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Grand Unified String Theories with $SU(3)$ Gauge Family Symmetry

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Abstract

In the framework of four dimensional heterotic superstring with free fermions we investigate the rank eight Grand Unified String Theories (GUST) which contain the $SU(3)_H$ -gauge family symmetry. GUSTs of this type accommodate naturally three fermion families presently observed and, moreover, can describe fermion mass spectrum without high dimensional representations of conventional unification groups. We explicitly construct GUST with gauge symmetry $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H \subset SO(16)$ in free complex fermion formulation. As the GUSTs originating from Kac - Moody algebras (KMA) contain only low- dimensional representations it is usually difficult to break the gauge symmetry. We solve this problem taking for the observable gauge symmetry the diagonal subgroup G^{sym} of rank 16 group $G \times G \subset SO(16) \times SO(16) \subset E(8) \times E(8)$. Such a construction effectively corresponds to a level two KMA, and therefore some higher dimensional representations of the diagonal subgroup appear. This (due to $G \times G$ - tensor Higgs fields) allows one to break GUST symmetry down to the $SU(3^c) \times U(1)_{em}$. In this approach the observed electromagnetic charge Q^{em} can be viewed as a sum of two $Q^{I'}$ - and $Q^{II'}$ charges of each G - group. In this case below the scale where $G \times G$ breaks down to G^{sym} the spectrum does not contain particles with exotic fractional charges.

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1 Introduction

Since a couple of years superstring theories and particularly the heterotic string theory, provide the efficient way to construct the Grand Unified Supersymmetric Theories (*GUST*) of all known interactions. Despite of that it is still difficult to construct unique and fully realistic low energy models resulting after decoupling of massive string modes. This is because it is only in 10- dimensional space-time that there exist just two consistent (invariant under reparametrization, superconformal, modular, Lorentz and SUSY transformations) theories with gauge symmetries $E(8) \times E(8)$ or $spin(32)/Z_2$ [1, 2] which after compactification of the six extra space coordinates (into the Calabi-Yau [3, 4] manifolds or into the orbifolds) can be used for constructing *GUSTs*. Unfortunately, the process of compactification to four dimensions is not unique and the number of possible low energy models is very large. On the other hand, starting the construction of the theory directly in 4- dimensional space-time requires including a considerable number of free bosons or fermions into the internal string sector of the heterotic superstring [5, 6, 7]. This leads to as large internal symmetry group as e.g. rank 22 group. The way of breaking this primordial symmetry is again not unique and leads to a huge number of possible models, each of them giving different low energy predictions.

On the other side, due to the presence of the affine Kac- Moody algebra (KMA) $\hat{g}$ (which is a 2- dimensional manifestation of gauge symmetries of the string itself) on the world sheet, string constructions yield definite predictions as to what representation of the symmetry group can be used for low energy models building. Therefore the following long-standing questions have the chance to be answered in this kind of unification schemes:

1. How are the chiral matter fermions assigned to the multiplets of the unifying group?
2. How is the GUT gauge symmetry breaking realized?
3. What is the origin and the form of the fermion mass matrices?

The first of these problems is, of course, closely connected with the quantization of the electromagnetic charge of matter fields. In addition, string constructions can shed some light on the questions about the number of generation and possible existence of mirror fermions which remain unanswered in conventional GUTs.

In this note starting from the rank 16 grand unified gauge group (which is the minimal rank allowed in strings [6, 10]) of the form $G \times G$ and making use of the KMA which select the possible gauge group representations we construct the string model based on the diagonal subgroup $G^{sym} \subset$

$G \times G \subset SO(16) \times SO(16)$ ($\subset E_8 \times E_8$). We discuss and consider $G^{sym} = SU(5) \times U(1) \times (SU(3) \times U(1))_H \subset SO(16)$ where the factor $(SU(3) \times U(1))_H$ is interpreted as the horizontal gauge family symmetry. We explain how the unifying gauge symmetry can be broken down to the Standard Model group. Furthermore, the horizontal interaction predicted in our model can give alternative description of the fermion mass matrices without invoking high dimensional Higgs representations. In contrast to other *GUST* constructions, our model does not contain particles with exotic fractional electric charges. This important virtue of the model is due to the symmetric construction of the electromagnetic charge Q_{em} from Q' and Q'' - the two electric charges of each of the $U(5)$ groups.

2 World-Sheet Kac-Moody Algebra And Main Features of Rank Eight GUST

Let's begin with a short review of the KMA results [8, 9]. The KMA $\hat{g}$ allow to grade the representations R of the gauge group by a level number x (a non negative integer) and by a conformal weight $h(R)$.

An irreducible representation of the affine algebra $\hat{g}$ is characterized by the vacuum representation of the algebra g and the value of the central term k , which is connected with the level number by the relation $x = 2k/\psi^2$ where ψ is the highest root. The value of the level number of the KMA determines the possible highest weight unitary representation which are present in the spectrum, in the following way:

$$x = \frac{2k}{\psi^2} \geq \sum_{i=1}^{rank\ g} n_i n_i, \quad (1)$$

where the sets of non-negative integers $\{n_i = m_1, \dots, m_r\}$ and $\{n_i = n_1, \dots, n_r\}$ define the highest root and the highest weight of a representation R respectively [8, 9, 10].

In fact, the KMA on the level one is realized in the 4-dimensional heterotic supersymmetric theories with free world sheet fermions [6, 7, 10]. For these models the level of KMA coincides with the Dynkin index of representation M to which free fermions are assigned:

$$x = x_M = \frac{Q_M \dim M}{\psi^2 \dim g} \quad (2)$$

(Q_M is a quadratic Casimir eigenvalue of representation M) and equals one in cases when real fermions form vector representation M of $SO(2N)$, or when the world sheet fermions are complex and M is the fundamental representation of $U(N)$ [8, 9].

Thus, in strings with KMA on the level one realized on the world-sheet, only very restricted set of unitary representations can arise in the spectrum [8, 9, 10]:

1. singlet and totally antisymmetric tensor representations of $SU(N)$ groups, for which $m_i = (1, \dots, 1)$;
2. singlet, vector and spinor representations of $SO(2N)$ groups, with $m_i = (1, 2, 2, \dots, 2, 1, 1)$;
3. singlet, $\underline{27}$, and $\underline{27}$ -plets of $E(6)$, corresponding to $m_i = (1, 2, 2, 3, 2, 1)$;
4. singlet of $E(8)$, with $m_i = (2, 3, 4, 6, 5, 4, 3, 2)$.

Therefore only these representations can be used to incorporate matter and Higgs fields in GUSTs.

For example, to describe chiral matter fermions in GUST with the gauge symmetry group $SU(5) \times U(1) \subset SO(10)$ the following sum of the level- one complex representations: $\underline{1}(-5/2) + \underline{\bar{5}}(+3/2) + \underline{10}(-1/2) = \underline{16}$ can be used. On the other side, as real representations of $SU(5) \times U(1) \subset SO(10)$, from which Higgs fields can arise, one can take for example $\underline{5} + \underline{\bar{5}}$ representations arising from real representation $\underline{10}$ of $SO(10)$. Also, real Higgs representations like $\underline{10}(-1/2) + \underline{10}(+1/2)$ of $SU(5) \times U(1)$ originating from $\underline{16} + \underline{\bar{16}}$ of $SO(10)$, which has been used in ref. [10] for further symmetry breaking, are allowed.

Another example is provided by the decomposition of $SO(16)$ representations under $SU(8) \times U(1) \subset SO(16)$. Here only singlet, $\underline{16}$, $\underline{128}$ and $\underline{128}'$ representations of $SO(16)$ are allowed by the KMA ($\underline{128}$ and $\underline{128}'$ are the two nonequivalent, real spinor representations). From the item 2. we can obtain the following $SU(8) \times U(1)$ representations: singlet, $\underline{8} + \underline{\bar{8}} (= \underline{16})$, $\underline{8} + \underline{56} + \underline{56} + \underline{\bar{8}} (= \underline{128})$ and $\underline{1} + \underline{28} + \underline{70} + \underline{28} + \underline{\bar{1}} (= \underline{128}')$.

However, as we will demonstrate, in each of the string sectors the requirement of the modular invariance (one of the GSO projections) necessarily eliminates either $\underline{128}$ or $\underline{128}'$. It is therefore important that, in order to incorporate chiral matter in the model, only one spinor representation is sufficient. Moreover, if one wants to solve the chirality problem applying further GSO projections (which break the gauge symmetry also the representation $\underline{10}$ which otherwise, together with $\underline{10}$, could form real Higgs representation, disappears from this sector. Therefore, the existence of $\underline{10}_{-1/2} + \underline{10}_{1/2}$, needed for breaking $SU(5) \times U(1)$ is incompatible with the possible solution of the chirality problem for the family matter fields.

Thus, in the rank eight group $SU(8) \times U(1) \subset SO(16)$ with Higgs representations from the level- one KMA only, one can not arrange for further symmetry breaking. Moreover, construction of the realistic fermion mass matrices seems to be impossible. In old-fashioned GUTs (see e.g. [11]), not

originating from strings, the representations of level-two (which are not the representations of level-one) were commonly used to solve these problems.

The way out from this difficulty is based on the following important observations. Firstly, all higher-dimensional representations of (simple laced) groups like $SU(N)$, $SO(2N)$ or $E(6)$, which belong to the level-two of the KMA (according to the equation (1)), appear in the direct product of the level- one representations:

$$R_G(x=2) \subset R_G(x=1) \times R_G'(x=1). \quad (3)$$

For example, the level-two representations of $SU(5)$: $\underline{15}$, $\underline{24}$, $\underline{40}$, $\underline{45}$, $\underline{50}$, $\underline{75}$ will appear in the direct products of: $\underline{5} \times \underline{\bar{5}}$, $\underline{5} \times \underline{\bar{5}} \times \underline{10}$, etc. respectively. In the case of $SO(10)$ the level two representations $\underline{45}$, $\underline{54}$, $\underline{120}$, $\underline{126}$, $\underline{210}$, $\underline{144}$ can be obtained by the suitable direct products: $\underline{10} \times \underline{10}$, $\underline{16} \times \underline{10}$, $\underline{16} \times \underline{16}$, $\underline{16} \times \underline{16}$. The level- two representations $\underline{78}$, $\underline{351}$, $\underline{351}'$, $\underline{650}$ of $E(6)$ are factors of the decomposition of the direct products of: $\underline{27} \times \underline{27}$ or $\underline{27} \times \underline{27}$. The only exception from this rule is the $E(8)$ group, two level- two representations ($\underline{248}$ and $\underline{3875}$) of which cannot be constructed as a product of level- one representations [12].

Secondly, the diagonal (symmetric) subgroup G^{symm} of $G \times G$ effectively corresponds to the level- two KMA $g(x=1) \oplus g(x=1)$ because taking the $G \times G$ representations in the form (R_G, R_G') of the $G \times G$, where R_G and R_G' belong to the level- one of G , one obtains representations of the form $R_G \times R_G'$ when one considers only the diagonal subgroup of $G \times G$. This observation is crucial, because such a construction allows one to obtain level- two representations. (This construction has implicitly been used in [13] where we have constructed some examples of GUST with gauge symmetry realized as a diagonal subgroup of direct product of two rank eight groups $U(8) \times U(8) \subset SO(16) \times SO(16)$.)

In strings, however, not all level- two representations can be obtained in that way because, as we will demonstrate, some of them become massive (with masses of order of the Planck scale). The condition ensuring that in the string spectrum states transforming as a representation R are massless reads:

$$h(R) = \frac{Q_R}{2k + Q_{ADJ}} = \frac{Q_R}{2Q_M} \leq 1, \quad (4)$$

where Q_i is the quadratic Casimir invariant of the corresponding representations, and M has been already defined before (see eq. (2)).

This condition, when combined with (1), gives a restriction at the rank of GUT's group ($r \leq 8$), whose representations can accommodate chiral matter fields. For example, for $G = SO(16)$ or $E(6) \times SU(3)$, representations $\underline{128}$, $(\underline{27}, \underline{3})$ ($h(\underline{128}) = 1$, $h(\underline{27}, \underline{3}) = 1$) respectively, satisfy both conditions. Obviously, these (important for incorporation of chiral matter) representations

will exist at the level- two KMA of the symmetric subgroup of the group $G \times G$.

In general, condition (4) severely constrains massless string states transforming as $(R_G(x=1), R_G'(x=1))$ of the direct product $G \times G$. For example, for $SU(8) \times SU(8)$ and for $SU(5) \times SU(5)$ constructed from $SU(8) \times SU(8)$ only representations of the form

$$R_{N,N} = ((\underline{N}, \underline{N}) + h.c.), ((\underline{N}, \underline{\bar{N}}) + h.c.); \quad (5)$$

with $h(R_{N,N}) = (N-1)/N$, where $N = 8$ or 5 respectively can be massless [13]. For $SO(2N) \times SO(2N)$ massless states are contained only in representations

$$R_{u,v} = (\underline{2N}, \underline{2N}) \quad (6)$$

with $h(R_{u,v}) = 1$. Thus, for the GUSTs based on a diagonal subgroup $G^{symm} \subset G \times G$, high dimensional representations, which are embedded in $R_G(x=1) \times R'_G(x=1)$ are also severely constrained by the condition (4).

For spontaneous breaking of $G \times G$ gauge symmetry down to G^{symm} (rank $G^{symm} = \text{rank } G$) one can use the direct product of representations $R_G(x=1) \times R_G(x=1)$, where $R_G(x=1)$ is the fundamental representation of $G = SU(N)$ or vector representation of $G = SO(2N)$. Furthermore, $G^{symm} \subset G \times G$ can subsequently be broken down to a smaller dimension gauge group (of the same rank as G^{symm}) through the VEVs of the adjoint representations which can appear as a result of $G \times G$ breaking. Alternatively, the real Higgs superfields (5) or (6) can directly break the $G \times G$ gauge symmetry down to a $G^{symm} \subset G^{symm}$ (rank $G^{symm} \leq \text{rank } G^{symm}$). For example when $G = SU(5) \times U(1)$ or $SO(10) \times U(1)$ $G \times G$ can directly be broken in that way down to $SU(3^c) \times G'_{EW} \times G''_{EW} \times \dots$

The above examples show clearly, that within the framework of GUSTs with the KMA one can get interesting gauge symmetry breaking chains including the realistic ones provided $G \times G$ gauge symmetry group is considered. However the lack of the higher dimensional representations (which are forbidden by (4)) on the level- two KMA prevents the construction of the realistic fermion mass matrices. That is why we consider an extended grand unified string model of rank eight.

There are good physical reasons for including the horizontal $SU(3)_H$ group into the unification scheme. Firstly, this group naturally accommodates three fermion families presently observed (explaining their origin) and, secondly, can provide correct and economical description of the fermion mass spectrum and mixing without invoking high dimensional representation of conventional $SU(5)$, $SO(10)$ or $E(6)$ gauge groups. Construction of a string model (GUST) containing the horizontal gauge symmetry provides additional, strong motivation to this idea. Moreover, the fact that in GUSTs

high dimensional representations are forbidden by the KMA is a very welcome feature in this context.

All this leads us naturally to consider possible forms for horizontal symmetry G_H , and G_H - quantum number assignments for quarks (anti-quarks) and leptons (anti-leptons) which can be realized within GUST's framework. To include the horizontal interactions with three known generations in the ordinary GUST it is natural to consider rank eight gauge symmetry. In this paper we concentrate on $SO(16)$ which is a maximal subgroup of $E(8)$ and which contains the rank eight subgroup $SO(10) \times (U(1) \times SU(3))_H$ (there are also other possibilities as eg. $E(6) \times SU(3)_H \subset E(8)$). We will be, therefore, concerned with the following chain:

$$\begin{aligned} E(8) &\longrightarrow SO(16) \longrightarrow SO(10) \times (U(1) \times SU(3))_H \longrightarrow \\ &\longrightarrow SU(5) \times U(1)_Y \times (SU(3) \times U(1))_H. \end{aligned}$$

According to this scheme one can get $SU(3)_H \times U(1)_H$ gauge family symmetry with $N_g = 3 + 1$ or $N_g = 3$ generations, where the possible, additional, fourth (massive) massive matter superfield is a singlet of $SU(3)_H$ and transforms as $\underline{16}$ under the $SO(10)$ group. The full chiral $SO(10) \times SU(3) \times U(1)$ matter multiplets can be constructed from $SU(8) \times U(1)$ -multiplets

$$(\underline{8} + \underline{56} + \underline{\bar{8}} + \underline{56}) = \underline{128} \quad (7)$$

of $SO(16)$. In the 4- dimensional heterotic superstring with free complex world sheet fermions, in the spectrum of the Ramond sector there can appear also representations which are factors in the decomposition of $\underline{128}$. In particular, $SU(5)$ -decouplets ($\underline{10} + \underline{10}$) from $(\underline{28} + \underline{28})$ of $SU(8)$. However their $U(1)_S$ hypercharge prevent using them for $SU(5) \times U(1)_S$ - symmetry breaking. Thus, in this approach we have only singlet and $(\underline{5} + \underline{\bar{5}})$ Higgs fields which can break the grand unified $SU(5) \times U(1)$ gauge symmetry. Therefore it is necessary (as we already explained) to construct rank eight GUST based on a diagonal subgroup $G^{symm} \subset G \times G$ primordial symmetry group, where in each rank eight group G the Higgs fields will appear only in singlets and in the fundamental representations as in (see 5).

A comment concerning $U(1)$ factors can be made here. Since the available $SU(5) \times U(1)$ decouplets have non- zero hypercharges with respect to $U(1)_S$ and $U(1)_H$, these $U(1)$ factors may remain unbroken down to the low energies in the model considered which seems to be very interesting.

3 Construction GUST with SU(3) Gauge Family Symmetry in Free Fermion Formulation

In heterotic string theories $[1, 2]$ ($N = 1$ $SUSY$)_{LEFT} ($N = 0$ $SUSY$)_{RIGHT} $\oplus \mathcal{M}_{c_L, c_R}$ with $d \leq 10$, the conformal anomalies of the space-time sector are canceled by the conformal anomalies of the internal sector $\mathcal{M}_{c_L, c_R}$, where $c_L = 15 - 3d/2$ and $c_R = 26 - d$ are the conformal anomalies in the left- and right- moving string sectors respectively.

In the fermionic formulation of the four-dimensional heterotic string theory in addition to the two transverse bosonic coordinates $X_\mu, \tilde{X}_\mu$ and their left- moving superpartners ψ_μ , the internal sector $\mathcal{M}_{c_L, c_R}$ contains 44 right- moving ($c_R = 22$) and 18- left- moving ($c_L = 9$) real fermions. The model is completely defined by a set Ξ of spin boundary conditions for all these world-sheet fermions. In a diagonal basis the vectors of Ξ are determined by the values of phases $\alpha(f) \in (-1, 1]$ fermions f acquire ($f \rightarrow -\exp(i\pi\alpha(f))f$) when parallel transported around the string. To construct the GUST according to the scheme outlined at the end of the previous section we consider the set Ξ which forms the $Z_2 \times Z_4 \times Z_2 \times Z_8 \times Z_2$ group under addition (mod 2) and is generated by the basis with six elements $B = b_1, b_2, b_3, b_4 \equiv S, b_5, b_6$ given in Table 1.

Table 1. Basis of the boundary conditions for all world- sheet fermions.

Vectors	$\psi_{1,2}$	$\chi_{1,...,6}$	$y_{1,...,6}$	$\omega_{1,...,6}$	$\varphi_{1,...,12}$	$\tilde{\psi}_{1,...,8}$	$\Phi_{1,...,8}$
b_1	11	111111	111111	111111	1^{12}	1^8	1^8
b_2	11	111111	000000	000000	0^{12}	$1/2^8$	0^8
b_3	11	111100	000011	000000	$0^4 1^8$	0^8	1^8
$b_4 \equiv S$	11	110000	001100	000011	0^{12}	0^8	0^8
b_5	11	001100	000000	110011	1^{12}	$1/4^5 - 3/4^3$	$-1/4^5 3/4^3$
b_6	11	110000	000011	001100	110011001111	1^8	0^8

Following [7] we construct the canonical basis in such a way that the vector $\bar{1}$, which belongs to Ξ , is the first element b_1 of the basis. The basis vector $b_i = S$ is the generator of supersymmetry [10] responsible for the conservation of the space- time $SUSY$.

We have chosen a basis in which all left movers ($\psi_\mu, \chi_i, y_i, \omega_i; i = 1, \dots, 6$) (on which the world sheet supersymmetry is realized nonlinearly) as well as 12 right movers ($\varphi_k; k = 1, \dots, 12$) are real whereas $(8 + 8)$ - right movers $\tilde{\psi}_A, \Phi_M$ are complex. Right- and left- moving real fermions can be used for breaking G^{comp} symmetry [6, 10]. In order to have a possibility to reduce the rank of the compactified group G^{comp} , we have to select the spin boundary conditions for the maximal possible number, $N_{LR} = 12$, of left- moving,

$\chi_{3,4,5,6}, y_{1,2,5,6}, \omega_{1,2,3,4}$, and right- moving, $\tilde{\phi}^{1,...,12}$ ($\tilde{\phi}^p = \tilde{\varphi}_p, p = 1, \dots, 12$) real fermions. The KMA based on 16 complex right moving fermions gives rise to the "observable" gauge group, G^{obs} , with:

$$rank(G^{obs}) \leq 16. \quad (8)$$

In our approach the basis vector b_2 is constructed as a complex vector with the $1/2$ spin- boundary conditions for the right- moving fermions $\tilde{\psi}_A, A = 1, \dots, 8$. Initially it generates chiral matter fields in the $\mathfrak{g} + \mathfrak{so} + \mathfrak{so} + \mathfrak{so}$ representations of $SU(8) \times U(1)$, which subsequently are decomposed under $SU(5) \times U(1) \times SU(3) \times U(1)$ to which $SU(8) \times U(1)$ gets broken by applying the the b_5 GSO projection.

Generalized GSO projection coefficients are originally defined up to fifteen signs some of which, are fixed by the supersymmetry conditions. Below, in Table 2, we present a set of numbers

$$\gamma \begin{bmatrix} b_i \\ b_j \end{bmatrix} = \frac{1}{i\pi} \log \mathcal{C} \begin{bmatrix} b_i \\ b_j \end{bmatrix}.$$

which we use as basis for our GSO projections.

Table 2. The choice of the GSO basis $\gamma \begin{bmatrix} b_i \\ b_j \end{bmatrix}$, b_i, i numbers rows and j - columns.

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	0	1	1	1	1	0
b_2	1	$1/2$	0	0	$1/4$	1
b_3	1	$-1/2$	0	0	$1/2$	0
b_4	1	1	1	1	1	1
b_5	0	1	0	0	$-1/2$	0
b_6	0	0	0	0	0	1

The physical states in the Hilbert space of a given sector α are obtained acting on the vacuum $|0\rangle_a$ with the bosonic and fermionic operators with frequencies

$$n(f) = 1/2 + 1/2\alpha(f), \quad n(f^*) = 1/2 - 1/2\alpha(f^*) \quad (9)$$

and subsequently applying the generalized GSO projections. The physical states satisfy the Virasoro condition:

$$M_L^2 = -1/2 + 1/8(\alpha_L \cdot \alpha_L) + N_L = -1 + 1/8(\alpha_R \cdot \alpha_R) + N_R = M_R^2, \quad (10)$$

where $\alpha = (\alpha_L, \alpha_R)$ is a sector in the set Ξ , $N_L = \sum_L(\text{frequencies})$ and $N_R = \sum_R(\text{frequencies})$.

We keep the same sign convention for the fermion number operator F as in [7, 10]. For complex fermions we have $F_a(f) = 1$, $F_a(f^*) = -1$ with the exception of the periodic fermions for which we get $F_{a=1}(f) = -1/2(1 - \gamma_5 f)$, where $\gamma_5 f / \Omega > = |\Omega >$, $\gamma_5 f b_0^+ / \Omega > = -b_0^+ / \Omega >$.

The full Hilbert space of the string theory is constructed as a direct sum of different sectors $\sum_i m_i b_i$, ($m_i = 0, 1, \dots, N_i$), where the integers N_i define additive groups $Z(b_i)$ of the basis vectors b_i . The generalized GSO projection leaves in sectors α those states, whose b_i -fermion number satisfies:

$$\exp(i\pi b_i F_\alpha) = \delta_\alpha C^* \begin{bmatrix} \alpha \\ b_i \end{bmatrix}, \quad (11)$$

where the space-time phase $\delta_\alpha = \exp(i\pi\alpha(\psi_\mu))$ is equal -1 for the Ramond sector and $+1$ for the Neveu-Schwarz sector. In our case of the $Z_2^4 \times Z_4 \times Z_8$ model, we initially have 256×2 sectors. After applying the GSO-projections we get only 49×2 sectors containing massless states, which depending on the vacuum energy values, E_L^{vac} and E_R^{vac} , can be naturally divided into some classes [13] and which determine the GUST representations.

The NS sector contains the $N = 1$ supergravity multiplet as well as gauge fields (with their $N = 1$ superpartners) of the observable and compactified gauge groups. For example, massless string states yielding gauge fields of the group

$$[SU(5) \times U(1)]_{GUT} \times [SU(3) \times U(1)]_H. \quad (12)$$

corresponding to $U(8)'$ are constructed as follows:

$$\begin{aligned} \psi_{1/2}^\mu |\Omega >_L \otimes \bar{\phi}_{1/2}^a \bar{\phi}_{1/2}^{b*} |\Omega >_R, \quad a, b = 1, \dots, 5; \\ \psi_{1/2}^\mu |\Omega >_L \otimes \bar{\phi}_{1/2}^i \bar{\phi}_{1/2}^{j*} |\Omega >_R, \quad i, j = 1, 2, 3, \end{aligned} \quad (13)$$

where $\bar{\phi}_{1, \dots, 5; 1, 2, 3} = \bar{\psi}_{1, \dots, 8}$. Similar states arise also for $U(8)'/I$ generated by $\bar{\phi}_{1/2}$ complex fermions. In the compactified $SO(12)$ sector there remains $SO(2)^3 \times SO(6)$ gauge group.

In NS sector the b_3 GSO projection leaves $(5, \bar{3}) + (\bar{5}, 3)$ Higgs superfields:

$$\chi_{1/2}^{3,1} |\Omega >_L \otimes \bar{\phi}_{1/2}^a \bar{\phi}_{1/2}^{i*}; \quad \bar{\phi}_{1/2}^{a*} \bar{\phi}_{1/2}^i |\Omega >_R \quad (14)$$

Four $(3_H + 1_H)$ generations of chiral matter fields from $(SU(5) \times SU(3))_I$ group forming $SO(10)$ -multiplets $(\underline{1}, \underline{3}) + (\bar{\underline{5}}, \underline{3}) + (\underline{10}, \underline{3})$; $(\underline{1}, \underline{1}) + (\bar{\underline{5}}, \underline{1}) + (\underline{10}, \underline{1})$ are contained in $b_2 + 6b_5$ and $3b_2 + 2b_5$ sectors. Applying b_3 GSO projection to the $3b_2 + 2b_5$ sector yields the following massless states:

$$\begin{aligned} b_{\psi_{12}}^+ b_{\chi_{36}}^+ |\Omega >_L \otimes \left\{ \bar{\phi}_{3/4}^{1*}, \quad \bar{\phi}_{1/4}^a \bar{\phi}_{1/4}^{b*}, \quad \bar{\phi}_{1/4}^a \bar{\phi}_{1/4}^{i*} \right\} |\Omega >_R, \\ b_{\chi_{36}}^+ |\Omega >_L \otimes \left\{ \bar{\phi}_{3/4}^{a*}, \quad \bar{\phi}_{1/4}^i \bar{\phi}_{1/4}^{b*}, \quad \bar{\phi}_{1/4}^i \bar{\phi}_{1/4}^{j*} \right\} |\Omega >_R \end{aligned} \quad (15)$$

with the space-time chirality $\gamma_5 \psi_{12} = -1$ and $\gamma_5 \psi_{12} = 1$, respectively. In these formulae the Ramond creation operators $b_{\psi_{12}}^+$ and $b_{\chi_{36}}^+$ of the zero modes are built of a pair of real fermions (as indicated by double indices): $\chi_{a,b}$, $(\alpha, \beta) = (3, 4), (5, 6), (7, 8)$. Here, as in (14) and (15) indices take values $a, b = 1, \dots, 5$ and $i, j = 1, 2, 3$ respectively.

We stress that without using the b_3 projection we would get matter supermultiplets belonging to real representations only i.e. "mirror" particles would remain in the spectrum. The b_6 projection instead, eliminates all chiral matter superfields from $U(8)'$ group.

Since the matter fields form the chiral multiplets of $SO(10)$, it is possible to write down $U(1)_{Y_5}$ -hypercharges of massless states. In order to construct the right electromagnetic charges for matter fields we must define the hypercharges operators for the observable $U(8)''$ group as

$$Y_5 = \int_0^r d\sigma \sum_a \Psi^{*a} \Psi^a, \quad Y_3 = \int_0^r d\sigma \sum_i \Psi^{*i} \Psi^i \quad (16)$$

and analogously for the $U(8)'$ group.

Then the following orthogonal combinations

$$\bar{Y}_5 = \frac{1}{4}(Y_5 + 5Y_3), \quad \bar{Y}_3 = \frac{1}{4}(Y_5 - 3Y_3), \quad (17)$$

play the role of the hypercharge operators of $U(1)_{Y_5}$ and $U(1)_{Y_H}$ groups, respectively. Below we shall write the hypercharges as $(Y_5', Y_3', Y_5'', Y_3'')$.

4 GUST Spectrum and Gauge Symmetry Breaking

In the GUST construction presented in the preceding section there exists a possibility to break the GUST group $(U(5) \times U(3)) \times (U(5) \times U(3))$ down to the symmetric group by the ordinary Higgs mechanism [13]:

$$G^I \times G^{II} \rightarrow G^{\text{symm}} \rightarrow \dots \quad (18)$$

To achieve such breaking one can use nonzero vacuum expectation values of the tensor Higgs fields (see eq.4) $\hat{\Phi}_{(\Gamma, N; \Gamma, N)} + h.c.$ with $(\Gamma = \underline{5}$ or $\underline{1}$ and $N = \underline{1}$ or $\underline{3})$, contained in the $2b_2 + 2(6)b_5(+S)$ sectors which transform under the $(SU(5) \times U(1) \times SU(3) \times U(1))_{\text{symm}}$ group in the following way:

$$\begin{aligned} (\underline{5}, \underline{1}; \underline{5}, \underline{1})_{(-1, 0; -1, 0)} &\rightarrow (\underline{24}, \underline{1})_{(0, 0)} + (\underline{1}, \underline{1})_{(0, 0)}; \\ (\underline{1}, \underline{3}; \underline{1}, \underline{3})_{(0, 1; 0, 1)} &\rightarrow (\underline{1}, \underline{8})_{(0, 0)} + (\underline{1}, \underline{1})_{(0, 0)}, \\ (\underline{5}, \underline{1}; \underline{1}, \underline{3})_{(-1, 0; 0, 1)} &\rightarrow (\underline{\bar{5}}, \underline{3})_{(1, 1)}; \\ (\underline{1}, \underline{3}; \underline{5}, \underline{1})_{(0, 1; -1, 0)} &\rightarrow (\underline{\bar{5}}, \underline{3})_{(-1, -1)}. \end{aligned} \quad (19)$$

The diagonal vacuum expectation values for the Higgs fields (19) break the the GUST group $(U(5) \times U(3))^I \times (U(5) \times U(3))^{II}$ down to the "skew"-symmetric group with the generators Δ_{symm} of the form:

$$\Delta_{\text{symm}}(t) = \theta(t) \times 1 + 1 \times t, \quad (21)$$

where $\theta(t) = -t^*$. The corresponding hypercharge of the symmetric group reads:

$$\bar{Y} = \bar{Y}^{II} - \bar{Y}^I. \quad (22)$$

Similarly, for the electromagnetic charge we get:

$$\begin{aligned} Q_{em} &= Q^{II} - Q^I = \\ &= (T_5^{II} - T_5^I) + \frac{2}{5}(\bar{Y}_5^{II} - \bar{Y}_5^I) = \bar{T}_5 + \frac{2}{5}\bar{Y}_5, \end{aligned} \quad (23)$$

where $T_5 = \text{diag}(\frac{1}{15}, \frac{1}{15}, \frac{1}{15}, \frac{2}{3}, -\frac{2}{3})$. Note, that this charge quantization does not lead to exotic states with fractional electromagnetic charges (e.g. $Q_{em} = \pm 1/2, \pm 1/6$).

Thus, in the breaking scheme (21) it is possible to avoid colour singlet states with fractional electromagnetic charges, to achieve desired GUT breaking and moreover to get the usual value for the weak mixing angle $\sin^2 \theta_W = 3/8$ at the unification scale (see [14]).

Adjoint representations which appear on the r.h.s of (19) can be used for further breaking of the symmetric group. This can lead to the final physical symmetry

$$(SU(3^c) \times SU(2_{EW}) \times U(1_Y) \times U(1)') \times (SU(3_H) \times U(1_H)) \quad (24)$$

with low- energy gauge symmetry of the quark – lepton generations with an additional $U(1)'$ - factor.

Note, that using the same Higgs fields as in (19), there exists also another, interesting way of breaking the $G^I \times G^{II}$ gauge symmetry:

$$\begin{aligned} G^I \times G^{II} &\rightarrow SU(3^c) \times SU(2)_{EW}^I \times SU(2)_{EW}^{II} \times U(1_Y) \times \\ &\times SU(3_H)^I \times SU(3_H)^{II} \times U(1_{Y_H}) \rightarrow \dots \end{aligned} \quad (25)$$

It is attractive because it naturally solves the Higgs doublet- triplet mass splitting problem with rather low energy scale of GUST symmetry breaking.

In turn, the Higgs fields from the NS sector (and fields (20)) which we denote as $\hat{h}_{(1,N)}$, originating from $\underline{63}$ of $SU(8)^{II}$ (or $SU(8)^I$), and transforming as

$$(\underline{5}, \underline{3})(-1, -1) + (\underline{5}, \underline{3})(1, 1) \quad (26)$$

can be used for constructing chiral fermion (see eq.27) mass matrices.

The B spin boundary conditions (Tabl.1) generate chiral matter and Higgs fields with the GUST gauge symmetry $G_{comp} \times (G^I \times G^{II})_{obs}$ (where $G_{comp} = SO(6) \times U(1)^3$ and $G^{I,II}$ have been already defined). The chiral matter spectrum, which we denote $\hat{\Psi}_{(1,N)}$ with $(\Gamma = \underline{1}, \underline{5}, \underline{10}; N = \underline{3}, \underline{1})$, consists of $N_g = 3_H + 1_H$ families with the following $((SU(5) \times U(1)) \times (SU(3) \times U(1)_H))^{symm}$ quantum numbers:

$$\begin{aligned} &(\underline{1}, \underline{3})_{(5/2, -1/2)}, \quad (\underline{5}, \underline{3})_{(-3/2, -1/2)}, \quad (\underline{10}, \underline{3})_{(1/2, -1/2)} \\ &(\underline{1}, \underline{1})_{(5/2, 3/2)}, \quad (\underline{5}, \underline{1})_{(-3/2, 3/2)}, \quad (\underline{10}, \underline{1})_{(1/2, 3/2)}. \end{aligned} \quad (27)$$

The $SU(3_H)$ anomalies of the matter fields (27) are naturally canceled by the chiral "horizontal" superfields coming in two copies and forming two sets: $\hat{\Psi}_{(1,N,1,N)}^H$ and $\hat{\Phi}_{(1,N,1,N)}^H$, $\Gamma = \underline{1}, N = \underline{1}, \underline{3}$, (with both $SO(2)_2$ chiralities). They transform respectively as:

$$\begin{aligned} &(\underline{1}, \underline{3}; \underline{1}, \underline{1})_{(0, -1/2, 0, -3/2)}, \quad (\underline{1}, \underline{1}; \underline{1}, \underline{3})_{(0, 3/2, 0, 1/2)}, \\ &(\underline{1}, \underline{1}; \underline{1}, \underline{1})_{(0, 3/2, 0, -3/2)}, \quad (\underline{1}, \underline{3}; \underline{1}, \underline{3})_{(0, -1/2, 0, 1/2)} \end{aligned} \quad (28)$$

and

$$\begin{aligned} &(\underline{1}, \underline{1}; \underline{1}, \underline{3})_{(0, -3/2, 0, 1/2)}, \quad (\underline{1}, \underline{1}; \underline{1}, \underline{1})_{(0, -3/2, 0, -3/2)}, \\ &(\underline{1}, \underline{3}; \underline{1}, \underline{1})_{(0, 1/2, 0, -3/2)}, \quad (\underline{1}, \underline{3}; \underline{1}, \underline{3})_{(0, 1/2, 0, 1/2)}. \end{aligned} \quad (29)$$

The horizontal fields (28) and (29) compensate all $SU(3)^{II}$ anomalies introduced by the chiral matter spectrum (27) of the $(U(5) \times U(3))^{II}$ group (due to b_6 GSO projection the chiral fields of the $(U(5) \times U(3))^I$ group disappear from the final string spectrum). Performing the decomposition of fields (28) under $(SU(5) \times SU(3))^{symm}$ we get (among other) three "horizontal" fields:

$$(\underline{1}, \underline{3})_{(0, -1)}, \quad (\underline{1}, \underline{1})_{(0, -3)}, \quad (\underline{1}, \underline{5})_{(0, 1)}, \quad (30)$$

coming from $\hat{\Psi}_{(\underline{1}, \underline{3}, \underline{1}, \underline{1})}^H$ (or $\hat{\Psi}_{(\underline{1}, \underline{1}, \underline{1}, \underline{3})}^H$), $\hat{\Psi}_{(\underline{1}, \underline{3}, \underline{1}, \underline{3})}^H$ and $\hat{\Psi}_{(\underline{1}, \underline{3}, \underline{1}, \underline{3})}^H$ respectively which make the low energy spectrum of the resulting model (25) $SU(3_H)^{symm}$ -anomaly free. The other fields arising from (28) and fields $\hat{\Phi}^H$ in (29) form real, anomaly- free representations of $(SU(3_H) \times U(1_H))^{symm}$:

$$2(\underline{1}, \underline{1})_{(0, 0)}, \quad (\underline{1}, \underline{3})_{(0, -1(2))} + (\underline{1}, \underline{3})_{(0, 1(-2))}, \quad (\underline{1}, \underline{5})_{(0, 0)}. \quad (31)$$

The real (Higgs) superfields $\hat{\phi}_{(1,N)}$ + h.c., where $(\Gamma = \underline{1}, \underline{5}; N = \underline{1}, \underline{3})$, from the sectors $1(3)b_2 + b_3 + 2(6)b_5 + b_6(+S)$, $1(3)b_2 + b_3 + 4b_5 + b_6(+S)$ forming

$$\begin{aligned} &(\underline{1}, \underline{1}, \underline{5}, \underline{1})_{(-5/4, 3/4; -1/4, 3/4)} \quad (\underline{5}, \underline{1}, \underline{1}, \underline{1})_{(-1/4, 3/4; -5/4, 3/4)} \\ &(\underline{1}, \underline{1}; \underline{1}, \underline{3})_{(-5/4, 3/4; -5/4, -1/4)} \quad (\underline{1}, \underline{3}; \underline{1}, \underline{1})_{(-5/4, -1/4; -5/4, 3/4)} \end{aligned} \quad (32)$$

representations of $(U(5) \times U(3))^{1/11}$ have either Q' or Q'' exotic fractional charges. Due to the strong $G_{\text{comp}}^{\text{comp}}$ gauge forces these fields may develop the double scalar condensate $\langle \hat{\phi}\hat{\phi} \rangle$, which can also serve for $U(5) \times U(5)$ gauge symmetry breaking. For example, the composite condensate $\langle \phi_{(5,1,1)}\phi_{(1,1,5)} \rangle$ can break the $U(5) \times U(5)$ gauge symmetry down to the symmetric diagonal subgroup with generators of the form

$$\Delta_{\text{symm}}(t) = t \times 1 + 1 \times t, \quad (33)$$

so for the electromagnetic charges we would have the form

$$Q_{em} = Q'' + Q'. \quad (34)$$

leading again to no exotic, fractionally charged states in the low-energy string spectrum.

The Higgs fields which transform nontrivially under the compactified group $G_{\text{comp}}^{\text{comp}} = SO(6) \times SO(2)^{\times 3}$, (denoted as $\hat{\sigma} + h.c.$), and which are singlets of $(SU(5) \times SU(3)) \times (SU(5) \times SU(3))$, arise in three sectors:

- a) $b_3 + 2(6)b_3(+5)$;
- b) $b_1 + 1(3)b_2 + 5(3)b_3 + b_6(+5)$,
- c) $b_1 + 2b_2 + 3(5)b_3 + b_6(+5)$;

The superfields $\hat{\sigma}$ form the spinor representations $\underline{4} + \bar{\underline{4}}$ of $SO(6)$ and they are also spinors of one of the $SO(2)$ groups. They have following hypercharges $\bar{Y}_5^{1/11}, \bar{Y}_3^{1/11}$:

$$\bar{Y} = (5/4, \mp 3/4; 5/4, \mp 3/4), \quad \bar{Y} = (5/4, 3/4; -5/4, -3/4). \quad (35)$$

With respect to the diagonal $G_{\text{symm}}^{\text{symm}}$ group with generators given by (21) or (33), the fields $\hat{\sigma}$ from sets a), b) or the set c), are of zero hypercharges and can, therefore, be used for breaking the $SO(6) \times SO(2)^{\times 3}$ group.

Note, that for the fields $\hat{\phi}$ (32) and for the fields $\hat{\sigma}$ any other electromagnetic charge quantization different than (23) or (34) would lead to "quarks" and "leptons" with the exotic fractional charges, for example, for the $\underline{5}$ - and $\bar{\underline{1}}$ - multiplets according to the values of hypercharges (see eqs.(32,35)) the generator Q'' (or Q') has the eigenvalues $(\pm 1/6, \pm 1/6, \pm 1/6, \pm 1/2, \mp 1/2)$ or $\pm 1/2$, respectively.

5 Discussion and Prospects

In this work we have considered the rank eight GUST with gauge symmetry $G = SU(5) \times U(1) \times (SU(3) \times U(1))_H \subset SO(16)$ which contains $N_g = \underline{3}_H + \underline{1}_H$ families. GUSTs originating from level- one KMA contain only low- dimensional representations of the unification group. It is, therefore, difficult to break the gauge symmetry. In order to solve this problem we have considered

as the observable gauge symmetry the diagonal subgroup $G_{\text{symm}}^{\text{symm}}$ of rank 16 group $G \times G \subset SO(16) \times SO(16) (\subset E(8) \times E(8))$. This construction allows to break the GUST symmetry to the low energy gauge group, which includes the G_H family gauge symmetry. Two mechanism of the gauge symmetry breaking have been considered. The virtue of the $GUST_{\text{symm}}^{\text{symm}}$ considered is that its low- energy spectrum does not contain particles with exotic charges. This GUST which is based on the maximal invariant $SO(16)$ subgroup of $E(8)$ leads to the $\underline{3}_H + \underline{1}_H$, therefore it will be interesting to consider GUST construction which is based on the $E(6) \times SU(3)_H \subset E(8)$ gauge symmetry [16] and which predicts only three quark- lepton families.

The ability to correctly describe the fermion masses and mixings will, of course, constitute the decisive criterion for selection of a model of this kind. Therefore, within our approach one has to

1. study the possible nature of the G_H horizontal gauge symmetry ($N_g = \underline{3}_H$ or $\underline{3}_H + \underline{1}_H$),
2. investigate the possible cases for G_H -quantum numbers for quarks (anti-quarks) and leptons (anti-leptons), i.e. whether one can obtain vector-like or axial- like structure (or even chiral $G_{III} \times G_{III}$ structure) for the horizontal interactions.
3. the structure of the sector of the matter fields which are needed for the $SU(3)_H$ anomaly cancellation (chiral neutral "horizontal" or "mirror" fermions),
4. write down all possible renormalizable and relevant non-renormalizable contributions to the superpotential W and their consequences for fermion mass matrices.

All these questions are currently under investigation [16]. Here we restrict ourselves to some general remarks only.

With the chiral matter and "horizontal" Higgs fields available in the model constructed in this paper, the possible form of the renormalizable (trilinear) part of the superpotential responsible for fermion mass matrices is well restricted by the gauge symmetry:

$$W_1 = g\sqrt{2} \left[\hat{\Psi}_{(1,3)} \hat{\Psi}_{(\bar{3},1)} \hat{h}_{(5,3)} + \hat{\Psi}_{(1,1)} \hat{\Psi}_{(\bar{3},3)} \hat{h}_{(5,3)} + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(\bar{3},3)} \hat{h}_{(5,3)} + \hat{\Psi}_{(10,3)} \hat{\Psi}_{(0,1)} \hat{h}_{(5,3)} \right] \quad (36)$$

From the above form of the Yukawa couplings follows that two (chiral) generations (27) have to be very light (comparing to M_W scale). The construction of realistic quarks and leptons mass matrices depends, of course, on the nature of the horizontal interactions. In the string construction described in

Sec.3 there is a freedom of choosing spin boundary conditions for $N_{LR}=12$ left and right fermions in the basis vectors $b_3, b_5, b_6, \dots$, which in the Ramond sector $2b_2$, may give rise to another Higgs fields, denoted as $\tilde{h}_{(1,N)}$ and transforming as $(\bar{5}, \bar{3})_{(-1,1)} + (\bar{5}, \bar{3})_{(1,-1)} \subset 2\bar{8} + 2\bar{8}$ of $SU(8)$. Using these Higgs fields we get the following alternative form of the renormalizable part of the superpotential W :

$$W'_1 = g\sqrt{2} \left[\tilde{\Psi}_{(1,3)} \tilde{\Psi}_{(5,3)} \tilde{h}_{(5,3)} + \tilde{\Psi}_{(10,1)} \tilde{\Psi}_{(5,3)} \tilde{h}_{(5,3)} + \right. \\ \left. + \tilde{\Psi}_{(10,3)} \tilde{\Psi}_{(10,3)} \tilde{h}_{(5,3)} + \tilde{\Psi}_{(10,3)} \tilde{\Psi}_{(5,1)} \tilde{h}_{(5,3)} \right] \quad (37)$$

However, since the above renormalizable interactions are not sufficient to construct the realistic fermion mass matrices, one has to use also the Higgs fields (19,20) and (33,36) and also to take into account all relevant non-renormalizable contributions [10] which may be responsible for masses of the fermions from the first two generations.

The Higgs fields (19) can be used for constructing Yukawa couplings of the horizontal superfields (29) and (30). The most general contribution of these fields to the superpotential is:

$$W_2 = g\sqrt{2} \left[\tilde{\Phi}_{(1,1,1,3)}^H \tilde{\Phi}_{(1,3,1,1)}^H \tilde{\Phi}_{(1,3,1,3)}^H + \tilde{\Phi}_{(1,1,1,1)}^H \tilde{\Phi}_{(1,3,1,1)}^H \tilde{\Phi}_{(1,3,1,3)}^H + \right. \\ \left. + \tilde{\Phi}_{(1,3,1,3)}^H \tilde{\Phi}_{(1,3,1,3)}^H \tilde{\Phi}_{(1,3,1,3)}^H + \tilde{\Psi}_{(1,3,1,1)}^H \tilde{\Psi}_{(1,3,1,3)}^H \tilde{\Phi}_{(1,3,1,3)}^H + \right. \\ \left. + \tilde{\Psi}_{(1,1,1,3)}^H \tilde{\Psi}_{(1,3,1,3)}^H \tilde{\Phi}_{(1,3,1,3)}^H \right] \quad (38)$$

From this expression it follows that some of the horizontal fields in (31) remain massless at the tree-level. This is a remarkable prediction: fields (31) interact with the ordinary chiral matter fields only through the $U(1_H)$ and $SU(3_H)$ gauge boson and therefore are very interesting in the context of the experimental searches for the new gauge bosons.

The superfields $\tilde{\Phi}_{(1,3,1,1)}^H$ and $\tilde{\Psi}_{(1,3,1,1)}^H$ (see (28) and (29)) can be used to construct the non-renormalizable contributions to the superpotential W . For example, the term

$$\Delta W_1 = \frac{cg^3}{M_{Pl}^2} \tilde{\Psi}_{(10,1)} \tilde{\Psi}_{(10,1)} \tilde{\Phi}_{(1,3,5,1)}^H \tilde{\Phi}_{(1,3,1,1)}^H \tilde{\Phi}_{(1,3,1,1)}^H \quad (39)$$

can give contribution to the mass to the fourth generation down-type quark ($c = O(1)$, see [10]). To get a reasonable value of the mass for this quark we must arrange for the $SU(3_H)'$ gauge symmetry breaking at the energy scale near the Planck scale, i.e. $< \tilde{\Phi}_{(1,3,1,1)}^H > = < \tilde{\Phi}_{(1,3,1,1)}^H > \sim M_{Pl}$. In this case one can get the $SU(3_H)''$ -family gauge group with a low energy breaking symmetry scale. Finally, we remark that the Higgs sector of our GUST allows

for conservation of the G_H gauge family symmetry down to the low energies ($\sim O(1\text{TeV})$ [15]). Thus we can expect at this energy region new interesting physics (new gauge bosons, new chiral matter fermions, superweak-like CP-violation in K, B, D -meson decays with $\delta_{KM} < 10^{-4}$ [15]).

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