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Burst avalanches in bundles of fibers:
Local versus global load-sharing

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Abstract

Bursts in bundles of many parallel fibers with stochastically distributed failure thresholds are studied. The distribution of the sizes Δ of burst avalanches has a power-law behavior, $\propto \Delta^{-\xi}$. When the load is shared equally among surviving fibers, the power-law exponent is $\xi = 2.5$. When, however, the increased stresses that result from a fiber failure are concentrated to the nearest-neighbour fibers, the exponent is considerable higher, for randomly distributed thresholds $\xi = 4.5$.

The strength of materials with stochastically distributed elements is important in applications. A bundle of many parallel elastic fibers, clamped at both ends, is a simple classical model in this field. The strengths $\{x_i\}$ — i.e., the maximum load the fibers $i = 1, 2, \dots, N$ are able to carry — vary from fiber to fiber, and are picked independently from a cumulative distribution $P(x)$ with corresponding probability density $p(x)$:

$$\text{Prob}(x_i < x) = P(x) = \int_0^x p(y) dy. \quad (1)$$

Ruptured fibers carry no load.

In the classical version of this model[1] *global* load-sharing is assumed, i.e., the increased stresses caused by a failed element is shared by all surviving fibers.

The load increase on the fibers, due to the rupture of one fiber, may cause further fibers to fail. Denoting the size of the burst by the number Δ of ruptured fibers, an interesting quantity is the expected number $D(\Delta)$ of burst avalanches of size Δ . In [2] the size distribution for the classical model with global load-sharing was shown to be

$$\lim_{N \rightarrow \infty} \frac{D(\Delta; N)}{N} = \frac{\Delta^{\Delta-1}}{\Delta!} \int_0^{x_0} (1 - f(x)) f(x)^{\Delta-1} e^{-f(x)\Delta} p(x) dx, \quad (2)$$

under mild conditions on the probability density $p(x)$. Here

$$f(x) = \frac{x p(x)}{1 - P(x)}, \quad (3)$$

while x_0 is determined by $f(x_0) = 1$.

The asymptotic large- Δ behaviour is found by evaluating the integral in Eq.(2) by the saddle-point method. This yields the universal power law

$$D(\Delta) \propto \Delta^{-\xi}, \quad (4)$$

with

$$\xi = \frac{5}{2}. \quad (5)$$

The fiber problem can be rephrased as a fuse problem[3]. The fiber bundle system is equivalent to a system of N electric fuses, each of unit resistance, coupled in parallel between two bus bars. The potential difference between the bus bars replaces fiber extension, the electric current in a fuse replaces the force on a fiber, and the stochastically distributed property is the maximum current a fuse may carry.

The assumption of global load-sharing among surviving fibers is often unrealistic, and the question arises if the power-law behaviour (4), and in particular the exponent value $\xi = 2.5$, is a result of this assumption. With this question in mind we replace in this note the equal load-sharing procedure with another, local, form of load-sharing at rupture.

The extreme form for local load redistribution is that all extra stresses caused by a fiber failure are taken up by the *nearest-neighbour* surviving fibers. The simplest geometry is one-dimensional, so that the N fibers are ordered linearly, with or without periodic boundary conditions. In this case precisely *two* fibers, one on each side, take up, and divide equally, the extra stress. This model has been discussed previously[4] for a different purpose.

In Fig.1 we exhibit simulation results for this local model, with threshold strengths randomly distributed in $[0,1]$. The number of fibers in the bundle is $N = 5000$, and the size distribution is averaged over 1000 bundles. Again a power-law distribution appears, now, however, with a much larger burst index

$$\xi \simeq 4.5 \quad (6)$$

Thus the relative frequency of long (non-fatal) burst avalanches is considerably reduced.

Is the result $\xi \simeq 4.5$ universal, i.e., under mild restrictions independent of the threshold distribution $P(x)$? We have investigated burst processes with distributions of the form

$$P(x) = x^\nu. \quad (7)$$

The burst indices ξ that we obtain are in all cases larger than 4. In Fig.1 we show the case $\nu = 1/5$, which leads to an apparent exponent ξ somewhat smaller than 4.5. This is a trend: The values of ξ increase somewhat with ν . Whether this trend is a real effect — thus signaling non-universality — or it is caused by not having reached its asymptotic behavior, is hard to judge from the numerical experiments.

The main conclusion is that systems with local load-sharing are *not* in the universality class of fiber bundles with global load redistribution. It is natural to draw an analogy with critical phenomena. Global load-sharing is similar to mean-field theories, valid for infinite-range interactions, with universal critical indices different from critical indices for models with short-range interactions.

The results of [2], in particular the power law (4,5) for the model with global load-sharing have already been applied in earth-quake theory[5] and for stick-slip models[6]. In view of the present results it is important that one in each application checks whether this assumption is realistic, or should be replaced by some form of local load-sharing.

These models with parallel fibers or fuses are essentially one-dimensional. What happens in higher dimensions? We have studied a square lattice of fuses with stochastically distributed current thresholds. Orientation of the lattice at 45° with respect to two parallel bus bars ensures that all fuses carry the same current when all are intact. As the voltage is increased, bursts of collective burn-outs of the fuses occur. Based on simulation of several hundred lattices of size up to 40×40 we find also in this case a power-law distribution (4), with $\xi = 2.7 \pm 0.3$ for the different distributions we studied. The precision is too low (due to the cost of these computations) to draw a firm conclusion about the possible universality of the exponent value. It is, however, very surprising that the burst exponent is much closer to the “mean-field” value (5) than to the higher value for the local load-sharing model since the breakdown process in two dimensions is a highly correlated one[7] in which

spatially close fuses tend to blow successively, especially near the end of the process. We conclude that the combined effect of dimensionality and range of load-sharing is yet to be understood.

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Fig.1 The distribution of bursts, $N(\Delta) = nD(\Delta)$, for the uniform threshold distribution, $P(x) = x$, where $0 \leq x \leq 1$ (squares) and $P(x) = x^{1/5}$ where $0 \leq x \leq 1$ (diamonds), with local load-sharing. The simulation results are based on n samples of bundles with $N = 5000$ fibers, with $n = 1000$ for the uniform distribution and $n = 700$ for $P(x) = x^{1/5}$. The slope of the straight line which is a best fit to the $P(x) = x$ data is -4.5 , and the slope of the straight line which is the best fit to the $P(x) = x^{1/5}$ data is -4.2 .

Fig.2 The distribution of bursts, $N(\Delta) = nD(\Delta)$, for square lattices of fuses, each of unit resistance, and with a current threshold picked from a threshold distribution $P(x)$ in the interval $(0 \leq x \leq 1)$, where $P(x) = x$ (squares) and $P(x) = x^2$ (diamonds). The slope of the straight lines is -2.7 . The simulation results are based on $n = 100$ samples of lattices of size 40×40 .