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TRANSVERSE ELECTROMAGNETIC FIELDS IN A DETUNED X-BAND ACCELERATING STRUCTURE*

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ABSTRACT

Results are presented of a study of the dipole electromagnetic fields in a detuned accelerating section which are excited by a pointlike bunch. The field-matching technique is used for the field calculations. The transverse coupling impedance, the kick factors, and the wake function are found.

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I. INTRODUCTION

The detuned accelerating section [1] designed to decrease the undesirable effects of the transverse wake field has previously been studied by approximating the structure with a chain of coupled equivalent circuits [2]. The parameters of the equivalent circuits have been obtained using numerical codes for periodic structures built out of different cells of the detuned section. For the Gaussian detuning (i.e., for the Gaussian distribution of the dipole field frequencies) with $\sigma \approx 2.5\%$, the transverse wakefield acting on a bunch travelling at the distance of 40 cm behind the previous one has been shown to be reduced by a factor of ≈ 100 . The high value of the reduction factor demands its accurate evaluation by other independent methods.

Here we present the results of a study of the deflecting dipole ($m = 1$) electromagnetic (EM) fields for the same structure excited by a pointlike charge, using computer code *PROGON* [3]. The code is based on the field-matching technique for the Fourier harmonics of the EM travelling waves. The calculation of the longitudinal monopole ($m = 0$) EM fields in that structure, including discussion of the choice of suitable couplers, can be found in [4,5]. The geometry of the considered section—built out of 204 cells ($N_c = 204$)—can be found in [3].

The present paper consists basically of two parts.

In the first part (Sec. 2), the results of calculation of the deflecting EM fields are presented for a detuned accelerating section.

For an infinitely periodic structure built out of connected rf cavities (cells), each eigenmode of a single cavity produces a passband with the continuum of allowed frequencies. Examples of dispersion curves are given for three different cells taken from the beginning, the middle, and the end of the accelerator section.

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For an aperiodic structure, the concept of the passband has little meaning. But loosely, we will call a passband for the whole structure the passband of one of the cells that is the widest in the frequency domain.

For an accelerating section consisting of a finite number of cells, its passband splits into an equal number of the EM field modes. These modes are found in the whole range of frequencies corresponding to the first passband of the accelerating section. For each mode, including the trapped ones, the field pattern, the group velocity, and the stored energy are calculated. The dipole coupling impedance is then found in this range of frequencies.

The second part of this paper (Secs. 3 and 4) is devoted to discussion of several different ways for evaluating the kick factors and the corresponding wake function. For a large number of cells, the transverse coupling impedance is represented by a large number of narrow interfering resonances. Under such conditions, the choice of the most accurate method to derive the wake function from the transverse coupling impedance, which is found numerically in a given finite number of frequency points, is a nontrivial problem.

A number of different authors have used the field-matching technique to obtain some of these results on various levels of development [6,7]. As we discussed in Sec. 5, our results match most of their results quite well. A computer program is described in [8] that allows calculation of the wake function directly in the time domain. Unfortunately, this program is not suitable for calculations of the wake function at the very long distance behind the bunch.

II. ELECTROMAGNETIC FIELDS AND COUPLING IMPEDANCE

Dispersion Curves

We first demonstrate the validity of the program *PROGON* by calculating the dispersion curves for an infinitely periodic structure corresponding to the three different geometries of the cells of the structure. They are labeled *C* for the cell at the beginning of the section, *D* for the cell in its middle, and *E* for the cell at its end. To facilitate the comparison, their labels and geometry are chosen to coincide with the labels and geometry considered in [2]. Let f be the frequency and ϕ the phase advance per cell of a periodic structure. The iris and cavity radii, and the synchronous f_s frequencies for the first and second passbands for the cells *C*, *D* and *E*, are given in Table 1. In Fig. 1 the solid curves labeled *C*, *D*, and *E* that are reproduced from [2] show the functions $f(\phi)$ obtained by the program *TRANSVRS* [9] for the first dipole passband, and the dashed curves [2] show the same for the second dipole passband. The symbols in Fig. 1 give the results of calculation using the codes *PROGON* (full symbols) and *URMEL* [10] (open circles). In this example, the matrices are truncated at 16 radial modes in a cavity and 30 modes in an iris. *PROGON* is in good agreement with both *TRANSVRS* and *URMEL*. Notice that there is a positive derivative of the dispersion curve *C* and a negative derivative of the dispersion curve *E* in the first passband. Since $df/d\phi$ is proportional to the group velocity, this is a necessary condition for the mode trapping.

Mode Patterns

Next, we illustrate the pattern of the EM field excited by a charge for three different frequencies that correspond to the excitation of the field in the beginning, middle, and end of the accelerating section. In Fig. 2, in the panels *a* and *b*,

respectively, the real and imaginary parts of the average synchronous component of the transverse EM force, i.e., the dipole coupling impedance Z_u , see below Eq. (1), are plotted for each frequency versus the cell number. The trapping of the field is exhibited for the frequencies which roughly correspond to the range 14.80 GHz $< f < 15.97$ GHz; cf., Table 1.

Stored Energy and Group Velocity

After the EM fields in the structure are found, the corresponding stored energy $W(n) = W_E(n) + W_M(n)$ in the n^{th} cell and the power flow $S_z(n)$ through the cavity cross section are found. These quantities define the relative group velocity $v_g(n)/c = S_z d/W$, where d is the cell length. The curves of the stored energy are plotted in panel *d* of Fig. 2 for each cell of the section, for the electric $W_E(n)$ and magnetic $W_M(n)$ stored energy separately. The two quantities are equal, with high accuracy, and are indistinguishable in Fig. 2. This is an indication that the fields are calculated correctly. Panel *c* of Fig. 2 illustrates the behavior of the relative group velocity along the section. The location of the (standing) trapped mode corresponds to the place inside the section where the group velocity is zero (the horizontal part of the curve).

Coupling Impedance

The dipole transverse coupling impedance Z_u is defined as integral of the transverse force from the dipole field excited by a charge Ne with offset r_0 acting on a test particle at the observation point z, r [11]:

$$Z_u(f) = -\frac{i}{eNr_0 \cos \theta} \sum_{n=1}^{N_c} \int_0^d dz e^{ikz} [E_r(z, r) - H_\theta(z, r)]. \quad (1)$$

Here k is the wave number $k = 2\pi f/c$. Each term in the sum is represented by a sum of two integrals—one in which the integration is performed over a cavity

gap of length g , and the other over an iris of thickness l , $d = g + l$. In each such integral, the integration of the $E_r - H_\theta$ can be substituted by an integration of the derivative over r of the longitudinal component of the electric field E_z , using the identity which follows from the Maxwell equations for an axial symmetric structure. For example, the identity for a cavity is

$$ik \int_0^g dz e^{-ikz} [E_r(z, r) - H_\theta(z, r)] = \int_0^g dz e^{-ikz} \frac{dE_z(z, r)}{dr} + E_r(0, r) - E_r(g, r) e^{-ikg}. \quad (2)$$

In the case when the last two terms in this identity are absent or cancel each other, expression (2) is the manifestation of the Panoisky–Wenzel theorem [12]. Expressions (1) and (2) are independent of the observation radius r .

The main contribution to the impedance comes from the modes that are localized inside the section. For most of them, the field extensions to the entrance and to the exit of the section are small, and hence, in a lossless structure, the energy deposited into them by a charge has no escape. In this case, the real part of the impedance is represented by a series of extremely narrow peaks. The frequency position of each of them is difficult to find numerically, and it is even more difficult to evaluate with any reasonable accuracy the area under a peak.

This difficulty can be alleviated by taking into consideration the energy loss either in the walls of the section or in the dielectric filling it. In the first approach rather complicated boundary conditions need to be considered. We choose the second approach: the section filled with a lossy dielectric. In the presence of such a dielectric, each resonance peak becomes lower and wider, preserving at the same time its area. The expressions for the field expansion in an axial symmetric structure filled with a dielectric with a complex constant permittivity ϵ is given in the Appendix.

In our approach, which implements the energy loss in the dielectric, the heights and widths of the resonance peaks have no relation to the actual loss of the EM energy. Nevertheless, all the relevant physical quantities, such as kick factors and wake functions, are expressed as integrals of the impedance and do not depend on the assumed imaginary part of the permittivity γ . This was confirmed numerically by showing that the area under the peaks is the same for different values of γ . A note of caution should be made in respect to the widths of the resonances. We discuss this problem below in Sec. 4.

At the same time, the choice of γ influences the choice of the frequency step size in the impedance calculations. Small γ decreases the needed frequency step and increases the time needed for accurate calculation of the impedance. If γ is chosen too big, the resonance peaks may strongly overlap. This would decrease the accuracy of the evaluation of the kick factors. As a reasonable compromise, we choose the permittivity to be $\epsilon = 1 + 0.5 \cdot 10^{-4} i$ and the frequency step 0.5 MHz.

The real and imaginary parts of the transverse coupling impedance $Z_{\text{tr}}(f)$ for the first dipole passband are presented in Figs. 3(a) and (b). The impedance is represented by a series of sharp peaks separated from each other by ≈ 6 MHz in the middle of the passband. By an order of magnitude this number is the width of the passband divided by the number of cells: $(f_{s1E} - f_{s1C})/N_c$ (cf., Table 1). The peak separation increases toward the edges of the passband.

III. KICK FACTORS

The analytic properties of the coupling impedance of any structure are defined by its poles and cuts in the complex plane of the variable $\omega = f/2\pi$. In the lowest passbands, the most important part of the impedance is given by a sum over poles, while the contribution of the cuts can be neglected. If the energy loss in

the structure is small in comparison to the energy stored in the EM field, the shift of a pole from the real axis is small in comparison to its real value. Each pole produces a Breit-Wigner resonance peak in the impedance. The interference of the neighboring resonances generates an offset of the impedance curve. Hence, the calculated impedance including the offset may be approximated by a sum of the Breit-Wigner terms.

Approximation of Impedance as a Sum of the Breit-Wigner Terms

The r^{th} resonance peak in the calculated transverse impedance $Z_{\text{tr}}(f)$ can be approximated by the Breit-Wigner formula

$$Z_{\text{approx}}^r(f) = i\kappa_r^{BW} \left(\frac{1}{f - f_r + i\Gamma_r} - \frac{1}{f + f_r + i\Gamma_r} \right). \quad (3)$$

Expression (3) with real resonance parameters κ_r^{BW} , f_r and Γ_r has the well known [13,14] symmetry property of the transverse impedance $Z(-f) = -Z^*(f)$, where the star means the complex conjugate value.

The offset in Z_{tr} should be taken out before a peak can be approximated by the Breit-Wigner curve. After that, the Breit-Wigner parameters for each resonance peak may be found, provided that the impedance is calculated for a sufficiently large number of frequency points.

Panel *a* in Fig. 4 illustrates the procedure. The whole frequency range was split into intervals between the subsequent two minima of the calculated function $\text{Re} Z_{\text{tr}}(f)$ (represented by the curve going through the open circles in Fig. 4). Let the frequencies of the minima on both sides of the r^{th} peak in $\text{Re} Z_{\text{tr}}$ be f_{1r} and f_{2r} . Then the difference $\text{Re} Z_{\text{tr}}(f) - \min[\text{Re} Z_{\text{tr}}(f_{1r}), \text{Re} Z_{\text{tr}}(f_{2r})]$, represented by the curve going through the full circles in Fig. 4, in the interval $f_{1r} \leq f \leq f_{2r}$ was approximated by the real part of the Breit-Wigner resonance function Eq. (3)

using the least square method to find the parameters κ_r^{BW} , f_r , and Γ_r . The real part of Z_{approx}^r for these parameters is shown without symbols in panel a of Fig. 4. The imaginary part of Z_{approx}^r (the curve without symbols) is compared with $\text{Im}Z_r$ (represented by a curve going through full circles) in panel b of Fig. 4.

In Figs. 5(a) and (b), the real and imaginary parts of the difference between the calculated impedance Z_r and the sum of the resonance terms $\Sigma_r Z_{\text{approx}}^r$ are plotted for comparison on the same scale as in Figs. 3(a) and (b). The two functions Z_r and $\Sigma_r Z_{\text{approx}}^r$ agree with each other for the whole frequency range. The agreement is slightly worse toward the edges of the passband.

Various Methods to Evaluate Kick Factors

Given the transverse impedance, there are several ways to derive the kick factors κ_r . Each of the methods has its own advantages and drawbacks:

Method 1: In the sum of the Breit-Wigner terms, Eq. (3), the parameter κ_r^{BW} has the meaning of the kick factor for the r^{th} resonance at frequency f_r . The parameter Γ_r is inversely proportional to the quality factor Q_r of the r^{th} resonance: $Q_r = f_r/2\Gamma_r$. For the main part of a passband, this is the most reliable estimate of the kick factors. The accuracy of such an estimate decreases at both edges of the passband, since the peaks $Z_{kr}(f)$ deviate there from the Breit-Wigner shape. In this case, the least square method does not give an accurate evaluation of the kick factor. The magnitude of the kick factors at the edge of the passband is smaller than in its interior, but their distribution in frequency affects the behavior of the wake function at large distances.

Method 2: The kick factor κ_r^{integ} can also be obtained by integration of the real part of the impedance $\text{Re } Z_r^r$ between points f_{1r} and f_{2r} :

$$\kappa_r^{\text{integ}} = 2 \int_{f_{1r}}^{f_{2r}} df \text{Re } Z_r^r(f). \quad (4)$$

Notice that this formula is derived from Eq. (3), but neglects the result of the peaks overlapping. As the consequence, the evaluation of the kick factor using this formula is reliable when the overlapping of the peaks is small.

Method 3: The kick factor can also be derived directly if the dipole EM field $E_z(z, r)$ is known. Define the voltage

$$V(r) = \int_0^L dz e^{-ik_z z} E_z(z, r), \quad (5)$$

where L is the total length of the section. Then calculate function

$$\kappa(f) = \frac{V^*(r_0)/r_0 dV(r)/dr}{4 kW}, \quad (6)$$

where W is the total stored energy for whole section $W = \Sigma_n W(n)$. Function $\kappa(f)$ can be calculated at the same time the impedance is calculated. Figure 3(c) presents this function.

Strictly speaking, formula (6) has been derived [13,14] to give the kick factor for a standing eigenmode of a closed cavity. Contrary to that, the EM modes we are dealing with are not the eigenmodes of the section. Nevertheless, by expanding the solution of nonhomogeneous Maxwell equations into the eigenmodes of the section, it can be shown that this formula gives an approximation to the kick factor κ_r for the r^{th} mode if it is evaluated at $f = f_r$ —i.e., in the vicinity of the r^{th} maxima of the real part of the transverse impedance. Hence, the kick factors κ_r^{WB} can be found by picking up the values of the function $\kappa(f)$ from Eq. (6) at $f = f_r$.

In Fig. 6, the kick factors obtained in three different ways are plotted as functions of frequency. Although, in general, all three curves follow each other, their detailed behavior is different—especially at the edges of the passband.

IV. WAKE FUNCTION

The wake function can be found by using its definition as the Fourier image of the coupling impedance.

Method 1

When the transverse impedance can be represented by the sum of the Breit-Wigner terms, the wake function

$$w_{tr}(s) = -i \int_{-\infty}^{\infty} df e^{-iks} \sum_r Z_{approx}^r(f) \quad (7)$$

is expressed in terms of the Breit-Wigner parameters κ_r , f_r , and Γ_r :

$$w_{tr}(s) = \begin{cases} \sum_r \kappa_r e^{-2\pi\Gamma_r s/c} \sin 2\pi f_r s/c & s \geq 0, \\ 0 & s < 0. \end{cases} \quad (8)$$

Notice that in this case the causality ($w_{tr}(s) = 0$ for $s < 0$) is fulfilled automatically.

Returning to the problem of the uncertainty of the widths of the resonance peaks that directly enter the expression Eq. (8), it looks like they might influence the magnitude of the wake function at a very long distance behind the bunch. But, in practice, such a distance for a detuned accelerating section is much larger than the length of the bunch train, ≈ 40 m in our case. Indeed, for the considered number of cells, the summation in Eq. (8) can be substituted by an integration [2]. (The distance S for such a substitution to be valid should be smaller than $c/\Delta f$, where $\Delta f \approx 6$ MHz is frequency difference between neighboring resonances; i.e., $S < 50$ m). The result of integration can then be represented as a product of two exponential factors: one coming from the relative frequency spread $\sigma/f \approx 2.5\%$, and the other from the field damping $Q = f/2\Gamma$. Evaluating the Q -factor for which the damping could overcome the frequency spread, we get $Q < f/\sigma \approx 40$.

Method 2

On the other hand, if the impedance is known in a sufficiently large frequency range, the wake function can be obtained by direct integration of the impedance, avoiding the necessity of evaluating the kick factors altogether:

$$w_{tr}(s) = 2 \int_0^{\infty} df [\text{Im } Z_{tr} \cdot \cos ks - \text{Re } Z_{tr} \cdot \sin ks]. \quad (9)$$

In this case, it follows from the causality that for $s < 0$

$$\int_0^{\infty} df \text{Im } Z_{tr} \cdot \cos ks = \int_0^{\infty} df \text{Re } Z_{tr} \cdot \sin ks \quad (10)$$

or

$$w_{tr}(s) = 4 \int_0^{\infty} df \text{Im } Z_{tr} \cdot \cos ks. \quad (11)$$

Equation (10) can be employed as a control of the correctness of calculations.

Figure 7 gives the close-range wake function calculated in four different ways, taking into account the impedance of the first passband only. The long-range wake is represented in Fig. 8.

V. CONCLUSIONS

The results obtained by using the program *PROGON*, which is based on the field-matching technique, is in good agreement with the results obtained by means of other codes. In particular, the dispersion curves obtained here agree with those obtained by the code *TRANSVRS*. The transverse coupling impedance agrees with that obtained in [7]. The kick factors are in good agreement with those obtained by applying the equivalent circuit method [2]. General behavior of the wake function calculated in several different ways is also in good agreement with [2].

Nevertheless, the detailed behavior of the wake function is somewhat different for different methods of its calculations. Moreover, the most significant parameter—the magnitude of the wake function reduction for the subsequent bunches—has rather different values for different methods of its calculation. It well may be that the method of calculating the transverse wake function by directly integrating the transverse impedance, Eq. (11), which avoids the calculation of the kick factors altogether, is the most accurate. In this case, indeed, the deflecting field acting on a bunch travelling at the distance of ≈ 40 cm behind the previous one is reduced by a factor of ≈ 100 , as has been found in Ref. [2]; see Fig. 7(c). Further study is needed to ensure reliable results for this quantity.

For evaluation of the wake function, the coupling impedance is needed in an infinite range of frequencies. In other words, the accuracy of the evaluation depends on the amount of the contribution from the higher passbands of the section. In particular, the values of the impedance in the second passband are two orders of magnitude smaller than the impedance in the first passband. On the other hand, the obtained reduction factor is also of the same order of magnitude. Hence, inclusion of the effects of the higher passbands might be important.

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APPENDIX

This appendix gives expressions for the Fourier components of the EM dipole ($m = 1$) fields in the axially symmetric structure of the radius b , filled by a dielectric with a complex constant permittivity ϵ .

Suppose that a charge Ne moves with the velocity of light c along a line parallel to the axis of the structure at distance r_0 . The EM field in the structure is the sum of the field of the charge and the radiation field excited by it. The expressions for the Fourier components at frequency f (and corresponding wave number $k = 2\pi f/c$) of the total fields in the region $r > r_0$ are

$$\begin{aligned} \frac{E_r}{\cos \theta} &= \frac{q}{\epsilon} e^{ikz} \left(\frac{1}{r^2} + \frac{1}{b^2} \right) \\ &+ \sum_{l=1}^{\infty} \left[\lambda_l \frac{dJ_l(\nu r/b)}{dr} y_l(z) - k \frac{J_l(\mu r/b)}{r} \tilde{y}_l(z) \right], \end{aligned} \quad (A.1)$$

$$\begin{aligned} \frac{E_\theta}{\sin \theta} &= \frac{q}{\epsilon} e^{ikz} \left(\frac{1}{r^2} - \frac{1}{b^2} \right) \\ &- \sum_{l=1}^{\infty} \left[\lambda_l \frac{J_l(\nu r/b)}{r} y_l(z) - k \frac{dJ_l(\mu r/b)}{dr} \tilde{y}_l(z) \right], \end{aligned} \quad (A.2)$$

$$\begin{aligned} \frac{E_z}{\cos \theta} &= -i \frac{q}{\epsilon} e^{ikz} k r \left(\frac{1}{r^2} - \frac{1}{b^2} \right) (1 - \epsilon) \\ &+ \sum_{l=1}^{\infty} \frac{\nu_l^2}{b^2} J_l \left(\frac{\nu r}{b} \right) x_l(z), \end{aligned} \quad (A.3)$$

$$\begin{aligned} \frac{H_r}{\sin \theta} &= -q e^{ikz} \left(\frac{1}{r^2} - \frac{1}{b^2} \right) \\ &+ i \sum_{l=1}^{\infty} \left[\epsilon k \frac{J_l(\nu r/b)}{r} x_l(z) - \sigma_l \frac{dJ_l(\mu r/b)}{dr} \tilde{x}_l(z) \right], \end{aligned} \quad (A.4)$$

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$$\begin{aligned} \frac{H_\theta}{\cos \theta} &= q e^{ikz} \left(\frac{1}{r^2} + \frac{1}{b^2} \right) \\ &- i \sum_{l=1}^{\infty} \left[\epsilon k \frac{dJ_l(\nu r/b)}{dr} x_l(z) - \sigma_l \frac{J_l(\mu r/b)}{r} \tilde{x}_l(z) \right], \end{aligned} \quad (A.5)$$

$$\frac{H_z}{\sin \theta} = i \sum_{l=1}^{\infty} \frac{\mu_l^2}{b^2} J_l \left(\frac{\mu r}{b} \right) \tilde{x}_l(z). \quad (A.6)$$

Here $q = 2\pi N e r_0 / c$, J_l is the Bessel function of the first kind and the first order, and $0 < \nu_1 < \nu_2 < \dots < \nu_l \dots < \infty$ are the roots of J_l (excluding 0) in ascending order. Likewise, $\mu_1 < \mu_2 < \dots < \mu_l \dots < \infty$ are the roots of $dJ_l(r)/dr$ in ascending order. The fields (A.1–A.6) satisfy the boundary conditions at the ideally conducting metal surface at $r = b$.

The propagation functions $x_l(z)$, $y_l(z)$ for the TM -modes and $\tilde{x}_l(z)$, $\tilde{y}_l(z)$ for the TE -modes are

$$x_l(z) = C_l^+ e^{i\lambda_l z} + C_l^- e^{-i\lambda_l z}, \quad (A.7)$$

$$y_l(z) = i \left[C_l^+ e^{i\lambda_l z} - C_l^- e^{-i\lambda_l z} \right], \quad (A.8)$$

$$\tilde{x}_l(z) = \tilde{C}_l^+ e^{i\sigma_l z} + \tilde{C}_l^- e^{-i\sigma_l z}, \quad (A.9)$$

$$\tilde{y}_l(z) = i \left[\tilde{C}_l^+ e^{i\sigma_l z} - \tilde{C}_l^- e^{-i\sigma_l z} \right], \quad (A.10)$$

where the constant complex amplitudes C_l^+ , C_l^- , $\tilde{C}_l^+$, and $\tilde{C}_l^-$ are unknown and should be defined from the boundary and continuity conditions at the interface of regions with different radii. Further, the propagation constants for the TM - and TE -modes, respectively, are

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$$\lambda_1 = \sqrt{\epsilon k^2 - \frac{\nu_1^2}{\beta^2}}, \quad (A.11)$$

and

$$\alpha_1 = \sqrt{\epsilon k^2 - \frac{\mu_1^2}{\beta^2}}. \quad (A.12)$$

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TABLE CAPTION

Table 1. Cell geometries for the three periodic structures shown in Figure 1. All cells have iris thickness $l = 0.146$ cm, gap length $g = 0.729$ cm, and period length $d = 0.875$ cm.

Label	a (cm)	b (cm)	$fs1$ (GHz)	$fs12$ (GHz)
C	0.5250	1.112	14.80	19.89
D	0.4625	1.083	15.34	20.26
E	0.4000	1.058	15.97	20.68

FIGURE CAPTIONS

1. The dispersion diagrams, reproduced from [2], for three periodic structures, labeled C , D and E , built of cells with geometries listed in Table 1. The solid curves, labeled C , D and E respectively, show the dispersion curves obtained by the program *TRANSVRS* for the first dipole passband, and the dashed curves show the same for the second dipole passband. The symbols give the results of calculations using the codes *PROGON* (full circle for the cell type C , diamond for the cell type D , and square for the cell type E) and *URMEL* (open circle). The dotted lines correspond to velocity of light c .

2. Three examples of the field pattern in the detuned structure showing the mode trapping. For each frequency shown at the top of the plot the row label means: (a) the real part of the transverse impedance $\text{Re } Z_{tr}(n)$; (b) the imaginary part of the transverse impedance $\text{Im } Z_{tr}(n)$; (c) the relative group velocity in the n^{th} cell $v_g(n)/c$ (the horizontal part of the curves corresponds to the zero group velocity); (d) the stored electric $W_E(n)$ and magnetic $W_M(n)$ energy in the n^{th} cell (both curves coincide).

3. (a) The real part of the transverse coupling impedance $\text{Re } Z_{tr}(f)$ for the first passband, (b) the imaginary part of the transverse coupling impedance $\text{Im } Z_{tr}(f)$ for the first passband, and (c) the kick function $\kappa(f)$ obtained by formula Eq. (6) for the first passband.

4. Illustration of the approximation of one of the impedance peaks by the Breit-Wigner curve Eq. (3). The resonance parameters found for this peak are: $\kappa_{BW} = 497.49 \text{ V/pC/m}$, $\Gamma = 0.31952 \cdot 10^{-3} \text{ GHz}$, the resonance frequency $f = 15.403 \text{ GHz}$, the vertical offset $= 0.10547 \cdot 10^2 \text{ M}\Omega/\text{m}$. The value of the impedance in the vicinity of the maximum of curve $\text{Re } Z_{approx}$ (open

Table 1

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square) is calculated independently by *PROGON*. It is added after the least-square calculation is performed in order to illustrate the accuracy of the approximation.

5. The difference of (a) the real parts of two functions Z_{r} and $\Sigma_r Z'_{\text{approx}}$ [cf., Fig. 3(a)], and (b) the imaginary parts of two functions Z_{r} and $\Sigma_r Z'_{\text{approx}}$ [cf., Fig. 3(b)].
6. Kick factors κ_r obtained in three different ways: [1] curve 1: interpolation of the peaks by the Breit-Wigner formula, Eq. (3); [2] curves 2 and 3: integration of the real parts of Z_{r} , Eq. (4), where two different integration routines give the same results; [3] curve 4: the values of the function $\kappa(f)$ according to Eq. (6) at $f = f_r$ (see text).
7. The absolute values of the close-range wake function calculated in four different ways: [1] panels (a,e) using direct integration of Z_{r} , Eq. (11); and using Equation (8) with the kicks calculated for [2] panels (b,f) from Eq. (3), [3] panels (c,g) from Eq. (6), and [4] panels (d,h) from Eq. (4). In panels e, f, g, h, the same functions are plotted on a scale ten times larger for the vertical axis than in panels a, b, c, d, respectively, to show the reduction of the wake function.
8. The absolute values of the long-range wake function calculated in four different ways: [1] panel (a) using direct integration of Z_{r} , Eq. (11); and using Equation (8) with the kicks calculated for [2] panel (b) from Eq. (3), [3] panel (c) from Eq. (6), and [4] panel (d) from Eq. (4). Notice that the scale for the vertical axis is enlarged to show the growth of the wake function at large distances.

Fig. 1

Fig. 2

Fig. 3(a)

Fig. 3(b)

Fig. 3(c)

Fig. 4

Fig. 5(a)

Fig. 5(b)

Fig. 6

Fig. 7

Fig. 8