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Recalculation of Electromagnetic Polarizabilities of the Nucleon in the Skyrme Model

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Abstract

We recalculate the electromagnetic polarizabilities of the nucleon within the Skyrme model in terms of the full field consisting both of the Skyrme field and the fluctuating pion field. We prove that the seagull-term contribution to the electric polarizability should vanish contrary to previous results, and that this holds for a wide class of models such as a linear sigma-model.

Electric and magnetic polarizabilities of a nucleon are important structure parameters, which characterize a dynamical response to the external electromagnetic fields. These parameters have long been studied through experimental and theoretical low-energy Compton scattering[1-4]. Recently, there are many theoretical investigations using various models including quarks[5-8], chiral soliton models[9-12], the chiral perturbation theory[13,14] and the dispersion theory[15,16].

In this paper we recalculate the electromagnetic polarizabilities within the original Skyrme model using the canonical quantization scheme with the full canonical pion fields. The scheme has been developed for pion-nucleon scattering[17] and applied to the pion-photoproduction process[18]. Applying the scheme to Compton scattering, we prove that the seagull-term contribution to the electric polarizability should vanish contrary to previous results in the Skyrme model, and that this holds for a wide class of models. This means that the electric polarizability does not come from the seagull term, but from pion-loop diagrams. Also we point out that there is a missing seagull term of $O(N_c)$ contributing to the diamagnetic polarizability, though the numerical size is not large.

At first we define the full canonical pion field Φ_a as

$$U = \frac{1}{f_\pi}(\Phi_0 + i\tau_a \Phi_a) \quad (1)$$

with

$$\Phi_0^2 = f_\pi^2 - \sum_{a=1}^3 \Phi_a^2, \quad (2)$$

where U is the $SU(2)$ matrix field, and f_π is the pion decay constant, which is 93 MeV empirically. The electromagnetic field $a_\mu(x)$ is introduced through replacing the derivatives by the covariant ones and gauging the Wess-Zumino term. The total Lagrangian exact up to $O(e^2)$ is written as

$$\begin{aligned} \mathcal{L}_{\text{tot}} = & \frac{1}{2} \dot{\Phi}_a K_{ab} \dot{\Phi}_b - \mathcal{V}[\Phi_a, \nabla \Phi_a] + e a_\mu(x) J^\mu(x) \\ & + \frac{1}{2} e^2 a_\mu a_\nu G^{\mu\nu}(x) + e^2 \varepsilon^{\mu\nu\rho\sigma} (\partial_\mu a_\nu) a_\rho W_\sigma(x) \\ & + O(e^3), \end{aligned} \quad (3)$$

where the first line is the original Skyrme Lagrangian, and J^μ , $G^{\mu\nu}$ and W_σ are functions of Φ_a and $\partial\Phi_a/\partial x^\mu$, the explicit expression of which are not given here. It should be noted that these functions except for K_{ab}

and $\mathcal{V}$ contain $\dot{\Phi}_a = \partial\Phi_a/\partial t$. Extracting the time-derivatives of the fields from J and others,

$$J^\mu = j^\mu + J_a^\mu \dot{\Phi}_a + \frac{1}{2} \dot{\Phi}_a J_{ab}^\mu \dot{\Phi}_b, \quad (4)$$

$$G^{\mu\nu} = \Gamma^{\mu\nu} + G_a^{\mu\nu} \dot{\Phi}_a + \frac{1}{2} \dot{\Phi}_a G_{ab}^{\mu\nu} \dot{\Phi}_b, \quad (5)$$

$$W_\sigma = g_{\sigma 0} W_a \dot{\Phi}_a + g_{\sigma i} W^i, \quad (6)$$

we rewrite the total Lagrangian up to $O(e^2)$ as follows:

$$\begin{aligned} \mathcal{L}_{\text{tot}} = & \frac{1}{2} \dot{\Phi}_a \widetilde{K}_{ab} \dot{\Phi}_b + \widetilde{L}_a \dot{\Phi}_a - \widetilde{\mathcal{V}} \\ & - \frac{1}{2} \partial_\nu a_\mu \partial^\nu a^\mu + e^2 \varepsilon^{0ijk} \dot{a}_i a_j W_k, \end{aligned} \quad (7)$$

where

$$\widetilde{K}_{ab} = K_{ab} + e a_\mu J_{ab}^\mu + \frac{1}{2} e^2 a_\mu a_\nu G_{ab}^{\mu\nu}, \quad (8)$$

$$\begin{aligned} \widetilde{L}_a = & e a_\mu J_a^\mu + \frac{1}{2} e^2 a_\mu a_\nu G_a^{\mu\nu} \\ & + e^2 \varepsilon^{ijk0} (\partial_i a_j) a_k W_a, \end{aligned} \quad (9)$$

$$\begin{aligned} \widetilde{\mathcal{V}} = & \mathcal{V} - e a_\mu j^\mu - \frac{1}{2} e^2 a_\mu a_\nu \Gamma^{\mu\nu} \\ & - e^2 \varepsilon^{i\mu\nu j} (\partial_i a_\mu) a_\nu W_j. \end{aligned} \quad (10)$$

The momenta, $\pi_\mu(x)$ and $\Pi_a(x)$ conjugate to $a_\mu(x)$ and $\Phi_a(x)$, respectively, are written as

$$\pi^\mu = -\dot{a}^\mu + e^2 \varepsilon^{0\mu jk} a_j W_k, \quad (11)$$

$$\Pi_a = \widetilde{K}_{ab} \dot{\Phi}_b + \widetilde{L}_a. \quad (12)$$

We obtain the Hamiltonian by using the standard canonical method as

$$\begin{aligned} H_{\text{tot}} = & \frac{1}{2} \int d^3x \{ (\Pi_a - \widetilde{L}_a) \widetilde{K}_{ab}^{-1} (\Pi_b - \widetilde{L}_b) + \widetilde{\mathcal{V}} \} \\ & + \frac{1}{2} \int d^3x \{ -(\pi^\mu - e^2 \varepsilon^{0\mu jk} a_j W_k)^2 + \partial_i a_\nu \partial^i a^\nu \} \\ \equiv & \widetilde{H}_{\text{sky}} + \widetilde{H}_{\text{elmg}}, \end{aligned} \quad (13)$$

where we put the first line as $\widetilde{H}_{\text{sky}}$ and the second line as $\widetilde{H}_{\text{elmg}}$.

Expanding the Hamiltonian in powers of the electric charge e up to $O(e^2)$, we have

$$\widetilde{H}_{\text{sky}} = H_{\text{sky}} + H_1 + H_2 + O(e^3), \quad (14)$$

where

$$H_{\text{sky}} = \int d^3x \{ \frac{1}{2} \Pi_a K^{-1}_{ab} \Pi_b + \mathcal{V} \} \quad (15)$$

$$\begin{aligned} H_1 = & \int d^3x \{ e a_\mu \{ -j^\mu - J_a^\mu K^{-1}_{ab} \Pi_b \\ & - \frac{1}{2} \Pi_a K^{-1}_{ac} J_{cd}^\mu K^{-1}_{db} \Pi_b \} \}, \end{aligned} \quad (16)$$

$$\begin{aligned} H_2 = & \int d^3x \{ \frac{1}{2} e^2 a_\mu a_\nu \{ -\Gamma^{\mu\nu} + J_a^\mu K^{-1}_{ab} J_b^\nu \\ & + 2 J_a^\mu K^{-1}_{ac} J_{cd}^\nu K^{-1}_{db} \Pi_b - G_a^{\mu\nu} K^{-1}_{ab} \Pi_b \} \\ & - \int d^3x \{ e^2 (\partial_i a_\mu) a_\nu \{ \varepsilon^{i\mu\nu 0} W_a K^{-1}_{ab} \Pi_b + \varepsilon^{i\mu\nu j} W_j \} \\ & + \int d^3x \{ \frac{1}{2} e^2 a_\mu a_\nu \Pi_a K^{-1}_{ac} J_{ce}^\mu K^{-1}_{ef} J_{fd}^\nu K^{-1}_{db} \Pi_b \\ & - \frac{1}{2} \Pi_a K^{-1}_{ac} G_{cd}^{\mu\nu} K^{-1}_{db} \Pi_b \} \}. \end{aligned} \quad (17)$$

The second term, $JK^{-1}J$ of $O(N_c)$, is often missed in previous papers, where the Lagrangian is written in terms of the classical Skyrmon fields. This term plays a crucial role in the calculation of the electromagnetic polarizabilities. In the above we discarded the ordering problem concerning to noncommuting operators, the effect of which is of higher order by $O(\hbar^2)$.

The electric current operator containing no time-derivatives is defined through the equation of motion:

$$\begin{aligned} \mathcal{J}^\mu(x) = & i^2 [H_{\text{tot}}, [H_{\text{tot}}, a^\mu(x)]] + \partial_i \partial^i a^\mu(x) \\ = & \mathcal{J}_1^\mu(x) + \mathcal{J}_2^\mu(x), \end{aligned} \quad (18)$$

where $\mathcal{J}_1^\mu$ is the current of $O(e)$ and $\mathcal{J}_2^\mu$ of $O(e^2)$,

$$\begin{aligned} \mathcal{J}_1^\mu = & -e \{ j^\mu + J_a^\mu K^{-1}_{ab} \Pi_b \\ & + \frac{1}{2} \Pi_a K^{-1}_{ac} J_{cd}^\mu K^{-1}_{db} \Pi_b \}, \end{aligned} \quad (19)$$

$$\begin{aligned} \mathcal{J}_2^\mu = & e^2 a_\nu(x) \{ -\Gamma^{\mu\nu} + J_a^\mu K^{-1}_{ab} J_b^\nu \\ & + J_a^\mu K^{-1}_{ac} J_{cd}^\nu K^{-1}_{db} \Pi_b + J_a^\nu K^{-1}_{ac} J_{cd}^\mu K^{-1}_{db} \Pi_b \\ & - G_a^{\mu\nu} K^{-1}_{ab} \Pi_b \\ & + \Pi_a K^{-1}_{ac} J_{ce}^\mu K^{-1}_{ef} J_{fd}^\nu K^{-1}_{db} \Pi_b \\ & - \frac{1}{2} \Pi_a K^{-1}_{ac} G_{cd}^{\mu\nu} K^{-1}_{db} \Pi_b \} \\ & - e^2 \{ a_\nu \partial_i [\varepsilon^{i\mu\nu 0} W_a K^{-1}_{ab} \Pi_b + \varepsilon^{i\mu\nu j} W_j] \\ & - 2(\partial_i a_\nu) [\varepsilon^{i\mu\nu 0} W_a K^{-1}_{ab} \Pi_b + \varepsilon^{i\mu\nu j} W_j] \\ & + \varepsilon^{0\mu\nu j} a_\nu \Pi_a K^{-1}_{ab} \frac{\partial W_j}{\partial \Phi_b} + 2\varepsilon^{0\mu\nu j} \pi_\nu W_j \}. \end{aligned} \quad (20)$$

We note that the current involves π^k . These are the building blocks for our calculations.

Now, we construct the Compton scattering amplitude using the standard reduction formula:

$$\begin{aligned} T^{\mu\nu} = & (2\pi)^3 i \int d^4z \{ e^{ik'z} \\ & < N(\mathbf{p}') | T(\mathcal{J}^\mu(z), \mathcal{J}^\nu(0)) + \delta(z^0) \{ [\dot{a}^\mu(z), \mathcal{J}^\nu(0)] \\ & - ik'^0 [a^\mu(y), \mathcal{J}^\nu(x)] \} | N(\mathbf{p}) > . \end{aligned} \quad (21)$$

The seagull term comes from the two equal-time commutators:

$$i[\dot{a}^\mu(\mathbf{z}, 0), \mathcal{J}^\nu(0)] = -\frac{\partial \mathcal{J}_2^\nu}{\partial a_\mu} \delta(\mathbf{z}) + e^2 \varepsilon^{0\mu\rho j} a_\rho^i [W_j, \mathcal{J}^\nu(0)]|_{z^0=0} \quad (22)$$

$$k'^0[a^\mu(\mathbf{z}, 0), \mathcal{J}^\nu(0)] = 2ik'^0 e^2 \varepsilon^{0\mu\nu j} W_j \delta(\mathbf{z}) \quad (23)$$

up to $O(e^2)$. Because the commutators are sandwiched between the single-baryon states, we discard the second term in Eq.(22), and then we have

$$\begin{aligned} S^{\mu\nu} = & e^2 \{ \Gamma^{\mu\nu} - J_a^\mu K^{-1}{}_{ab} J_b^\nu \\ & - J_a^\mu K^{-1}{}_{ac} J_{cd}^\nu K^{-1}{}_{db} \Pi_b - J_a^\nu K^{-1}{}_{ac} J_{cd}^\mu K^{-1}{}_{db} \Pi_b \\ & + G_a^{\mu\nu} K^{-1}{}_{ab} \Pi_b - \Pi_a K^{-1}{}_{ac} J_{ce}^\mu K^{-1}{}_{ef} J_{fd}^\nu K^{-1}{}_{db} \Pi_b \\ & - \frac{1}{2} \Pi_a K^{-1}{}_{ac} G_{cd}^{\mu\nu} K^{-1}{}_{db} \Pi_b \\ & - \partial_i [\varepsilon^{i\mu\nu 0} W_a K^{-1}{}_{ab} \Pi_b + \varepsilon^{i\mu\nu j} W_j] \\ & - \varepsilon^{0\mu\nu j} \Pi_a K^{-1}{}_{ab} \frac{\partial W_j}{\partial \Phi_b} + 2ik'^0 \varepsilon^{0\mu\nu j} W_j \}. \end{aligned} \quad (24)$$

as the seagull term. The full pion fields above are reduced to the classical Skyrme fields with the collective coordinates, the center of the Skyrme $\mathbf{R}(t=0)$ and the rotation angles expressed in terms of the rotation matrices R_{ai} in the tree approximation, when they are sandwiched between the one-nucleon states[17]:

$$\begin{aligned} & \langle B(\mathbf{p}') | S^{\mu\nu} [\Phi_a(0)] | N(\mathbf{p}) \rangle \\ & = \int d^3x e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} \langle B | S^{\mu\nu} [\Phi_S^a(\mathbf{x})] | N \rangle \end{aligned} \quad (25)$$

with

$$\begin{aligned} \Phi_S^a(\mathbf{x}) &= f_\pi \sin F(r) R_{ai} \hat{x}_i, \\ \Phi_S^0(\mathbf{x}) &= f_\pi \cos F(r), \end{aligned} \quad (26)$$

where $F(r)$ is the profile function of the hedgehog ansatz and $\hat{x}_i = x_i/r$. Here we take only the rotational modes of the classical Skyrme fields.

The time-ordered product term of currents consist of the Born term with the discrete single-baryon states as the intermediate states and the continuous term with the baryon-plus-pion states induced by pion-photoproduction. The single-baryon states denoted as B are restricted to the states given as rotational levels of the Skyrme, that is the nucleon and Δ state, here. The current in the time-ordered product term should be $\mathcal{J}_1^\mu$, because the scattering amplitude is correct up to $O(e^2)$.

According to the definition through the low-energy Compton amplitude by Friar and Petrun'kin[2,4], the electric polarizability is given as

$$\alpha = 2 \sum_{B \neq N} \frac{|\langle B | \int d^3x x_3 \mathcal{J}^0(\mathbf{x}) | N \rangle|^2}{M_B - M_N}. \quad (27)$$

On the other hand, the definition employed in previous works in the chiral soliton models[10-12] differs from the above definition, because there is the contribution from the seagull term:

$$\begin{aligned} \alpha = & 2 \sum_{B \neq N} \frac{|\langle B | \int d^3x x_3 \mathcal{J}^0(\mathbf{x}) | N \rangle|^2}{M_B - M_N} \\ & + \langle N | \int d^3x x_3^2 S^{00}(\mathbf{x}) | N \rangle, \end{aligned} \quad (28)$$

where only $\Gamma^{00}[\Phi_a^a]$ of $O(N_c)$ out of the seagull terms has been taken into account. Is this due to the difference among the models used? In the following we prove that the seagull-term contribution to the electric polarizability vanishes for a wide class of models.

The seagull term contains many terms: The leading order terms of $O(N_c)$ consist of Γ^{00} and the term

$$-J_a^0 K^{-1}{}_{ab} J_b^0, \quad (29)$$

which is missed in previous papers on the chiral soliton models. This term arises from the full quantization procedure of both the electromagnetic and the pion fields. It is easy to see that this term cancels Γ^{00} completely as the operator equation: Since $J_a^0 = \varepsilon_{3cd} \Phi_c K_{da}$, we have

$$\begin{aligned} J_a^0 K^{-1}{}_{ab} J_b^0 &= \varepsilon_{3ca} \varepsilon_{3db} K_{ab} \Phi_c \Phi_d = \Phi_\perp^2 \\ &+ \frac{1}{\kappa^2 f_\pi^2} [\Phi_\perp^2 \partial_i \Phi_a X_{ab} \partial_i \Phi_b - (\vec{\Phi} \times \partial_i \vec{\Phi})_3^2], \end{aligned} \quad (30)$$

where $\vec{\Phi}_\perp^2 = \sum_{a=1,2} \Phi_a^2$, $X_{ab} = \delta_{ab} + \Phi_a \Phi_b / \Phi_0^2$ and $\kappa = f_\pi \epsilon$ with ϵ being the Skyrme constant. This is just equal to Γ^{00} , because explicit calculation shows that

$$\begin{aligned} \Gamma^{\mu\nu} &= g^{\mu\nu} \{ \vec{\Phi}_\perp^2 + \frac{1}{\kappa^2 f_\pi^2} [\vec{\Phi}_\perp^2 \partial_i \Phi_a X_{ab} \partial_i \Phi_b - (\vec{\Phi} \times \partial_i \vec{\Phi})_3^2] \} \\ &+ g^{\mu i} g^{\nu j} \frac{1}{\kappa^2 f_\pi^2} [\vec{\Phi}_\perp^2 \partial_i \Phi_a X_{ab} \partial_j \Phi_b \\ &- (\vec{\Phi} \times \partial_i \vec{\Phi})_3 (\vec{\Phi} \times \partial_j \vec{\Phi})_3]. \end{aligned} \quad (31)$$

Other higher order terms in Eq.(24) vanish for $\mu = \nu = 0$. Thus, the seagull term does not contribute to the electric polarizability.

The vanishing of the seagull terms is one of our main results. Further, we should notice that this result is also valid in the model which satisfies the commutation relation

$$[a_\mu(y), \mathcal{J}_\nu(x)] \delta(y^0 - x^0) = 0. \quad (32)$$

because the gauge invariance $T_{\mu\nu}k^\nu = 0$ implies the result $S_{\mu 0} = 0$: Define the Compton scattering amplitude as

$$\begin{aligned} \widetilde{T}_{\mu\nu} &= i^2 \int d^4y d^4x e^{ik'y - ikx} \\ &\times \langle \mathbf{p}' | T(\mathcal{J}_\mu(y) \mathcal{J}_\nu(x)) \\ &+ \delta(y^0 - x^0) [\dot{a}_\nu(y), \mathcal{J}_\nu(x)] | \mathbf{p} \rangle, \end{aligned} \quad (33)$$

where we assumed Eq.(32). Then, we have

$$\begin{aligned} \widetilde{T}_{\mu\nu}k^\nu &= i \int d^4y d^4x e^{ik'y - ikx} \\ &\times \langle \mathbf{p}' | -\delta(y^0 - x^0) [\mathcal{J}_\mu(y), \mathcal{J}_0(x)] \\ &+ (\partial_x^0 \delta(y^0 - x^0)) [\dot{a}_\mu(y), \mathcal{J}_0(x)] | \mathbf{p} \rangle, \end{aligned} \quad (34)$$

where used is $\partial^\nu \mathcal{J}_\nu(x) = 0$. Now, using

$$\begin{aligned} \mathcal{J}_\mu(y) &= \ddot{a}_\mu(y) - \Delta a_\mu(y), \\ \partial_x^0 \delta(y^0 - x^0) &= -\partial_y^0 \delta(y^0 - x^0), \end{aligned} \quad (35)$$

and Eq.(32), we get

$$\begin{aligned} \widetilde{T}_{\mu\nu}k^\nu &= i \int d^4y d^4x e^{ik'y - ikx} i k'^0 \\ &\times \langle \mathbf{p}' | \delta(y^0 - x^0) [\dot{a}_\mu(y), \mathcal{J}_0(x)] | \mathbf{p} \rangle = 0, \end{aligned} \quad (36)$$

which just means $S_{\mu 0}$. The gauge invariance therefore implies $S_{\mu 0}(-\mathbf{k}', \mathbf{k}) = 0$. This result is easily shown to be consistent with the commutation relation Eq.(32) by integrating the commutator multiplied by k^ν like $\widetilde{T}_{\mu\nu}$.

Thus the vanishing of the seagull term holds for a wide class of models containing a linear sigma-model with quarks, since it satisfies Eq.(32). When the commutation relation (32) does not hold, the gauge invariance cannot imply any constraint on the seagull term. Though in the Skyrme model Eq.(32) is satisfied only for $\mu = 0$, $S_{00} = 0$ holds from Eq.(32).

The electric dipole interaction cannot excite the Δ isobar from the nucleon. The whole contributions to the electric polarizability come from the continuous term with pion-loop diagrams as discussed in the dispersion theory[16] and the chiral perturbation theory[13].

The cancellation among the seagull terms does not occur in the case of the magnetic polarizability. The magnetic polarizability is given as

$$\begin{aligned} \beta &= 2 \sum_{B \neq N} \frac{|\langle B | \int d^3x \frac{1}{2} \mathbf{x} \times \vec{J}(\mathbf{x}) | N \rangle|^2}{M_B - M_N} \\ &+ \langle N | \int d^3x \frac{1}{2} \varepsilon_{3li} \frac{1}{2} \varepsilon_{3mj} x^l x^m S^{ij} | N \rangle \\ &+ \{\text{continuous term}\}. \end{aligned} \quad (37)$$

There is also the term $-J_a^i K^{-1}{}_{ab} J_b^j$ of $\mathcal{O}(N_c)$, which was not taken into account in a previous treatment of the seagull term. The spatial component of J_a^i comes from the anomalous baryonic current:

$$J_a^i = -\frac{1}{2} \frac{N_c}{3} \frac{1}{4\pi^2 f_\pi} \varepsilon^{ijk} \varepsilon_{bcd} t_{ad} \ell_{bj} \ell_{ck}, \quad (38)$$

where t_{ad} and ℓ_{bj} are defined as $U \partial^0 U = i\tau_d t_{ad} \dot{\Phi}_a / f_\pi$ and $U \partial^j U = i\tau_b \ell_{bj}$, respectively, and we factored out $N_c/3$ to show the N_c dependence of the anomalous current[19]. The missing term, β'_{dia} , contributing to the diamagnetic polarizability is written as

$$\begin{aligned} \beta'_{\text{dia}} &= -\frac{8\pi}{3} \left(\frac{N_c}{24\pi^2 f_\pi} \right)^2 \\ &\times \int dr r^2 \frac{\sin^2 F \cdot F'^2}{1 + (F'^2 + \sin^2 F / r^2) / \kappa^2}. \end{aligned} \quad (39)$$

The numerical size of the missing term is not so large for $N_c = 3$: For the parameter set I of ref.[20], $f_\pi = 0.054$ GeV, $\epsilon = 4.84$ and $m_\pi = 0.138$ GeV, we have $\beta'_{\text{dia}} = -0.46 \times 10^{-4} \text{ fm}^3$, and for the parameter set II of ref.[12], $f_\pi = 0.093$ GeV, $\epsilon = 3.55$ and $m_\pi = 0.138$ GeV, we have $\beta'_{\text{dia}} = -0.13 \times 10^{-4} \text{ fm}^3$.

The total diamagnetic polarizability β_{dia} is given as

$$\begin{aligned} \beta_{\text{dia}} &= -13.1 \times 10^{-4} \text{ fm}^3 \quad \text{parameter set I,} \\ \beta_{\text{dia}} &= -16.3 \times 10^{-4} \text{ fm}^3 \quad \text{parameter set II,} \end{aligned}$$

where we used the values of ref.[12].

The paramagnetic polarizability is written as

$$\beta_{\text{para}} = 2 \frac{\mu_{\Delta N}^2}{(M_\Delta - M_N)}, \quad (40)$$

where $\mu_{\Delta N}$ is the transition magnetic moment from N to Δ . Since there is the relation $(M_\Delta - M_N)\mu_{\Delta N} = \sqrt{2}e/4$ in the Skyrme model, $\beta_{\text{para}} \propto (M_\Delta - M_N)^{-3}$ depends strongly on the parameters f_π and ϵ . If we take the experimental values $\mu_{\Delta N} = 3.29e/2M_N$ and $M_\Delta - M_N = 293 \text{ MeV}$ [20], we have $\beta_{\text{para}} = 11.7 \times 10^{-4} \text{ fm}^3$. The total magnetic polarizability is then $\beta = -1.4 \times 10^{-4} \text{ fm}^3$ for the parameter set I and $-4.6 \times 10^{-4} \text{ fm}^3$ for the parameter set II.

Finally, we discuss about the question on the consistent $1/N_c$ expansion of the magnetic polarizability[8]. We first notice that the polarizabilities are the coefficient of the terms proportional to ω^2 in the expansion of the low energy Compton scattering amplitude in powers of ω . Expanding the matrix element of the spatial current $\langle B | \vec{\epsilon}_i \vec{J}(\mathbf{k}) | N \rangle$ in powers of $\mathbf{k}$, we get the term,

$$(\vec{\epsilon}_j \times \hat{k}')(\vec{\epsilon}_i \times \hat{k}) \omega^2 \frac{2(M_B - M_N + \frac{\omega^2}{2M_B}) |\langle B | \mu_3 | N \rangle|^2}{(M_B - M_N + \frac{\omega^2}{2M_B})^2 - \omega^2} \quad (41)$$

in the expansion of the numerator of the Born terms for forward scattering in the laboratory system, where we put the final photon momentum $\mathbf{k}' = \omega \hat{\mathbf{k}}'$ and the initial momentum $\mathbf{k} = \omega \hat{\mathbf{k}}$. If we define the paramagnetic polarizability as

$$\tilde{\beta}_{\text{para}} = \sum_B \frac{2(M_B - M_N) | \langle B | \mu_3 | N \rangle |^2}{(M_B - M_N)^2 - \omega^2 (1 - \frac{M_B - M_N}{M_B})}, \quad (42)$$

$\tilde{\beta}_{\text{para}}$ is of $O(N_c)$ for finite ω , since the numerator is of $O(N_c)$ owing to the factor of the mass difference, while the denominator is of $O(1)$ because of the presence of the photon energy squared, that is of $O(1)$. When we put $\omega = 0$, $\tilde{\beta}_{\text{para}}$ reduces to β_{para} , and it jumps to the quantity of $O(N_c^2)$, because the mass difference squared of $O(N_c^{-2})$ remains, but ω^2 of $O(1)$ is made to vanish. Thus, the $1/N_c$ expansion is meaningful only for finite ω , while N_c must be finite for the expansion in powers of ω around $\omega = 0$. We point out, here, that our

formalism did not use the $1/N_c$ expansion when we got the Hamiltonian and the Compton scattering amplitude, but it involved higher $1/N_c$ terms within the restriction up to e^2 .

In this paper we recalculated the electromagnetic polarizabilities within the Skyrme model applying the canonical quantization scheme to the Compton scattering amplitude. We showed that the electric polarizability from the seagull term should vanish in the Skyrme model, and that the vanishing of the seagull term holds for a wide class of models consistently with the gauge invariance, and that there is the missing term in the magnetic polarizability, though it is not large numerically. Since contributions from the continuous term, which consists of pion-loop contributions, are not fully treated within the chiral soliton models, the calculations of the electromagnetic polarizabilities are incomplete yet in such models.

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