Elastic Diffraction and Non–Perturbative Gluons

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Abstract

We consider a simple model for the propagator of a non–perturbative gluon and derive the rules determining its coupling to perturbative gluons and quarks. With these rules, we evaluate the fourth and sixth–order amplitude for quark–quark scattering at high energies.

In this framework, numerical predictions are presented for high energy nucleon–nucleon scattering and compared with experimental data.
Fig. 2
and

\[ (x) \cdot (y) = \int_\mathbb{R} \, f(x) \cdot g(x) \, dx \]

0

\[ \Delta(x - \epsilon) = \begin{cases} 1 & \text{if } x = \epsilon \\ 0 & \text{otherwise} \end{cases} \]

Let \( G \) be the correction factor for the shot propagator.

For identifications of the parameters.

The shot propagator will then be used to determine the solution function.

The error in the solution function is the difference between the actual function and the approximation.

The error is defined as the difference between the error function and the approximation function.

We consider the data points and choose the least-squares fit.

We start from the following structure function of the transverse photon propagator.
\[ K(x, y) = y \int_{0}^{\pi} d\theta \; \text{sen} \theta \frac{D(k^2)}{k^2} \frac{3\pi}{8} D(y) \]

where

\[ k^2 = x + y - 2(x y)^{1/2} \cos \theta \]

If, instead of \( D(k^2) \), we insert eq. (1) we obtain a kernel semidefinite negative. This suggests a very small value for \( G(-p^2) \) at the scale \( \mu^2 \). A numerical calculation, following the method of ref. [13], confirms this result and a previous finding obtained for \( \mu^2 = 0 \) [14].

Tentatively we will put

\[ G(-\mu^2) = \gamma \sim \mu^2 / M^2 \]

and consider the Slavnov-Taylor identity for the three-gluon vertex. In the limit of any one of the incoming momenta going to zero we obtain, due to the smallness of \( \mu^2 \), the approximate equation

\[ T^{\mu\nu\alpha}(0, -r, -r) = -i \gamma \frac{\partial}{\partial r_\mu} \left[ (g^{\rho\sigma} - \frac{r_\rho r_\sigma}{r^2}) D^{-1}(-r^2) \right] \]

When \( r^2 \to \infty \), we get from eq. (1)

\[ T^{\mu\nu\alpha}(0, -r, -r)|_{r^2 \to \infty} \to -i \gamma (g^{\rho\sigma} - 2g^{\rho\sigma} \rho^\mu + g^{\rho\sigma} \rho^\alpha) \]

and the emission of a non-perturbative gluon from a perturbative one is strongly suppressed.

If \( r^2 = -\mu^2 \)

\[ T^{\mu\nu\alpha}(0, -r, -r)|_{r^2 = -\mu^2} \approx -i \gamma (g^{\rho\sigma} - 2g^{\rho\sigma} \rho^\mu + g^{\rho\sigma} \rho^\alpha) \frac{4\mu^4}{c} \]

that is the perturbative answer multiplied by \( 4\mu^4 / (cM^2) \).

This factor changes the non-perturbative propagator to a "perturbative" one

\[ \frac{4\mu^4}{cM^2} \left( \frac{c}{r^2} - \frac{\mu^2}{r^2} \right) |_{r^2 = -\mu^2} \to \frac{1}{M^2} \]

while the vertex remains perturbative.

The non-perturbative gluon is abelian-like; it can emit or absorb perturbative gluons but otherwise there is no interaction.

This rule forbids many diagrams in quark–quark scattering. Otherwise, one diagram with \( n \) internal gluon lines would give rise to \( 2^n \) different diagrams when gluons of two kinds are considered.

Another simplification concerns the quark–gluon vertex. The emission of two gluons with zero momentum from a quark is suppressed since

\[ \sqrt{S_R} \Gamma_R S_R \sqrt{S_R} = \frac{1}{2i} \sqrt{S} \Gamma S \sqrt{S} \]

and

\[ \tilde{Z}_2 \approx 1 / \gamma \]

Here \( S \) and \( \Gamma \) refer to the quark propagator and to the quark–gluon vertex while \( \tilde{Z}_2 \) is the renormalization constant of the ghost propagator (\( \tilde{Z}_1 = 1 \)). As a consequence, a quark cannot emit or absorb two successive non-perturbative gluons.

All the rules for the gluon of "mass" \( M \) remain the usual Feynman ones.

This approach presents many features in common with the Mandelstam mechanism of confinement [7] with one important difference. The potential derived from eq. (1), in the lowest order, reproduces the potential of the Schwinger model that provides for screening but not for confinement.

In the next Section we will use these rules to calculate fermion–fermion scattering at high-energies. It is not a trivial fact that Feynman gauge can be used, instead of Landau gauge, without changing the asymptotic behaviour of the diagrams we consider. This conclusion is quite natural for the channel gluons. For \( s \)-channel gluons such a simplification occurs because also the discontinuity of the spin structure

\[ g^{\rho\sigma} = \frac{k^\rho k^\sigma}{k^2 + i\epsilon} \]

contributes.
leading real part of the amplitude and
maximun power of $\gamma$ areaddock of $\gamma$ and $\phi$.

Hence, for $\gamma\phi$ of the form $\gamma=\gamma' N, \phi=\phi'$, the

\( (\gamma'N)^2 \gamma' \phi' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi' \gamma' \phi'}
where \( \mathcal{N} \) is the number of colours.

We close this Section by noticing that some of the \( \text{"terms proportional to } c^n \text{" refer to radiative diagrams with only one non perturbative gluon. They diverge and the divergence does not cancel with analogous terms (\( \propto c \)) in the ladder diagram where they do not appear at this order.}

However, when the variation of \( c \) with the transverse momentum is properly taken into account, they converge and become negligible, \( O(\mu^2/M^2) \), with respect to other sixth-order contributions.

4. Nucleon-nucleon scattering

In order to obtain measurable cross sections from the above results we must describe the emission of two gluons by a proton. We follow an approach [18] based on the non-relativistic quark model. Consequently we multiply the amplitude by the numbers of quarks in both incoming hadrons, nine in our case, and include in the integrals of eqs. (4) the factor

\[
\frac{w^4}{(w^2 - t/4)^2} \cdot \left( \frac{k_2^2 + t/4}{w^2 + k_1^2} \right)^2
\]

(10)

where

\[
w^2 = 2/ < r_p^2 > \sim .113 \quad (\text{GeV})^2.
\]

Consider now an energy region, around \( \sqrt{s} \sim 20 \text{ GeV} \), where the Pomeron contribution is nearly constant.

Let \( \hat{P}(t) \) and \( \hat{N}(t, f) \) denote the integrals (4a) and (4b) with the factor (10) included under the integral sign. Then the amplitude for the exchange of a Pomeron, at the lowest order, will be

\[
T^{(\text{pom})} = 4\pi g^8 s |\hat{P}(t) + 2c\hat{N}(t, 0)|
\]

(11)

valid for small \( t \leq 0 \) and asymptotic \( s \).

We add the contribution of the \( f \) and \( \omega \) reggeons in the standard way [19] and use the experimental data for total and differential p-p cross-section at \( \sqrt{s} = 19.42 \text{ GeV} [20] \). The total cross-section (\( \sigma_{\text{tot}} = 38.97 \pm 0.04 \text{ mb} \)) fixes the only parameter left free from spectroscopy

\[
\alpha_s \simeq 0.425 \quad \text{or} \quad c \simeq 0.79(\text{GeV})^2
\]

since, from eq. (2), \( \alpha_s c \simeq .336 \). Regge poles at this energy contribute by 8.6 mb.

Fig. (1) shows that the theoretical differential cross-section is in good agreement with experiment till \( |t| \leq 0.1 \text{ (GeV)}^2 \). Probably the value of \( \alpha_s \) is overestimated since other contributions to the amplitude, decreasing with \( s \), are not negligible at this energy. Eq. (11) is the leading term at \( s \to \infty \) and we should add to \( T^{(\text{pom})} \) a term which is constant in \( s \) but \( t \)-dependent. Moreover a fit at lower energies would produce a smaller coupling and a larger value for \( c \).

At higher energies the cross-section increases with \( s \) and a log \( s \) contribution comes from the \( g^8 \) diagrams evaluated in the previous section.

We face now the following problems:

1) we do not know how to describe in a simple way the block for the emission of three gluons from a proton. Radiative diagrams and three gluon exchange in the \( t \)-channel must be treated separately from the ladder.

2) Terms going like \( s \) in the amplitude cannot be recovered at sixth order with the method of ref.[15]. They have been estimated numerically only for the ladder [21].

There are however cancellations between radiative corrections and the three-gluon term

\[-2c^2 Q(t, \mu, 2),\]

appearing in eqs. (7) and (9). Numerically the cancellation is almost complete for \( t = 0 \), leaving only the ladder diagram contribution to the amplitude:

\[
12\pi c^2 g^8 s \ln s[4\hat{N}(0, 1)\hat{N}(0, 0) - (c - M^2)(\hat{N}(0, 0))^2]
\]

(13)

Here \( \hat{N} \) includes only one block for the emission of two gluons; i.e. the square root of the factor (10). Eq.(13) holds only for \( t = 0 \). When \( t \neq 0 \), it becomes difficult to set up a theoretical model for the \( t \)-dependence of the amplitude.

In order to overcome the second difficulty we introduce an unknown constant to represent terms proportional to \( s \). Moreover to obtain a better fit at medium – low
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Figure captions

Fig.1 – The differential cross section for p–p at $\sqrt{s} = 19.42$ GeV (from [20]), with calculation described in the text.

Fig.2 – Data for $p - \bar{p}$ total cross-sections calculated from eq.(14) of the text.
Fig. 1