included. The central values and the first errors quoted refer to $M_H = 300$ GeV. The second errors correspond to the variation of the central value when varying M_H in the interval $60 \text{ GeV} < M_H < 1000 \text{ GeV}$. The values quoted for $\sin^2\theta_{eff}^{\text{lept}}$, $\sin^2\theta_W$ and M_W are derived results from the Standard Model fits above.

Without Lepton Universality:	
Γ_{ee}	83.86 ± 0.30
$\Gamma_{\mu\mu}$	83.78 ± 0.40
$\Gamma_{\tau\tau}$	83.50 ± 0.45
With Lepton Universality:	
$\Gamma_{\ell\ell}$	83.82 ± 0.27
Γ_{had}	1740.3 ± 5.9
Γ_{inv}	497.6 ± 4.3

Table 18: Z^0 partial decay widths in MeV.

M_Z	91.187 ± 0.007
M_t	$171^{+17}_{-19} {}^{+19}_{-26} \text{ GeV}$
α_s	0.124 ± 0.006
Σ_x	$-8.2 \pm 8.8^{+2.3}_{-0.2} \text{ MeV}$
$\Sigma_{x, \ell\ell}$	$-0.18 \pm 0.32^{+0.15}_{-0.06} \text{ MeV}$
$\Sigma_{x, \text{had}}$	$-4.4 \pm 7.3^{+1.2}_{-0.0} \text{ MeV}$

Table 19: Result of a fit which allows for non Standard Model contributions to the inclusive partial widths. The central values and the first errors quoted refer to $M_H = 300 \text{ GeV}$, fixed. The second error reflects the variation of the central value when varying M_H in the interval $60 \leq M_H [\text{GeV}] \leq 1000$. The value of the strong coupling constant has been constraint by $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ [89].

	$\hat{g}_a^\ell$	$\hat{g}_v^\ell$
e^+e^-	-0.50096 ± 0.00093	-0.0373 ± 0.0031
$\mu^+\mu^-$	-0.5013 ± 0.0012	-0.0288 ± 0.0064
$\tau^+\tau^-$	-0.5005 ± 0.0014	-0.0372 ± 0.0032
$\ell^+\ell^-$	-0.50093 ± 0.00082	-0.0359 ± 0.0018

Table 20: The effective vector and axial-vector couplings of charged leptons determined in a fit with the following input data: M_Z , Γ_Z , σ_h^0 , R_e , R_μ , R_τ , $A_{\text{FB}}^{0,\ell}(e^+e^-)$, $A_{\text{FB}}^{0,\ell}(\mu^+\mu^-)$ and $A_{\text{FB}}^{0,\ell}(\tau^+\tau^-)$ with their correlation matrix, together with $\mathcal{P}_\tau$ and $\mathcal{A}_e$ from tau polarization measurements. Free parameters of the fit are also M_Z , Γ_Z and σ_h^0 constrained by their input values. Without the hypothesis of lepton universality we obtain $\chi^2/(d.o.f.) = 2.5/(11-9)$, and $\chi^2/(d.o.f.) = 4.5/(11-5)$ with this hypothesis.

	LEP			
$\hat{g}_a^\ell$	-0.50059 ± 0.00084			
$\hat{g}_t^\ell$	-0.0359 ± 0.0018			
$\hat{g}_a^{\nu^2} + \hat{g}_t^{\nu^2}$	0.5023 ± 0.0044			
$\hat{g}_a^b$	$-0.510^{+0.048}_{-0.024}$			
$\hat{g}_v^b$	$-0.335^{+0.038}_{-0.063}$			
$\hat{g}_a^c$	$0.495^{+0.027}_{-0.032}$			
$\hat{g}_t^c$	$0.201^{+0.057}_{-0.047}$			

	LEP	Standard Model [71]		νN [100]
$\hat{g}_L^\ell$	-0.26824 ± 0.00093	$-0.2675^{+0.0030}_{-0.0034}$		
$\hat{g}_R^\ell$	0.2324 ± 0.0010	$0.2333^{+0.0016}_{-0.0020}$		
$\hat{g}_L^{\nu^2} + \hat{g}_R^{\nu^2}$	0.2511 ± 0.0022	$0.2514^{+0.0015}_{-0.0013}$		
$\hat{g}_L^b$	-0.4224 ± 0.0081	$-0.4209^{+0.0010}_{-0.0009}$	$\hat{g}_L^d$	-0.437 ± 0.012
$\hat{g}_R^b$	$0.088^{+0.031}_{-0.055}$	$0.0777^{+0.0006}_{-0.0006}$	$\hat{g}_R^d$	$-0.023^{+0.077}_{-0.047}$
$\hat{g}_L^c$	0.348 ± 0.020	$0.3454^{+0.0028}_{-0.0024}$	$\hat{g}_L^u$	0.329 ± 0.015
$\hat{g}_R^c$	$-0.147^{+0.041}_{-0.033}$	$-0.1555^{+0.0013}_{-0.0011}$	$\hat{g}_R^u$	-0.178 ± 0.013

Table 21: Results of a fit of quark and lepton effective vector and axial-vector couplings and the transformation to chiral couplings. For leptons universality is assumed, but not for quarks. Input data to the fit are: M_Z , Γ_Z , σ_h^0 , R_ℓ and $A_{\text{FB}}^{0,\ell}$ with their correlation matrix, together with P_{ℓ} , A_{ℓ} , $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$, $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$, $A_{\text{FB}}^{b\bar{b}}$ and $A_{\text{FB}}^{c\bar{c}}$. Free parameters of the fit are also M_Z and R_ℓ , constrained by their input values. For this fit we obtain $\chi^2/(d.o.f.)=2.3/(16-10)$.

Fit	$\epsilon_1 \cdot 10^3$	$\epsilon_3 \cdot 10^3$	$\epsilon_b \cdot 10^3$	$\alpha_s(M_Z^2)$	Γ_Z	σ_h^0	$\chi^2/(d.o.f.)$
1	3.6 ± 3.4	4.5 ± 3.5	—	—	c	c	0.4/(7-5)
2	3.6 ± 3.4	4.5 ± 3.5	2.6 ± 5.5	c	c	c	0.4/(9-7)
3	2.2 ± 3.1	3.9 ± 3.4	-2.0 ± 4.5	c	—	—	2.7/(9-5)
4	1.7 ± 3.1	3.5 ± 2.9	-1.3 ± 4.5	c	—	—	3.9/(17-5)
5	2.4 ± 3.2	3.9 ± 3.0	4.8 ± 6.8	0.107 ± 0.013	—	—	2.5/(16-5)

Table 22: Fit results for the ϵ parameter analysis (see Subsection 5.4). The ‘c’ refers to free parameters which are constrained by input data. In all fits M_Z is treated as additional free parameter constrained to $M_Z = 91.187 \pm 0.007$ MeV.

Figure Captions

Figure 1: An example of a LEP detector.

Figure 2: Lowest order diagrams for the cross sections of the process $e^+e^- \rightarrow f\bar{f}$.

Figure 3: Definition of the scattering angle θ in e^+e^- annihilation.

Figure 4: Radiative corrections to the process $e^+e^- \rightarrow f\bar{f}$.

Figure 5: The measured hadronic cross section (top) and the distribution of residuals to a fit of the Standard Model prediction (bottom). The solid lines are the results of the Standard Model fit to the combined e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$ and hadronic data. The solid squares show the 1992 data, the solid circles the 1991 data and the open circles the 1990 data. Only statistical errors are shown.

Figure 6: Leptonic cross sections (top) and forward-backward asymmetries (bottom) as functions of centre-of-mass energy for: a) $e^+e^- \rightarrow e^+e^-$, b) $e^+e^- \rightarrow \mu^+\mu^-$, c) $e^+e^- \rightarrow \tau^+\tau^-$. The solid lines are the results of a Standard Model fit to the combined e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$ and hadronic data. The solid squares show the 1992 data, the solid circles the 1991 data and the open circles the 1990 data. The plots in the middle display the residuals to the Standard Model fit to the cross sections. Only statistical errors are shown. The cross section for $e^+e^- \rightarrow e^+e^-$ is integrated over $|\cos\theta| < 0.70$ and corrected for efficiency within the geometrical acceptance; cross sections for $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ are corrected for acceptance. The forward-backward asymmetries are within $|\cos\theta| < 0.70$, $|\cos\theta| < 0.95$ and $|\cos\theta| < 0.90$ for $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$, respectively.

Figure 7: One standard deviation contours (39% probability) in the R_ℓ - $A_{\text{FB}}^{0,\ell}$ plane. Also shown as dotted symbol is the Standard Model prediction for $M_Z = 91.187$ GeV, $M_t = 150$ GeV, $M_H = 300$ GeV, $\alpha_s = 0.123$. The lines with arrows correspond to the variation of the Standard Model prediction when M_t , M_H or α_s are varied in the intervals $50 < M_t(\text{GeV}) < 250$, $60 < M_H(\text{GeV}) < 1000$ and $\alpha_s(M_Z^2) = 0.123 \pm 0.006$, respectively. The arrows point in the direction of increasing values for M_t , M_H and α_s .

Figure 8: Helicity considerations for the $\tau \rightarrow \pi(K)\nu$ decay.

Figure 9: The spectrum of $\tau \rightarrow \pi(K)\nu$ decays as a function of the normalized energy E_π/E_{beam} (L3 collaboration). Also shown is the fitted contribution from each helicity of the τ lepton including backgrounds for that helicity. The hatched histogram shows the total background.

Figure 10: The two possible spin configurations for the $\tau \rightarrow \rho\nu$ and $\tau \rightarrow a_1\nu$ decays.

Figure 11: The angular distribution of the τ polarization.

Figure 12: Sources of inclusive leptons in hadronic events.

Figure 13: The distribution of momentum p (top) and transverse momentum p_T (bottom) for muon candidates passing all the selection cuts except that on p or p_T respectively.

Figure 14: Definition and sign convention for the decay length d and the impact parameter δ . The decay length d is defined as the distance between primary and secondary vertex. The impact parameter δ is defined for each track as closest approach to the primary vertex and does not need the reconstruction of a secondary vertex. The definition of a sign needs the reconstruction of an estimated b direction (which not necessarily has to join primary and secondary vertex).

Figure 15: Hemisphere tag efficiencies and purities as function of the cut on $\mathcal{P}_H$, which is constructed from the measured impact parameters in one hemisphere, and represents the probability that the collection of tracks in the hemisphere does not contain any decay products of long-lived particles.

Figure 16: LEP results for $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ [4]. Where three errors are quoted, the first is statistical, the second from uncorrelated systematic errors and the third from common systematic errors.

Figure 17: $B^0\bar{B}^0$ -mixing as source of wrong-sign leptons.

Figure 18: The measurements of $A_{\text{FB}}^{b\bar{b}}$, corrected for mixing, from the individual LEP experiments and the LEP average. For each measurement the statistical and uncorrelated systematic errors are given, followed by the correlated error [4].

Figure 19: Reconstruction of exclusive D^* decays. a) The $K\pi$ invariant mass distribution for all events with $143.5 < \Delta M = M(K\pi\pi) - M(K\pi)$ [MeV] < 147.5 . b) The distribution of ΔM for all events with a $K\pi$ combination with an invariant mass $1790 < M(K\pi)$ [MeV] < 1940 . The solid line represents a fit to the signal over a smooth background. The background contribution is indicated as dashed line.

Figure 20: LEP results for $\Gamma_{c\bar{c}}/\Gamma_{\text{had}}$ [4]. The first error is statistical, the second systematics. Common systematics is ignored in the average.

Figure 21: The measurements of $A_{\text{FB}}^{c\bar{c}}$ from the individual LEP experiments and the LEP average. For each measurement the statistical and uncorrelated systematic errors are given, followed by the correlated error [4].

Figure 22: a) and b): Comparison of LEP measurements with the Standard Model prediction as a function of M_t . The cross-hatched area shows the variation of the Standard Model prediction with M_H spanning the interval $60 < M_H$ (GeV) < 1000 and the singly-hatched area corresponds to a variation of $\alpha_s(M_Z^2)$ within the interval $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. For the ratios of partial widths, $\Gamma_{b\bar{b}}/\Gamma_{\text{had}}$ and $\Gamma_{d\bar{d}}/\Gamma_{\text{had}}$, this variation nearly cancels. The experimental errors on the parameters are indicated as vertical bands.

Figure 23: Comparison of the value of $\sin^2\theta_{\text{eff}}^{\text{lept}}$ from several LEP measurements. The $\chi^2/(d.o.f.)$ of the average is 0.5/5.

Figure 24: Comparison of the direct measurement of M_Z at LEP with indirect determinations from various observables assuming the Standard Model prediction. The bands display the ± 1 standard deviation variation when fixing the value of M_H to 300 GeV and allowing the strong coupling constant to vary within $\alpha_s(M_Z^2) = 0.123 \pm 0.006$.

Figure 25: The χ^2 curves for the Standard Model fit in Table 17, column 3 to the electroweak precision measurements listed in Table 16 as a function of M_t for three different Higgs mass values spanning the interval $60 \leq M_H$ [GeV] ≤ 1000 .

Figure 26: The χ^2 curves as a function of the sum of possible non Standard Model contributions to the hadronic, leptonic and invisible Z^0 partial widths, Σ_x , from a fit to the electroweak precision measurements listed in Table 16 for three different Higgs mass values. The minimum value of χ^2 from the $M_H = 60$ GeV curve has been subtracted from all curves.

Figure 27: One standard deviation contours (39% probability) in the $\hat{g}_v^\ell - \hat{g}_a^\ell$ plane. The shaded band represents the Standard Model prediction [71].

Figure 28: Effective couplings from a combined analysis of hadronic and leptonic couplings. For b and c quarks the one standard deviation contours (39% probability) are shown. These are compared to the constraints obtained from $\mathcal{C}_d$ and $\mathcal{C}_u$, derived from an analysis of final state radiation in hadronic events, which are represented by circles in the $\hat{g}_v^f - \hat{g}_a^f$ plane. Similarly the fit result for $\hat{g}_v^{\nu^2} + \hat{g}_a^{\nu^2}$ corresponds to a circle in this plane. The small rectangle for $\ell^+\ell^-$ corresponds to the size of the enlarged view of lepton couplings given in Figure 27.

Figure 29: The one standard deviation contours (39% probability) among the ϵ parameters, and of the ϵ parameters with $\alpha_s(M_Z^2)$, for a fit which uses the full set of LEP electroweak measurements and treats $\alpha_s(M_Z^2)$ as a free parameter. For comparison we also show as dashed ellipses the one standard deviation contours of a fit based on the same input data imposing in addition the constraint $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. Also indicated is the Standard Model prediction [71] for the ϵ parameters. The symbols refer to $M_t = 90$ GeV, 150 GeV and 200 GeV, where the symbol size increases with M_t . Circular, box and triangular symbols discriminate between $M_H = 60$ GeV, 300 GeV and 1000 GeV, respectively.

Figure 30: Comparison of the direct measurement of M_Z at LEP with an indirect determination from the sum of all other electroweak precision tests summarized in Table 16, assuming the Standard Model. This band displays the ± 1 standard deviation variation when fixing the value of M_H to 300 GeV and allowing the strong coupling constant to vary within $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. Also shown are the bands which would result from a measurement of M_W at LEP 200 with an accuracy of 37 MeV for two different Higgs masses. The vertical band corresponds to a hypothetical direct measurement of M_t with an accuracy of 5 GeV.

Figure 1:

a) $\sigma(e^+e^- \rightarrow f\bar{f} \text{ with } f \neq e) \propto$

$$\left| \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} \gamma \\ \text{---} \end{array} \begin{array}{c} \diagup \\ \bar{f} \end{array} + \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} Z^0 \\ \text{---} \end{array} \begin{array}{c} \diagup \\ \bar{f} \end{array} \right|^2$$

b) $\sigma(e^+e^- \rightarrow e^+e^-) \propto$

$$\left| \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} \gamma \\ \text{---} \end{array} \begin{array}{c} \diagup \\ e^+ \end{array} + \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} Z^0 \\ \text{---} \end{array} \begin{array}{c} \diagup \\ e^+ \end{array} + \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} \gamma \\ \text{---} \end{array} \begin{array}{c} \diagup \\ e^- \end{array} + \begin{array}{c} e^+ \\ \diagdown \end{array} \begin{array}{c} Z^0 \\ \text{---} \end{array} \begin{array}{c} \diagup \\ e^- \end{array} \right|^2$$

Figure 2:

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7:

Figure 8:

Figure 9:

Figure 10:

Figure 11:

Figure 12:

Figure 13:

Figure 14:

Figure 15:

Γ_{bb}/Γ_{had}

lepton tags

ALEPH, 90-92 prel.
 $0.2223 \pm 0.0042 \pm 0.0033 \pm 0.0046$

DELPHI, 90-92 prel.
 $0.222 \pm 0.009 \pm 0.002 \pm 0.004$

L3, 91-92 prel.
 $0.2184 \pm 0.0081 \pm 0.0045 \pm 0.0059$

OPAL, 90-91, 90-91 prel.
 $0.221 \pm 0.004 \pm 0.008 \pm 0.006$

lepton tags, LEP average
 $0.221 \pm 0.003 \pm 0.002 \pm 0.005$

event shape tags

ALEPH, 90-91
 $0.228 \pm 0.005 \pm 0.005 \pm 0.001$

DELPHI, 90-91
 $0.232 \pm 0.005 \pm 0.011 \pm 0.013$

L3, 91
 $0.222 \pm 0.003 \pm 0.002 \pm 0.006$

event shape tags, LEP average
 $0.226 \pm 0.002 \pm 0.003 \pm 0.004$

lifetime tags

ALEPH, 92
 $0.2192 \pm 0.0022 \pm 0.0020 \pm 0.0023$

DELPHI, 91-92 prel
 $0.206 \pm 0.0066 \pm 0.0037 \pm 0.0047$

DELPHI, 91 prel
 $0.222 \pm 0.007 \pm 0.0063 \pm 0.003$

OPAL, 90
 $0.222 \pm 0.007 \pm 0.008 \pm 0.002$

OPAL, 91-92 prel
 $0.2133 \pm 0.0041 \pm 0.0027 \pm 0.0033$

lifetime tags, LEP average
 $0.2169 \pm 0.0018 \pm 0.0015 \pm 0.0027$

global LEP average

0.2200 ± 0.0027

Figure 16:

Figure 17:

Figure 18:

DELPHI

Figure 19:

Figure 20:

Figure 21:

Figure 22: a)

Figure 22: b)

Figure 23:

Figure 24:

Figure 25:

Figure 26:

Figure 27:

Figure 28:

Figure 29:

Figure 30: