SHADOWING IN THE DEUTERON AND THE NEW $F_2^n/F_2^p$ MEASUREMENTS

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Abstract

The quantity $2F_2^d(x,Q^2)/F_2^p(x,Q^2) - 1$ is calculated in the region of low $x$ and low- and moderate $Q^2$ relevant for recent NMC and E665 measurements as well as for the expected final results of the precise NMC analysis of their low $x$ data. The calculations include nuclear shadowing effects and a suitable extrapolation of the structure functions of free nucleons to the low $Q^2$ region. The theoretical results are in a good agreement with the NMC data. The shadowing correction to the experimental estimate of the Gottfried sum is quantified.

A new determination of $F_2^n-F_2^p$ at $Q^2 = 4$ GeV$^2$ by the NMC, [1], as well as the ongoing NMC data analysis which will result in extending the precise $F_2^d/F_2^p$ measurements down to $x = 0.0008$ [2] are the main reasons for revisiting the problem of nuclear shadowing in the deuteron and in particular the way shadowing affects determination of the Gottfried sum $S_G$:

$$S_G(Q^2) = \int_0^1 \frac{dx}{x} \left[ F_2^d(x,Q^2) - F_2^p(x,Q^2) \right]$$

(1)

where $F_2^d$ and $F_2^p$ are the proton and neutron structure functions. The NMC performed precise measurements of the ratio $F_2^d/F_2^p$ which permitted to obtain the ratio $(F_2^n/F_2^p)_{NMC}$ defined as $2F_2^d/F_2^p - 1$, i.e. no shadowing was assumed. Taking the shadowing into account the deuteron structure function $F_2^d(x,Q^2)$ is related in the following way to the $F_2^n$, $F_2^p$ and to the shadowing term $\delta F_2^d(x,Q^2)$, which is non-negligible for $x$ less than, say, 0.1 or so:

$$2F_2^d(x,Q^2) = F_2^n(x,Q^2) + F_2^p(x,Q^2) - 2\delta F_2^d(x,Q^2).$$

(2)

Throughout this paper $F_2^d$ and $\delta F_2^d$ will be normalised per nucleon. It follows from the eq. (2) that the quantity $2F_2^d(x,Q^2)/F_2^p(x,Q^2) - 1$ which in absence of nuclear shadowing would be equal to the ratio of the neutron and proton structure functions, contains also the shadowing effects i.e.:

$$2\frac{F_2^d(x,Q^2)}{F_2^p(x,Q^2)} - 1 = \frac{F_2^n(x,Q^2)}{F_2^p(x,Q^2)} - \frac{2\delta F_2^d(x,Q^2)}{F_2^p(x,Q^2)}$$

(3)

The difference between the proton and neutron structure functions or the integrand in the Gottfried sum, eq.(1), is thus the following:

$$F_2^n(x,Q^2) - F_2^p(x,Q^2) = \left( F_2^n(x,Q^2) - F_2^p(x,Q^2) \right)_{NMC} - 2\delta F_2^d(x,Q^2)$$

(4)
The \((F_p^0(x, Q^2) - F_p^0(x, Q^2))_{NMC}\) term which in the NMC analyses was obtained as

\[
(F_p^0 - F_p^0)_{NMC} = 2 F_p^d \left[ \frac{1 - (F_p^0)^2_{NMC}}{1 + (F_p^0)^2_{NMC}} \right]
\]

(5)

and which is used by the NMC for determination of the Gottfried sum has to be corrected for shadowing effects in order to obtain the difference of the structure functions of free nucleons. Thus the shadowing leads to smaller \(S_G(Q^2)\) than that determined experimentally assuming no shadowing.

In the original NMC determination of \(S_G\), [3], \(F_p^d\) in eq.(5) was taken from a global fit to the results of earlier experiments; in the recent determination, [1], the NMC used the \(F_p^d\) resulting from a fit to their own precise measurements and to the data of the SLAC and BCDMS experiments, [4]. The results of this fit differed substantially from the parametrisation used in [3], especially at low \(x\). Also the ratio \(F_p^d/F_2^p\) was newly determined in [1]: the statistics was increased and the Dubna scheme instead of the Mo and Tsai method was used for radiative corrections determination [5]. The measurements of the \(F_2^d/F_2^p\) ratio have also been reported recently by the E665 Collaboration at Fermilab, [6]. The data reach \(x = 0.00005\) at \(Q^2 = 0.03\) GeV\(^2\). Precision of these measurements is however very poor.

The re-evaluated NMC data of ref.[1] and the E665 results [6] will be compared with our model calculations which include nuclear shadowing effects in the deuteron and a suitable extrapolation of the high \(Q^2\) structure functions of free nucleons to the low \(Q^2\) region. Predictions will also be given for the \((x, Q^2)\) region relevant for the (expected) final results of the NMC analysis which will extend their \(F_2^d/F_2^p\) measurements down to \(x = 0.0008\), [2] and low values of \(Q^2\) \((< Q^2 > \approx 0.25\) GeV\(^2\)).

In our model [7] the shadowing phenomenon in the inelastic lepton–deuteron scattering is analysed using the double interaction formalism in which we relate shadowing to inclusive diffractive processes. Both the vector meson and parton contributions are considered for low and high \(Q^2\) values. The QCD corrections with parton recombination are included for high \(Q^2\).

The shadowing term \(\delta F_2^d(x, Q^2)\) is related through the (double) pomeron exchange to the diffractive structure function, \(\partial^2 F_2^{dij}/\partial \xi \partial t\)

\[
\delta F_2^d(x, Q^2) = \int d^2 k_\perp \int_{\xi_0}^{1} d\xi S(k^2) \frac{\partial^2 F_2^{dij}}{\partial \xi \partial t}
\]

(6)

where \(S(k^2)\) is the deuteron form factor. We define \(\xi = 2kq/p_dq\) where \(k, q\) and \(p_d\) are the four momenta of the pomeron, virtual photon and of the deuteron respectively; \(\xi_0 = x(1 + M_{p_0}^2/Q^2)\) where \(M_{p_0}\) is the lowest mass squared of the diffractively produced hadronic system. We also have \(t \simeq -k_\perp^2 - k_\parallel^2\) where \(k_\parallel^2 = M^2 \xi^2\) with \(M\) being equal to the nucleon mass. The integration over \(\xi\) corresponds to the integration over \(M_{\pi}^2\) where \(M_{\pi}\) is the mass of the diffractively produced system. The region of low \(M_{\pi}^2\) is dominated by the diffractive production of low mass vector mesons which is assumed to be described by the vector meson dominance mechanism, VMD. In this model the contribution of the vector mesons to the nuclear shadowing is

\[
\delta F_2^d = \frac{Q^2}{4\pi} \sum_{v} \frac{M_v^4 \delta \sigma_v^d}{\gamma_v^2 (Q^2 + M_v^2)^2}
\]

(7)
where the sum extends over the low mass vector mesons $\rho, \omega$ and $\phi$. $M_v$ are the masses of the vector mesons $v$ and $\gamma_v$ can be obtained from the leptonic widths of the vector mesons [8]. The double scattering cross section $\delta \sigma_v^d$ is obtained from the Glauber model, [9] with the energy-dependent vector meson-nucleon cross sections. The contribution of large $M^2_\rho$ corresponds to the partonic mechanism of shadowing which is related to the partonic content of the pomeron. For the detailed formulation of the model see references [7].

In [7] we have only calculated the shadowing term $\delta F_2^d(x,Q^2)$ alone. In order to make the theoretical prediction for the quantity $2 F_2^d(x,Q^2)/F_2^0(x,Q^2) - 1$ and compare it with the existing data one also needs the elementary structure functions $F_2^0(x,Q^2)$ and $F_2^n(x,Q^2)$ in particular in the region of small $x$ and small $Q^2$. To this aim we shall use the structure function model developed by us in [10] which satisfactorily reproduces the structure function data and in which the contributions from both the parton model with QCD corrections suitably extended to the low $Q^2$ region and from the low mass vector mesons were taken into account. In this model the structure function $F_2(x,Q^2)$ is given by the following formula:

$$F_2^{p,n}(x) = \frac{Q^2}{4\pi} \sum_v \frac{M_v^4 \sigma_v(W^2)}{\gamma_v^2(Q^2 + M_v^2)} + \frac{Q^2}{(Q^2 + Q_0^2)} F_{2,AS}^{p,n}(\bar{x}, Q^2 + Q_0^2) = F_{2,d}(x,Q^2) + F_{2,n}(x,Q^2)$$

where the first term corresponds to the VMD contribution and the second one to the partonic mechanism. The cross sections $\sigma_v(W^2)$ are vector meson-nucleon cross sections, $W$ being the effective mass of the electroproduced system. The structure functions $F_{2,AS}^{p,n}(x,Q^2)$ are assumed to be given. They are obtained from the structure function analysis in the large $Q^2$ region and are constrained by the Altarelli-Parisi evolution equations. The variable $\bar{x}$ is defined as

$$\bar{x} = x \frac{Q^2 + Q_0^2}{Q^2 + xQ_0^2}$$

The parameter $Q_0^2$ was set equal to 1.2 GeV$^2$. It should be observed that apart from the parameter $Q_0^2$ which is constrained by physical requirements the representation (8) does not contain any other free parameters except of course those which are implicitly present in the parametrisation of parton distributions defining $F_{2,AS}^{p,n}$.

The shadowing effects in the deuteron have also been considered in refs [12, 13] using the formalism similar to ours, [7]. In [12] the two gluon exchange model of the inelastic diffraction was used; the VMD contribution to shadowing, corresponding to the double scattering of vector mesons – an important part of the shadowing effect at low $Q^2$ – was however ignored. This model has recently been used to estimate the shadowing effects in the kinematical region of the reanalysed NMC results, [14]. In [13] our model [7] was extended by including the meson exchange effects.

In order to do the comparison with the data we computed the values of the quantity $2 F_2^d(x,Q^2)/F_2^0(x,Q^2) - 1$, eq.(3), for the $(x, Q^2 = 4 \text{ GeV}^2)$ values measured by the NMC, [1]. The $\delta F_2^d$ was calculated using the formulae (6) and (7), [7], and $F_2^0$ and $F_2^n$ were calculated according to eqs (8,9), [10]. As a large $Q^2$ structure functions $F_{2,AS}^{p,n}$ required by the model as an input, eq.(8), we used the MRS D-$^1$ parametrisation [11], resulting from the analysis which includes the NMC $F_2$ measurements [4] corrected for nuclear shadowing in the large $Q^2$ region.
The results of our calculations are shown in table 1 and in figures 1 and 2. Only $x$ values smaller than about 0.1 where the shadowing is non-negligible were considered. Table 1 contains results corresponding to the re-evaluated NMC data, [1]. In fig.1 results of the calculations are presented together with the NMC [1] and E665 [6] data. Plotted is the quantity $2F_2^p(x,Q^2)/F_2^p(x,Q^2) - 1$ which in the absence of shadowing would be just equal to $F_2^n(x,Q^2)/F_2^p(x,Q^2)$, cf. eq. (3). The average $Q^2$ values for the E665 measurements range from 0.03 GeV$^2$ at 0.00005 to 28.8 GeV$^2$ at $x = 0.15$, i.e. the very low $x$ data were taken at very low $Q^2$. The continuous curve is a result of our calculations which were done in the $(x,Q^2 = 4 \text{GeV}^2)$ points corresponding to the NMC results and the broken one to the $(x,Q^2)$ points corresponding to the E665 measurements.

It can be noticed from the table 1 and figure 1 that the quantity $2F_2^p(x,Q^2)/F_2^p(x,Q^2) - 1$ is close to one in the region of small $x$ and its difference from unity has to be attributed to the shadowing effect. The reason is that the difference between elementary structure functions $F_2^p$ and $F_2^n$ calculated from equations (8) and (9) is very small in the low $x$, low $Q^2$ region. The function $F_2^p - F_2^n$ should of course vanish in the limit $x = 0$ (for fixed $Q^2$) since it is controlled by the $A_2$ Regge exchange in this limit, i.e.

$$F_2^p - F_2^n \sim x^{1 - \alpha_2}$$  (10)

where $\alpha_{A_2} \approx 1/2$ is the $A_2$ reggeon intercept. The expectations coming from Regge theory are incorporated in the parametrisation of parton distributions defining $F_{2\text{IAS}}^p$ and of $\sigma_0(W^2)$.

The model predicts saturation of shadowing, i.e. its approximate $x$ independence. This comes from the fact that the $x$ dependence of the $\delta F_2^n(x,Q^2)$ and of the elementary structure function $F_2^p(x,Q^2)$ is similar in the region of $x$ and $Q^2$ relevant for the data. The saturation persists to be present for moderately large values of $Q^2$. Our models reproduce well the NMC measurements; the E665 results, reaching down to very low values of $x$ are not sufficiently precise to conclude about the shadowing. Observe that in the NMC/E665 data overlap region the theoretical prediction practically coincide. We

<table>
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<th>$x$</th>
<th>$F_2^p$</th>
<th>$(F_2^p + F_2^n)/2$</th>
<th>$2F_2^p/F_2^p-1$</th>
<th>$\delta F_2^p$</th>
<th>$\Delta S_G$</th>
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Table 1: The proton structure function, $F_2^p$, the structure function $(F_2^p + F_2^n)/2$, the quantity $2F_2^p/F_2^p-1$, eq.(3), the shadowing contribution to the deuteron structure function $\delta F_2^d$, [7, 10] and the shadowing contribution to the Gottfried sum, $\Delta S_G(x_{min} - 0.8) = 2 \int_{x_{min}}^{0.8} \delta F_2^d dx/x$ where $x_{min}$ is equal to the lower limit of the each $x$ bin. The MRS D’ parametrisation [11] of the high $Q^2$ structure function was used in the calculations. All values are at $Q^2 = 4 \text{ GeV}^2$; the $x$ values are the same as for the reanalysed NMC results [1] except that only points with $x$ smaller than about 0.1 are listed.
also made calculations for the \((x, Q^2)\) points relevant for the extended sample of the NMC \(F_2^d/F_2^p\) measurements, \([2]\). The \(x\) values reach down to \(x = 0.0008\) at \(< Q^2 > \approx 0.25\) GeV\(^2\). The \(< Q^2 >\)there increases with increasing \(x\) up to \(< Q^2 > \approx 10\) GeV\(^2\) at \(x = 0.11\). These predictions were practically indistinguishable from those for the E665, fig.1.

In fig.2 we show the shadowing contribution to the deuteron structure function, \(\delta F_2^d\), calculated according to the model \([7]\) in the \((x, Q^2)\) points corresponding to the extended NMC measurements (preliminary data \([2]\)). At \(x \sim 0.001\) the shadowing accounts for approximately 2.5\% of the deuteron structure function. Both the partonic and the vector meson contributions are shown in the figure. The main contribution to the \(\delta F_2^d\) comes from the vector meson rescattering, cf. eq.\((7)\); the parton contributions become comparable only for \(Q^2 \sim 2\) GeV\(^2\) corresponding to \(x \sim 0.01\).

The nuclear shadowing effect in the deuteron lowers the NMC Gottfried sum estimate, cf. eqs \((4)\) and \((1)\). The numerical results of the shadowing correction to the Gottfried sum,

\[
\Delta S_G(x_{\text{min}} - 0.8) = 2 \int_{x_{\text{min}}}^{0.8} \frac{dx}{x} \delta F_2^d
\]

where \(x_{\text{min}}\) is equal to the lower limit of the measured \(x\) interval, are given in table 1. This implies that the Gottfried sum, estimated by the NMC \([1]\) to be equal to \(S_G(0.004 - 0.8) = 0.221 \pm 0.008\) (stat.)\(\pm 0.019\) (syst.) is thus further decreased by \(\Delta S_G = 0.0264\), i.e. by about 12\%. For a different estimate of the shadowing effect in the Gottfried sum, \([15]\), the relevant number is about 17\%.

To sum up we have calculated the quantity \(2F_2^d(x, Q^2)/F_2^p(x, Q^2) - 1\) in the region of low \(x\) and low- and moderate \(Q^2\) relevant for the NMC and E665 measurements and for the expected results of the final NMC analysis of their low \(x\) data. In this region both the shadowing as well as the elementary structure functions are dominated by the VMD mechanism. The model predicts saturation, i.e. \(x\) independence of the quantity \(2F_2^d(x, Q^2)/F_2^p(x, Q^2) - 1\) in the very low \(x\) and \(Q^2\) region. The fact that this quantity stays systematically below unity has to be attributed to nuclear shadowing in the deuteron. The amount of shadowing predicted from the model \([7, 10]\) is in agreement with the recently re-evaluated NMC data at \(Q^2 \approx 4\) GeV\(^2\). The shadowing decreases the experimentally estimated value of the Gottfried sum by about 12\%.

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**References**


[2] For the preliminary results see:


[5] See e.g. B. Badelek et al., *to be submitted to Z. Phys. C* and references therein.


Figure 1: The quantity $2F_2^d/F_2^p - 1$ calculated from our models [7, 10] which include the shadowing in the deuteron, eq.(3) compared with the re-evaluated NMC measurements, [1], (cf. table 1) and with the Fermilab E665 data, [6]. The errors are statistical.
Figure 2: The shadowing contribution to the deuteron structure function, $\delta F_2^d$, calculated according to the model [7] in the $(x,Q^2)$ points corresponding to the extended NMC measurements, [2]. The partonic and the vector meson scattering contributions are also shown.