POLARIZED NEUTRON MATTER WITH SKYRME FORCES

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Abstract:

It is shown that Skyrme forces with a commonly used parameter $t_2 < 0$ lead to instability of polarized neutron matter. This instability has a form of the ferromagnetic spin ordering. There is, however, no ground state and the system displays a singular behaviour. Physically consistent description of polarized neutron matter requires that the exchange parameter x_2 , which was neglected in spin-saturated systems, is in the range $-5/4 < x_2 \leq -1$. We find that Skyrme forces with x_2 in this range provide very good parameterization of realistic neutron matter calculations.

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Simplicity of the Skyrme potential [1] makes it a useful tool in nuclear structure calculations [2,3] and in studying properties of dense matter in neutron stars [4,5]. Here we are concerned with the latter case.

The Skyrme potential used in astrophysical calculations is [5]

$$v(\mathbf{r}) = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \frac{1}{2}t_1(1 + x_1 P_\sigma)[\mathbf{k}'^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{k}^2] \\ + t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r})\mathbf{k} + v_3. \quad (1)$$

Here $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. The operator $\mathbf{k} = \frac{i}{2}(\nabla_1 - \nabla_2)$ acts on the right of the delta-function and $\mathbf{k}'$ is the same operator acting on the left of $\delta(\mathbf{r})$. P_σ is the spin exchange operator.

The 3-body term $v_3 = t_3\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3)$ is equivalent to a two-body density dependent term [2] $(1/6)t_3(1 + P_\sigma)n(\mathbf{R})\delta(\mathbf{r})$, where $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$. In the following we shall use a modified form due to Ravenhall, Bennet and Pethick [4] who included in this expression an exchange term proportional to a new parameter x_3 .

The energy density in a general case is

$$H(\mathbf{r}) = \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0[(1 + \frac{1}{2}x_0)\rho^2 - (x_0 + \frac{1}{2})(\rho_N^2 + \rho_P^2)] + \frac{1}{4}[t_1(1 + \frac{1}{2}x_1) + t_2(1 + \frac{1}{2}x_2)]\rho\tau \\ + \frac{1}{4}[t_2(\frac{1}{2} + x_2) - t_1(\frac{1}{2} + x_1)](\rho_N\tau_N + \rho_P\tau_P) - \frac{1}{16}[t_2(1 + \frac{1}{2}x_2) - 3t_1(1 + \frac{1}{2}x_1)](\nabla\rho)^2 \\ - \frac{1}{16}[t_2(\frac{1}{2} + x_2) + 3t_1(\frac{1}{2} + x_1)][(\nabla\rho_N)^2 + (\nabla\rho_P)^2] + \frac{1}{4}t_3(\rho_N\rho_P + \frac{1}{4}x_3(\rho_N - \rho_P)^2)\rho. \quad (2)$$

Here τ_N and τ_P are kinetic energy densities of, respectively, neutrons and protons, ρ_N and ρ_P are their number densities, $\tau = \tau_N + \tau_P$ and $\rho = \rho_N + \rho_P$.

The expression (2) contains 8 parameters $t_0, t_1, t_2, t_3, x_0, x_1, x_2$ and x_3 which are to be adjusted to fit properties of nuclei and nuclear matter. In most fits exchange parameters x_1 and x_2 are neglected, $x_1 = x_2 = 0$.

Choice of the parameters is briefly described below. An important feature of various parameter sets [1,2,3] is that often the parameter t_2 is negative, $t_2 < 0$, for $x_2 = 0$. In

Table I we give two sets of Skyrme force parameters: The first one, *SI*, due to Vautherin and Brink [2] and the second one, *SII*, due to Beiner et al. [3]. There exist other sets in Ref.[3] with $t_2 > 0$ and very weak three-body force. In astrophysical calculations [4,5], however, negative values of t_2 were commonly used.

In this note we point out that restricted Skyrme potential, i.e. with $x_1 = x_2 = 0$ and with $t_2 < 0$ is inadequate to describe spin-polarized neutron matter. It leads to a singular behaviour of a polarized neutron matter. Any finite density solution is therefore metastable and there is no ground state. This is due to suppressing the exchange term in p-wave interaction by putting $x_2 = 0$. We show that the Hamiltonian is stabilized by including this term with a properly chosen exchange parameter x_2 .

The instability becomes manifest if we consider fully polarized neutron matter with a uniform density. In this case the energy density is

$$\epsilon_f = \frac{\hbar^2}{2m} \tau_u + \frac{1}{2} t_2 \tau_u \rho_N^u, \quad (3)$$

where $\tau_u = (3/5)(6\pi^2 \rho_N^u)^{2/3} \rho_N^u$ and $\rho_N^u = \rho_N$ is the density of neutrons with spin up. This formula shows that for $t_2 < 0$ and $x_2 = 0$, as in both parameter sets in Table I, the energy has no lower bound and the system collapses to a singularity. The energy per particle for ferromagnetic neutron matter in this case is shown in Fig.1 (dashed line with $x_2 = 0$). Thus the Hamiltonian we use is not well defined. It should be improved by including a new term making the system stable.

The physical origin of this instability is rather obvious. The Skyrme potential is a contact potential with momentum-dependent terms representing s-wave and p-wave interactions. All these terms contribute in spin-saturated systems. However, for ferromagnetic neutron matter the Pauli principle suppresses completely all s-wave terms, and the only term surviving is the p-wave contribution proportional to t_2 . Since this term has stronger density dependence than kinetic energy term it always wins at high densities and leads to a singularity when $t_2 < 0$.

In order to make the Hamiltonian well defined one should include the exchange term in the p-wave term which was neglected by putting $x_2 = 0$. That is the only way to stabilize ferromagnetic neutron matter with Skyrme potential (1) since p-wave interaction is the only term which contributes in the ferromagnetic state.

Including x_2 the energy density of the ferromagnetic neutron matter becomes

$$\epsilon_f = \frac{\hbar^2}{2m}\tau_u + \frac{1}{2}t_2(1 + x_2)\tau_u\rho_N^u. \quad (4)$$

This formula shows that $x_2 \leq -1$ is required in order to make the Hamiltonian stable, for negative values of t_2 . Of course, no x_2 is needed for $t_2 > 0$.

Let us now briefly describe how the parameter x_2 can be determined. The Skyrme force parameters are adjusted in first place to fit properties of symmetric nuclear systems where $\rho_N = \rho_P = \rho/2$. For such systems the Hamiltonian simplifies to

$$\begin{aligned} H(\mathbf{r}) = & \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}[3t_1 + 5t_2(1 + \frac{4}{5}x_2)]\rho\tau \\ & + \frac{1}{64}[9t_1 - 5t_2(1 + \frac{4}{5}x_2)](\nabla\rho)^2 + \frac{1}{16}t_3\rho^3. \end{aligned} \quad (5)$$

One can notice that this formula contains only one exchange parameter x_2 . All other exchange parameters, x_0 , x_1 and x_3 cancel out in symmetric nuclear systems and hence cannot be determined from properties of these systems. Let us also note that the values of parameters t_2 and x_2 cannot be determined separately since these appear in two terms above in the same product

$$\bar{t}_2 \equiv t_2(1 + \frac{4}{5}x_2). \quad (6)$$

By fitting properties of symmetric nuclear systems one can determine the values of t_0 , t_1 , t_3 and $\bar{t}_2$. In calculations shown in Figs.1 and 2 we adopt the values obtained by Vautherin and Brink [2] who fit saturation properties of nuclear matter and energies of

^{16}O and ^{208}Pb nuclei (SI force in Table I). The latter nucleus is slightly asymmetric, however its asymmetry parameter is small.

Saturation properties of nuclear matter give the following relations:

$$t_0\rho_0 = \frac{4}{9}K + \frac{40}{3}W - \frac{16}{5}T_F, \quad (7)$$

$$\frac{3}{16}t_3\rho_0^2 = K + 15W - \frac{9}{5}T_F, \quad (8)$$

$$\frac{1}{16}[3t_1 + t_2(5 + 4x_2)]k_F^2\rho_0 = 2T_F - 15W - \frac{5}{6}K, \quad (9)$$

where the saturation density $\rho_0 = 0.155\text{fm}^{-3}$ and the saturation energy $W = -16\text{MeV}$ are input parameters in the fit of Ref.[2]. Corresponding value of the incompressibility modulus is $K = 370\text{MeV}$, somewhat above the empirical value of 220MeV . It is a general feature of Skyrme forces that the incompressibility is a little too high.

Requirement of stability of polarized neutron matter and the empirical result that $\bar{t}_2 < 0$ allow us to constrain the range of x_2 . The parameter x_2 should satisfy inequalities

$$\bar{t}_2 \equiv t_2(1 + \frac{4}{5}x_2) < 0 \quad \text{and} \quad t_2(1 + x_2) \geq 0. \quad (10)$$

These two conditions can be fulfilled only if $t_2 < 0$ and $-5/4 < x_2 \leq -1$. We find thus a remarkably narrow range for physically allowed values of x_2 for $\bar{t}_2 < 0$.

Remaining exchange parameters, x_0 , x_1 and x_3 , enter the formulae for asymmetric nuclear systems. We shall focus on uniform density systems. The symmetry energy is

$$E_{sym} = \frac{1}{3}T_F - \frac{1}{4}t_0(x_0 + \frac{1}{2})\rho_0 - \frac{1}{16}t_3(1 - x_3)\rho_0^2 + \frac{1}{24}[-3t_1x_1 + t_2(4 + 5x_2)]k_F^2\rho_0, \quad (11)$$

where $k_F = (1.5\pi^2\rho_0)^{1/3}$ and $T_F = k_F^2/2m$.

The energy density of a uniform neutron matter reads

$$\begin{aligned}
H(\mathbf{r}) = & \frac{\hbar^2}{2m}\tau_N + \frac{1}{4}t_0(1-x_0)\rho_N^2 + \frac{1}{8}[t_1(1-x_1) + 3t_2(1+x_2)]\rho_N\tau_N \\
& + \frac{3}{32}[t_1(1-x_1) - t_2(1+x_2)](\nabla\rho_N)^2 + \frac{1}{16}t_3x_3\rho_N^3.
\end{aligned} \tag{12}$$

Here $\tau_N = (3/5)k_N^2\rho_N$ with $k_N = (3\pi^2\rho_N)^{1/3}$. Since the parameter x_1 is irrelevant to both ferromagnetic neutron matter and symmetric nuclear matter in the following we shall put $x_1 = 0$ for simplicity.

In Ref.[2] it is assumed that $x_1 = x_2 = 0$ and $x_3 = 1$. From eq.(11) the value of x_0 is then obtained, assuming empirical value $E_{sym} = 30\text{MeV}$. Resulting equation of state of neutron matter is, however, too stiff as compared with realistic neutron matter calculations. To obtain better description of neutron matter a lower value $x_3 = 0.52$ was adopted in Ref.[4]. This leads to different value for x_0 .

In this paper we adjust the exchange parameters x_2 and x_3 to obtain an overall good reproduction of neutron matter calculations of Wiringa, Fiks and Fabrocini [6]. As can be seen in Fig.2, we reproduce the energies rather well. Particularly interesting is the case of $UV14 + TNI$ interaction, which is very nicely reproduced with $x_2 = -1.08$ and $x_3 = 0$, solid line in Fig.2. For comparison we show the dashed curve corresponding to $x_2 = 0$ and $x_3 = 0.3$ ($x_0 = 0.42$). Clearly, the curve with no three-body force, $x_3 = 0$, and $x_2 = -1.08$ fits the calculations of Ref.[6] much better. The $UV14 + UVII$ interaction has stronger three-body force contribution and to reproduce the energy of neutron matter the exchange parameter $x_3 = 0.24$ is used.

In Table II we give the values of the exchange parameters x_0 , x_2 and x_3 adjusted to reproduce the $UV14 + TNI$ and $UV14 + UVII$ results from Ref.[6] (solid curves in Fig.2). They are obtained for the SI Skyrme force in Table I. Also, the exchange parameters corresponding to the SII force from Table I, are given.

The energy of the ferromagnetic neutron matter is shown in Fig.1 for $x_2 = -1.08$ as a dashed line. It increases monotonically with density and lies above energy of normal

neutron matter with $UV14 + TNI$ interactions. This means that normal neutron matter is the ground state of the system at any density. The reason of this behaviour is the fact that there is no three-body interaction in neutron matter, $x_3 = 0$, in this case.

For other interactions from Ref.[6] $x_3 > 0$ and at high enough densities the ferromagnetic neutron matter becomes the ground state of the system. This occurs always when there is a three-body contribution present in normal neutron matter energy since the energy of ferromagnetic matter has weaker density dependence.

In conclusion, consistent description of polarized neutron matter with Skyrme forces requires that for negative values of the parameter t_2 corresponding exchange parameter x_2 is in a narrow range $-5/4 < x_2 \leq -1$. Skyrme forces with properly chosen exchange parameters x_2 and x_3 provide very good representation of realistic neutron matter calculations.

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REFERENCES

- [1] T. H. R. Skyrme, Nucl. Phys. **9**, 615 (1959).
- [2] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
- [3] M. Beiner, H. Flocard, Nguyen Van Giai and P. Quentin, Nucl. Phys. **A238**, 29 (1975).
- [4] D. G. Ravenhall, C. D. Bennett and C. J. Pethick, Phys. Rev. Lett. **28**, 978 (1972).
- [5] J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. **31**, 337 (1981).
- [6] R. B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. C **38**, 1010 (1988).

TABLE I

Skyrme force parameters: *SI* from Ref.[2] and *SII* from Ref.[3]

Force	$t_0[MeV fm^3]$	$t_1[MeV fm^5]$	$\bar{t}_2[MeV fm^5]$	$t_3[MeV fm^6]$
SI	-1057.3	235.9	-100.0	14463.5
SII	-1128.8	395.0	-95.0	14000.0

TABLE II

Exchange parameters of Skyrme forces *SI* and *SII* in Table I

Force	Equation of state	x_0	x_2	x_3
SI	UV14+TNI	0.19	-1.08	0.00
	UV14+UVII	0.17	-1.02	0.24
SII	UV14+TNI	0.28	-1.00	0.00
	UV14+UVII	0.16	-1.06	0.07

FIGURE CAPTIONS

Fig.1

Energy per particle of normal neutron matter (solid line) and ferromagnetic neutron matter (dashed lines) as a function of neutron density. Open squares represent calculations of Ref.[6] for the $UV14 + TNI$ interaction. Solid line is the SI Skyrme force fit with parameters given in Table I and exchange parameters from Table II. Upper dashed line is for stable ferromagnetic matter whereas the lower one corresponds to singular case with $x_2 = 0$.

Fig.2

The same as in Fig.1 for normal neutron matter results from Ref.[6]. Solid lines are SI Skyrme force fits with parameters in Tables I and II. For comparison results with $x_2 = 0$ are shown as dashed lines. Corresponding values of remaining exchange parameters for the $UV14 + TNI$ (squares) and $UV14 + UVII$ (triangles) interactions are, respectively, $x_0 = 0.42$, $x_3 = 0.30$ and $x_0 = 0.32$, $x_3 = 0.48$.

Fig. 1

Fig. 2