On the Search for Weak Decays of Heavy Quarkonium in Dedicated Heavy-Quark Factories

Miguel Angel Sanchis-Lozano

*Departamento de Física Teórica
and
Instituto de Física Corpuscular (IFIC)
Centro Mixto Universidad de Valencia-CSIC*

Dr. Moliner 50, E-46100 Burjassot, Valencia (Spain)

December 2, 1993

Abstract

The possibility of detecting weak decays of charmonium and bottomonium in dedicated heavy-quark factories is examined. First, semi-leptonic and two-body non-leptonic decays of heavy quarkonium are analyzed theoretically on the basis of the spin symmetry of heavy hadrons. Finally, possible procedures for conducting the experimental search are proposed and the requested detector requirements are discussed.

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1 Introduction

At present, two general trends may be distinguished in accelerator particle physics. On one hand, the foreseen construction of huge accelerators like LHC in Europe or SSC in USA will permit to explore the frontier line of high-energy physics. On the other one, smaller installations reaching lower energies but with very large statistics and low backgrounds (particle factories) would allow to perform precise tests and accurate determinations of many parameters of the Standard Theory. Moreover, the scrutiny of rare processes may enlight new physics in a complementary fashion to high-energy colliders.

Among those physical processes that high-luminosity dedicated heavy-quark factories would afford to examine [1] [2], we shall focus on weak decays of heavy quarkonium: charmonium and bottomonium, which would permit for the first time to observe the weak decay of an also strongly decaying particle. The fact that the latter decay mode is OZI-suppressed for those states below the open flavour threshold causing the narrowness of their full widths, could render the weak decay mode observable. Besides, the detection of a single explicitly flavoured meson coming from a weak process is bias-free from inefficient double tagging of flavoured meson pairs produced through conventional decays of quarkonium occuring only above threshold [3]. Moreover, let us point out that the observation of an anomalous production rate of single charmed or bottom mesons in the final state of e^+e^- collisions would be a hint of new physics either in the continuum via flavour-changing neutral currents [3] or in the decays of resonances due to unexpected effects of quark dynamics.

In particular, the physics covered by the analysis of weak decays of quarkonium mainly concerns QCD effects in hadronic systems composed of two heavy quarks by comparison to the analogous decays of hadrons containing a single heavy quark like D or B mesons.

Furthermore, cross-comparisons might be extended to baryons consisting of two heavy quarks and a light quark to be produced copiously in LHC and SSC machines [4]. In addition, the weak decay of ψ and Υ resonances offers the unique opportunity of observing the weak decay of a *vector* meson, where polarization effects can be used as tests of the underlying structure and dynamics of hadrons and currents.

In fact, considerable attention has been paid in recent years both from the experimental and theoretical sides to transitions between hadrons containing a single heavy quark (Q) with the help of new symmetries emerging when its mass is much greater than the characteristic scale Λ_{QCD} of the strong interactions. Below, we outline the main features of this formalism in order to establish a later comparison with heavy quarkonium and its weak decay.

In a hadron built up by a heavy quark and one or more light constituents, the former practically does not feel the soft binding interaction with the latter, acting as a pure static source of colour with chromomagnetic interaction vanishing at this limit. Thus its flavour (*i.e.* mass) and spin decouple from the light degrees of freedom. This rather intuitive picture of such spin-flavour symmetry has been cast into the language of a heavy quark effective theory (HQET) [5] by means of an effective Lagrangian for QCD in the limit $m_Q \rightarrow \infty$ and four-velocity fixed, adequate to describe almost on-shell quarks at rest in the hadron rest frame. In the case of a heavy but finite mass quark, the breaking terms of the (now approximate) spin-flavour symmetry are of order Λ_{QCD}/m_Q and $\alpha_s(m_Q)/\pi$ (perturbative QCD).

When the hadron containing the massive quark undergoes a weak transition as a semi-leptonic (s.l.) decay into another hadron still containing a heavy quark, the "immovable" center of chromoelectric field is "replaced" by another one, so that as far as the latter remains at the same velocity than the former, no change would be noticed by the light quarks and gluon cloud (the so-called "brown muck"). Even away from this non-recoil kinematic point,

the "brown muck" will only know that its center of force is recoiling now with a different velocity, independently of the mass and spin of the initial and final heavy quarks, at least to leading order in the inverse heavy quark mass and α_s expansions. Thus, the application of the HQET spin-flavour symmetry simplifies considerably the analysis of this process, reducing the large set of hadronic form factors to a single universal function (the Isgur-Wise function) which, besides, is normalized to unity at zero recoil. Therefore, HQET has relaunched at present the interest in the underlying essential idea of the spectator behaviour, offering besides a systematic expansion in the inverse mass of the heavy quark and allowing a solid, nearly model-independent basis for an analysis of many weak decay processes.

In section 2.1 we shall argue that the full heavy quark spin-flavour symmetry does not apply completely in the description of the s.l decays of heavy quarkonium. Nevertheless, an approximate spin symmetry should hold in the mass region corresponding to the charmonium and bottomonium states leading also to a simpler treatment, though no heavy quark flavour symmetry is expected to relate initial to final mesons.

The terminology used in this paper is the following: a heavy-heavy (h-h) meson is built up by a heavy quark-antiquark pair, *i.e.* $(Q\bar{Q})$ where $Q = c, b$. A heavy-semiheavy (h-s) meson is composed of a heavy quark and a semiheavy antiquark (in the sense that this constituent is considered rather heavy than light) or viceversa, *i.e.* $(Q\bar{q})$ where $q = s, c$ will be called semiheavy compared to the heavy $Q = c, b$ respectively. Finally, a heavy-light (h-l) meson is composed of a heavy quark and a light antiquark (u, d) or viceversa. Treating the charm quark as heavy compared to the strange quark, which occupies a borderline position and can be considered either as semiheavy or as light, lies inside the accuracy of this work.

2 Weak decays of heavy quarkonium

Firstly, we shall estimate the branching ratios for *inclusive* weak decays of heavy quarkonia (charmonium and bottomonium) ¹ via a single quark decay of either the constituent quark or antiquark relying on a simple spectator picture. For the J/ψ one gets: ²

$$BR \simeq \frac{2/\tau_D}{\Gamma_{J/\psi}} \simeq (2 - 4) \times 10^{-8} \quad (1)$$

where D denotes either the charged or neutral charmed meson and $\Gamma_{J/\psi}$ stands for the total width of the resonance (86 KeV [6]). The uncertainty in using the charged/neutral D meson to estimate the weak decay width of the J/ψ is a consequence of assuming or not the (destructive) interference to account for the well-known difference of lifetimes and s.l. branching ratios of charmed mesons. Notice that whether such interference mechanism is or not the main responsible for these differences, it should not be operative in charmonium weak decays. Indeed, neither identical final quarks nor equal two-body final states will appear via either internal or external spectator non-leptonic decays, conversely to the charged D . Therefore it is reasonable to assume the expected BR to be closer to 4×10^{-8} , *i.e.* using the D^0 lifetime and ignoring any W -exchange contribution to its decay rate.

For weak decays of the $\Upsilon(1S)$, using the experimental values $\tau_B = 1.3 \text{ ps}$ and the full width $\Gamma_{\Upsilon(1S)} = 52 \text{ keV}$, the expected BR is $\simeq 2 \times 10^{-8}$. In the $\Upsilon(3S)$ case with $\Gamma_{\Upsilon(3S)} = 24 \text{ keV}$, one gets the slightly higher branching fraction 4×10^{-8} . All these numbers are rough estimates and only indicative of the expected order of magnitude.

Such small BR's should make the observation of weak decays of heavy quarkonium ex-

¹To include toponium in our analysis would require a mass of the top quark less than $\sim 110 \text{ GeV}$. A larger mass induces the weak decay even before significant QCD binding occurs

²Natural units will be used throughout

tremely difficult despite the foreseen cleanness of events. Obviously, due to the huge data sample to be analyzed to detect these rare processes appropriate requirements and kinematical constraints must be put on events ³.

Semi-leptonic decays offer several advantages over the purely hadronic ones both from the experimental and theoretical points of view. In particular, we will analyze the s.l. decays of quarkonium $(Q\bar{Q})_{1-}$ induced by the quark-level weak transition $Q \rightarrow qW^*$ taking into account a possible spin flip: $(Q\bar{Q})_{1-} \rightarrow (q\bar{Q})_{0-}$, and $(Q\bar{Q})_{1-} \rightarrow (q\bar{Q})_{1-}$. Hence, let us consider the Cabibbo allowed exclusive channels:

$$\begin{aligned} \psi(nS) &\rightarrow D_s \ell \nu \quad , \quad \psi(nS) \rightarrow D_s^* \ell \nu \quad (n = 1, 2 ; \ell = e, \mu) \\ \Upsilon(nS) &\rightarrow B_c \ell \nu \quad , \quad \Upsilon(nS) \rightarrow B_c^* \ell \nu \quad (n = 1, 2, 3 ; \ell = e, \mu, \tau) \end{aligned} \quad (2)$$

where the prompt charged lepton can be useful for tagging purposes, removing a large amount of conventional hadronic decays. Besides, one may employ experimental methods involving missing energy/momentum (*e.g.* missing mass) due to the escaping ν . Finally, the invariant mass reconstruction of an explicitly flavoured meson in the final state should provide an unambiguous signature for such process.

Nevertheless, the very low decay rate can still make unclear the observation of such rare processes due to the huge background from conventional decays of the resonance. Therefore, radiative decays of the excited mesons D_s^* , B_c^* can provide a useful extra signal ⁴ and a powerful constraint in the mass reconstruction of D_s , B_c mesons to detect weak decays of charmonium and bottomonium.

³In a τ -charm factory with a peak luminosity of $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, the foreseen number of $J/\psi(1S)$'s is $\sim 10^{11}$ after few years of data taking with conventional optics or just one year by using monochromators [7], leading to $\simeq 4000$ weak decays when combined with the expected BR

⁴Yielding in the Lab system detectable photons [8] in the energy interval $\sim 90 - 200 \text{ MeV}$ and $\sim 40 - 110 \text{ MeV}$ due to the recoil of the D_s^* and B_c^* respectively

2.1 Spin symmetries

As is well-known long time ago, mesons built up by heavy quarks can be reasonably well described by a single non-relativistic Shrödinger equation with a static (QCD-inspired) potential [9]. This description is justified in charmonium and bottomonium since quarks tend to be non-relativistic ($\langle \vec{v}_Q \cdot \vec{v}_Q \rangle \sim O(1/4)$, $O(1/10)$ respectively) and due to the impressive empirical success of its predictions on their mass spectra [10]. Quark velocities are, however, sizeable and should lead to corrections to a non-relativistic picture of hadronic transitions though sufficiently small to make sense a perturbative approach [11] [12]. In the hypothetical infinite mass limit, both quarks would be almost static in the low-lying states, with velocities inside quarkonium of order $\alpha_s(m_Q) \rightarrow 0$ (i.e. asymptotic freedom) mainly governed by the Coulomb-like part (one gluon exchange) of the $Q\bar{Q}$ potential. Despite this, one cannot get rid of the internal motion of heavy quarks in h-h (or h-s) bound systems so easily as in h-l ones. This is because the virial theorem relates the average potential and kinetic energies, implying that the latter cannot be neglected even in the formal limit $m_Q \rightarrow \infty$ [13]⁵. Thereby, heavy quark masses are involved in the dynamics of the h-h (or h-s) systems, conversely to h-l systems.

In a weak transition of heavy quarkonium into a h-s or a h-l meson, the velocities of all constituents should change appreciably even at zero-recoil and both hadrons should be described by different wave functions in sharp contrast to transitions between h-l mesons. Thus *no heavy quark flavour symmetry* must be expected to hold relating initial h-h and final h-s (or h-l) hadron states.

On the other hand, as commented in the Introduction, the net effect of neglecting the

⁵Then the quark velocity, r.m.s radius of the meson and kinetic energy of the constituents behave as: $v_Q \sim \alpha_s \rightarrow 0$,

$r_{(Q\bar{Q})} \sim (m_Q \alpha_s)^{-1} \rightarrow 0$, $T \sim m_Q \alpha_s^2 \rightarrow \infty$

spin interactions between constituent quarks inside a hadron is the appearance of a spin symmetry in the description of its state. This approach, as a leading order approximation in the inverse heavy quark mass, is justified in mesons composed of two heavy quarks since they are essentially non-relativistic, whereas spin forces may be accounted for as a relativistic effect in potential models [9]. The actual mass splittings between the spin-singlet and spin-triplet states like η_c and J/ψ or D_s and D_s^* are of the same order than the breaking effects of the spin symmetry in h-l mesons, for example between D and D^* .

Moreover, spin symmetry has been recently discussed in mesons containing two heavy quarks in the context of HQET: the kinetic term to be kept in the effective Lagrangian to balance the potential energy breaks explicitly the heavy flavour symmetry but leaves the heavy quark spin symmetry. Even though no asymptotic limit for the heavy quark symmetries seems to exist in h-h or h-s bound states, there is an agreement that the spin symmetry should hold as an useful approximation excluding the flavor symmetry in the preasymptotic mass region corresponding to the charmonium and bottomonium states [14].

Furthermore, as pointed out in [15] heavy quarks in weakly bound systems are not far off-shell to permit the use of a covariant description of the spin wave function of hadrons belonging to a spin multiplet. This amounts to consider the constituent quarks moving collinearly and/or that the hadron mass is just the sum of the masses of all of them as stressed in the quoted reference [15].

Therefore, we shall follow the covariant description of the spin wave functions of h-h and h-s (s -wave) mesons according to: [15] [16]

$$J^P = 0^- : H(v) = \sqrt{m} \left(\frac{1 + \not{v}}{2} \right) \gamma_5 ; \quad J^P = 1^- : H(v, \epsilon) = \sqrt{m} \left(\frac{1 + \not{v}}{2} \right) \not{\epsilon} \quad (3)$$

where ϵ^μ denotes the polarization four-vector of the vector particle. Let us remark that we are not completely ignoring the relative motion of heavy quarks inside the hadron since it

will be taken into account in the weak transition form factor to appear below.

2.2 Non-recoil approximation

In order to estimate the hadronic matrix elements for weak transitions of heavy quarkonium we shall use the non-recoil approximation, that is, in the rest frame of the initial hadron the final hadron is at rest too. This approximation is strongly suggested by the fact that at

$$q^2 = (p_1 - p_2)^2 = 0,$$

$$(v_1 \cdot v_2)_{max} = \frac{m_1^2 + m_2^2}{2m_1 m_2} \simeq 1.1 \quad (1.07) \quad (4)$$

where m_1 , v_1 , p_1 and m_2 , v_2 , p_2 stand for the mass, four-velocity and four-momentum of the initial and final mesons respectively. The above numerical values correspond to the $J/\psi \rightarrow D_s(D_s^*) \ell \nu$ decays, respectively. With regard to the Cabibbo suppressed decays $J/\psi \rightarrow D(D^*) \ell \nu$, $(v_1 \cdot v_2)_{max} \simeq 1.13$ (1.1). Moreover, Υ transitions to B_c mesons yield similar values for a reasonable mass of the latter [17]. Therefore, the kinetic energy of the final meson will never exceed 10% of its mass in the quarkonium rest frame, since $(\gamma_2 - 1)_{max} = (m_1 - m_2)^2 / 2m_1 m_2$. Let us point out that there is a rather distinct situation in s.l. decays of D mesons leading to light final hadrons $K(K^*)$ with $(v_1 \cdot v_2)_{max} \simeq 2$ (1.3), whereas $(v_1 \cdot v_2)_{max} \simeq 1.6$ (1.5) in $B \rightarrow D(D^*)$ transitions.

Henceforth, we shall adopt the non-recoil approximation: $v_1 \cdot v_2 = 1$, when calculating the amplitudes for weak hadronic transitions of heavy quarkonium. This limit for J/ψ and Υ is somewhat equivalent to the condition $(m_1 - m_2)^2 / (m_1 + m_2)^2 \ll 1$ as proposed in the Voloshin and Shifman's paper [18] for h-l mesons. Let us remark that in the decays (2) of charm and bottom resonances this inequality is much better fulfilled than in B s.l. decays (0.04 – 0.05 versus 0.23).

The amplitudes of the hadronic transitions of vector mesons into pseudoscalar and vector

mesons will be estimated according to the well-known trace formalism: [19]

$$\begin{aligned}
\langle H_2(v_2) | \bar{q} \Gamma Q | H_1(v_1, \epsilon_1) \rangle &= -\eta_{12}(v_1 \cdot v_2) \text{Tr}[\bar{H}_2(v_2) \Gamma H_1(v_1, \epsilon_1)] \\
\langle H_2(v_2, \epsilon_2) | \bar{q} \Gamma Q | H_1(v_1, \epsilon_1) \rangle &= -\eta_{12}(v_1 \cdot v_2) \text{Tr}[\bar{H}_2(v_2, \epsilon_2) \Gamma H_1(v_1, \epsilon_1)] \quad (5)
\end{aligned}$$

where the initial and final hadron states are characterized by their four-velocities, instead of four-momenta; Γ is a combination of γ matrices and η_{12} plays the role of the reduced matrix element in the Wigner-Eckart theorem that expression (5) actually represents. It is the analogue of the dimensionless Isgur-Wise function for transitions between h-l mesons [5] and measures the overlap of the initial and final hadronic wave functions. Note, however, that $\eta_{12}(v_1 \cdot v_2)$ depends on the type of hadrons involved in the process and it is not an universal function of the velocity difference only. On the other hand, the spin symmetry ensures a single η_{12} (reduced) form factor for hadronic transitions, irrespective of Γ , between different spin-states. Finally, let us stress that η_{12} is not absolutely normalized to unity at zero recoil via the vector current since h-h and h-s or h-l systems do not belong to a multiplet of a heavy quark flavour symmetry group (there should be a mismatch between the initial and final hadron wave functions even at zero recoil).

The relevant hadronic matrix elements in the non-recoil approximation ($v_1 = v_2$) are given by:

$$\begin{aligned}
\langle (q\bar{Q})_{0-} | A^\mu | (Q, \bar{Q})_{1-, \epsilon_1} \rangle &\simeq 2 \eta_{12} \sqrt{m_1 m_2} \epsilon_1^\mu \\
\langle (q\bar{Q})_{1-, \epsilon_2} | V^\mu | (Q, \bar{Q})_{1-, \epsilon_1} \rangle &\simeq -2 \eta_{12} \sqrt{m_1 m_2} (\epsilon_1 \cdot \epsilon_2^*) v_1^\mu \\
\langle (q\bar{Q})_{1-, \epsilon_2} | A^\mu | (Q, \bar{Q})_{1-, \epsilon_1} \rangle &\simeq -2 \eta_{12} \sqrt{m_1 m_2} i \epsilon^{\mu\nu\alpha\beta} v_{1\nu} \epsilon_{1\alpha} \epsilon_{2\beta}^*
\end{aligned} \quad (6)$$

where η_{12} is taken at $v_1 \cdot v_2 = 1$. Observe that the vector current $\bar{q} \gamma^\mu Q$ does not change the spin state of the "active" quark, since $\epsilon_1 \cdot \epsilon_2^* = -\delta_{\lambda_1, \lambda_2}$ at this approximation, where λ_i denotes the polarization state of the initial or final meson, whereas the axial-vector current

$\bar{q}\gamma^\mu\gamma^5 Q$ can do it.

In the appendix A, full expressions for the hadronic matrix elements are presented in the general case $v_1 \neq v_2$ and the accuracy of the non-recoil approximation is discussed. Here, let us just mention that the theoretical uncertainty due to this approximation in the evaluation of the s.l. decay widths is of order $< 40\%$.

In (6) the motion of the final meson was ignored. Besides, leptonic masses will be neglected since we do not consider the decay $\Upsilon \rightarrow B_c(B_c^*)\tau\nu$, further suppressed by phase space and providing a less clean tagging. In the following we shall use, however, exact kinematic factors as phase space to compute the decay rates.

Next, we contract the hadronic currents of (6) with the leptonic current $\bar{\ell}\gamma_\mu(1 - \gamma_5)\nu$. Squaring and averaging over initial polarization states (assuming unpolarized quarkonium) and summing over final polarization states, one gets for each lepton species and charge mode

$$\frac{d\Gamma}{dq^2}(1^- \rightarrow 0^-)|_{axial} \simeq \frac{1}{2} \frac{d\Gamma}{dq^2}(1^- \rightarrow 1^-)|_{axial} \simeq \frac{G_F^2(\lambda^{3/2} + 12q^2 m_1^2 \lambda^{1/2})}{576 \pi^3 m_1^3} f_{12}^2 |V_{Qq}|^2 \quad (7)$$

$$\frac{d\Gamma}{dq^2}(1^- \rightarrow 1^-)|_{vector} \simeq \frac{G_F^2 \lambda^{3/2}}{192 \pi^3 m_1^3} f_{12}^2 |V_{Qq}|^2 \quad (8)$$

where $\lambda \equiv \lambda(m_1^2, m_2^2, q^2)$ denotes the Källén function, V_{Qq} is the KM mixing matrix element involved in the weak decay of the active quark and the (single) hadronic form factor f_{12} is related to η_{12} by:

$$f_{12} = \sqrt{\frac{m_2}{m_1}} \eta_{12}(v_1 \cdot v_2 = 1) \quad (9)$$

assumed nearly constant over the whole available q^2 range. This introduces an uncertainty of $O[(m_1 - m_2)^2/m_2^2]$ according to a dipole q^2 -dependence of the form factor dominated by the nearest pole ($\sim m_2$). In the most general case $v_1 \neq v_2$, one should deal with a set of form factors, all of them related to η_{12} due to the quoted spin symmetry. Notice, however, that suppression factors of order $\sim 20\%$ at the amplitude level as shown in the appendix A, would

permit to ignore all of them except the leading form factor (9) as a first approach. Observe also that we have considered the same f_{12} for $1^- \rightarrow 0^-$ and $1^- \rightarrow 1^-$ decays. In fact, both η_{12} 's should not differ largely as suggested by the small breaking of the mass degeneracy between spin-singlet and spin-triplet meson states. Then, expression (9) amounts to a theoretical uncertainty lying within the framework of the approximations made in this work.

Integrating over q^2 between 0 and $q_{max}^2 = (m_1 - m_2)^2$,

$$\Gamma(1^- \rightarrow 0^-)|_{axial} \simeq \frac{1}{2} \Gamma(1^- \rightarrow 1^-)|_{axial} \simeq \frac{G_F^2 m_1^5}{576 \pi^3} f_{12}^2 |V_{Qq}|^2 [\varphi(x) + \phi(x)] \quad (10)$$

$$\Gamma(1^- \rightarrow 1^-)|_{vector} \simeq \frac{G_F^2 m_1^5}{192 \pi^3} f_{12}^2 |V_{Qq}|^2 \varphi(x) \quad (11)$$

where φ and ϕ are phase space factors depending on $x = m_2/m_1$. In the appendix B their full expressions are shown. In the limit $x \rightarrow 1$, corresponding to the inequality $(m_1 - m_2)^2 \ll (m_1 + m_2)^2$, *i.e.* small four-momentum transfer compared to the hadronic masses, phase space factors behave as:

$$\phi(x)/2 \rightarrow \varphi(x) \simeq \frac{16}{5} (1 - x)^5 \quad (12)$$

In this extreme non-relativistic limit,

$$\Gamma(1^- \rightarrow 0^-)|_{axial} \simeq \frac{1}{2} \Gamma(1^- \rightarrow 1^-)|_{axial} \simeq \Gamma(1^- \rightarrow 1^-)|_{vector} \simeq \frac{G_F^2 (m_1 - m_2)^5}{60 \pi^3} f_{12}^2 |V_{Qq}|^2 \quad (13)$$

2.3 e^+e^- factories

In e^+e^- factories charmonium and bottomonium are produced resonantly with transverse polarization only, *i.e.* helicity ± 1 along the direction defined by the colliding beams (z). Therefore all the previous calculations must be reexamined taking into account this fact. Averaging over the initial (two) transverse polarizations along z and summing over the (three) polarizations of the final vector meson along its line of motion, the double differential decay

rate in the non-recoil approximation again turns out to be:

$$\begin{aligned}
\frac{d^2\Gamma_T}{dq^2 d\cos\theta}(1^- \rightarrow 0^-)|_{axial} &\simeq \frac{G_F^2 [\sin^2\theta\lambda^{3/2} + 8q^2 m_1^2 \lambda^{1/2}]}{768 \pi^3 m_1^3} f_{12}^2 |V_{Qq}|^2 \\
\frac{d^2\Gamma_T}{dq^2 d\cos\theta}(1^- \rightarrow 1^-)|_{axial} &\simeq \frac{G_F^2 [(1 + \cos^2\theta)\lambda^{3/2} + 16q^2 m_1^2 \lambda^{1/2}]}{768 \pi^3 m_1^3} f_{12}^2 |V_{Qq}|^2 \\
\frac{d^2\Gamma_T}{dq^2 d\cos\theta}(1^- \rightarrow 1^-)|_{vector} &\simeq \frac{G_F^2 \lambda^{3/2}}{384 \pi^3 m_1^3} f_{12}^2 |V_{Qq}|^2
\end{aligned} \tag{14}$$

where θ is the (polar) angle defined by the outgoing hadron and the beams' direction. Now, integrating over all the possible outgoing directions, the set of equations (7) and (8) are recovered. In fact, the net effect is to "depolarize" quarkonium and any initial polarization effect disappears.

2.4 Semileptonic widths and branching ratios

From expressions (10) and (11), the widths for the s.l. decays (2) are:

$$\Gamma[H_1(1^-) \rightarrow H_2(0^-)\ell\nu] = \frac{G_F^2 m_1^5}{288 \pi^3} [\varphi(\mathbf{x}) + \phi(\mathbf{x})] f_{12}^2 |V_{Qq}|^2 \tag{15}$$

$$\rightarrow \frac{G_F^2 (m_1 - m_2)^5}{30 \pi^3} f_{12}^2 |V_{Qq}|^2 \tag{16}$$

$$\Gamma[H_1(1^-) \rightarrow H_2(1^-)\ell\nu] = \frac{G_F^2 m_1^5}{288 \pi^3} [5\varphi(\mathbf{x}) + 2\phi(\mathbf{x})] f_{12}^2 |V_{Qq}|^2 \tag{17}$$

$$\rightarrow \frac{G_F^2 (m_1 - m_2)^5}{10 \pi^3} f_{12}^2 |V_{Qq}|^2 \tag{18}$$

for *each* leptonic generation and *both* charge conjugate modes altogether.

The right hand side limits, *i.e.* equations (16) and (18), correspond to the non-relativistic limit $\mathbf{x} \rightarrow 1$ in the phase space factors, retaining only the leading terms. Then a close analogy shows up with the pioneering result by Voloshin and Shifman [18] in s.l. decays of h-l mesons, except for the form factor f_{12} . Setting formally $f_{12} = 1$ the total decay rate is dual to a quark-parton description of the s.l. decays of quarkonium in a free spectator picture. Indeed

in the $(m_Q - m_q)^2 \ll (m_Q + m_q)^2$ limit, one arrives at:

$$\Gamma[(Q\bar{Q}) \rightarrow (q\bar{q}) \text{ or } (\bar{q}Q) + \ell\nu] \simeq \frac{2 G_F^2 (m_1 - m_2)^5}{15 \pi^3} |V_{Qq}|^2$$

since $m_Q - m_q \simeq m_1 - m_2$. Then, under all these assumptions 25% of s.l. decays would yield a pseudoscalar particle (D_s or B_c) and the remaining 75% would yield a vector particle (D_s^* or B_c^*). Let us observe that for $f_{12} \neq 1$ (as expected even in the non-recoil approximation) this relative branching ratio r would be still equal to 3 although the saturation of the s.l. decay mode of quarkonium by both exclusive channels will be no longer complete.

The above discussion corresponds to an ideal physical situation in the extreme non-relativistic limit and besides degenerate final hadrons. Notice, however, the sharp dependence on the mass difference in the phase space factors. Thereby, taking into account the actual mass splitting between pseudoscalar and vector particles, the expressions (15) and (17) (or even (16) and (18)) furnish the more realistic relative branching ratio: $r \simeq 1.5$ in charmonium decays. Instead in bottomonium decays one expects r to be closer to the "magic" value 3.

In order to get absolute estimates of branching ratios for s.l. decays one can estimate the overlap factor η_{12} at q_{max}^2 by means of the ISGW model [20] according to: $\eta_{12} \simeq (2 \beta_1 \beta_2 / \beta_1^2 + \beta_2^2)^{3/2}$, where β_i essentially measures the average transverse quark momentum in the i -meson. In particular for the J/ψ decays setting $\beta_{J/\psi} = 0.6$ GeV and $\beta_{D_s} \simeq \beta_{D_s^*} = 0.5$ GeV [21] as inputs in equations (15) and (17), we have

$$\Gamma(J/\psi \rightarrow D_s \ell \nu) \simeq 2.3 \times 10^{-13} \text{ GeV} \quad ; \quad \Gamma(J/\psi \rightarrow D_s^* \ell \nu) \simeq 3.6 \times 10^{-13} \text{ GeV}$$

where ℓ stands for the e and μ altogether ⁶. Combining these results with the full width of

⁶The extreme non-relativistic expressions (16) and (18) give slightly higher values within the accuracy of our estimates

the resonance, $\Gamma_{J/\psi} = 86 \text{ KeV}$ [6], one arrives at

$$BR(J/\psi \rightarrow D_s \ell \nu) \simeq 0.26 \times 10^{-8} \quad ; \quad BR(J/\psi \rightarrow D_s^* \ell \nu) \simeq 0.42 \times 10^{-8}$$

Summing over both modes one gets a $BR \simeq 0.7 \times 10^{-8}$, representing about 20% of the expected total weak decay rate when combined the result from equation (1). Taking into account the overall theoretical uncertainty ($\simeq 40\%$), the expected BR lies in the interval $(0.4 - 1.0) \times 10^{-8}$.

With regard to the bottomonium family, the most favourable is the $\Upsilon(3S)$ from the viewpoint of the (weak) signal to (conventional decay) background ratio due to its smallest full width (24 keV [6]). Setting $|V_{cb}| = 0.04$ an equivalent calculation gives:

$$\Gamma(\Upsilon(3S) \rightarrow B_c \ell \nu) \simeq 2.0 \times 10^{-13} \text{ GeV} \quad ; \quad \Gamma(\Upsilon(3S) \rightarrow B_c^* \ell \nu) \simeq 5.1 \times 10^{-13} \text{ GeV}$$

Then, the expected branching ratios are:

$$BR(\Upsilon(3S) \rightarrow B_c \ell \nu) \simeq 0.82 \times 10^{-8} \quad ; \quad BR(\Upsilon(3S) \rightarrow B_c^* \ell \nu) \simeq 2.1 \times 10^{-8}$$

that combined lead to $BR \simeq 3 \times 10^{-8}$.

3 Extension to other weak decays

In this section we apply both the spin symmetry and the non-recoil approximation to decays other than (2) in a straightforward way. Let us address the non-leptonic two-body decays:

$$J/\psi \rightarrow D_s(D_s^*) \pi \quad , \quad \Upsilon \rightarrow B_c(B_c^*) \pi \tag{19}$$

which can be related to s.l. ones at $q^2 = m_\pi^2$ assuming factorization, as suggested by Bjorken [22] in B decays. The relative velocity of the outgoing pion with respect to the initial hadron

can be easily calculated by means of the invariant quantity: ⁷

$$v_1 \cdot v_3 = \frac{m_1^2 + m_3^2 - m_2^2}{2m_1 m_3}$$

providing the $\gamma = (1 - v_3^2)^{-1/2}$ of the pion, labelled as 3, in the rest frame of the initial particle 1. The numerical values are $\gamma \simeq 6.5$, 20 for the J/ψ and Υ decays respectively, while $\gamma \simeq 16.6$ in the analogous $B \rightarrow D\pi$ decays.

Keeping the non-recoil approximation in the hadronic transition amplitudes, from equations (7) and (8) one arrives at the relative branching ratio:

$$r = \frac{\Gamma(J/\psi \rightarrow D_s^* \pi)}{\Gamma(J/\psi \rightarrow D_s \pi)} \simeq \frac{d\Gamma(J/\psi \rightarrow D_s^* \ell \nu)/dq^2|_{q^2=m_\pi^2}}{d\Gamma(J/\psi \rightarrow D_s \ell \nu)/dq^2|_{q^2=m_\pi^2}} \simeq 3.5 \quad (20)$$

and $r \simeq 5$ for the Υ decays in (19).

If a ρ substitutes the π one gets $r \simeq 1.4$ for the J/ψ and $\simeq 4$ for the Υ . Notice that the corresponding values of $\gamma \simeq 1.3$ (4) will probably make factorization less reliable now.

All the above estimates, showing an overall enhancement of final vector particles with respect to pseudoscalar ones (which could be extended to other exclusive channels with a set of light mesons in the final state) suggest once more the use of D_s^* or B_c^* as signals to search for weak decays of heavy quarkonium even in non-leptonic decays.

4 Event selection

Let us consider first the weak decays of the J/ψ resonance in a τ -charm factory. Assuming a sample of 10^{11} J/ψ 's to be collected after few years of data taking or even just one year

⁷It seems sensible to assume that factorization works more properly for energetic pions [23], according also to the intuitive argument of time-dilation permitting the pion to reach its asymptotic state faraway from the colour fields existing in the neighbourhood of quarkonium [22]

time if monochromator optics is used, the central value $\text{BR} \simeq 0.7 \times 10^{-8}$ leads to $\simeq 700$ s.l. decays of the type:

$$\psi(1S) \rightarrow D_s(D_s^*) \ell \nu \quad (\ell = e, \mu)$$

$\simeq 400$ of which with a final state D_s^* . Thus, the following hunting strategy could be applied by tightly focusing on such exclusive s.l. channels:

- a) The prompt charged lepton can tag the weak decay, removing a large amount of ordinary hadronic decays of the resonance. In order to decrease cascade decay sources, the tagging lepton momentum could be restricted, say, to be greater than 0.5 GeV and less than 1 GeV⁸, close to the upper kinematic limit for the decay under consideration. A high-quality lepton identification from charged pions is of the utter importance.
- b) The mass reconstruction of a D_s meson can provide an unambiguous signature for the weak decay of the J/ψ , lying below the open charm threshold. Let us note that reconstructing a single charmed meson should be easier and cleaner than reconstructing a pair of charmed mesons in the strong decays of ψ'' , ... whose decay products are completely intermixed yielding a combinatorial background. Notice also that the expected contribution from the e^+e^- continuum to a single charmed meson in the final state via a flavour-changing neutral current process is completely negligible [3] and hence no background of this type must be subtracted.
- c) Once the D_s is fully reconstructed, selected candidates should have a missing mass consistent with zero, due to the initial neutrino. Moreover, the momentum of the D_s candidate (*i.e.* the vectorial sum of the momenta of all the detected particles in each

⁸Let us point out that consequently the weak decay sub-sample would be enriched with D_s^* modes: the $1^- \rightarrow 1^-$ transition via the vector current does not flip the decaying quark spin. It is the converse than in D decays into K and K^* , where lower energy charged leptons are predominantly emitted along with the final vector meson

event except the tagging lepton) must be smaller than 0.9 GeV due to the kinematics of the primary decay.

- d) The detection of a photon within the energy interval (90 – 200) MeV can be combined with the above criteria: the additional constraint of an intermediate D_s^* state should confirm the D_s identification reducing the surviving background.

With regard to hadronic modes the BR's corresponding to exclusive processes like $J/\psi(1S) \rightarrow D_s(D_s^*)M$ ($M = \pi, \rho, \dots$) are probably too tiny to look for a particular decay channel. Thereby, it seems more sensible to make an inclusive search for $\psi(1S) \rightarrow D_s^* + X$ in a τ -charm factory. The γ from the excited meson should be useful as a kinematic constraint for the mass reconstruction of the D_s meson once again.

Similar experimental procedures might be extended to s.l. weak decays of bottomonium in a B-factory, although now the perspective would be somewhat more complicated due to the longer cascade decay of the B_c meson, the higher final multiplicity and therefore the difficulty for its unambiguous mass reconstruction. However, the intermediate B_c meson could be useful for partial reconstruction of the decay, especially when it yields a J/ψ resonance. Therefore the decay chain:

$$\Upsilon \rightarrow B_c(B_c^*) \ell \nu \rightarrow J/\psi X \ell \nu \rightarrow (\ell'^+ \ell'^-) \ell \nu X$$

could provide a signature of the bottomonium weak decay according to the following criteria:

- a) A fast high- p_t charged lepton with energy within the interval $\simeq (1 - 3)$ GeV. The lower limit should suppress cascade decay sources while the upper limit is close to the kinematic limit for the s.l. decay of the $\Upsilon(3S)$. The additional requirement of missing transverse momentum (always with respect to the beams' direction) due to the prompt neutrino could be imposed.

- b) The detection of two more oppositely-charged leptons (electron or muon pairs). *All three* (the prompt lepton and lepton pair) *should point to the e^+e^- interaction point.*
- c) The invariant mass of the lepton pair should be compatible with a J/ψ .
- d) The possibility of disentangling the J/ψ vertex from the primary vertex can be considered with the help of microvertex detectors especially in an asymmetric collider [2] due to a larger average flight path. By primary vertex we mean the point where the Υ undergoes the weak decay which would not coincide with the e^+e^- interaction point.

Let us observe that an alternative process which could fake the weak decay signal is the hadronic direct decay of the resonance: $\Upsilon \rightarrow J/\psi + X$ where the J/ψ can be created by gluon fusion, gluon or photon split up into a $c\bar{c}$ pair [24]. The discrimination between the weak and strong decay modes could be based on the correlation between the J/ψ and the high- p_t prompt lepton.

Next, let us address additional/alternative criteria depending on the decay mode of the intermediate B_c meson. If the B_c proceeds to decay semileptonically [21] yielding the J/ψ the requirement would be then *four* rather than three fast leptons pointing to the same point. If the B_c undergoes a non-leptonic decay (as for example $B_c \rightarrow J/\psi \pi$) and could be fully reconstructed, the additional requirement of a γ providing an invariant mass compatible with a B_c^* should confirm the weak decay. Finally, considering the purely leptonic decay via the direct annihilation $B_c \rightarrow \tau \nu$ with $\text{BR} \simeq 5\%$ [21]⁹, a large amount of missing energy/momentum must be expected in such events. Moreover, the subsequent "1-prong" decay of the τ occurring with $\text{BR} \simeq 86\%$ [6] would lead to very clean events with only two final p_t -unbalanced charged particles.

⁹This is in contrast to the leptonic decay of the D_s , whose BR is expectedly smaller [6]

5 Conclusions

The possible experimental detection of weak decays of quarkonium in future dedicated heavy-quark factories would offer the unique opportunity to observe the weak decay of a vector meson. Besides, interesting cross-comparisons of decay rates can be done between mesons or baryons containing a single heavy quark and resonances consisting of a heavy quark-antiquark pair and, once LHC or SSC machines become operative, baryons containing two or even three heavy quarks.

In this work, we have analyzed the weak decays of heavy quarkonium with the help of the (approximate) spin symmetry of heavy mesons. However, no heavy quark flavour symmetry relating initial (h-h) to final (h-s or h-l) mesons should hold. The non-recoil approximation was adopted in the calculation of the hadronic matrix elements, amounting to a theoretical uncertainty of $\simeq 40\%$ in the evaluation of the decay widths. Therefore, our results at this stage should be viewed as estimates suggesting experimental strategies for the detection of such rare process, rather than definite predictions for branching ratios.

We have focused our analysis on charmonium and bottomonium states beneath the open flavour (charm and bottom respectively) threshold. Toponium has not been included since a (foreseen) very massive top quark would decay weakly even before strong binding takes place .

Charmonium: A relative branching fraction of $\psi(1S) \rightarrow D_s^* \ell \nu$ with respect to $\psi(1S) \rightarrow D_s \ell \nu$ of $\simeq 1.5$ is expected as a leading order calculation. Let us stress that this is a model-independent evaluation based on the spin symmetry of heavy hadrons. We have also made a model-dependent estimate of the respective absolute branching ratios obtaining $\text{BR} \simeq (0.4 - 1.0) \times 10^{-8}$ summing over D_s , D_s^* , e , μ and both charge conjugate modes.

A possible experimental detection procedure was proposed in section 4 by focusing on

such exclusive s.l. decay channels taking advantage of the prompt charged lepton, missing mass and lastly radiative decay of the D_s^* . The mass reconstruction of the D_s should provide the actual signature of the weak process since such final state is forbidden via the strong or radiative decay of the resonance lying below the open charm threshold.

Bottomonium : In a B factory, the primary aim is running on the Υ family above open bottom threshold to produce a huge amount of $B\bar{B}$ pairs affording very interesting physics. However, after enough integrated luminosity had been collected, the machine will certainly run both on the nearby continuum and at other resonances like the $\Upsilon(3S)$. When expectedly so, our prospects to detect the weak decays of bottomonium could be applied.

For bottomonium s.l. decays of Eq. (2) our estimate of the branching fraction for the $\Upsilon(3S)$ resonance in particular is $\simeq (2-4)\times 10^{-8}$ with a relative branching fraction of $\simeq 3$ in favour of final B_c^* mesons. A search based on the decay chain:

$$\Upsilon \rightarrow B_c(B_c^*) \ell \nu \rightarrow J/\psi X \ell \nu \rightarrow (\ell'^+\ell'^-) \ell \nu X$$

can give a signal of the weak decay, where the mass reconstruction of the J/ψ by combining two oppositely-charged leptons together with an additional energetic high- p_t charged lepton should furnish the tagging of the weak decay. If the B_c decays semileptonically the signal would be then four charged leptons pointing to the e^+e^- interaction point. The use of microvertex detectors to disentangle the J/ψ decay vertex from the first weak decay of the Υ would be helpful to establish the signature of the process.

The leptonic decay of the B_c into $\tau\nu$ deserves also attention due to the large amount of missing energy/momentum in such events. Besides the "1-prong" decay of the τ should give rise to a topology of only two final charged particles clearly distinct kinematically from conventional $\ell^+\ell^-$ events.

Finally, if the B_c (non-leptonic) decay could be fully reconstructed the detection of a γ

providing an invariant mass compatible with a B_c^* should confirm the weak decay of the Υ resonance.

Acknowledgments I have benefitted from discussions with N. Horwitz. This work was partially supported by CICYT under contract AEN 93-0234 and by IVEI.

Appendices

A

In this appendix we examine the accuracy of the non-recoil approximation which means that in the evaluation of the hadronic transition amplitudes, the final hadron is supposed at rest in the decaying particle rest frame. Under the assumption of the full validity of the heavy quark spin symmetry, the expressions relative to the weak hadronic transitions: $1^- \rightarrow 0^-$ and $1^- \rightarrow 1^-$ for the general case $v_1 \neq v_2$, are: [25]

$$\langle (q\bar{Q})_{0^-} | V^\mu | (Q, \bar{Q})_{1^-}, \epsilon_1 \rangle = -\eta_{12}(v_1 \cdot v_2) \sqrt{m_1 m_2} i \epsilon^{\mu\nu\alpha\beta} \epsilon_{1\nu} v_{1\alpha} v_{2\beta} \quad (\text{A.1})$$

$$\langle (q\bar{Q})_{0^-} | A^\mu | (Q, \bar{Q})_{1^-}, \epsilon_1 \rangle = \eta_{12}(v_1 \cdot v_2) \sqrt{m_1 m_2} [(1 + v_1 \cdot v_2) \epsilon_1^\mu - (\epsilon_1 \cdot v_2) v_1^\mu] \quad (\text{A.2})$$

$$\begin{aligned} \langle (q\bar{Q})_{1^-}, \epsilon_2 | V^\mu | (Q, \bar{Q})_{1^-}, \epsilon_1 \rangle &= -\eta_{12}(v_1 \cdot v_2) \sqrt{m_1 m_2} [(\epsilon_1 \cdot \epsilon_2^*) (v_1 + v_2)^\mu - (\epsilon_2^* \cdot v_1) \epsilon_1^\mu \\ &\quad - (\epsilon_1 \cdot v_2) \epsilon_2^{*\mu}] \end{aligned} \quad (\text{A.3})$$

$$\langle (q\bar{Q})_{1^-}, \epsilon_2 | A^\mu | (Q, \bar{Q})_{1^-}, \epsilon_1 \rangle = -\eta_{12}(v_1 \cdot v_2) \sqrt{m_1 m_2} i \epsilon^{\mu\nu\alpha\beta} (v_1 + v_2)_\nu \epsilon_{1\alpha} \epsilon_{2\beta}^* \quad (\text{A.4})$$

where $V^\mu = \bar{q}\gamma^\mu Q$ and $A^\mu = \bar{q}\gamma^\mu\gamma^5 Q$.

In the non-recoil approximation ($v_1 = v_2$) it is readily seen that the first transition (A.1) does not contribute. The remaining three matrix elements lead at once to the set of equations (6).

Let us observe that we are neglecting terms containing factors $v_{1,2} \cdot (v_2 - v_1)$ and $\epsilon_{1,2} \cdot v_{2,1}$ in the differential rate, bounded by:

$$|v_{1,2} \cdot (v_2 - v_1)| \leq \frac{(m_1 - m_2)^2}{2m_1 m_2}, \quad |\epsilon_1 \cdot v_2| \leq \frac{m_1^2 - m_2^2}{2m_1 m_2}, \quad |\epsilon_2^* \cdot v_1| \leq \frac{m_1^2 - m_2^2}{2m_1 m_2}$$

where we have set $q^2 = 0$. This implies an uncertainty in the amplitude of $\simeq 20\%$ for each term neglected compared to the leading one at $q^2 = 0$, and $< 40\%$ in the width after integration over the allowed range of q^2 in s.l. decays.

Moreover, let us point out that in (A.3), for unpolarized quarkonium:

$$\frac{1}{3} \sum_{\lambda_1, \lambda_2} \epsilon_1(\lambda_1) \cdot \epsilon_2^*(\lambda_2) = -1 + O\left[\frac{(m_1 - m_2)^2}{2m_1 m_2}\right]$$

Thereby, taking $v_1 = v_2$ will amount again to an unaccuracy of $< 40\%$ in the final s.l. width.

The hadronic tensors formed from the hadronic amplitudes in the non-recoil approximation after averaging and summing over polarizations are:

$$\begin{aligned} T^{\mu\nu}(1^- \rightarrow 0^-)|_{axial} &\simeq \eta_{12}^2 \frac{4}{3} \frac{m_1 m_2}{m_1 m_2} (-g^{\mu\nu} + v_1^\mu v_1^\nu) \\ T^{\mu\nu}(1^- \rightarrow 1^-)|_{vector} &\simeq \eta_{12}^2 4 \frac{m_1 m_2}{m_1 m_2} v_1^\mu v_1^\nu \\ T^{\mu\nu}(1^- \rightarrow 1^-)|_{axial} &\simeq \eta_{12}^2 \frac{8}{3} \frac{m_1 m_2}{m_1 m_2} (-g^{\mu\nu} + v_1^\mu v_1^\nu) \end{aligned} \quad (\text{A.5})$$

to be contracted with the symmetric part of the (zero mass) leptonic tensor expressed as

$$L_{\mu\nu}^{sym} = 8/3 (q_\mu q_\nu - q^2 g_{\mu\nu}) \text{ in order to get the expressions shown in Eqs. (7) and (8).}$$

B

Defining $x = m_2/m_1$, phase-space integration over q^2 of $\lambda^{3/2}(m_1^2, m_2^2, q^2)$ and $12 m_1^2 q^2 \lambda^{1/2}(m_1^2, m_2^2, q^2)$ between 0 and $(m_1 - m_2)^2$ yields respectively:

$$\begin{aligned} \varphi(x) &= \frac{1}{4} [(1 - x^4)(1 - 8x^2 + x^4) - 24 x^4 \ln x] \\ \phi(x) &= 2 (1 - x^2) [(1 + x^2)^2 + 8x^2] + 24 (1 + x^2) x^2 \ln x \end{aligned}$$

besides a common m_1^8 multiplicative factor. In the limit $x \rightarrow 1$, the leading terms of both are the same except for a factor 2: $\phi(x)/2 \simeq \varphi(x) \simeq 16 (1 - x)^5/5$.

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