AD

DFPD 93 TH 71

ju 8404

Elastic Diffraction and Non-Perturbative Gluons

L.Jenkowszky⁽¹⁾, *A.Kotikov*⁽²⁾, *F.Paccanoni*⁽³⁾

1) Institute for Theoretical Physics, Kiev, Ukraine

2) Joint Institute for Nuclear Research, Dubna, Russia

3) Dip. di Fisica, Padova – INFN Sezione di Padova, Italy

Abstract

We consider a simple model for the propagator of a non-perturbative gluon and derive the rules determining its coupling to perturbative gluons and quarks. With these rules, we evaluate the fourth and sixth-order amplitude for quark-quark scattering at high energies.

In this framework, numerical predictions are presented for high energy nucleon-nucleon scattering and compared with experimental data.

DFPD 93/TH/71

November 1993

Fig. 2

1. Introduction

The continuous interest in searching for a non-perturbative model of the transverse gluon propagator [1,2] threw open, recently the problem of high energy hadron scattering to a new field theoretical description. Applications to elastic scattering range from models based on a phenomenological ansatz [3] to more ambitious attempts [4].

We can compare two proposals. The perturbative approach to the Pomeron, first discussed by Lipatov [5], has a sound basis. It gives reliable results where the leading logarithmic approximation holds. The derivation of a non-perturbative gluon propagator from QCD contains instead many assumptions to be proven such as, for example, the truncation of the Schinger-Dyson equations.

In many of these studies, based on QCD, the low-momentum singularity of the gauge field propagator is softened. A soft infrared behaviour can be obtained in a Yang-Mills theory with Higgs mechanism, where the gluon acquires a mass. A similar situation is realized for some solutions of the Schwinger-Dyson equations or simply in the presence of a cutoff. The softening of the $1/k^2$ pole seems required also on phenomenological grounds [3].

These assertions are in conflict with a popular suggestion of the confinement mechanism where vacuum instability implies that low frequency modes are enhanced [6-8]. The same mechanism leads to an indefinitely rising linear potential between quarks and it is successful in determining the meson and baryon masses.

As matters stand it seems that spectroscopy and scattering need different and, in a sense opposite, descriptions. Our aim is to show that this dichotomy can be overcome.

In this paper we consider a model for the Pomeron based on the analyticity properties of the gluon propagator in QCD. The non-perturbative gluon propagator will be discussed in Sec. 2 together with the derivation of the Green functions necessary for the description of quark-quark scattering. Sec. 3 collects the calculations of the scattering amplitude up to sixth order of perturbation theory. Applications to nucleon-nucleon scattering will be investigated in Sec. 4.

2. The model and its rules

We start from the following structure function of the transverse gluon propagator

$$D(-k^2) = \frac{c}{(k^2 - \mu^2)^2} - \frac{1}{k^2 - M^2}, \quad \mu^2 < M^2 \quad (1)$$

that is the simplest realization of the analyticity requirement [9,10]. The parameters c, μ^2 and M^2 depend on the coupling and vanish when $g \rightarrow +0$. In the model they will be considered as constants and their values being determined from the predictions implied by eq.(1). Gluon condensate [11] and heavy quark spectroscopy [12] fix these parameters:

$$\mu \simeq 50 \text{ MeV}, \quad M \simeq 750 \text{ MeV}, \quad \frac{g^2 c}{6\pi} \simeq .224 (\text{GeV})^2 \quad (2)$$

In ref. [12] the non relativistic potential associated with (1) has been considered. With the parameters of eq. (2) the mass spectrum of charmonium and bottomium is successfully reproduced.

The first term in the r.h.s. of eq. (1) clearly enhances low frequency modes and, as we shall see, it shares other properties with a well known confinement mechanism [6,7]. In the following we will refer to this term as to the non-perturbative gluon propagator.

We consider now other relevant Green functions and choose the Landau gauge where eq. (1) has been obtained. First we will show that ghosts are not important in our calculations but they will play a role in the determination of the triple-gluon vertex. The ghost propagator will then be used like an auxiliary function for identifying renormalization constants.

Let $G(p^2)$ be the correction factor for the ghost propagator

$$\bar{D}(p) = -G(-p^2)/p^2$$

since $\bar{Z}_1 = 1$ in Landau gauge, the renormalized Schwinger-Dyson equation for the ghost propagator can be written as (with euclidean momenta)

$$\frac{1}{G(x)} = \frac{1}{G(0)} + \frac{3}{(2\pi)^3} \int_0^\infty dy g^2 K(x,y) G(y) \quad (3)$$

and

$$K(x, y) = y \int_0^\pi d\theta \sin^4 \theta \frac{D(k^2)}{k^2} - \frac{3\pi}{8} D(y)$$

where

$$k^2 = x + y - 2(xy)^{1/2} \cos \theta$$

If, instead of $D(k^2)$, we insert eq. (1) we obtain a kernel semidefinite negative. This suggests a very small value for $G(-p^2)$ at the scale μ^2 . A numerical calculation, following the method of ref. [13], confirms this results and a previous finding obtained for $\mu^2 = 0$ [14].

Tentatively we will put

$$G(-\mu^2) \equiv \gamma \sim \mu^2/M^2$$

and consider the Slavnov-Taylor identity for the three-gluon vertex. In the limit of any one of the incoming momenta going to zero we obtain, due to the smallness of μ^2 , the approximate equation

$$T^{\mu\nu\alpha}(0, -r, -r) = -i\gamma \frac{\partial}{\partial r_\mu} \left[(g^{\alpha\nu} - \frac{r^\alpha r^\nu}{r^2}) D^{-1}(-r^2) \right]$$

When $r^2 \rightarrow \infty$, we get from eq. (1)

$$T^{\mu\nu\alpha}(0, -r, -r)|_{r^2 \rightarrow \infty} \rightarrow -i\gamma(g^{\mu\alpha}r^\nu - 2g^{\alpha\nu}r^\mu + g^{\nu\mu}r^\alpha)$$

and the emission of a non-perturbative gluon from a perturbative one is strongly suppressed.

If $r^2 = -\mu^2$

$$T^{\mu\nu\alpha}(0, -r, -r)|_{r^2 = -\mu^2} \approx -i\gamma(g^{\mu\alpha}r^\nu - 2g^{\alpha\nu}r^\mu + g^{\nu\mu}r^\alpha) \frac{4\mu^2}{c}$$

that is the perturbative answer multiplied by $4\mu^4/(cM^2)$.

This factor changes the non-perturbative propagator to a "perturbative" one

$$\frac{4\mu^4}{cM^2} \left[\frac{c}{(r^2 - \mu^2)} \right]_{r^2 = -\mu^2} \rightarrow \frac{1}{M^2}$$

while the vertex remains perturbative.

The non-perturbative gluon is abelian-like; it can emit or absorb perturbative gluons but otherwise there is no interaction.

This rule forbids many diagrams in quark-quark scattering. Otherwise, one diagram with n internal gluon lines would give rise to 2^n different diagrams when gluons of two kinds are considered.

Another simplification concerns the quark-gluon vertex. The emission of two gluons with zero momentum from a quark is suppressed since

$$\sqrt{SR}\Gamma_R\Gamma_R\sqrt{SR} = \frac{1}{Z_2^2}\sqrt{STST}\sqrt{S}$$

and

$$\tilde{Z}_3 \approx 1/\gamma$$

Here S and Γ refer to the quark propagator and to the quark-gluon vertex while $\tilde{Z}_3$ is the renormalization constant of the ghost propagator ($\tilde{Z}_1 = 1$). As a consequence, a quark cannot emit or absorb two successive non-perturbative gluons. All the rules for the gluon of "mass" M remain the usual Feynman ones.

This approach presents many features in common with the Mandelstam mechanism of confinement [7] with one important difference. The potential derived from eq. (1), in the lowest order, reproduces the potential of the Schwinger model that provides for screening but not for confinement.

In the next Section we will use these rules to calculate fermion-fermion scattering at high-energies. It is not a trivial fact that Feynman gauge can be used, instead of Landau gauge, without changing the asymptotic behaviour of the diagrams we consider. This conclusion is quite natural for t -channel gluons. For s -channel gluons such a simplification occurs because also the discontinuity of the spin structure

$$g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 + i\epsilon}$$

contributes.

3. The high-energy amplitude

Many calculations of the fermion-fermion scattering amplitude in non-Abelian gauge theories have been done in the past [5, 15 - 17]. The methods of calculation are different but results coincide at least at order g^6 .

We follow the method of ref. [15] since it allows to describe the transition from quark to nucleon scattering in an easier way. We decompose the amplitude into a part with colour flip in the t-channel ($\propto \lambda^{(1)}\lambda^{(2)}$) and a colour-nonflip part ($\propto \delta^{(1)}\delta^{(2)}$).

The latter will be interpreted as the Pomeron contribution, a colour singlet formed by two or more gluons. The superscripts (1) and (2) refer to the upper and lower quark respectively. Since the calculation follows ref. [15] we will not enter in technical details but will comment afterwards on the main differences.

Let $\tilde{\Delta}$ be the momentum transfer, with $t = -\tilde{\Delta}^2$, and define

$$P(t) = \int \frac{d^2 k_{\perp}}{(2\pi)^3} \left[\left(\vec{k}_{\perp} - \frac{\tilde{\Delta}}{2} \right)^2 + M^2 \right]^{-1} \left[\left(\vec{k}_{\perp} + \frac{\tilde{\Delta}}{2} \right)^2 + M^2 \right]^{-1} \quad (4a)$$

$$N(t, j) = \int \frac{d^2 k_{\perp}}{(2\pi)^3} (\vec{k}_{\perp}^2)^j \left(\left(\vec{k}_{\perp} - \frac{\tilde{\Delta}}{2} \right)^2 + M^2 \right)^{-1} \left[\left(\vec{k}_{\perp} + \frac{\tilde{\Delta}}{2} \right)^2 + \mu^2 \right]^{-2} \quad (4b)$$

$$Q(t, z, j) = \int \frac{d^2 k_{1\perp}}{(2\pi)^3} \int \frac{d^2 k_{2\perp}}{(2\pi)^3} (\vec{k}_{1\perp}^2 + z^2)^{-j} (\vec{k}_{2\perp}^2 + z^2)^{-j} \left[\left(\vec{k}_{1\perp} + \vec{k}_{2\perp} - \tilde{\Delta} \right)^2 + M^2 \right]^{-1} \quad (4c)$$

Then we obtain the amplitudes (for quarks with zero mass):

$$T^{(2)} = -\frac{1}{2} g^4 \lambda^{(1)} \lambda^{(2)} \left\{ \frac{1}{M^2 - t} + \frac{c}{(\mu^2 - t)^2} \right\} \quad (5)$$

$$T^{(4)} = \frac{4}{3} \left(\frac{1}{2} \right)^5 g^4 s \{ 4 \lambda^{(1)} \lambda^{(2)} \left(\frac{9}{2} \ln s - i\pi \right) + i\pi \frac{32}{3} \delta^{(1)} \delta^{(2)} \} \cdot [P(t) + 2cN(t, 0)] \quad (6)$$

$$T^{(6)} = -4 \left(\frac{1}{2} \right)^5 g^6 s \{ 2 \lambda^{(1)} \lambda^{(2)} \left(\frac{9}{4} \ln^2 s - i\pi \ln s \right) f_1(t) + i\pi \frac{32}{3} \delta^{(1)} \delta^{(2)} (\ln s) f_2(t) \} \quad (7)$$

6

where

$$f_1(t) = \left(\frac{3}{2} M^2 - t \right) P^2(t) + c^2 (c + M^2 - \frac{3}{2} t) N^2(t, 0) - 2c^2 N(t, 1) N(t, 0) + \text{terms proportional to } c \quad (8)$$

and

$$f_2(t) = f_1(t) - Q(t, M, 1) - 2c^2 Q(t, \mu, 2) \quad (9)$$

The leading terms at any order appear in the colour-flip amplitudes, proportional to $\lambda^{(1)}\lambda^{(2)}$. The power of c counts the number of non-perturbative gluons.

Our results differ from the perturbative calculations of ref. [15] for the following reasons:

- i) the weight of the diagrams is different; for example, the sixth order ladder and the crossed one give rise to eight different graphs according to our rules of Sec. 2;
- ii) scalar fields, that give a mass to the vector meson through the Higgs mechanism, are absent in our model;
- iii) the discontinuity formula gives, for the structure function in eq. (1), the contribution

$$2\pi i [c\delta(k^2 - \mu^2) + \delta(k^2 - M^2)]$$

iv) for all amplitudes, a factor four comes from the spin sum over final states and an average over initial states;

v) the last term in eq. (9) comes from the graphs with three gluons in the t-channel. It should have a factor three in front but one of the possible graphs is forbidden by our rules. This will have important consequences in what follows.

Reggeisation is lost for the non-perturbative (abelian) gluon and it is only approximate for the perturbative one because of ii). Resummation of all orders is possible only for particular classes of diagrams; it becomes easy for those with the maximum power of c . These are ladders and, at order $2n$, they contribute to the leading real part of the amplitude as

$$-2s g^{2n} c^{2n-3} \left(\frac{-N}{4} \right)^{n-1} \frac{(\ln s)^{n-1}}{(n-1)!} [N(t, 0)]^{n-1} \quad (n \geq 2)$$

7

where $\mathcal{N}$ is the number of colours.

We close this Section by noticing that some of the "terms proportional to c^n " refer to radiative diagrams with only one non perturbative gluon. They diverge and the divergence does not cancel with analogous terms ($\propto c$) in the ladder diagram where they do not appear at this order.

However, when the variation of c with the transverse momentum is properly taken into account, they converge and become negligible, $O(\mu^2/M^2)$, with respect to other sixth-order contributions.

4. Nucleon-nucleon scattering

In order to obtain measurable cross sections from the above results we must describe the emission of two gluons by a proton. We follow an approach [18] based on the non-relativistic quark model. Consequently we multiply the amplitude by the numbers of quarks in both incoming hadrons, nine in our case, and include in the integrals of eqs. (4) the factor

$$\frac{w^4}{(w^2 - t/4)^2} \cdot \left(\frac{\vec{k}_\perp^2 + t/4}{w^2 + \vec{k}_\perp^2} \right)^2 \quad (10)$$

where

$$w^2 = 2 / < r_p^2 > \sim .113 \quad (\text{GeV})^2.$$

Consider now an energy region, around $\sqrt{s} \sim 20$ GeV, where the Pomeron contribution is nearly constant.

Let $\tilde{P}(t)$ and $\tilde{N}(t, j)$ denote the integrals (4a) and (4b) with the factor (10) included under the integral sign. Then the amplitude for the exchange of a Pomeron, at the lowest order, will be

$$T^{(pom)} = 4\pi i g^4 s [\tilde{P}(t) + 2c\tilde{N}(t, 0)] \quad (11)$$

valid for small $t \leq 0$ and asymptotic s .

We add the contribution of the f and ω reggeons in the standard way [19] and use the experimental data for total and differential p-p cross-section at $\sqrt{s} =$

19.42 GeV [20]. The total cross-section ($\sigma_{tot} = 38.97 \pm 0.04 \text{ mb}$) fixes the only parameter left free from spectroscopy

$$\alpha_s \simeq 0.425 \quad \text{or} \quad c \simeq 0.79 (\text{GeV})^2 \quad (12)$$

since, from eq. (2), $\alpha_s c \simeq .336$. Regge poles at this energy contribute by 8.6 mb. Fig. (1) shows that the theoretical differential cross-section is in good agreement with experiment till $|t| \lesssim 0.1 (\text{GeV})^2$. Probably the value of α_s is overestimated since other contributions to the amplitude, decreasing with s , are not negligible at this energy. Eq. (11) is the leading term at $s \rightarrow \infty$ and we should add to T^{pom} a term which is constant in s but t -dependent. Moreover a fit at lower energies would produce a smaller coupling and a larger value for c .

At higher energies the cross-section increases with s and a log s contribution comes from the g^4 diagrams evaluated in the previous section.

We face now the following problems:

1) we do not know how to describe in a simple way the block for the emission of three gluons from a proton. Radiative diagrams and three gluon exchange in the t -channel must be treated separately from the ladder.

2) Terms going like s in the amplitude cannot be recovered at sixth order with the method of ref.[15]. They have been estimated numerically only for the ladder [21]. There are however cancellations between radiative corrections and the three-gluon term

$$-2c^2 Q(t, \mu, 2),$$

appearing in eqs. (7) and (9). Numerically the cancellation is almost complete for $t = 0$, leaving only the ladder diagram contribution to the amplitude:

$$12\pi c^2 g^4 s \ln s [4\tilde{N}(0, 1)\tilde{N}(0, 0) - (c - M^2)(\tilde{N}(0, 0))^2] \quad (13)$$

Here $\tilde{N}$ includes only one block for the emission of two gluons; i.e. the square root of the factor (10). Eq.(13) holds only for $t = 0$. When $t \neq 0$, it becomes difficult to set up a theoretical model for the t -dependence of the amplitude.

In order to overcome the second difficulty we introduce an unknown constant to represent terms proportional to s . Moreover to obtain a better fit at medium - low

energies a contribution decreasing as ζ/s has been introduced phenomenologically. ζ represents the neglected non-leading contributions present in the g^4 and g^6 amplitudes. We neglect the small perturbative term proportional to $P^2(t)$.

The final form of the total cross-section is

$$\sigma_{tot} = Regge[19] + \alpha + \beta \ln s + \zeta/s \quad (14)$$

and a comparison with experimental data [22] gives

$$\alpha = 0.365 \text{ mb}, \quad \beta = 4.895 \text{ mb}, \quad \zeta = 210.03 \text{ mb}(\text{GeV})^2 \quad (15)$$

Fig.2 shows the result of the fit. Agreement with data can be improved and the value of the parameters can change slightly. But more important are the approximate values of c and α_s obtained by equating eq. (13) to the value of β in eq. (15).

With the help of eq. (2) we get

$$\alpha_s = 0.386 \quad \sigma = 0.87(\text{GeV})^2 \quad (16)$$

and c is near to its upper bound imposed by the positivity of eq. (13). This "fine tuning" is an unpleasant, but necessary at this order, feature of the model. Comparing eqs. (16) and (12) we see that α_s decreased. We noticed that α_s has been overestimated in the fit at $\sqrt{s} \sim 20 \text{ GeV}$. A value $\alpha_s \simeq .4$ is compatible with the crude approximations adopted and makes the model consistent.

5. Conclusion

We have considered a model of quark interaction that successfully predicted the masses of heavy quark bound states. In this framework we computed both the leading real and imaginary parts of the scattering amplitude at high energy to sixth order. The agreement with experimental cross sections shows that the model is consistent. It is worth noticing at this point that our scheme is rather restrictive. All the parameters of the model are fixed except α_s .

Many problems are left open because of technical difficulties. In particular, the differential cross-section at very high energies requires the calculation of non-leading terms. The same is true for lower energies at large $|t|$.

As it stands, the total cross-section (14) recalls a dipole Pomeron model [23] whose extension to a perturbative series of multipoles has been considered recently (see [19] and references therein). It is possible that higher orders in this model will reproduce a multipole Pomeron. There is however another possibility. At higher orders regularities will become more evident. For the ladder these regularities begin with the g^8 diagram making possible a resummation at all orders of the leading terms.

The final answer could well satisfy asymptotically the Froissart bound or even lead to an amplitude of the form $i(\alpha + \beta \ln s)$ for $t = 0$. The latter situation would represent a self-consistent solution of s -channel unitarity.

References

1. A. Hädick: J. Mod. Phys. A, 6 (1991) 3321 and references quoted there.
2. J.M. Namyslawski: "Non perturbative QCD methods", Warsaw Univ. preprint. IFT-92-08 (1992) and references quoted there.
3. P.V. Landshoff and O. Nachtmann: Z. Phys. C 35 (1987) 405.
4. J.R. Cudell and D.A. Ross: Nucl. Phys. B 359 (1991) 247; F. Halzen, G. Krein and A.A. Natale, Phys. Rev. D 47 (1993) 295; M.B. Gay Ducati, F. Halzen and A.A. Natale, Phys. Rev. D 48 (1993) 2324;
5. L.N. Lipatov: Yad. Fiz. 23 (1976) 642 [Sov. J. Nucl. Phys. 23 (1976) 338].
6. H. Nielsen and P. Olesen: Nucl. Phys. B 144 (1978) 376;
7. J. Ambjorn and P. Olesen: Nucl. Phys. B 170 (1980) 60.
7. S. Mandelstam: Phys. Rev. D 20 (1979) 3223
8. H. Pagels: Phys. Rev. D 15 (1977) 2991;
9. J.S. Ball and F. Zachariasen: Nucl. Phys. B 143 (1978) 148; ibidem B 186 (1981) 531, 560;
- R. Delbourgo: J. Phys. G 5 (1979) 603.
9. K. Nishijima: Progr. Theor. Phys. 74 (1985) 889; Progr. Theor. Phys. 77 (1987)

1035.

10. R. Oehme: Phys. Lett. B 195 (1987) 60; Phys. Lett. B 232 (1989) 498.
11. Z.E. Chikovani, L.L. Jenkovszky and F. Paccanoni: Yad. Fiz. 53 (1991) 526 [Sov. J. Nucl. Phys. 53 (1991) 329].
12. Z.E. Chikovani, L.L. Jenkovszky and F. Paccanoni: Mod. Phys. Lett. A 6 (1991) 1409; A.V. Kholodkov et al.: J. Phys. G 18 (1992) 985
13. C.D. Roberts and B.H.J. Mc. Kellar: Phys.Rev. D41 (1990) 672
14. F. Paccanoni: Nuovo Cimento 88A (1985) 427
15. B.M. Mc Coy and T.T. Wu: Phys. Rev. D12 (1975) 3257.
16. L.L. Frankfurt and V.E. Sherman: Yad. Fiz. 23 (1976) 1099 [Sov. J. Nucl. Phys. 23 (1976) 581].
17. L. Tyburski: Phys. Rev. D13 (1976) 1107.
18. J.F. Gunion and H. Soper: Phys. Rev. D15 (1977) 2617; E.M. Levin and M.G. Ryskin: Sov. J. Nucl. Phys. 34 (1985) 619.
19. P. Desgrolard, M. Giffon and L. Jenkovszky: Z. Phys. C55 (1992) 637; Institut de Physique Nucleaire de Lyon, preprint Lyceen (9245), Nov.1992
20. A.S. Carroll et al: Phys. Lett. 61B (1976) 303; A.Schiz et al.: Phys. Rev. D 24 (1981) 26.
21. F. Paccanoni: Proceedings of the Workshop Hadrons - 92, Kiev (1992) 40.
22. C. Augier et al., UA4/2 Coll.: Phys. Lett. B 315 (1993) 503, where recent available data are listed.
23. L.L. Jenkovszky: Fortschr. Phys.34 (1986) 791.

Figure captions

Fig.1 - The differential cross section for p-p at $\sqrt{s} = 19.42$ GeV (from [20]), with calculation described in the text.

Fig.2 - Data for $p - \bar{p}$ total cross-sections calculated from. eq.(14) of the text.

Fig. 1