1

Introduction

Circular steered instabilities in curved
perturbations around these nucleated strings both in the end of the de Sitter phase and after having entered the radiation dominated era. The evolution of circular cosmic strings in a radiation dominated universe has also been considered in [11, 12, 13]. Furthermore, circular strings have recently been discussed in the context of a more systematic investigation of string dynamics in curved spacetimes [14, 15, 16, 17, 18], without considering perturbations around the rings, however. Finally we mention that superconducting charge-current carrying circular strings in black hole backgrounds have been considered in [19, 20, 21].

Our work is a natural continuation of the analysis of Lousto and Sánchez [1] and we hope it will give more insight into the connection between curved spacetimes and string instabilities. We will confirm that in some cases (for instance de Sitter spacetime at \(r \to \infty\) [22, 23]) the instabilities are really due to general features of the underlying curved background, while in other cases (for instance Reissner-Nordström black hole for \(r \to 0\)) they are just artifacts of the dynamics of the special unperturbed solution considered.

The paper is organized as follows: In section 2 we will derive the equations of motion for the unperturbed circular string in the Schwarzschild, Reissner-Nordström and de Sitter backgrounds. Then we use the general formalism of Ref.[3] to obtain the linearized equations determining the physical (transverse) perturbations. For simplicity we will take only 4-dimensional spacetimes; it is of course trivial to include more "angular" space coordinates but concerning the perturbations they will all behave in the same way in our analysis. For both physical perturbations (one "radial" and one "angular") we get Schrödinger-like equations of the form:

\[
d^2 f + V(r(\tau))f = 0,
\]

where \(f\) is the comoving perturbation, \(r(\tau)\) is the radius of the unperturbed circular string and the string time \(\tau\) plays the role of the spatial coordinate.

In section 3 we analyse these equations, taking \(r\) as a parameter. From the sign of \(V(r)\) we find the regions where we expect that the perturbations develop imaginary frequencies and eventually grow infinitely, in the 3 backgrounds considered. While in section 3 we only get indications of the emergence of string-instabilities, in section 4 we consider the exact time-evolution of the perturbations in the regions \(r \to 0\) (ring-collapse) and \(r \to \infty\). This provides a connection between the less strictly obtained results of section 3 and the question of bounded/unbounded perturbations.

The details of our results are presented in sections 3 and 4, and are summarized in Fig.2.

For \(r \to 0\) the perturbations in the direction perpendicular to the string plane are bounded in all 3 backgrounds, while the perturbations in the plane of the string grow infinitely.

For \(r \to \infty\) both physical perturbations are bounded in the case of Schwarzschild and Reissner-Nordström black holes (which is not surprising since these spacetimes are asymptotically flat), but unbounded in the case of de Sitter spacetime.

Throughout the paper we use sign-conventions of Misner-Thorne-Wheeler [24] and units where \(G = 1, c = 1\) and the string tension \((2\pi \alpha')^{-1} = 1\).

2 Equations for the string-perturbations

The classical equations of motion for the bosonic string are in the conformal gauge given by:

\[
\ddot{x}^u - \dot{x}^u \dot{x}^v \Gamma^w_{uv}(\dot{x}^2 + \ddot{x}^a \ddot{x}^a) = 0, \quad (2.1)
\]

where dot and prime denote derivative with respect to the string coordinates \(\tau\) and \(\sigma\), respectively. As usual these equations are supplemented with the 2 gauge constraints:

\[
g_{\mu\nu} \ddot{x}^{\mu} \ddot{x}^{\nu} = g_{\mu\nu} (\dot{x}^2 + \ddot{x}^a \ddot{x}^a) = 0. \quad (2.2)
\]

In this paper we will consider perturbations around a circular string configuration embedded in Schwarzschild, Reissner-Nordström and de Sitter spacetimes. In static coordinates these spacetimes are all special cases of the line element:

\[
ds^2 = -a(r)c^2 dt^2 + \frac{dr^2}{a(r)} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (2.3)
\]

so in the first place we will keep \(a(r)\) as an arbitrary function. The components of the Christoffel symbol and of the Riemann tensor (that we will need later) corresponding to the metric (2.3) are listed in the appendix.
\[ \nabla_\mu \mu^\mu = \partial_\mu \mu^\mu + \Gamma^\mu_{\nu \mu} \mu^\nu, \quad \text{etc.} \] (2.19)

Finally, \( R_{\mu \nu \rho \sigma} \) represents the spacetime Riemann tensor.

This extremely complicated system of 2 coupled linear second order partial differential equations fortunately simplifies enormously for the special cases considered here. One can show that all components of the normal fundamental form vanish, while the only non-vanishing components of the second fundamental form are:

\[ \Omega_{\mu \nu} = \Omega_{\rho \sigma} = -E. \] (2.20)

The non-vanishing components of the relevant projections of the Riemann tensor become:

\[ h^{-2} n^\mu n^\nu R_{\mu \nu} n^\rho n^\sigma = \frac{r}{2} (\sigma_{\rho \sigma} + a_{\rho}), \] (2.21)

\[ h^{-2} n^\mu n^\nu R_{\mu \nu} n^\rho n^\sigma = a - 1 + \frac{r}{2} a_{\rho}. \] (2.22)

Finally, the d’Alambertian reduces to (conformal gauge) \( \Box = \partial_r^2 - \partial_t^2 \) and \( G^+ = h^{-1} G_0^+ = 2r^2 \). The original system (2.15) now decouples and leads to the 2 equations:

\[ (\partial_r^2 - \partial_t^2) \delta x_1 - \frac{1}{2} (\sigma_{\rho \sigma} + a_{\rho}) \delta x_1 = 0, \]
\[ (\partial_r^2 - \partial_t^2) \delta x_2 - (a - 1 + \frac{r}{2} a_{\rho}) \delta x_2 = 0. \] (2.23)

These equations can further be reduced to ordinary differential equations by Fourier transforming the comoving perturbations:

\[ \delta x_1 = \sum_{n=0}^{\infty} C_{n1}(\tau) e^{-inr}, \]
\[ \delta x_2 = \sum_{n=0}^{\infty} C_{n2}(\tau) e^{-inr}, \] (2.24)

where \( C_{n1} = C_{-n1}, C_{n2} = C_{-n2} \) and the tilde denotes summation for \( | n| \neq 0, 1 \) only. The zero modes and the \( | n| = 1 \) modes are excluded from the summations since they do not correspond to "real" perturbations on a circular string [10]. They describe spacetime translations and rotations that do not change the shape of the string. They therefore correspond to simply "jumping" from one unperturbed circular string to another unperturbed circular string. We are then left with the 2 equations (\( | n| \geq 2 \)):

\[ \dot{C}_{n1} + \left( n^2 + \frac{r}{2} a_{\rho} + \frac{r}{2} a_{\rho} \right) C_{n1} = 0, \]
\[ \dot{C}_{n2} + \left( n^2 + a - 1 - \frac{r}{2} a_{\rho} - 2 \frac{r}{2} a_{\rho} \right) C_{n2} = 0. \] (2.25)

Until now we have kept the function \( a \) in the line element (2.3) as an arbitrary function of \( r \). As announced in the abstract we will however only consider the 3 cases of Schwarzschild, Reissner-Nordström and de Sitter. These spacetimes are essentially the scalar curvature flat cases of (2.3) since, from the appendix, the condition \( R = \text{const} \equiv K \) is:

\[ R = \frac{2}{r^2} (1 - a) - 4 \frac{a}{r^2} - a_{\rho \rho} = K, \] (2.26)

that is integrated to:

\[ a(r) = 1 + \frac{\alpha}{r} + \frac{\beta}{r^2} - \frac{K}{2}, \] (2.27)

where \( \alpha \) and \( \beta \) are constants. This expression covers the cosmologically and gravitationally interesting cases of de Sitter (\( \alpha = \beta = 0, K = 12H^2 \)), Schwarzschild (\( \beta = K = 0, \alpha = -2M \)) as well as Reissner-Nordström (\( K = 0, \alpha = -2M, \beta = Q^2 \)) spacetimes. In these cases (2.26) leads to:

\[ \dot{C}_{n1} + \left( n^2 + \frac{r}{2} a_{\rho} + \frac{r}{2} a_{\rho} \right) C_{n1} = 0, \]
\[ \dot{C}_{n2} + \left( n^2 + a - 1 - \frac{r}{2} a_{\rho} - 2 \frac{r}{2} a_{\rho} \right) C_{n2} = 0, \] (2.28)

and \( r(\tau) \) is determined by (2.9):

\[ r - r_0 = \pm \int_{r_0}^{r} \frac{dz}{\sqrt{K z^2 - a z + (E^2 - \beta)}}. \] (2.29)

For \( K = 0 \) (the black hole case) \( r(\tau) \) is then a trigonometric function, while for \( K \neq 0 \) (de Sitter case) is generally, but not always, elliptic.

3 Analysis of string-perturbations

In this section we analyse the equations for the perturbations (2.28) taking \( \tau \) as the parameter. Both equations are Schrödinger-like equations of the form \( \int V(\tau)f = 0 \), and we will then say that the solutions are oscillatory in time \( r \) if \( V(\tau) \) is positive but non-oscillatory in time \( r \), developing imaginary frequencies, if \( V(\tau) \) is negative. We are using the words oscillatory and non-oscillatory in a weak (and sloppy) sense, that should not be confused with the more strict use of the words in the mathematical literature,
\[ \begin{align*}
\left( \frac{dW}{dte} < \alpha \right), \frac{dW}{dte} > \alpha \\
\text{and we move to different cases namely,} \\
(\text{Case 1}) & \\
\frac{dW}{dte} < \alpha \\
& \text{and the equation reduces to} \\
& \frac{dW}{dte} = \alpha \\
& \text{and the solution is given by} \\
& \frac{dW}{dte} < \alpha \\
\end{align*} \]

Therefore, the solution for each case is determined by the condition on \( \frac{dW}{dte} \).

### 2.3.2.6 Schematic Explanation of the Problem

In this case, the differential equation is

\[ \frac{dW}{dte} < \alpha \]

### 3.1 Schematic Explanation

Let us now consider the partial solutions in the case.

The equation is of the form:

\[ \frac{dW}{dte} = \alpha \]

The solution is given by:

\[ W(t) = \alpha t + W_0 \]

where \( W_0 \) is the initial condition.

### 3.3.6 Schematic Explanation

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\[ \frac{dW}{dte} = \alpha \]

The solution is given by:

\[ W(t) = \alpha t + W_0 \]

where \( W_0 \) is the initial condition.
(i), $E^2 > Q^2$: The string has its maximal radius $r_{\text{max}} = M + \sqrt{M^2 + E^2 - Q^2}$ for $\tau = 0$. It first contracts through the horizon $r_{\text{h}} = M + \sqrt{M^2 - Q^2}$ for $\tau = \arccos \frac{\sqrt{M^2 - Q^2}}{\sqrt{M^2 + E^2 - Q^2}} \in [0, \pi/2]$ and then through the inner horizon $r_{\text{i}} = M - \sqrt{M^2 - Q^2}$ for $\tau = \arccos \frac{\sqrt{M^2 - Q^2}}{\sqrt{M^2 + E^2 - Q^2}} \in [\pi/2, \pi]$. It eventually falls into the Reissner-Nordström singularity $r = 0$ for $\tau = \arccos \frac{M}{\sqrt{M^2 + E^2 - Q^2}} \in [\pi/2, \pi]$. What the equations determining the modes of $\delta \phi_1$ and $\delta \phi_0$, respectively, are:

\[
\begin{align*}
\ddot{C}_{n+} + (n^2 - \frac{M}{r} + 2\Omega_0^2)C_{n+} &= 0, \\
\ddot{C}_{n0} + (n^2 - \frac{M}{r} - 2\Omega_0^2)C_{n0} &= 0.
\end{align*}
\] (3.5)

Formally the $C_{n0}$ equation is identical to the $C_{n1}$ equation in the Schwarzschild case, but one should remember that in this case the charge of course is modified by the expression for $r(\tau)$.

We now find that $\delta \phi_1$ is oscillatory for all $r$ if $Q^2 \geq M^2/32$. If $Q^2 < M^2/32$ the first mode ($|n| = 2$) is non-oscillatory in the interval $r \in [(M + \sqrt{M^2 - 32Q^2})/8, (M - \sqrt{M^2 - 32Q^2})/8]$ and the higher modes are non-oscillatory in smaller and smaller sub-intervals (for smaller $Q^2$). These intervals of non-oscillatory behaviour are furthermore located between the outer and inner horizons, so in both cases (i) and (ii) $\delta \phi_1$ is oscillatory at the horizons, and when $r \to 0$ and $r \to r_{\text{max}}$, respectively.

For the radial perturbations it turns out that the situation is quite complicated. We find that $\delta \phi_2$ is oscillatory for $r > (M + \sqrt{M^2 + 32Q^2})/8 \equiv r_{\text{crit}}$, i.e. the first mode ($|n| = 2$) becomes non-oscillatory for $r \to r_{\text{crit}}$ and the higher modes become non-oscillatory for smaller and smaller $r$. Note that $r_{\text{crit}} < r_{\text{max}}$ so that $\delta \phi_2$ is always oscillatory when the string has its maximal size. The exact location of this critical radius compared to the 2 horizons and $r_{\text{max}}$ (for $E^2 > Q^2$) is however very complicated since we have 2 parameters ($E^2$ and $Q^2$) to play with, so almost all situations are possible. The result is most easily visualized by Fig. 1. accompanied by the following comments:

In region (i1) we have $E^2 > Q^2$ and $r_{\text{crit}} > r_{\text{s}}$, so that $\delta \phi_2$ is non-oscillatory at the (outer) horizon and all the way towards the singularity $r = 0$.

In region (i2) we have $E^2 > Q^2$ and $r_{\text{s}} > r_{\text{crit}} > r_{\text{h}}$, so that $\delta \phi_2$ is oscillatory at the horizon but becomes non-oscillatory before the inner horizon, from which it is non-oscillatory all the way towards the singularity $r = 0$.

In region (ii3) we have $E^2 > Q^2$ and $r_{\text{s}} > r_{\text{crit}}$, so that $\delta \phi_2$ is oscillatory at both horizons but becomes non-oscillatory before the inner horizon. It is then non-oscillatory all the way to $r_{\text{max}}$.

Finally in (ii3) we have $E^2 < Q^2$ and $r_{\text{crit}} > r_{\text{s}}$, so this is a very interesting region since $\delta \phi_2$ is always oscillatory! That possibility did not exist for $E^2 \geq Q^2$.

3.3 de Sitter spacetime

In this case $r(\tau)$ is in general given by a Weierstrass elliptic function. The detailed dynamics of unperturbed circular strings has been discussed elsewhere [6, 8, 15, 16, 18] so we shall not go into it here. We will consider only the following 3 types of solutions, whose existence is clear from (2.8) when $a = 1 - H^2$:

(i): For $4H^2E^2 \leq 1$ there is a solution starting with a maximal radius $r_{\text{max}}^2 = (1 - \sqrt{1 - 4H^2E^2})/2H^2$ and then collapsing to $r = 0$. It is always inside the horizon $r_{\text{h}} = 1/H$.

(ii): Still for $4H^2E^2 \leq 1$ there is another solution starting with a minimal radius $r_{\text{min}}^2 = (1 + \sqrt{1 - 4H^2E^2})/2H^2$ and then expanding through the horizon towards infinity.

(iii): For $4H^2E^2 > 1$ there is a solution starting at $r = 0$ and then expanding through the horizon towards infinity.
4. TIME-EVOLUTION AND ASYMMETRIC BEHAVIOR

The results of the experiment on combined or uncombined composite materials
in the next section will provide some of these critical results in the development of
the experimental program. Some cases of these materials are shown in the
scanning electron micrographs. The experimental results are shown in
Figure 3. An example of this is shown in Figure 3.

\[ 0 = \frac{1}{2} (x - 1) + \frac{1}{2} y \]

The two equations have the same number of variables.
in the potential $\alpha(\tau - \tau_0)^{-3}$ [1, 26, 27], in the sense that they are described by a stationary Schrödinger equation with $\tau$ playing the role of the spatial parameter. It is an elementary observation that if $\alpha < 0$ and $\beta \geq 2$ there are singular solutions for $\tau \to \tau_0$. Therefore, as soon as we have obtained the potential with the two parameters $\alpha$ and $\beta$ we can conclude whether the perturbations blow up, indicating that the underlying circular string is unstable. For completeness we will however give the full solutions in the asymptotic regions, demonstrating explicitly if and how the perturbations blow up.

### 4.1 Schwarzschild black hole

In this case we have $r(\tau) = M + \sqrt{M^2 + E^2 \cos \tau}$ with $r \to 0$ corresponding to $\tau \to \tau_0 \equiv \arccos \sqrt{\frac{M^2}{M^2 + E^2}}$ from below (cf. subsection 3.1). For $r \to 0$ we then have approximately:

$$r(\tau) \approx |E| (\tau - \tau_0), \quad (4.1)$$

and the 2 equations determining the perturbations (3.3) become approximately:

$$\tilde{C}_{n1} - \frac{M}{(\tau_0 - \tau)^2} \tilde{C}_{n0} = 0, \quad (4.2)$$

Let us first consider the perturbations in the angular direction (the $C_{n1}$'s). Keeping in mind that $\tau_0 - \tau$ is positive in the relevant range of $\tau$ we find the 2 real independent solutions in terms of Bessel functions [28]:

$$f = \sqrt{\tau_0 - \tau} J_1(2\sqrt{M^2/|E|} \sqrt{\tau_0 - \tau}), \quad g = \sqrt{\tau_0 - \tau} N_1(2\sqrt{M^2/|E|} \sqrt{\tau_0 - \tau}). \quad (4.3)$$

The most interesting feature of these solutions is that they are actually bounded [28]:

$$f \to \sqrt{M^2/|E|} (\tau_0 - \tau), \quad g \to -\frac{1}{2} \sqrt{E/|M|}; \quad \tau \to \tau_0. \quad (4.4)$$

This therefore provides an example where the solutions were classified as non-oscillatory (according to section 3.1), but where the actual time evolution demonstrates that the solutions are bounded, and they are in fact oscillatory in the strict mathematical sense of having infinitely many zeroes. This is however an exceptional case; in the other cases under consideration here we will find that non-oscillatory behaviour at $r \to 0$ or $r \to \infty$ leads to unbounded solutions.

For the perturbations in the radial direction (the $C_{n0}$'s) we find the complete solution:

$$C_{n0}(\tau) = \alpha_0(\tau_0 - \tau)^2 + \frac{\beta_0}{(\tau_0 - \tau)^2}, \quad (4.5)$$

where $\alpha_0$ and $\beta_0$ are arbitrary constants. This solution is indeed unbounded for $\tau \to \tau_0$.

### 4.2 Reissner-Nordström black hole

Here we only consider the case where $E^2 > Q^2$ to ensure that we have solutions collapsing to $r = 0$. Then the unperturbed string is determined by $r(\tau) = M + \sqrt{M^2 + E^2 - Q^2 \cos \tau}$ and $r \to 0$ corresponds to $\tau \to \tau_0 \equiv \arccos \sqrt{\frac{M^2}{M^2 + E^2 - Q^2}}$ from below (cf. subsection 3.2). The approximate solution for $r \to 0$ is then:

$$r(\tau) \approx \sqrt{E^2 - Q^2}(\tau_0 - \tau); \quad E^2 > Q^2 \quad (4.6)$$

and:

$$r(\tau) \approx \frac{M}{2} (\tau_0 - \tau)^2; \quad E^2 = Q^2 \quad (4.7)$$

The 2 equations determining the perturbations (3.5) become:

$$\tilde{C}_{n1} + \frac{Q^2}{(E^2 - Q^2)(\tau_0 - \tau)} \tilde{C}_{n0} = 0; \quad E^2 > Q^2 \quad (4.8)$$

and:

$$\tilde{C}_{n1} - \frac{Q^2}{(E^2 - Q^2)(\tau_0 - \tau)} \tilde{C}_{n0} = 0; \quad E^2 = Q^2 \quad (4.9)$$

The solution of (4.8) is:

$$C_{n1}(\tau) = \alpha_0(\tau_0 - \tau)^{1 + n/2} + \beta_0(\tau_0 - \tau)^{1 - n/2},$$

$$C_{n0}(\tau) = \gamma_0(\tau_0 - \tau)^{1 + n/2} + \delta_0(\tau_0 - \tau)^{1 - n/2}, \quad (4.10)$$
Conclusion

In this section, we have presented the results obtained in this subsection.

For example, let us consider the following expression:

\[ \frac{\partial^2}{\partial t^2} - \Delta u = f(x,t) \]

where \( u \) is the unknown function and \( f(x,t) \) is a given function.

This equation is a second-order, linear, partial differential equation.

Solving this equation, we find that the solution is given by:

\[ u(x,t) = \sum_{n=1}^{\infty} A_n \sin \left( \frac{n\pi x}{L} \right) \cos \left( \frac{n\pi a}{L} t \right) \]

where \( A_n \) are constants determined by the initial and boundary conditions.

The solution represents a superposition of sine waves, each with a frequency determined by the equation.

Finally, we conclude that the solution is obtained by superimposing the individual solutions.

If the initial conditions are specified, the constants \( A_n \) can be determined uniquely.

The solution is valid for all times \( t \geq 0 \).

For bounded solutions, the behavior of the solution in the region \( t \to \infty \) and \( x \to \pm \infty \) is determined by the boundary conditions.

In particular, if the boundary conditions are such that \( u \to 0 \) as \( x \to \pm \infty \), then the solution is said to be exponentially bounded.

The asymptotic behavior of the solution can be studied using techniques such as the method of characteristics or the method of Fourier transforms.

In conclusion, the solution obtained is valid for all times and space variables.

For more details, please refer to the section on boundary value problems.
6 Appendix

In this appendix we give the explicit expressions for the non-vanishing components of the Christoffel symbol and Riemann tensor corresponding to the line element (2.3).

The metric:

$$g_{tt} = -a, \quad g_{rr} = \frac{1}{a}, \quad g_{\theta\theta} = r^2, \quad g_{\phi\phi} = r^2 \sin^2 \theta; \quad a = a(r).$$  \hspace{1cm} (6.1)

The Christoffel symbol:

$$\Gamma^r_{tt} = \frac{a''}{a}, \quad \Gamma^r_{rr} = \frac{a''}{a}, \quad \Gamma^r_{\theta\theta} = \frac{1}{2} a a_{rr},$$

$$\Gamma^r_{\theta\theta} = -ar, \quad \Gamma^r_{\phi\phi} = -a r \sin^2 \theta, \quad \Gamma^r_{\phi\theta} = \frac{1}{r},$$

$$\Gamma^r_{\phi\phi} = \cot \theta, \quad \Gamma^{\theta}_{t\theta} = \frac{1}{r}, \quad \Gamma^{\theta}_{r\theta} = -a \sin \theta \cos \theta.$$  \hspace{1cm} (6.2)

The Riemann tensor:

$$R_{tt\theta} = \frac{1}{2} a_{rr}, \quad R_{tt\theta\phi} = \frac{1}{2} a_{rr}, \quad R_{tt\phi\phi} = \frac{1}{2} a_{rr} \sin^2 \theta,$$

$$R_{t\theta\theta\phi} = \frac{1}{2} a a_{rr}, \quad R_{t\theta\phi\phi} = \frac{1}{2} a a_{rr} \sin^2 \theta, \quad R_{t\phi\phi\phi} = r^2 (1 - a) \sin^2 \theta.$$  \hspace{1cm} (6.3)

The Ricci tensor:

$$R_{tt} = -a^2 R_{rr} = a \left( \frac{a_{rr}}{2} + \frac{a_{\phi\phi}}{r} \right), \quad R_{\theta\theta} = \sin^2 \theta R_{\phi\phi} = \left( 1 - a - r a_{rr} \right) \sin^2 \theta.$$  \hspace{1cm} (6.4)

Finally the scalar curvature is:

$$R = -a_{rr} + \frac{2}{r^2} (1 - a) - \frac{4}{r} a_{r}.$$  \hspace{1cm} (6.5)
Figure Captions

Fig. 1. The location of the critical radius where the non-oscillatory behaviour sets in for the \(| n | = 2\) mode in the case of Reissner-Nordström black hole. The details of this figure are explained at the end of subsection 3.2. Note that we only consider \(Q^2 \leq M^2\).

Fig. 2. This diagram summarizes the results obtained in this paper. \(\delta x_\perp\) and \(\delta x_\parallel\) corresponds to the comoving angular and radial perturbations, respectively.
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**Fig. 3.**