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Soft Gluon Radiation in Photon plus Single Jet Events at LEP

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Abstract

'Isolated' photon events at LEP have been defined using either a cone algorithm (where the photon is isolated in a cone and the remaining particles clustered with a jet algorithm) or a democratic algorithm (where all particles are clustered and the cluster containing the photon is identified as the photon). By studying the flow of soft gluon radiation in 'isolated' photon + single jet events at LEP we give a qualitative explanation of the size of radiative corrections in the different schemes.

Recently e^+e^- annihilation events with hadronic final states containing an ‘isolated’ photon have attracted both experimental [1, 2, 3, 4] and theoretical interest [5, 6, 7, 8]. The physics motivation for studying such events is that the photon preferentially couples to up-type quarks, and therefore this event sample is enriched in up-type quarks. A measurement of the quark couplings to the Z boson [9] is thus possible. However, since a perfectly isolated photon is not an infrared safe quantity, it is necessary to discuss exactly what is meant by ‘isolation’.

Experimentally, the photon candidate has been defined in two ways. In the CONE algorithm, the photon and all particles in a cone of half-angle θ_c are removed from the event to form the photon cluster. The remaining particles are then scanned for jets using a jet algorithm such as the E0 [10] or Durham [11] schemes with a jet resolution parameter y_{cut} . In the democratic (DEMO) algorithm, the jet finding algorithm is applied to all particles in the event. The ‘jet’ containing the photon is then interpreted as the photon candidate. In both cases, in order to reduce the background from π^0 decay, the photon cluster is required to contain little or no hadronic contamination and the variable

$$z = \frac{E_\gamma}{E_\gamma + E_h} \quad (1)$$

where E_h is the hadronic energy in the cluster, is constrained to be close to unity. Since it is very hard to detect very low energy particles, for practical purposes an ‘isolated’ photon may be accompanied by $O(500 \text{ MeV})$ of hadronic energy.

Theoretical calculations have used both the democratic [5] and cone-type [6, 7, 8] algorithms. In order to cancel the infrared singularities associated with soft gluons, the soft gluon must be allowed within the photon cluster. However, if all coloured partons are to be treated equally, then the quarks within the event should also be allowed inside the photon cluster and may be collinear with the photon. The quark-photon collinear singularity is regularised by the as yet unmeasured photon fragmentation function. Therefore, to eliminate the uncertainty due to this fragmentation function, the theoretical calculations have compromised by allowing a small amount of gluonic energy into the photon cluster, but not allowing quarks to cluster with the photon. Since different calculations allow different amounts of gluon energy inside the photon cluster, the calculations are not in complete agreement [12]. To reconcile these differences it is necessary to treat the quark and gluon equally and to involve the fragmentation function properly [8, 13]. Even then, the region round $z = 1$ is problematic since radiating additional soft gluons may change z significantly.

The issue we wish to address in this letter is the size of radiative corrections in the different photon algorithms for photon + 1 jet events. It has been noted [5] that in a democratic scheme the corrections are small. In other words, the ratio of next-to-leading to leading order cross sections ($\mathcal{K}$ factor) is close to unity. On the other hand, in the cone type scheme, the corrections are quite large and negative [6, 7], $\mathcal{K} < 1$. Clearly the absolute size of the correction is determined by how the soft gluon region is treated, nevertheless, the relative size of the correction is determined by how the isolation cuts affect gluons with an energy above the energy allowed within the photon cluster. The $\mathcal{K}$ factors in both CONE and DEMO schemes for both the E0 and Durham jet algorithms are shown in Fig. 1 using

Figure 1: The ratio of next-to-leading order to leading-order cross sections for $e^+e^- \rightarrow 1\text{-jet} + \text{'isolated' photon}$ production at LEP energies in the CONE scheme (with $\theta_c = 15^\circ$) for the Durham (solid) and E0 (dotted) jet algorithms and in the DEMO scheme also for the Durham (dashed) and E0 (dash-dotted) jet algorithms.

the theoretical definition of ‘isolation’ as described in ref. [7]. We see that for all y_{cut} and for both jet algorithms,

$$\mathcal{K}_{\text{DEMO}} > \mathcal{K}_{\text{CONE}}.$$

By dressing the lowest order $e^+e^- \rightarrow q\bar{q}\gamma$ matrix elements with a soft gluon and examining the radiation pattern, we hope to show why this is so.

The leading order contribution from the $e^+e^- \rightarrow q\bar{q}\gamma$ process to the photon + 1 jet rate is given by integrating the differential cross section (for massless quarks of charge e_q),

$$\frac{1}{\hat{\sigma}_0} \frac{d^2\sigma}{dx dx'} = \sum_q \left(\frac{\alpha e_q^2}{2\pi} \right) \frac{x^2 + x'^2}{(1-x)(1-x')}, \quad (2)$$

over the allowed region of the Dalitz plot. Here x and x' are the quark energy-fractions given by $x = 2E_q/\sqrt{s}$, $x' = 2E_{\bar{q}}/\sqrt{s}$ and $x_\gamma = 2 - x - x'$. In terms of these energy fractions, the scaled pair invariant masses are given by,

$$y_{q\gamma} = 1 - x', \quad y_{\bar{q}\gamma} = 1 - x, \quad y_{q\bar{q}} = 1 - x_\gamma, \quad (3)$$

where $y_{ij} = (p_i + p_j)^2/s = s_{ij}/s = 2E_i E_j (1 - \cos \theta_{ij})/s$.

The Dalitz plots for the CONE and DEMO schemes are shown in Fig. 2 for both the E0 and Durham algorithms. In the CONE scheme, the quark-photon singularities at $\theta_{q\gamma} = 0$ and $\theta_{\bar{q}\gamma} = 0$ which lie along the sides of the Dalitz plot are regulated by the photon isolation cut θ_c . In the DEMO scheme, the effective cone size for the photon cluster is much larger and consequently the cross section is much smaller [7]. Note that as is the case for pure jet analysis the Durham scheme [11] affords more phase space to the analysis for a given y_{cut} than that of the E0 scheme.

Figure 2: The Dalitz plots of isolated 1-jet + photon events with $y_{\text{cut}} = 0.1$ for (a) the CONE algorithm with $\theta_c = 30^\circ$ and (b) the DEMO algorithm. The 1-jet phase space is the region enclosed by the main diagonal, the isolation cut (dashed line) and the clustering algorithm - dotted line for E0 clustering and the dash-dotted line for the Durham algorithm.

The topology of the average (planar) event in the CONE and DEMO algorithms are quite different. As shown in Table 1, in the DEMO scheme, the average energy of the softest quark is larger than in the CONE scheme while the average angle between the quark and antiquark is much smaller. In the CONE algorithm, the softest quark typically lies relatively close to the isolation cone so that $y_{\gamma q \text{ soft}}$ is minimised. Momentum conservation then requires $\theta_{q\bar{q}}$ to be large while the requirement that the q and $\bar{q}$ coalesce to form a jet forces $E_{q \text{ soft}}$ to be relatively small. On the other hand, in the DEMO scheme, because the photon is in principle allowed to cluster with the quark/antiquark, the isolation criterion forces the $\theta_{\gamma q \text{ soft}}$ to be large. As a consequence, $\theta_{q\bar{q}}$ is much smaller and $E_{q \text{ soft}}$ somewhat larger. Within a given scheme, we see that $\theta_{q\bar{q}}$ is smaller with the E0 jet algorithm than in the Durham scheme. This reflects the extra phase space admitted by the Durham algorithm which is principally at large angles. The difference in the average angle between quark and antiquark will influence the colour flow in the event at higher orders.

Scheme	$E_\gamma/\sqrt{s}$	$E_{q \text{ hard}}/\sqrt{s}$	$E_{q \text{ soft}}/\sqrt{s}$	$\theta_{\gamma q \text{ hard}}$	$\theta_{\gamma q \text{ soft}}$	$\theta_{q\bar{q}}$
CONE Durham	.448	.475	.077	173	61	126
CONE E0	.486	.468	.047	176	81	103
DEMO Durham	.475	.419	.106	169	126	65
DEMO E0	.487	.427	.085	173	132	55

Table 1: The average value of energies and angles for 1 jet + ‘isolated’ photon events with $y_{\text{cut}} = 0.06$ and $\theta_c = 15^\circ$ for the different schemes. All angles are in degrees.

The next-to-leading order corrections involve the addition of a gluon to the event in its real and virtual forms. The physical cross section is the result of a cancellation between the divergent (and positive) bremsstrahlung process and the divergent (and negative) virtual process. The isolation cuts will play an important role in determining the size of the bremsstrahlung process since they will veto the allowed configuration of the $q\bar{q}\gamma g$ final state. The radiative cross section is largest when the gluon is soft. In this case, the matrix elements undergo the usual soft factorisation in the limit $E_g \rightarrow 0$,

$$|M_{q\bar{q}\gamma g}|^2 \rightarrow S_F |M_{q\bar{q}\gamma}|^2, \quad (4)$$

where the soft eikonal factor S_F is given by,

$$S_F \sim \frac{y_{q\bar{q}}}{y_{qg}y_{g\bar{q}}}. \quad (5)$$

One can therefore study the effect of the isolation cuts on the bremsstrahlung process by fixing the energy of the gluon to be soft and examining the colour flow with respect to the partons of the underlying $q\bar{q}\gamma$ event. The more that the isolation cuts restrict the allowed phase space for the soft gluon, the smaller the radiative contribution will be. The $\mathcal{K}$ factor will be correspondingly smaller.

In Fig. 3 we show radial plots, in the plane of the underlying event, of the soft eikonal factor, S_F . The $q\bar{q}\gamma$ configurations chosen are the average final state energies and angles, appropriate to (a) the CONE scheme and (b) the DEMO scheme for both the E0 and Durham algorithms as given in Table 1. The shaded regions of the plots indicate the forbidden region for gluon emission as dictated by the isolation cuts of the different schemes.

The quark and antiquark form a coloured antenna that radiates gluons [14]. The radiation is greatest in the direction of the quark (antiquark) corresponding to the singularity in $1/s_{qg}$ ($1/s_{g\bar{q}}$). Nevertheless, there is significant soft gluon radiation at other angles. The radiation patterns clearly reflect the differences between the CONE and DEMO schemes. In the CONE scheme, the antenna is bent to one side of the exclusion cone surrounding the photon. As a result, the radiation pattern is stretched on the side opposite the soft quark and compressed between the quark and antiquark. The colour flow in such events is still relatively spread out, and soft gluons are able to identify sources of colour flow all around phase space. In the E0 scheme the average angle between quark and antiquark is smaller and this effect is slightly enhanced relative to the CONE Durham scheme.

In contrast, in the DEMO scheme, the antenna has been bent much more - the average angle between quark and antiquark is less than 90° and the compression/stretching of the radiation pattern is much more significant. Together the quark and antiquark form a colour singlet antenna out of which only hard gluons, with a small wavelength, are able to radiate; i.e. soft gluons are unable to resolve the macroscopic dimensions of the jet. The physical consequence of this is that the soft gluons are radiated in the region *between* the two quarks where they can see two colour sources. In the limit that $y_{\text{cut}} \rightarrow 0$, $\theta_{q\bar{q}} \rightarrow 0$ and the antenna closes up. The jet has no net colour and there should be no soft gluon radiation outside of the jet. This is precisely what we are seeing in Fig. 3. As is true in the CONE case, the effect is more marked with the E0 jet algorithm than the Durham scheme.

Figure 3: Eikonal factor plots for a soft gluon lying in the plane of the underlying 1-jet + photon event for $y_{\text{cut}} = 0.06$ in (a) the CONE scheme (with $\theta_c = 15^\circ$) for the Durham (solid) and E0 (dotted) jet algorithms and (b) the DEMO scheme for the Durham (dashed) and E0 (dash-dotted) jet algorithms. The $q\bar{q}\gamma$ configuration represents the average configuration as described in Table 1. For reference, the energy vectors for the Durham scheme are also plotted. The scale is radially logarithmic and normalised to the minimum value of the eikonal factor in the DEMO E0 scheme. The shaded regions indicate the appropriate photon isolation cut for each scheme. In (b) the darker shaded region corresponds to the E0 isolation cut.

We can now interpret these radiation plots in terms of the photon isolation cuts. Whenever there is a significant overlap in the energy flow of soft gluons into the isolation cone, those events will be vetoed by the isolation criterion. We can see that in the CONE algorithm, there is a significant overlap in the energy flow of soft gluons into the isolation cone. In contrast to this the soft gluon flow in the democratic events is almost exclusively *away* from the photon and relatively few soft gluon events will be eliminated by the isolation cuts. We therefore expect that

$$\mathcal{K}^{DEMO} > \mathcal{K}^{CONE}.$$

We would also expect that hadronisation effects (and their influence on isolation) would be relatively smaller in the DEMO scheme precisely because the colour flow is largely contained within the jet. Experimentally, jet + photon events in the DEMO scheme should look rather clean in the hemisphere containing the photon. Indeed, it would be an interesting test of QCD to compare the energy flow (particle multiplicity) in ordinary two jet events where each jet is nominally a colour triplet with the energy flow (particle multiplicity) in democratic jet + photon events where the jet is almost a colour singlet.

Finally, we can make some comments about the jet algorithm dependence of the $\mathcal{K}$ factors within a given photon definition. In the democratic analysis, we see that (a) $\theta_{q\bar{q}}$ is smaller and (b) the angular region excluded by the isolation cuts is smaller in the E0 scheme compared to the Durham scheme. Crudely speaking, the E0 scheme is better at reaching into the wrong hemisphere to pull the gluon away from the photon and combine it with the quark-antiquark pair. This is precisely the feature that made the E0 scheme unattractive for purely hadronic final states. These arguments indicate that the E0 scheme will be more stable to radiative corrections so that,

$$\mathcal{K}_{E0}^{DEMO} > \mathcal{K}_D^{DEMO},$$

as is seen in Fig. 1. In fact, unlike ordinary purely hadronic jet events (and the CONE type photon + jet events where the colour antenna is relatively straight and the colour flow is more spread out), as $y_{\text{cut}} \rightarrow 0$, we do not resolve more jets because the radiation is largely within the jet and the $\mathcal{K}$ factor does not diminish for small y_{cut} . This is more marked in the DEMO E0 scheme because the average value of $\theta_{q\bar{q}}$ is smaller than in the DEMO Durham scheme.

In the CONE type algorithm, these simple arguments do not tell the whole story. We see in Fig. 3 that isolation cuts do play a more significant role in the Durham scheme, however, contrary to our naive expectations, we see from Fig. 1 that,

$$\mathcal{K}_D^{CONE} > \mathcal{K}_{E0}^{CONE}.$$

The reason for this discrepancy is that the typical energy of the soft quark is much lower in the CONE E0 scheme than in all the other schemes. This is shown in Fig. 4 as a function of the jet resolution parameter y_{cut} . Typically, the soft quark in this scheme has an energy of only a few GeV, significantly less than in the other schemes. To understand how the isolation cuts affect the isolated photon cross section, we have focussed on soft gluon radiation with an energy that is in principle resolvable by an experiment, which is typically of $O(1 \text{ GeV})$. This is not much less than the average energy of the softest quark in the CONE E0 scheme.

Figure 4: The average energy of the softest quark at leading order for $e^+e^- \rightarrow 1\text{-jet} +$ ‘isolated’ photon production at LEP energies as a function of y_{cut} in the CONE scheme (with $\theta_c = 15^\circ$) for the Durham (solid) and E0 (dotted) jet algorithms and in the DEMO scheme also for the Durham (dashed) and E0 (dash-dotted) jet algorithms.

Therefore, we might expect that the soft gluon approximation is not very reliable in this case and, although the gross features are correct, the radiation pattern does not accurately describe the distribution of soft gluons for this particular scheme. Consequently, the detailed ordering of $\mathcal{K}$ -factors in the CONE scheme cannot be explained by the rather qualitative discussion presented here.

In this letter we have attempted to understand the relative sizes of QCD corrections to ‘isolated’ photon + 1 jet events at LEP which depend quite dramatically upon the scheme used to define the ‘isolated’ photon. To do this we have examined the colour flow of soft gluons with respect to the underlying hard scattering and how the isolation cuts restrict the allowed phase space of the soft gluon. While these arguments are rather qualitative, they do describe the more quantitative results of Fig. 1, namely,

$$\mathcal{K}^{DEMO} > \mathcal{K}^{CONE}.$$

At the same time, we can understand the hierarchy of radiative corrections in the DEMO scheme.

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