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The laboratory and astrophysical constraints on the neutrino charge radius

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Abstract

If the properties of neutrino magnetic moments have been extensively studied, the consequences of a non-vanishing charge radius have drawn little attention so far. The laboratory bounds are reviewed. They are fairly comparable to the constraints set by the cooling of red-giant stars. The relevance of anapole neutrinos to the explosion mechanism of massive stars is also discussed. As a matter of fact, if neutrinos have an anomalous charge radius, they can drag some fraction of the gravitational energy released during the collapse of a compact object. As they transfer it to the envelope of the supernova, neutrinos revitalize the stalled shock wave.

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1 - Introduction.

In the Standard Model of the electroweak interactions, neutrinos are massless and have no magnetic or electric dipole moment. However, simple extensions [1] of the minimal scheme lead to a magnetic moment for massive Dirac species, of the order of

$$\mu_\nu = \frac{3G_F m_e m_\nu}{4\sqrt{2}\pi^2} \simeq 3.2 \times 10^{-19} \mu_B \left(\frac{m_\nu}{1 \text{ eV}} \right) , \quad (1)$$

where $\mu_B = e/2m_e$ is the Bohr magneton. The incorporation of additional physics may imply significantly larger magnetic moments [2]. In left-right symmetric models, for instance, the neutrino mass m_ν of equation (1) is replaced by $m_L \sin 2\xi$, where ξ denotes the rotation angle between the propagation and interaction eigenstates of the two weak vector bosons W and W' , whereas m_L is the charged lepton mass. Even more exotic options have been proposed [3], where μ_ν may reach up to a few $10^{-10} \mu_B$, and may help solving the solar neutrino puzzle [4]. The phenomenology of neutrino magnetic moments has been thoroughly investigated. If the theory tends to predict fairly small values, the limits set by neutrino-electron scattering in beam dump or reactor experiments are on the order of $2 - 4 \times 10^{-10} \mu_B$, much larger than the theoretical expectations. The best bound from studies of $\bar{\nu}_e e^-$ elastic scattering is [5, 6]

$$\mu_{\bar{\nu}_e} < 4 \times 10^{-10} \mu_B , \quad (2)$$

while for the electron-neutrino, the limit is [7]

$$\mu_{\nu_e} < 10.8 \times 10^{-10} \mu_B . \quad (3)$$

Similarly, muon-neutrino beams and the resulting $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ interactions have been used to set the bounds [7, 8]

$$\mu_{\nu_\mu, \bar{\nu}_\mu} < 7.4 \text{ or } 8.5 \times 10^{-10} \mu_B . \quad (4)$$

For tau-neutrinos, the laboratory constraints are fairly less restrictive in so far as a pure ν_τ or $\bar{\nu}_\tau$ beam has not yet been produced. However, by searching for electrons scattered in the forward direction by neutrinos produced by dumping a 400 GeV proton beam on a cooper target, and by estimating the flux of tau-neutrinos originating from the decays of the D_s mesons along the dump, A. M. Cooper-Sarkar and collaborators [9] have derived the limit

$$\mu_{\nu_\tau, \bar{\nu}_\tau} < 5.4 \times 10^{-7} \mu_B . \quad (5)$$

Astrophysics provides even stronger constraints. During a supernova explosion, for instance, a large magnetic moment would induce the helicity flip between the left-handed

neutrinos produced by the cooling of the residual neutron star, and the right-handed sterile species which would drag the bulk of the gravitational energy, weakening and even suppressing the conventional neutrino emission. This argument leads to [10]

$$\mu_\nu < (1 - 20) \times 10^{-13} \mu_B , \quad (6)$$

and applies as long as the right-handed neutrinos do not interact after their emission. If their mass reaches, say, 1 MeV and becomes comparable to their energy, right-helicity Dirac states start to interact through the left-handed chiral electroweak coupling, and the previous argument does not hold. By analyzing the cooling pattern of red-giant stars, G. Raffelt [11] has derived the constraint

$$\mu_\nu < (2 - 3) \times 10^{-12} \mu_B , \quad (7)$$

valid for neutrinos lighter than the typical plasma frequency at the cores of helium stars, *i.e.*, $\sim 10 - 20$ keV. As regards neutrino magnetic moments, the theoretical predictions are therefore remarkably smaller than the bounds derived from astrophysical considerations which, in turn, are two orders of magnitude below the laboratory limits.

The situation is not quite the same for the charge radius (or anapole moment) of neutrinos where a detailed investigation of the experimental constraints as well as of the stellar signatures is still in the air. First, the theoretical situation in the framework of the Standard Model is quite confusing. Some authors [12] have concluded that the electroweak radiative corrections to the photon-neutrino vertex lead to a divergent and gauge dependent charge radius. For instance, by performing a calculation in the R_ξ gauge [13], the off-shell electromagnetic coupling of the neutrino was claimed to vary with the weak gauge parameter and the authors concluded that the charge radius of neutrinos could not be a physical observable of Nature. In order to circumvent the conceptual difficulties affecting the definition of the neutrino electromagnetic form factor, Deggrasi and Sirlin [14] considered the amplitude of the neutrino-lepton elastic scattering. They showed that radiative corrections, in the renormalization scheme in which $\sin^2 \theta_W = 1 - m_W^2/m_Z^2$, could be separated into two finite and gauge-invariant parts. One part does not depend on the target on which the neutrino scatters and the associated function $f(q^2)$ was readily interpreted as an effective electromagnetic form factor of the neutrino in the framework of the low-energy theory derived from the Standard Model, at very small scales q^2 with respect to m_W^2 . Deggrasi and Sirlin derived an effective electroweak charge radius

$$\langle r_{\nu L}^2 \rangle = 6 \left. \frac{\partial f(q^2)}{\partial q^2} \right|_{q^2=0} \sim \frac{G_F}{\sqrt{2}\pi^2} \text{Log} \left(\frac{m_W}{m_L} \right) \quad L = e, \mu, \tau \quad (8)$$

on the order of $0.1 - 1$ am^{*}. Their result satisfies the crucial requirement of being finite and gauge invariant, and depends sensitively on the mass of the top quark. More precisely, for $m_t = 90$ GeV and $m_H = 100$ GeV, they obtained

$$\langle r^2 \rangle \simeq \begin{cases} -(0.134 \pm 0.036) \text{ am}^2 & \text{for } \nu_e \\ +(0.185 \pm 0.036) \text{ am}^2 & \text{for } \nu_\mu \\ +(0.354 \pm 0.036) \text{ am}^2 & \text{for } \nu_\tau \end{cases} \quad (9)$$

On the other hand, some theorists claimed to have either estimated [15] or calculated [16] the neutrino charge radius in the framework of the Standard Model. Very recently, G3nrgora-T. and Stuart [17] defined the charge radius and anapole moment of a free fermion as being respectively the vector and axial-vector contact interaction that couples that particle to an external electromagnetic current. This physical external electromagnetic current serves as a tag for contributions to the charge radius and anapole moment as opposed to the unphysical off-shell photon field, which was generally used in previous works. They derived an expression which, despite earlier conclusions of the contrary, was claimed to be demonstrably gauge-invariant and observable in principle.

The scope of this article is to investigate the experimental and astrophysical implications of an anomalous neutrino charge radius, irrespective of its nature and regardless of the physical processes responsible for its magnitude. For some unknown reasons associated to exotic physics beyond the Standard Model, neutrinos may well undergo electromagnetic interactions. As a matter of fact, they could be composite objects [18]. As is the case for nuclei or molecules [19], the existence of inner structure would lead to an actual charge radius or anapole moment whose magnitude could be considerably larger than the value implied by the radiative corrections of the Standard Model. In section 2, we discuss the laboratory limits on the neutrino charge radius. The best bounds are derived from the elastic scattering of neutrinos on electrons in beam dump experiments. Note that for ν_e and ν_μ , these limits are on the order of the attometer, just an order of magnitude above what is expected conventionally. The difference between diagonal and transition moments is emphasized. The single photon production in e^+e^- collisions, and the corresponding ASP and CELLO constraints, are also reviewed. Section 3 is devoted to the cooling of red-giant stars through the decay of plasmons into neutrino pairs. The inferred astrophysical bound on the charge radius or the anapole moment of neutrinos is found to be fairly comparable to the ASP and CELLO limits. Finally, in section 4, the possibility of a large ν_τ anapole moment is investigated. It may help a significant fraction of the gravitational energy released by the core collapse of massive stars to be deposited in the

^{*}The natural unit of length which will be used throughout this article is the attometer (am) where 1 attometer = 10^{-18} m = 10^{-16} cm.

exploding envelopes of supernovae. In the state of the art numerical simulations, the shock wave does not work its way throughout the star up to the surface. An anomalous anapole moment would help this stalled shock wave to be turned into a successful explosion.

2 - The laboratory limits.

The Lagrangian which describes the possible interactions of neutrinos with an electromagnetic plane wave of four-momentum q , may be expressed as a sum of three terms

$$\begin{aligned} \mathcal{L}_{em}^{(\nu)} = & - e F_{CR}(q^2) \bar{\psi} \gamma_\mu \psi A^\mu - e \bar{\psi} \sigma_{\mu\lambda} \{ F_{MD}(q^2) + i \gamma_5 F_{ED}(q^2) \} \psi F^{\mu\lambda} \\ & - e F_A(q^2) \bar{\psi} \gamma_\mu \gamma_5 \psi \partial_\lambda F^{\lambda\mu} . \end{aligned} \quad (10)$$

The first expression ($F_{CR}(q^2)$) stands for the distribution of the electric charge associated to the neutrino. Since the latter is globally neutral, $F_{CR}(0)$ vanishes. However, the neutrino may be the superimposition of two distributions of opposite electric charge, with the same centers, but with different spatial extensions. The associated charge radius is defined as the derivative of the form factor

$$\langle r^2 \rangle = 6 \left. \frac{\partial F_{CR}(q^2)}{\partial q^2} \right|_{q^2=0} , \quad (11)$$

where $\langle r^2 \rangle$ may be positive (r_c^2) or negative ($-r_c^2$), depending on the charge of the most extended distribution. Note that, at low-energy, the coupling is proportional to q^2 so that neutrinos do not interact with real photons. Classically, this first term is equivalent to the interaction $\vec{j} \cdot \vec{A}$ and is therefore even under parity and time-reversal transformations. The second expression in the right-hand side of equation (10) refers to the possibility of magnetic (F_{MD}) and electric (F_{ED}) dipole moments. These interactions have been extensively discussed in the literature. Suffice it to say that the magnetic moment corresponds to the parity even and time-reversal even classical coupling $\vec{S} \cdot \vec{B}$ between the neutrino spin $\vec{S}$ and the magnetic field $\vec{B}$. On the contrary, since an electric dipole interaction may be translated into the coupling between the spin and the electric field, it violates both parity and time-reversal symmetries. Finally, the last expression in (10) describes the neutrino anapole moment which Zel'dovich introduced in 1957 [20], when he was interested in electromagnetic interactions with parity violation. It corresponds classically to the coupling between the spin and the vector potential $\vec{A}$ and therefore is parity-odd but does not violate the time-reversal invariance. A fairly simplified representation of the anapole is provided by a doughnut-like distribution of current interacting with a uniform magnetic field. In the lorentz gauge, the anapole term is quite similar to the charge radius interaction in so far as both are proportional to q^2 . Note that a Majorana neutrino

may only interact with the electromagnetic field through an anapole moment, hence the relevance of such a coupling. For the sake of clarity, the electromagnetic interactions of left-handed Dirac neutrinos will be described by the current

$$\langle p' | J_{em}^\alpha | p \rangle = e \frac{\langle r^2 \rangle}{6} q^2 \{ \bar{u}(p') \gamma^\alpha L u(p) \} - i \frac{e}{2m_e} \mu_\nu \{ \bar{u}(p') \sigma^{\alpha\beta} q_\beta L u(p) \} , \quad (12)$$

where $q = p' - p$ and L is the projection operator $L = (1 - \gamma_5)/2$. In this section, the magnetic moment μ_ν and the square $\langle r^2 \rangle$ of the charge radius are the relevant parameters of our analysis. Finally, if neutrinos are Majorana species, their electromagnetic interactions further simplify into the anapole coupling

$$\mathcal{L}_{em}^{(\nu)} = e \frac{\langle r^2 \rangle}{12} q^2 \{ \bar{\psi}_M \gamma_\alpha \gamma_5 \psi_M \} A^\alpha . \quad (13)$$

A constraint on the charge radius of neutrinos is provided by reactor and beam dump experiments. When a neutrino beam with energy E_ν interacts with matter, some electrons are kicked out into the forward direction, with kinetic energy T . The associated differential cross section may be expressed as [6]

$$\begin{aligned} \frac{d\sigma}{dT} = & \frac{G_F^2 m_e}{2\pi} \left\{ (g_V + X + g_A)^2 + (g_V + X - g_A)^2 \left(1 - \frac{T}{E_\nu} \right)^2 \right. \\ & + \left. \left[g_A^2 - (g_V + X)^2 \right] \frac{m_e T}{E_\nu^2} \right\} \\ & + \frac{\pi \alpha^2 \mu_\nu^2}{m_e^2} \left\{ \frac{1 - T/E_\nu}{T} \right\} , \end{aligned} \quad (14)$$

where the vector (g_V) and axial (g_A) couplings are respectively defined by

$$g_V = \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \quad (15)$$

and

$$g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau. \end{cases} \quad (16)$$

The difference between electron-neutrinos and the other species is due to the presence of charged current interactions. The existence of an anomalous charge radius or anapole moment merely results into the rescaling of the vector coupling g_V which must be changed into the effective parameter $g_V + X$, where X is defined as

$$X = \frac{\sqrt{2} \pi \alpha \langle r^2 \rangle}{3G_F} . \quad (17)$$

A charge radius $r_c \sim 6.5$ am corresponds to $X = 1$, and starts to significantly modify the interactions of neutrinos with electrons and nuclei. It is not surprising, therefore, if the experimental limits are on the order of a few attometers.

A) Electron-neutrinos, from muon decay at rest, have been observed to scatter elastically on the electrons of a 15-ton fine-grained tracking calorimeter [21]. The detection of $236 \pm 35 \nu_e e^-$ events was translated into a limit on the vector coupling $g_V = 2 \sin^2 \theta_W - 1/2$

$$-0.216 < g_V < 0.214 \quad . \quad (18)$$

Assuming the value $\sin^2 \theta_W = 0.2325$ for the weak mixing angle, this 90% confidence interval yields the bound

$$-0.181 < X < 0.249 \quad , \quad (19)$$

and finally the constraint

$$-7.62 \text{ am}^2 < \left(\langle r^2 \rangle \right)_{\nu_e} < 10.5 \text{ am}^2 \text{ (90\% CL)} \quad (20)$$

on the charge radius of electron-neutrinos. Note that the result quoted in reference [21], *i.e.*,

$$-3.56 \text{ am}^2 < \left(\langle r^2 \rangle \right)_{\nu_e} < 5.44 \text{ am}^2 \text{ (90\% CL)} \quad (21)$$

relies on the value $\sin^2 \theta_W = 0.227$ as well as on another definition of the charge radius (or anapole moment) of neutrinos. As a matter of fact, our normalization of $\langle r^2 \rangle$ is based on relation (12) – see also [14] – whereas an alternative convention, where the charge radius is twice as small as in this article, is defined according to [22]

$$\langle \bar{\nu} | J_\gamma^\mu | \nu \rangle = e F(q^2) \{ \bar{u}_\nu \gamma^\mu u_\nu \} \quad . \quad (22)$$

B) Intense beams of electron-antineutrinos may be produced by nuclear reactors. The limit set by a recent Russian experiment [23] on the charge radius or anapole moment of $\bar{\nu}_e$ is

$$r_c(\bar{\nu}_e) < 2.7 \text{ am (90\% CL)} \quad . \quad (23)$$

C) The CHARM experiment [24] detected $83 \pm 16 \nu_\mu e^-$ events and $112 \pm 21 \bar{\nu}_\mu e^-$ reactions. The inferred value of the vector coupling is

$$g_V = -0.06 \pm 0.07 (\text{stat.}) \pm 0.02 (\text{syst.}) \quad , \quad (24)$$

and may be readily translated into the 90% confidence range

$$-0.18 < g_V < 0.06 \quad . \quad (25)$$

By comparing this interval with the Standard Model value of $g_V = -0.035$, a bound on the charge radius of muon-neutrinos obtains

$$-6.1 \text{ am}^2 < \left(\langle r^2 \rangle \right)_{\nu_\mu, \bar{\nu}_\mu} < 4.0 \text{ am}^2 \text{ (90\% CL)} . \quad (26)$$

The CHARM experiment was upgraded into the CHARM II collaboration [25]. The statistics were considerably increased with $2105 \pm 69 \nu_\mu e^-$ and $2215 \pm 76 \bar{\nu}_\mu e^-$ elastic scatterings. The vector coupling g_V was determined with greater accuracy, *i.e.*,

$$g_V = -0.025 \pm 0.019 \text{ (stat. and syst.)} . \quad (27)$$

Note that final results are obtained with a level of confidence of 90%. Therefore, the error of ± 0.019 in the determination of g_V must be multiplied by a statistical factor of 1.64, hence the constraint on the charge radius of muon-neutrinos

$$-0.9 \text{ am}^2 < \left(\langle r^2 \rangle \right)_{\nu_\mu, \bar{\nu}_\mu} < 1.7 \text{ am}^2 \text{ (90\% CL)} . \quad (28)$$

Finally, an equivalent limit has been set by the experiment E734 [8] at the Brookhaven National Laboratory, with a muon-neutrino beam of fairly low-energy ($\sim 1 \text{ GeV}$). The $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ reactions totalled respectively a mere 160 and 97 events. With our definition of the charge radius, but keeping the original value for $\sin^2 \theta_W$, the bound turns out to be

$$-4.22 \text{ am}^2 < \left(\langle r^2 \rangle \right)_{\nu_\mu, \bar{\nu}_\mu} < 0.48 \text{ am}^2 \text{ (90\% CL)} . \quad (29)$$

To summarize the above-mentioned experimental results, the upper limit on the charge radius of electron-neutrinos is of order $2 - 3 \text{ am}$ whereas, for muon-neutrinos, the constraint is slightly more stringent, with $\sim 1 \text{ am}$. These bounds have been derived with the assumption that $\mu_\nu = 0$. Notice that both charge radius and magnetic moment have combined effects on neutrino-electron elastic scattering, as is clear from relation (14). Such a correlation should definitely be addressed in the forthcoming analysis of data [26], and lead to exclusion plots in the diagram (r_c, μ_ν) .

In beam dump or reactor experiments, a neutrino stream interacts with matter and probes the nucleon structure. This beam is wide-spread in energy but, for the sake of clarity, we will consider that the incoming neutrinos have the energy E_ν . Most of the interactions take place with the quarks inside nuclei. However, one of the by-products of these experiments is a fairly small number of collisions with electrons, *i.e.*, typically a few hundreds of reactions. The cut-off on the energy E_e of recoiled electrons, set by the detector, is denoted by E_{min} . The effective cross section for neutrino-electron interactions is therefore

$$\sigma_{exp} = \int_{E_{min}}^{E_{max}} \frac{d\sigma}{dT} dE_e , \quad (30)$$

where the maximum energy of the kicked electrons is

$$E_{max} = m_e + \frac{2E_\nu^2}{2E_\nu + m_e} \simeq E_\nu . \quad (31)$$

Neglecting the electron mass m_e with respect to E_ν and E_{min} leads to

$$\begin{aligned} \sigma_{exp} = & \frac{G_F^2 m_e}{6\pi E_\nu^2} (E_\nu - E_{min}) \left\{ 3E_\nu^2 (g_V + X + g_A)^2 + (E_\nu - E_{min})^2 (g_V + X - g_A)^2 \right\} \\ & + \frac{\pi \alpha^2 \mu_\nu^2}{m_e^2 E_\nu} \left\{ E_\nu \text{Log} \left(\frac{E_\nu}{E_{min}} \right) - E_\nu + E_{min} \right\} . \end{aligned} \quad (32)$$

Limits on r_c and μ_ν originate mostly from the uncertainty ΔN on the measurement of the number N_0 of collected neutrino-electron elastic scatterings. Suppose that N_0 corresponds to vanishing charge radius and magnetic moment

$$N_0 = N [r_c = 0, \mu_\nu = 0] . \quad (33)$$

Since N_0 is measured with some degree of accuracy, the error $\Delta N \sim \sqrt{N_0}$ readily translates into uncertainties in the determination of r_c and μ_ν which, as pointed out recently [23], are correlated. However, the analysis which Gurevich and collaborators performed is somewhat crude. In order to refine it and to study how both Δr_c and $\Delta \mu_\nu$ depend on the error ΔN , let us consider, for instance, $\nu_\mu e^-$ scatterings. The bench-mark value for the total number of collected events is $N_0 = 1000$ while the uncertainty ΔN is set equal to 50. In fig. 1a, the density contrast of the propability function

$$\text{Prob}(r_c, \mu_\nu) = \exp \left\{ - (N[r_c, \mu_\nu] - N_0)^2 / \Delta N^2 \right\} \quad (34)$$

is presented in the plane $(\langle r^2 \rangle, \mu_\nu)$. The allowed region, compatible with the above-mentioned simulated results, appears in dark-grey and is superimposed over the white excluded domain. The energy of incoming neutrinos has been set equal to the typical value of $E_\nu = 25$ GeV, while the cut-off on the energy of recoiled electrons is $E_{min} = 5$ GeV. This density contrast plot clearly exhibits a strong correlation between the charge radius and the magnetic moment. For instance, the point where $\langle r^2 \rangle = 15 \text{ am}^2$ and $\mu_\nu = 8 \times 10^{-9} \mu_B$ is compatible with the total number N_0 of collected events. It is as probable as the situation where both charge radius and magnetic moment vanish. In fig. 1b, the cut-off energy E_{min} has been decreased from 5 GeV to 100 MeV. The net effect is a better determination of the magnetic moment μ_ν whereas the uncertainty on the charge radius r_c tends to increase. The cross section (14) is indeed divergent for the small values of the recoiled energy T , a typical behavior associated to the presence of the magnetic moment μ_ν . The allowed region has still the same shape as in fig. 1a. Charge radii as large as $\langle r^2 \rangle \sim 25 \text{ am}^2$ cannot be ruled out a priori. Note, furthermore,

that negative values of $\langle r^2 \rangle$ are excluded in both plots 1a and 1b. By considering now muon-antineutrinos and still assuming $N_0 = 1000$ with $\Delta N = 50$, a different part of the $(\langle r^2 \rangle, \mu_\nu)$ plane is forbidden. In fig. 1c, the beam energy and the electron energy threshold are respectively $E_\nu = 25$ GeV and $E_{min} = 100$ MeV. Because we deal with antineutrinos, the only difference with the previous case is a change in the sign of the axial coupling g_A . A significant portion of the allowed domain spreads now over the negative values of $\langle r^2 \rangle$. By combining both data from $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ interactions, most of the $(\langle r^2 \rangle, \mu_\nu)$ plane may be excluded, hence the stringent constraints set by the experiment E734 of BNL [8], and by the CHARM II Collaboration [25].

A neutrino interacts with electrons through the exchange of Z^0 and possibly W . If the neutrino has a non-vanishing charge radius, an additional process ensues with the exchange of a photon, and a possible interference with the Standard weak reaction obtains. As a consequence, the vector coupling g_V is renormalized and must be replaced by the effective parameter $g_V + X$. Both Z^0 and γ exchanges preserve the neutrino helicity. In the case of a non-vanishing magnetic moment, the helicity is flipped and the corresponding cross section merely adds up to the other contributions. The situation is not quite the same if the charge radius (or the anapole moment) leads to transitions among neutrino flavors. Such a non-diagonal interaction occurs for the electromagnetic current

$$\langle p', a | J_{em}^\alpha | p, b \rangle = \frac{e}{6} q^2 \left\{ \bar{u}_a(p') (\langle r^2 \rangle)_{ab} \gamma^\alpha L u_b(p) \right\} , \quad (35)$$

where a and b stand respectively for one of the leptonic flavors e, μ and τ . The interference between the photon exchange (vector coupling X) and the Z^0 mediated process (parameter g_V) disappears and relation (14) becomes

$$\begin{aligned} \frac{d\sigma}{dT} &= \frac{G_F^2 m_e}{2\pi} \left\{ (g_V + g_A)^2 + (X)^2 + ([g_V - g_A]^2 + [X]^2) \left(1 - \frac{T}{E_\nu}\right)^2 \right. \\ &\quad \left. + [g_A^2 - g_V^2 - X^2] \frac{m_e T}{E_\nu^2} \right\} \\ &\quad + \frac{\pi \alpha^2 \mu_\nu^2}{m_e^2} \left\{ \frac{1 - T/E_\nu}{T} \right\} . \end{aligned} \quad (36)$$

For the same values of E_ν and E_{min} , fig. 1a, 1b and 1c are respectively replaced by plots 2a, 2b and 2c. The same trends appear in both diagrams 1 and 2. The new exclusion plots are nevertheless symmetrical in $\langle r^2 \rangle$ in so far as the transition $\nu_\mu e \rightarrow \nu_X e$ does not interfere now with the standard weak interaction. As the cut-off energy E_{min} decreases, the limit on μ_ν improves at the expense of the determination of r_c . Furthermore, the interaction mediated by the transition moments (35) contributes the same factor to neutrino and antineutrino events, as is clear from the comparison between fig. 2b and 2c. Combining

neutrino and antineutrino data does not improve the bounds. With typical statistics, the limit on such transition couplings is of order $r_c \sim 2$ to 3 am, irrespective of the neutrino generation.

The anomalous single photon experiment (ASP [27]) was designed to search for the reaction

$$e^+e^- \rightarrow \gamma + \text{invisible particles} , \quad (37)$$

in electron-positron collisions at $\sqrt{s} = 29$ GeV. The experimental signature is the production of a single photon, with energy E_γ , polar angle θ_γ with respect to the beam, and unbalanced transverse momentum. The experiment found a total of 1.6 events, in agreement with the expectations of the electroweak Standard Model whose contribution to reaction (37) is the radiative production of neutrino-antineutrino pairs

$$e^+e^- \rightarrow \gamma\nu_L\bar{\nu}_L . \quad (38)$$

The corresponding cross section, in the local-limit approximation where $\sqrt{s}$ is negligible with respect to m_W and m_Z , is [28]

$$\frac{d^2\sigma_{\gamma\nu\bar{\nu}}}{dx dy} = \frac{2\alpha}{\pi} \left\{ \frac{\left(1 - \frac{x}{2}\right)^2 + \frac{x^2 y^2}{4}}{x(1 - y^2)} \right\} \sigma_{\nu\bar{\nu}}(s') , \quad (39)$$

where $y = \cos\theta_\gamma$ and x is the ratio $2E_\gamma/\sqrt{s}$. The total cross section $\sigma_{\nu\bar{\nu}}$ for neutrino pair production is evaluated for the reduced center-of-mass energy $s' = s(1 - x)$ and simplifies, in the four-point approximation, into

$$\sigma_{\nu\bar{\nu}}(s') \simeq \frac{G_F^2}{12\pi} s' \Sigma . \quad (40)$$

The sum Σ runs over the neutrino generations

$$\Sigma = \sum_\nu \left\{ g_A(\nu)^2 + g_V(\nu)^2 \right\} , \quad (41)$$

where the vector (g_V) and axial (g_A) couplings have been previously defined in relations (15) and (16). The ASP collaboration constrained the cross section for the single photon production (37) to be less than 0.072 pb. This result was translated into a limit on the number N_ν of neutrino generations as well as on the properties of supersymmetric particles, such as the mass of the selectron. With N_ν neutrino families, expression (41) becomes

$$\Sigma = N_\nu (g_A^2 + g_V^2) + 2(g_A + g_V) + 2 , \quad (42)$$

with $g_A = -1/2$ and $g_V = 2\sin^2\theta_W - 1/2$. The ASP analysis resulted into the bound $N_\nu < 12.3$ while CELLO [29] obtained $N_\nu < 13.9$, both limits being set at the 95%

confidence level. By combining the results of both experiments, the limit $N_\nu < 8.1$ ensued. Our bench-mark value of

$$N_\nu < 10 \quad (43)$$

will be used when the ASP bound is translated into a limit on the charge radius of tau-neutrinos. The total decay width of the Z^0 boson has been accurately measured at LEP [30]. There are only three neutrino families [31], in perfect agreement with the Standard Model. The ASP bound may nevertheless be useful if tau-neutrinos have non-vanishing anapole moments. If so, the sum (41) becomes

$$\Sigma = \{(g_A + 1)^2 + (g_V + 1)^2\} + \{g_A^2 + g_V^2\} + \{g_A^2 + (g_V + X_\tau)^2\} , \quad (44)$$

where the three terms in the right-hand side of the previous expression respectively refer to the electron, muon and tau-neutrinos. The presence of a non-vanishing charge radius for the tau species implies the additional contribution X_τ to the vector coupling. Therefore, the experimental limit (43) may be readily expressed as the bound

$$X_\tau \leq \frac{\sqrt{N_\nu - 3}}{2} , \quad (45)$$

hence the constraint

$$r_c(\nu_\tau) < 8 - 9 \text{ am} . \quad (46)$$

If, on the one hand, beam dump and reactor experiments set limits on the electromagnetic properties of electron and muon-neutrinos, the absence of an anomalous single photon production in e^+e^- collisions constrains the charge radius (or anapole moment) of tau-neutrinos. Note finally that at LEP, electron-positron annihilations are tuned on the Z^0 resonance. If neutrinos have an anomalous charge radius, the reaction $e^+e^- \rightarrow \nu\bar{\nu}$ is also mediated by the exchange of a virtual photon. However, its contribution to the total annihilation cross section turns out to be completely swamped in the Z peak. Therefore, no useful information on the charge radius or anapole moment of neutrinos may be derived from the LEP experiments.

3 - The stellar evolution constraint.

The core of a red-giant star is made of pure helium, with density $\rho \sim 10^6 \text{ g cm}^{-3}$ and temperature $T \sim 10^8 \text{ K}$. Stellar matter consists of a plasma in which the Fermi momentum of electrons is

$$p_F = (3\pi^2 n_e)^{1/3} \sim 410 \text{ keV} , \quad (47)$$

where n_e denotes the electron density. Since p_F exceeds the temperature T by an order of magnitude, the plasma is quasi-degenerate. In such a medium, electromagnetic waves do not propagate as in the vacuum, but follow dispersion relations where their energy ω is no longer equal to their momentum $Q = \|\vec{Q}\|$. As already predicted by Maxwell, the transverse modes, with two helicities, correspond to an electric field $\vec{E}$ orthogonal to the direction $\vec{Q}$ of propagation. For non-relativistic electrons, the dispersion relation for transverse propagation is given by the classical expression

$$\omega_T^2 = \omega_P^2 + Q^2 . \quad (48)$$

In the non-relativistic regime, the plasma frequency ω_P is related to the density n_e and the mass m_e of electrons

$$\omega_P = \left(\frac{4\pi\alpha n_e}{m_e} \right)^{1/2} \sim 20 \text{ keV} . \quad (49)$$

There is also a single longitudinal mode where the electric field $\vec{E}$ points in the same direction as the propagation $\vec{Q}$, with dispersion

$$\omega_L = \omega_P , \quad (50)$$

where Q does not exceed the plasma frequency ω_P . The quanta associated to plasma waves are called *plasmons* and their properties have been extensively analyzed. Quite recently, the dispersion relations of longitudinal and transverse plasmons [32] have been derived in the light of thermal field theory [33], for all temperatures and densities of a plasma, in the perfect gas limit. Note that inside a red-giant star, the Fermi momentum p_F is comparable to the electron mass m_e , so that neither the classical nor the ultra-relativistic limits apply. The plasma frequency may be expressed as

$$\omega_P^2 = \frac{4\alpha}{3\pi} p_F^2 v_F , \quad (51)$$

where $v_F = p_F/E_F$ denotes the velocity of the states at the surface of the Fermi sea. The dispersion relations are slightly more elaborate than in the classical approximation. For transverse plasmons, the energy ω_T and the momentum $\vec{Q}$ are related by

$$\omega_T^2 - Q^2 = \left\{ \frac{3\omega_T^2\omega_P^2}{2v_F^2 Q^2} \right\} \left\{ 1 - \left(\frac{\omega_T}{2v_F Q} \right) \left(1 - \frac{v_F^2 Q^2}{\omega_T^2} \right) \text{Log} \left(\frac{\omega_T + v_F Q}{\omega_T - v_F Q} \right) \right\} . \quad (52)$$

This branch extends up to $Q \rightarrow \infty$. The dispersion relation for the longitudinal mode is

$$\omega_L^2 = \left\{ \frac{3\omega_L^2\omega_P^2}{v_F^2 Q^2} \right\} \left\{ \left(\frac{\omega_L}{2v_F Q} \right) \text{Log} \left(\frac{\omega_L + v_F Q}{\omega_L - v_F Q} \right) - 1 \right\} , \quad (53)$$

with the spatial momentum varying from $Q = 0$ up to the point where the branch crosses the light cone, at $Q = Q_{max}$, with

$$Q_{max}^2 = \left\{ \frac{3\omega_P^2}{v_F^2} \right\} \left\{ \left(\frac{1}{2v_F} \right) \text{Log} \left(\frac{1+v_F}{1-v_F} \right) - 1 \right\} . \quad (54)$$

Note that plasmons follow Bose-Einstein statistics with, at the cores of red-giants, a ground state energy ω_P of the same order of magnitude as the temperature $T \sim 10$ keV.

Just like plasmons, neutrinos which propagate inside an ionized gas are also affected by thermal collective processes. By surfing on the free electrons of the medium, they behave as if they had the non-vanishing charge radius [34]

$$(< r^2 >)_{th} = \frac{3G_F}{\sqrt{2}\pi\alpha} g_V \simeq (6.5 \text{ am})^2 g_V , \quad (55)$$

regardless of the properties of the plasma. This thermally induced electromagnetic form factor is responsible for the effective coupling between neutrinos and plasmons. Since $(< r^2 >)_{th}$ is proportionnal to g_V , electron-neutrinos are mostly involved at the expense of the ν_μ and ν_τ species. Plasmons may therefore decay into neutrino pairs which flow out of the star and cool down its core. If, in addition to this standard reaction, muon or tau-neutrinos have an anomalous charge radius, the decay of plasma waves is enhanced, as well as the neutrino emission and the subsequent stellar energy loss. This extra-heat flow ϵ_{ext} results from the propensity of plasmons for decaying, and depends also on the statistical distribution of plasma waves. Assuming the anomalous charge radius (or anapole moment)

$$\mathcal{L}_{em}^{(\nu)} = \pm \frac{e}{6} r_c^2 \left\{ \bar{\psi} \gamma_\mu L \psi \right\} \partial_\nu F^{\nu\mu} , \quad (56)$$

a calculation performed in the Lorentz gauge leads to the decay rate

$$\Gamma_{anom}(\gamma_P \rightarrow \nu\bar{\nu}) = \frac{\alpha}{216} r_c^4 \frac{(q^2)^{3/2}}{\omega} (q^2 - m_\nu^2) (q^2 - 4m_\nu^2)^{1/2} . \quad (57)$$

Details of the calculation may be found in [34]. The energy and momentum of plasmons are respectively denoted by ω and Q , while $q^2 = \omega^2 - Q^2$. The mass m_ν of neutrinos does not vanish a priori – note that the upper limit on the ν_τ mass is 35 MeV. The decay rate (57) is the same for both transverse and longitudinal plasma waves. Once it is integrated over the Bose-Einstein distribution $\eta_B(\omega)$ of plasmons, the rate of the extra-energy loss ensues

$$\epsilon_{ext} = \int \frac{d^3\vec{Q}}{8\pi^3} g_h \eta_B(\omega) \omega Z \Gamma_{anom} . \quad (58)$$

The helicity factor g_h respectively takes the values 1 and 2 for longitudinal and transverse modes. The additional term Z , in relation (58), corrects for the fact that plasmon propagation follows intricate dispersion relations $\omega(Q)$. In a degenerate gas, and for transverse

plasmons, this normalization factor may be expressed as

$$Z_T = \frac{2 \omega_T^2 (\omega_T^2 - v_F^2 Q^2)}{3 \omega_P^2 \omega_T^2 + (\omega_T^2 + Q^2) (\omega_T^2 - v_F^2 Q^2) - 2 \omega_T^2 (\omega_T^2 - Q^2)} , \quad (59)$$

while for the longitudinal branch

$$Z_L = \frac{2 \omega_L^2}{q^2} \left\{ \frac{(\omega_L^2 - v_F^2 Q^2)}{3 \omega_P^2 - (\omega_L^2 - v_F^2 Q^2)} \right\} . \quad (60)$$

In the non-relativistic regime where $v_F \rightarrow 0$, Z_T merely reduces to unity whereas Z_L tends to ω_P^2/q^2 .

The energy flow ϵ_{ext} is presented in fig. 3, as a function of the density ρ , with temperature $T = 10^8$ K, vanishing neutrino mass m_ν and charge radius $r_c = 10$ am. Note that for typical conditions at the centers of red-giant stars, i.e., for a density $\rho \sim 10^6$ g cm $^{-3}$, this energy loss is ~ 190 erg g $^{-1}$ s $^{-1}$, six times larger than the conventional rate for which plasmon decay is mediated by weak interactions. The curve of fig. 3 exhibits a maximum for the density $\rho_c \sim 10^7$ g cm $^{-3}$. Below ρ_c , the larger the density, the larger the plasma frequency ω_P and therefore, the faster the decay as is clear from relation (57). If we neglect the longitudinal states and assume that electrons are non-relativistic so that relation (48) applies, the heat loss (58) considerably simplifies in the regime where the plasma frequency is much smaller than the temperature

$$\epsilon_{\text{ext}} \simeq \frac{\alpha}{216} \frac{n_\gamma}{\rho} r_c^4 \omega_P^6 . \quad (61)$$

The density of plasmons n_γ may be approximated by the value corresponding to a gas of real photons, for which $n_\gamma/T^3 \sim 20$ K $^{-3}$ cm $^{-3}$. Expression (61) leads therefore to

$$\epsilon_{\text{ext}} \sim 770 \text{ erg g}^{-1} \text{ s}^{-1} \left(\frac{T}{10^8 \text{ K}} \right)^3 \left(\frac{r_c}{10 \text{ am}} \right)^4 \left(\frac{\rho}{10^6 \text{ g cm}^{-3}} \right)^2 , \quad (62)$$

and is in fair agreement with fig. 3, for the small values of the density ρ . Above ρ_c , the plasma frequency, and therefore the energy ω , are way larger than the temperature. Plasma waves start to be depopulated as implied by Bose-Einstein statistics. Hence, at fixed temperature, the plasmon decay rate decreases with increasing density. For a very dense medium, the rate ϵ_{ext} is indeed suppressed by the factor $\exp(-\omega_P/T)$.

As shown by G. Raffelt [11], if neutrino cooling is enhanced during the red-giant (RG) stage, observational modifications in the color-magnitude diagrams of globular clusters obtain. Once their cores have exhausted their hydrogen supply, stars evolve out of the main sequence and start to ascend the so-called red-giant branch of the Hertzsprung-Russell

(HR) diagram. Their interiors are quite modified. They are now degenerate, with the Fermi pressure of electrons balancing the crushing gravity. The helium core is surrounded by a hydrogen shell where nuclear fusion takes place, hence an increased luminosity and an expanded envelope whose surface mostly radiates in the red. As hydrogen burning proceeds, the mass of the helium core increases. The centers of red-giant stars lose energy mostly because plasmons decay into neutrino pairs. Since the overall stellar energy decreases, as a result of this neutrino emission, the centers go on shrinking with increasing temperature and density. At some point, the stellar material reaches the conditions under which the triple-alpha reaction is ignited. Since matter is degenerate, the helium flash ensues. Then, red-giant stars evolve on the horizontal branch (HB) of the HR diagram, where their helium cores are burnt into carbon and oxygen. The magnitude of the tip of the RG branch, as well as the luminosity of the subsequent HB stage, depend critically on the mass $\mathcal{M}_{\text{He}}$ of the helium core at the flash. For low-mass stars of population II, which typically pervade globular clusters, $\mathcal{M}_{\text{He}}$ is believed to be $\sim 0.45 - 0.5 \mathcal{M}_{\odot}$. The magnitude difference ΔM_{RR}^{tip} between the tip of the RG branch and the RR Lyrae stars of the HB sequence may be obtained from the color-magnitude diagrams of globular clusters. This luminosity shift has been inferred from measurements of the bolometric magnitude of the brightest red-giants in 26 globular clusters

$$\Delta M_{RR}^{tip} = (4.19 \pm 0.03) + (0.41 \pm 0.06)([\text{Fe}/\text{H}] + 1.3) \text{ mag} . \quad (63)$$

Any change $\delta \mathcal{M}_{\text{He}}$ in the helium core mass translates into the variation

$$\delta(\Delta M_{RR}^{tip}) \simeq 4.0 \left(\frac{\delta \mathcal{M}_{\text{He}}}{\mathcal{M}_{\odot}} \right) . \quad (64)$$

If the decay rate of plasmons is anomalously large, which is the case for a non-vanishing neutrino charge radius or anapole moment, red-giant stars are overcooled. Their ascension along the RG branch is slowed down. At a given stage, *i.e.*, for fixed $\mathcal{M}_{\text{He}}$, the larger the neutrino emission, the smaller the central temperature. Conversely, if the standard neutrino cooling rate is varied by an overall factor $F_{\nu} > 1$, red-giant stars reach helium ignition slightly later than in the conventional scheme, with a slightly more evolved, and therefore, heavier core. Sweigart and Gross [35] computed the core mass at helium flash as a function of F_{ν} , for a star with mass $0.7 \mathcal{M}_{\odot}$, helium content 30% and low metallicity $Z = 0.001$. Their results are well-described by the relation

$$\left(\frac{\mathcal{M}_{\text{He}}}{\mathcal{M}_{\odot}} \right) \simeq 0.447 + 0.031 F_{\nu}^{1/2} . \quad (65)$$

Whence the larger the neutrino losses F_{ν} , the heavier the core at helium flash and the larger the brightness difference ΔM_{RR}^{tip} . By measuring the latter, G. Raffelt [11] has constrained the amount of any extra-cooling not to exceed the magnitude of the conventional

neutrino emission

$$\epsilon_{\text{ext}} < \epsilon_{\text{std}} \sim 30 \text{ erg g}^{-1} \text{ s}^{-1} . \quad (66)$$

This astrophysical bound readily translates into the exclusion plot of fig. 4 where the upper limit on the charge radius r_c is presented as a function of the neutrino mass m_ν . The area above the curve is forbidden. For vanishing neutrino mass, the charge radius cannot exceed 6 to 7 am, under penalty of overcooling red-giant stars. Note that when the neutrino mass exceeds 9 keV ($\sim \omega_P/2$), plasmon decay is suppressed and the limit on r_c relaxes. Stellar arguments lead therefore to a constraint fairly similar to the ASP and CELLO bounds.

4 - Anapoles and supernovae.

A massive star, with $M > 13 - 15 M_\odot$, burns very rapidly its nuclear fuel and builds up a degenerate iron core, whose mass exceeds the Chandrasekhar limit of instability. The star evolves towards increasing central temperature and central density. At some point, the iron starts to be photodissociated, as required by statistical mechanics. This reaction is endo-energetic, hence a slight temperature decrease which induces a pressure deficit. The latter initiates the collapse of the core. Electron conversion on the free nucleons produced by further iron dissociation accelerates the falling-in of the star. Its interiors may be decomposed into an homologous core which collapses as a whole, surrounded by outer-layers in free fall. The homologous core reaches supra-nuclear densities, bounces back and expands. As it collides on the infalling outer-layers, a shock wave bursts out and starts its ascent throughout the star. As the shock emerges from the neutrinosphere, an intense flash of neutrinos is produced, with luminosity

$$L(\nu_e + \bar{\nu}_e) \simeq 2.8 \times 10^{52} \text{ ergs s}^{-1} , \quad (67)$$

and approximate equal share among the three flavors of neutrinos. Within 2 seconds, most of the gravitational energy of the proto-neutron star ($\sim 1.7 \times 10^{53}$ ergs) is radiated away. As it proceeds outwards, the shock photodissociates the iron of the collapsing core, is considerably weakened and is eventually stalled. Unable to make it alone to the surface, it is expected to be revitalized by energy deposition in the mantle. The strong neutrino wind (67) should indeed dump some fraction α of its energy in the hot bubble region which extends from ~ 100 up to ~ 1000 km. However, recent numerical calculations [36] have shown that this energy deposition is slightly too small so that either the shock is marginally successful, or, in most of the simulations, it does not work its way throughout the star. In this section, we assume the existence of a non-vanishing ν_τ charge radius (or

anapole moment if neutrinos are Majorana species), and investigate the consequences on the revival of the shock wave.

The neutrinosphere extends up to $R_\nu \sim 20$ km. The structure of the mantle which surrounds the proto-neutron star has been described in detail in [37]. During the critical 2 seconds which follow the collapse, it may be decomposed into several layers, extending from R_ν up to ~ 1000 km. Energy deposition in the mantle by the strong neutrino wind, supplemented by convection, maintain a strong pressure behind the shock.

A) Between the neutrinosphere ($R_\nu \sim 20$ km) and the gain radius ($R_g \sim 100$ km), matter is completely dissociated into free nucleons, with roughly the same amount of protons and neutrons ($Y_p \simeq 0.48$). Hydrostatic equilibrium holds approximately. This layer is characterized by the equality between the energy Ψ gained by neutrino absorption and the heat Λ lost by electron capture. The total entropy per nucleon increases up to $S \simeq 15$ k_B, with $S_n \simeq 12$ k_B whereas the entropy of the radiation reaches $S_r \sim 3$ units. The temperature decreases from $T_\nu \sim 5$ MeV down to $T_g \sim 1.5 - 2$ MeV.

B) Above R_g up to R_α , matter is still made of free protons and neutrons. This region is completely convective so that the entropy remains fixed at ~ 15 units per nucleon. The temperature varies from T_g down to $T_\alpha \sim 1$ MeV. The density is quite low, with $\rho \sim 10^8$ g cm⁻³. The mantle which surrounds the nascent neutron star is indeed a hot bubble, with a plasma of relativistic ($T > m_e$) and non-degenerate ($\eta = \mu_e/T \sim 1$) electrons and positrons. The latter are captured by the free nucleons immersed in this radiation, with an average rate of

$$\Gamma = \{\Gamma(n \rightarrow p) + \Gamma(p \rightarrow n)\} / 2 = \frac{G_F^2}{2\pi^3} (g_V^2 + 3g_A^2) T^5 \phi(\eta) , \quad (68)$$

where an equal density of protons and neutrons has been assumed. Neglecting m_e and the proton-neutron mass gap Q with respect to the temperature T leads to simplifications

$$\phi(\eta) = \frac{1}{2} \int_0^{+\infty} x^4 dx \left\{ \left[e^{(x+\eta)} + 1 \right]^{-1} + \left[e^{(x-\eta)} + 1 \right]^{-1} \right\} . \quad (69)$$

Each capture produces a neutrino whose escape from the mantle translates into the energy loss

$$\Lambda(\text{per nucleon}) = \frac{G_F^2}{24\pi^3} (g_V^2 + 3g_A^2) T^6 \left\{ \frac{31\pi^6}{21} + 7\pi^4 \eta^2 + 5\pi^2 \eta^4 + \eta^6 \right\} . \quad (70)$$

For $\eta \sim 1$, a value implied by $Y_p \sim 0.5$ and $S_r/k_B \sim 3$, electron capture produces the heat flow

$$\Lambda \simeq (3.2 \times 10^{18} \text{ ergs g}^{-1} \text{ s}^{-1}) \left(\frac{T}{1 \text{ MeV}} \right)^6 . \quad (71)$$

Conversely, the rate of neutrino absorption on free neutrons and protons at distance r from the center of the star is

$$\Psi = \frac{AL}{4\pi r^2} \text{ ergs g}^{-1} \text{ s}^{-1} , \quad (72)$$

where L stands for the total luminosity of ν_e and $\bar{\nu}_e$ (relation 67), whereas A depends on the neutrino capture cross section ($\sigma_a = 9 \times 10^{44} \text{ cm}^2$) and on the average energy E_ν of the neutrino wind

$$A = \frac{1}{2} N_A \sigma_a E_\nu^2 . \quad (73)$$

The Avogadro number is denoted by N_A . With $E_\nu^2 \simeq 250 \text{ MeV}^2$, the value $A \simeq 6.7 \times 10^{-18} \text{ cm}^2 \text{ g}^{-1}$ ensues. Assuming hydrostatic equilibrium as well as the conservation of entropy throughout the layer leads to the relation

$$\left(\frac{T}{1 \text{ MeV}} \right) = \frac{K}{r_7} + C , \quad (74)$$

where r_7 denotes the radius in units of $10^7 \text{ cm} = 100 \text{ km}$. The constant K depends on η and Y_p . Bethe finds $K = 2.87$. The constant C is fixed by the boundary condition $T = T_\alpha \sim 1 \text{ MeV}$ at $r = R_\alpha \sim 200 \text{ km}$. The gain radius R_g obtains directly from the requirement that $\Lambda = \Psi$ at the bottom of the stratum. If neutrino absorption compensates exactly for electron capture below the gain radius, there is a net energy deposition for $r > R_g$. Notice that the above-mentioned processes are only relevant as long as there are free nucleons.

C) As a matter of fact, above R_α , the stellar gas is not completely dissociated but is a mixture of neutrons, protons and ^4He nuclei. As the radius r increases, the fraction of free nucleons drops. It is just one-half at the distance $R_{1/2} \sim 300 \text{ km}$. Energy input by neutrino absorption as well as the antagonistic electron capture are no longer effective above $R_{1/2}$. Therefore, the fraction of the luminosity (67) which is dropped by the neutrino wind in the mantle between R_g and $R_{1/2}$ is

$$\alpha_1 = A \int_{R_g}^{R_{1/2}} \rho F dr , \quad (75)$$

where ρ is the density and F is a global factor which accounts for both neutrino absorption and electron capture

$$F = 1 - \frac{\Lambda}{\Psi} = 1 - \left[\left(\frac{T}{T_g} \right)^6 \left(\frac{r}{R_g} \right)^2 \right] . \quad (76)$$

Numerically, the integrated fraction adds up to

$$100 \alpha_1 = 0.67 \int_{R_g}^{R_{1/2}} \rho_8 F dr_7 , \quad (77)$$

where the density ρ_8 is expressed in units of 10^8 g cm^{-3}

$$\rho_8 = 0.72 \left(\frac{\eta}{Y_p} \right) \left(1 + \frac{\eta^2}{\pi^2} \right) \left(\frac{T}{1 \text{ MeV}} \right)^3 . \quad (78)$$

D) Above $R_{1/2}$ and up to $R_{max} \sim 1000 \text{ km}$, the stellar gas is made mostly of helium and carbon nuclei. The temperature decreases quite slowly as a result of the intense convection which brings up the energy dumped by the neutrino wind inside the inner layers. This stratum behaves therefore as a hot bubble of radiation, with a fairly small density and a temperature $T > m_e$. The plasma is still dominated by non-degenerate and relativistic electrons on which neutrinos diffuse. The relevant cross section is a function of the vector (g_V) and axial (g_A) couplings

$$\sigma = \frac{G_F^2}{4\pi} s \left\{ (g_V + g_A)^2 + \frac{1}{3} (g_V - g_A)^2 \right\} . \quad (79)$$

The neutrino stream scatters elastically on both electrons and positrons whose densities are roughly the same

$$n_{e-} \simeq n_{e+} \simeq (15 \text{ cm}^{-3}) \left(\frac{T}{1 \text{ K}} \right)^3 . \quad (80)$$

the velocity factor is on average $4/3$ while the square of the energy in the center-of-mass frame is $s \sim 2E_\nu(3.15 k_B T)$. As shown by Tubbs and Schramm [38] who performed the numerical integration of this process over the statistical distributions of electrons and positrons, half of the neutrino energy E_ν is transferred to the plasma. Therefore, over a path of length dr , the neutrino wind deposits a fraction $d\alpha_2$ of its energy

$$\frac{d\alpha_2}{dr} = (1.7 \times 10^{-11} \text{ cm}^{-1}) (g_V^2 + g_A^2) \left(\frac{T}{1 \text{ MeV}} \right)^4 . \quad (81)$$

Muon and tau-neutrinos may also diffuse elastically on electrons. If we assume an equal share of the total luminosity among the three species of neutrinos, the integrated fraction α_2 turns out to be proportional to Σ , a factor previously defined in relation (41)

$$100 \alpha_2 = 0.017 \Sigma \int_{R_g}^{R_{max}} \left(\frac{T}{1 \text{ MeV}} \right)^4 dr_7 . \quad (82)$$

The antagonistic reaction is the annihilation of e^+e^- pairs into neutrinos which restricts the amount of heat dumped in the plasma by the elastic scatterings. When the cross section (40) is integrated over the thermal distributions of electrons and positrons, the annihilation loss obtains [39]

$$\epsilon_{ann} = - (6.17 \times 10^{24} \text{ ergs cm}^{-3} \text{ s}^{-1}) \Sigma \left(\frac{T}{1 \text{ MeV}} \right)^9 . \quad (83)$$

It leads to the energy fraction

$$100 \alpha_3 = -7.7 \times 10^{-4} \Sigma \left(\frac{10^{52} \text{ ergs s}^{-1}}{L} \right) \int_{R_g}^{R_{max}} \left(\frac{T}{1 \text{ MeV}} \right)^9 r_7^2 dr_7 , \quad (84)$$

where L is the luminosity of the $\nu_e + \bar{\nu}_e$ emission. Unless the temperature is significantly larger than ~ 1 MeV, pair annihilations induce a negligible depletion of heat.

The total amount of energy which neutrinos deposit in the mantle is therefore

$$Q = (\alpha_1 + \alpha_2 + \alpha_3) L \tau . \quad (85)$$

The duration of the peak activity of the neutrino emission is estimated to be $\tau \sim 2$ s. Neutrino absorption and electron capture occur in the inner layers. A fraction $\sim 2/3$ of the corresponding energy is believed to be brought by convection upwards to the distant shock. On the contrary, neutrino-electron scatterings and pair-annihilations take place much further out, all over the hot bubble region. We therefore estimate that, in this case, the energy feeds more efficiently the shock behind which the heat is directly deposited. The effective amount of energy supplied by the neutrino wind to the shock adds up to

$$Q_{shock} = \left(\frac{2\alpha_1}{3} + \alpha_2 + \alpha_3 \right) L \tau . \quad (86)$$

In fig. 5, the variation of Q_{shock} with respect to the anomalous charge radius of tau-neutrinos is presented for two models of the mantle. In both cases, the temperature T_α is set equal to 1.2 MeV at a distance $R_\alpha = 200$ km from the center of the star. With this boundary condition, we infer a gain radius $R_g = 151$ km, with a temperature $T_g = 1.66$ MeV. In model A (lower curve), the temperature decreases like $r^{-1/3}$ in the region above $r = R_\alpha$, up to $R_{max} = 1200$ km. In model B (upper curve), it is constant throughout the layer which extends from R_α to $R_{max} = 900$ km. Both models lead to the same conclusion. As the charge radius $\langle r^2 \rangle$ departs from 0, the factor Σ increases according to relation (44). So do the fraction α_2 and the energy Q_{shock} . The mantle is slightly hotter in model B than in model A. As electrons and positrons are more energetic, they have an enhanced chance to interact with the neutrino stream. In model A, Q_{shock} is equal to 0.86 foe for $\langle r^2 \rangle = 0$ (1 foe = 10^{51} ergs). This value may be compared to the observed energy of 1.4 ± 0.4 foe in the case of SN 1987A. If tau-neutrinos have an anomalous charge radius of order 7 am, Q_{shock} reaches up to 1 foe. In model B, the increase is even more spectacular. The energy transferred from the neutrino wind to the shock wave evolves from 1 foe (the shock barely makes it to the surface) up to 1.3 foe (the explosion disrupts the star). The existence of an anomalous ν_τ charge radius (or anapole moment) results into the supply of 0.2 - 0.3 additional foe to the mantle. Such a modest increase is nevertheless enough to turn a stalled shock into a successful explosion.

5 - Conclusions and prospects.

An effective form factor $F(q^2)$ is a convenient description of possible electromagnetic interactions of neutrinos, especially if those species are composite objects. If it reduces to q^2 at low energy, it should be kept in mind that neutrino-photon interactions cannot diverge in the limit where $q^2 \rightarrow +\infty$. At high energy, the form factor $F(q^2)$ should therefore depart from the q^2 -behavior. An unconventional electromagnetic coupling of the neutrino is most likely to show up at low energy, where it should appear as a distortion of the expected signal in reactor or beam dump experiments, and also as a slight modification of stellar evolution.

Current laboratory limits on the charge radius (or anapole moment) of neutrinos are on the order of ~ 1 am for muon-neutrinos and ~ 3 am for electron-neutrinos. Future experiments should perform a detailed analysis of the data in the diagram (r_c, μ_ν) . They should combine both parameters in order to derive limits on the electromagnetic properties of neutrinos. As shown in fig. 1, the correlation between the charge radius and the magnetic moment is fairly strong. Future analysis should furthermore distinguish between diagonal and transition moments for which the interpretation of the results may be quite different, as shown by the exclusion plots contained in fig. 1 et 2. Finally, a direct analysis of the energy spectrum of the recoiled electrons is mandatory in order to obtain the best possible bound on the magnetic moment μ_ν . The lower the energy threshold E_{min} , the better that limit. Once information on μ_ν is derived, further analysis of the data should lead to a fairly accurate determination of r_c . Note that the experiment E734 of BNL [8] obtains a limit on the charge radius of muon-neutrinos by a direct inspection of the recoiled spectrum. That bound (29) compares well with the result (5) of the CHARM II Collaboration, derived with many more events (~ 20 times more). The MUNU Collaboration [40] also plans to follow the above-mentioned method. Finally, as regards tau-neutrinos, the limit of 8 to 9 am is derived from ASP and CELLO.

Those laboratory bounds are supplemented by astrophysical considerations. The charge radius of neutrinos cannot exceed 6 to 7 am under penalty of overcooling stars and of the subsequent modifications of the color-magnitude diagrams of globular clusters. This constraint holds equally for the three species of neutrinos, provided their masses do not exceed 9 keV. It improves a previous analysis by Dolgov and Zel'dovich [15] who found $r_c < 15$ am. They applied to white dwarves the same line of arguments as in section 3. However, neutrino cooling only dominates the very specific period during which hot white dwarves rapidly cool down, slightly after their formation, in a regime where statistics are scarce. Even if the frequency ω_P of the plasma inside these stars is larger than at the

cores of red-giants, the constraint on r_c is not as stringent as what may be derived from ASP and CELLO. The asteroseismology of hot white dwarves seems nevertheless very promising. These objects do pulsate, especially the DA white dwarves whose hydrogen envelopes are fairly hot ($\sim 13,000$ K). The main pulsation periods have been observed to vary in time [41], with a rate of change on the verge to be accurately measured. The latter is directly related to the temperature decrease, hence an observational handle on the neutrino losses which drive the early cooling of these stars.

The charge radius of neutrinos may also play a role in supernovae explosions. As shown by Grifols and Masso [42], if *right-handed* neutrinos have a non-zero charge radius, a significant fraction of the gravitational binding energy should be released in the form of a strong emission of these right-handed species, at the expense of the more conventional flux of left-handed neutrinos. A limit of

$$\langle r^2 \rangle_R \leq 0.2 \text{ am}^2 \quad (87)$$

was derived. Our analysis, presented in section 4, is quite different. If left-handed tau-neutrinos have an anomalous charge radius (or anapole moment), they transfer some additional heat to the mantle of the star, strengthening the shock and making it reach the surface. A value of $r_c \sim 7 \text{ am}$ turns a stalled shock into a successful explosion. As a matter of fact, a non-vanishing $\langle r^2 \rangle$ is generated by collective effects, as shown in [34]. An exciting possibility, yet to be investigated, is that electron-neutrinos could acquire a thermal charge radius by diffusing coherently on the free electrons and positrons of the hot bubble plasma. If this charge radius turns out to be large, explosions of massive stars would naturally obtain.

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Figure captions

Figure 1a : The dark region in the plane $(\langle r^2 \rangle, \mu_\nu)$ is compatible with the assumed 1000 ± 50 neutrino-electron events. This allowed domain is superimposed on the light excluded area. The incoming neutrinos have the energy $E_\nu = 25$ GeV, while the threshold on the energy of recoiled electrons is $E_{min} = 5$ GeV. The process under scrutiny is the elastic scattering $\nu_\mu e \rightarrow \nu_\mu e$.

Figure 1b : Exclusion plot with $E_\nu = 25$ GeV and $E_{min} = 100$ MeV.

Figure 1c : Exclusion diagram with $E_\nu = 25$ GeV, $E_{min} = 100$ MeV and muon-antineutrinos.

Figure 2a : Exclusion plot with a transition moment instead of a diagonal coupling. The relevant process is the lepton flavor changing scattering $\nu_\mu e \rightarrow \nu_L e$, where L stands for the electron or tau families. Parameters are the same as in fig. 1a.

Figure 2b : Same diagram as in fig. 2a with $E_{min} = 100$ MeV.

Figure 2c : Same exclusion plot as in fig. 2b with muon-antineutrinos. The interaction under scrutiny is the transition $\bar{\nu}_\mu e \rightarrow \bar{\nu}_L e$, where L stands for the electron or tau species.

Figure 3 : The extra energy emission ϵ_{ext} , resulting from the decays of plasmons into pairs of massless neutrinos, is mediated by an anomalous charge radius (or anapole moment) r_c of 10 am. Its is presented as a function of the stellar density ρ , for a temperature $T = 10^8$ K.

Figure 4 : In this exclusion plot, the upper limit on the neutrino charge radius (or anapole moment) r_c is presented as a function of the neutrino mass m_ν . This bound is derived from arguments of stellar evolution presented in section 3.

Figure 5 : The variation of the energy Q_{shock} , which is transferred from the neutrino wind to the envelope of the star, is presented as a function of the charge radius of tau-neutrinos (or anapole moment in the case of Majorana fermions), for two models of the mantle of the supernova.

Figure 1a

Figure 1b

Figure 1c

Figure 2a

Figure 2b

Figure 2c

$\mathcal{E}_{ext} \text{ (erg g}^{-1} \text{ s}^{-1}\text{)}$

$\rho \text{ (g cm}^{-3}\text{)}$

Figure 3

r_C (am)

m_v (keV)

Figure 4

Q_{shock} (foes)

Figure 5