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Three-Pion Reactions in Dilepton Production from Hadron Gas

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Abstract

Kinetic theory formulae are derived for the dilepton production rate from processes of the type $1+2+3 \rightarrow l^+l^-$ and $1+2 \rightarrow 3+(l^+l^-)$ in hadronic or partonic systems. The formalism is applied to 3-pion reactions $\pi^+\pi^-\pi^0 \rightarrow \mu^+\mu^-$, $\pi^\pm\pi^0 \rightarrow \pi^\pm\mu^+\mu^-$ and $\pi^+\pi^- \rightarrow \pi^0\mu^+\mu^-$ in a thermalized hadron gas. The dilepton mass spectra are compared to those from previously considered reactions at different temperatures. The three-pion processes amplify the $\omega(783)$ and $\phi(1020)$ peaks in the dilepton mass spectrum. The consequences for using resonance dileptons as a tool for measuring mean lifetime and temperature of hadronic fireballs are discussed.

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I. INTRODUCTION

Unlike-sign lepton pairs are often considered one of the most sensitive probes of the dynamics of nuclear and subnuclear matter in high energy collisions [1–8]. In order to be able to distinguish experimentally among different scenarios, we must have reliable predictions of dilepton yield from each of them [9].

A general formalism for dilepton emission from a quark–gluon plasma in processes with arbitrary numbers of partons in initial and final states was developed in [5]. In recent years substantial progress has been achieved in perturbative calculations of the lepton pair production rate from a quark gluon plasma [10,11]. In some kinematical regions the α_s corrected rate may be higher by several orders of magnitude than the Born term corresponding to QED annihilation $q + \bar{q} \rightarrow l^+ l^-$. The calculations in the first order of α_s include 3-parton processes $q + \bar{q} \rightarrow g + l^+ l^-$, $q + g \rightarrow q + l^+ l^-$, $\bar{q} + g \rightarrow \bar{q} + l^+ l^-$, and $q + \bar{q} + g \rightarrow l^+ l^-$. The bremsstrahlung of lepton pairs in the processes involving four partons has recently been evaluated in the soft photon approximation [12,13].

Most calculations from a hadron gas have been done considering only the $\pi^+ \pi^-$ annihilation. The $K^+ K^-$ and $N \bar{N}$ annihilation was included in [14]. All possible pairwise reactions $P + P \rightarrow l^+ l^-$, $P + V \rightarrow l^+ l^-$, and $V + V \rightarrow l^+ l^-$ between the pseudoscalar $P \equiv \{\pi, K, \bar{K}, \eta, \eta'\}$ and vector $V \equiv \{\rho, K^*, \bar{K}^*, \omega, \phi\}$ mesons have been explored only recently [15].

The most important contribution at low dilepton masses comes from hadron decays. Dalitz decays of η and π^0 were investigated in [16], all possible decays of the type $V \rightarrow P + l^+ l^-$ and $P \rightarrow V + l^+ l^-$ in [15,17]. The basic difference between [16] and [17] on the one side and [15] on the other is the following: The former works calculated the number of dileptons from decays of hadrons which are released from hadronic fireball after it decoupled, whereas the latter considered decays which occur during its existence.

Thermal emission rates for dileptons produced by the bremsstrahlung of participating hadrons were calculated in the soft virtual photon approximation. For neutron–proton collisions it was done in [18], for pion–pion ones in [16] (here only for initial states with one neutral pion) and in [12,17,19]. To our knowledge, three-hadron processes have not been on attention at all.

The purpose of this paper is to present kinetic theory formulae for calculating the dilepton production rate from three-particle processes $1 + 2 + 3 \rightarrow l^+ l^-$ and $1 + 2 \rightarrow 3 + l^+ l^-$ as a function of dilepton mass and momentum, and to assess how important the processes of this kind may be in comparison with the processes considered so far in a thermalized hadron gas. For the sake of simplicity we will consider a truly neutral gas with vanishing chemical potentials, although our basic formulae contain unspecified mean occupation number functions. Because most of our formulae involve multidimensional quadratures, numerical calculations are done by means of Monte Carlo method. It allows extension to cases with non-vanishing chemical potentials easily.

The dilepton production rates are certainly only a part of the whole story. To get the quantities that are more related to what is measured in experiments, they have to be inserted into a model that describes the evolution of heavy ion collisions. Many models of this kind have been used [7,8,11,15,16,20–25]. But only few of them went beyond the framework given by the Born term $q + \bar{q} \rightarrow l^+ l^-$ in a quark–gluon plasma or by the pion annihilation

$\pi^+\pi^- \rightarrow l^+l^-$ in a hadron gas. The merging of “realistic” rates with “realistic” model(s) still remains to be done.

In Section II we use the independent particle approximation of kinetic theory to derive formulae for dilepton production rate in processes of the type $1 + 2 + 3 \rightarrow l^+l^-$ and $1 + 2 \rightarrow 3 + (l^+l^-)$. The formalism is applied to three-pion processes $\pi^+\pi^-\pi^0 \rightarrow \mu^+\mu^-$, $\pi^\pm\pi^0 \rightarrow \pi^\pm\mu^+\mu^-$, and $\pi^+\pi^- \rightarrow \pi^0\mu^+\mu^-$ in Section III. The necessary matrix element is found using the Vector Meson Dominance model and a simple Lagrangian that describes the interactions between vector mesons and pions. Its parameters are fixed by means of experimental data on some decay widths of vector mesons. The differential dilepton rates in M^2 are compared to “standard” reactions at different temperatures. In Section IV the implications for using resonance dileptons as hadronic fireball clock and thermometer are discussed. We conclude with summarizing our main points in Section V. Technicalities are deferred to Appendixes. Appendix A contains an evaluation of the s -constrained thermal phase space integrals, which are useful in calculating the differential dilepton rates from reaction with 2 particles in initial state. The correct quantum statistics (Bose–Einstein and/or Fermi–Dirac) is used and generalization to non-vanishing chemical potentials shown. Appendix B contains some decay width formulae.

II. GENERAL FORMALISM

A. Three body initial states

Let us consider a many particle system and the reaction

$$1 + 2 + 3 \rightarrow l^+ + l^- \quad (2.1)$$

in which the polarizations λ_a, λ_b (to simplify the notation we will use $a \equiv l^+, b \equiv l^-$ in what follows) of leptons and all quantum numbers of incoming particles are fixed (no degeneracy). Following the usual procedure of relating the transition probability to observable quantities (decay rate, cross section) we arrive at the expression for the dilepton production rate (number of dileptons produced in unit volume per unit time) in the form

$$r_{\lambda_a\lambda_b} = \int d\Phi_{123} \frac{d^3p_a}{(2\pi)^3 2E_a} \frac{d^3p_b}{(2\pi)^3 2E_b} (2\pi)^4 \delta(p_1 + p_2 + p_3 - p_a - p_b) |\mathcal{M}_{123 \rightarrow ab}|^2, \quad (2.2)$$

where

$$d\Phi_{123} = W(\mathbf{p}_1, \mathbf{x}; \mathbf{p}_2, \mathbf{x}; \mathbf{p}_3, \mathbf{x}; t) \prod_{i=1}^3 \frac{d^3p_i}{2E_i} \quad (2.3)$$

includes the 3-particle Wigner function of the many particle system. It reduces to the space (momentum) density when integrated over the momentum (space) variables. For a thermalized free gas it is simply

$$W(\mathbf{p}_1, \mathbf{x}; \mathbf{p}_2, \mathbf{x}; \mathbf{p}_3, \mathbf{x}; t) = \prod_{i=1}^3 \frac{1}{(2\pi)^3} f_i(E_i) \quad (2.4)$$

within the space region occupied by the gas and zero elsewhere. The f 's are the Fermi-Dirac or Bose-Einstein mean occupation numbers as appropriate. In one-photon approximation the matrix element for the process (2.1) has the structure (see Fig. 1)

$$\mathcal{M}_{123 \rightarrow ab} = A_\mu(p_1, p_2, p_3, p) B^\mu(p, p_a, p_b), \quad (2.5)$$

where p is the 4-momentum of the virtual photon. The ambiguity in allocating the p -dependent factors to different pieces in (2.5) is resolved by requiring $\mathcal{M}_{123 \rightarrow \gamma^*} = A_\mu \epsilon^\mu(\lambda)$ be the matrix element for producing a massive photon with polarization λ . Summing the rate (2.2) over the lepton polarizations and making use of (2.5) we obtain

$$\sum_{\lambda_a \lambda_b} r_{\lambda_a \lambda_b} = 2\pi \int d\Phi_{123} (-A_\mu A_\nu^*) T^{\mu\nu},$$

where we have introduced the tensor

$$T^{\mu\nu} = \frac{1}{(2\pi)^3} \int \frac{d^3 p_a}{2E_a} \frac{d^3 p_b}{2E_b} \delta(p - p_a - p_b) \sum_{\lambda_a, \lambda_b} (-B^\mu B^{*\nu}).$$

Recalling the explicit form of B^μ

$$B^\mu = -\frac{e}{p^2} \bar{u}(p_b, \lambda_b) \gamma^\mu v(p_a, \lambda_a)$$

it is easy to show that

$$T^{\mu\nu} = \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2} \right) \mathcal{T}(M^2).$$

Here $M^2 = p^2$ is the dilepton mass and

$$\mathcal{T}(M^2) = \frac{\alpha}{3\pi} \frac{M^2 + 2m^2}{M^4} \left(1 - \frac{4m^2}{M^2} \right)^{1/2} \quad (2.6)$$

with m being the lepton mass. Using the gauge invariance relation $A_\mu p^\mu = 0$ and summing over quantum numbers of incoming particles we finish with the total dilepton rate in the form

$$R = 2\pi \mathcal{N} \int d\Phi_{123} \overline{|\mathcal{M}_{123 \rightarrow \gamma^*}|^2} \mathcal{T}(M^2); \quad (2.7)$$

The overscore signifies the sum over the massive photon polarizations and averaging over the same quantum numbers of initial particles which are accounted for in the degeneracy factor $\mathcal{N}$.

The general formula (2.7) can be simplified in the free-gas approximation (2.4) by making explicit use of the spherical symmetry. Then it reads:

$$R = \frac{\mathcal{N}}{16(2\pi)^6} \int \frac{dE_3}{p_3} f_3(E_3) \int dE_2 f_2(E_2) \int dE_1 f_1(E_1) \\ \times \int ds_{23} \int ds_{13} \int d\varphi \overline{|\mathcal{M}_{123 \rightarrow \gamma^*}|^2} \mathcal{T}(M^2). \quad (2.8)$$

The s_{ij} is the square of invariant energy of i -th and j -th particle and φ the angle between $(\vec{p}_1, \vec{p}_3)$ and $(\vec{p}_2, \vec{p}_3)$ planes. The singularity in the first integral is only apparent because the integration regions in both s_{13} and s_{23} shrink linearly with decreasing p_3 . An attempt to proceed further and derive a formula for differential rate in M^2 leads to

$$\begin{aligned} \frac{dR}{dM^2} &= \frac{\mathcal{N}}{4(2\pi)^6} \mathcal{T}(M^2) \int \prod_{i=1}^3 dE_i p_i f_i(E_i) \\ &\times \int d\cos\theta_{23} \int d\cos\theta_{13} D^{-1/2} |\mathcal{M}_{123 \rightarrow \gamma^*}|^2 \end{aligned} \quad (2.9)$$

with

$$\begin{aligned} D &= 4p_1^2 p_2^2 \sin^2 \theta_{13} \sin^2 \theta_{23} - [p_1^2 + p_2^2 + p_3^2 + M^2 \\ &- (E_1 + E_2 + E_3)^2 + 2p_1 p_3 \cos \theta_{13} + 2p_2 p_3 \cos \theta_{23} + 2p_1 p_2 \cos \theta_{13} \cos \theta_{23}]^2. \end{aligned}$$

The boundary of integration region is given by the equation $D = 0$ in conjunction with the ordinary bounds for cosine. Due to the complexity of formula (2.9) it is more convenient to compute dR/dM^2 (and any other differential distribution) by a Monte Carlo method based on Eq. (2.8).

B. Reaction $1 + 2 \rightarrow 3 + (l^+ l^-)$.

To derive the basic equation for dilepton production rate in this reaction we start with

$$\begin{aligned} r_{\lambda_a \lambda_b} &= \int d\Phi_{12} \frac{d^3 p_a}{(2\pi)^3 2E_a} \frac{d^3 p_b}{(2\pi)^3 2E_b} \frac{d^3 p_3}{(2\pi)^3 2E_3} \\ &\times (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_a - p_b) F_3(E_3) |\mathcal{M}_{12 \rightarrow 3ab}|^2, \end{aligned} \quad (2.10)$$

where $d\Phi_{12}$ is a two-particle analogue of (2.3). The function $F_3(E_3)$ describes a possible influence of the medium on the outgoing hadron (Bose-Einstein enhancement or Pauli-Dirac suppression). Following closely the procedure from the previous subsection we get

$$R = 2\pi \mathcal{N} \int d\Phi_{12} \int_{\Omega_3} \frac{d^3 p_3}{(2\pi)^3 2E_3} F_3(E_3) |\mathcal{M}_{12 \rightarrow 3\gamma^*}|^2 \mathcal{T}(M^2). \quad (2.11)$$

In contrary to (2.7) the integration region in $\vec{p}_3$ is bounded by the energy-momentum conservation to $\Omega_3 \equiv \{\vec{p}_3; E_3 < E_1 + E_2 \wedge (p_1 + p_2 - p_3)^2 \geq 4m_l^2\}$. The differential cross section for the binary reaction $1 + 2 \rightarrow 3 + (l^+ l^-)$ is given by

$$\frac{d\sigma_{l^+ l^-}}{dM^2 dt} = \frac{1}{64\pi s |\vec{p}_1^*|^2} |\mathcal{M}_{12 \rightarrow 3\gamma^*}|^2 \mathcal{T}(M^2).$$

As usual, the Mandelstam variables are defined by $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$. The triangle function

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \quad (2.12)$$

enters the formula for the center-of-mass momentum of an incoming particle:

$$|\vec{p}_1^*|^2 = \frac{1}{4s} \lambda(s, m_1^2, m_2^2). \quad (2.13)$$

Expressing the invariant d^3p_3/E_3 by means of the quantities in the centre-of-mass frame of particles 1 and 2 we obtain

$$R = \frac{2}{\pi} \mathcal{N} \int d\Phi_{12} \sqrt{s} |\vec{p}_1^*| \int dM^2 \int dt \frac{d\sigma_{l+l-}}{dM^2 dt} \int d\varphi^* F_3(E_3). \quad (2.14)$$

The energy of outgoing hadron comes out after a little kinematics exercise as (we will use a simplified notation for 3-momentum magnitudes wherever possible in what follows)

$$E_3 = \frac{1}{\sqrt{s}} \left\{ (E_1 + E_2) E_3^* + \frac{p_3^*}{p_1^*} \left[(E_1 E_2^* - E_2 E_1^*) \cos \theta_{13}^* - \sqrt{p_1^2 p_2^2 - (\vec{p}_1 \cdot \vec{p}_2)^2} \sin \theta_{13}^* \sin \varphi^* \right] \right\},$$

where

$$p_3^* = \frac{1}{2\sqrt{s}} \lambda^{1/2}(s, m_3^2, M^2),$$

$$\cos \theta_{13}^* = \frac{1}{2p_1^* p_3^*} (t + 2E_1^* E_3^* - m_1^2 - m_3^2).$$

In the free gas approximation (2.4) this formula reduces to a six-dimensional integral

$$R = \frac{\mathcal{N}}{(2\pi)^5} \int dE_1 f_1(E_1) \int dE_2 f_2(E_2) \int ds p_1^* \sqrt{s} \times \int dM^2 \int dt \frac{d\sigma_{l+l-}}{dM^2 dt} \int d\varphi^* F_3(E_3) \quad (2.15)$$

with $F_3(E_3) = 1 \pm f_3(E_3)$, the upper sign holding for bosons, lower for fermions. Eq. (2.15) can serve as a starting point for Monte Carlo calculations of arbitrary differential rate(s). It may not be convenient for dR/dM^2 in a region where the latter behaves wildly. To get a formula better tailored to this particular distribution we first rewrite Eq. (2.14) into

$$\frac{dR}{dM^2} = \frac{2}{\pi} \mathcal{N} \int ds \sqrt{s} |\vec{p}_1^*| \int d\Phi_{12} \delta[s - (p_1 + p_2)^2] \int dt \frac{d\sigma_{l+l-}}{dM^2 dt} \int d\varphi^* F_3(E_3),$$

and then (in the free gas approximation) make use of the expression (A3) for the s -constrained momentum-space integral (A1). We thus get

$$\frac{dR}{dM^2} = \frac{\mathcal{N}T}{2(2\pi)^5} \int_{s_0}^{\infty} ds \frac{\lambda(s, m_1^2, m_2^2)}{\sqrt{s}} \tilde{K}(\sqrt{s}, m_1, m_2, T) \int_{t_-}^{t_+} dt \frac{d\sigma_{l+l-}}{dM^2 dt} \int_0^{2\pi} d\varphi^* F_3(E_3), \quad (2.16)$$

where $s_0 = \max[(m_1 + m_2)^2, (m_3 + M)^2]$ and $t_{\pm} = m_1^2 + m_3^2 - 2E_1^* E_3^* \pm 2p_1^* p_3^*$. Function $\tilde{K}$ contains a one-dimensional integral. Its concrete form depends on the statistics the initial particles obey. See Appendix A for details.

If we set $F_3(E_3) \equiv 1$ and assume the free gas approximation, from Eq. (2.14) we get the well-known kinetic theory formula for the rate of two-body reactions with no medium influence on final state particles

$$R = \mathcal{N} \int \prod_{i=1,2} \frac{d^3 p_i f_i(E_i)}{(2\pi)^3 E_i} p_1^* \sqrt{s} \sigma(s). \quad (2.17)$$

Similarly, if we replace the quantum statistics by the classical Boltzmann one [$F_3 \equiv 1$, $\tilde{K} \rightarrow K_1(\sqrt{s}/T)$, see Eq. (A5)], the formula (2.16) reduces to Eq. (C9) of [12].

III. THREE PION REACTIONS IN HADRON GAS

A. Formulae and parameters

The 3-pion initial state process which operates in a hadron gas is depicted in Fig. 2. To calculate the matrix element for massive photon production we describe the interaction among π 's, ρ 's, ω 's and ϕ 's by the following Lagrangian (a, b, c are isospin indices)

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \frac{f_{\omega\rho\pi}}{m_\pi} \epsilon_{\mu\nu\rho\sigma} (\partial^\mu \rho_a^\nu)^+ \partial^\rho \omega^\sigma \varphi_a + \frac{f_{\phi\rho\pi}}{m_\pi} \epsilon_{\mu\nu\rho\sigma} (\partial^\mu \rho_a^\nu)^+ \partial^\rho \phi^\sigma \varphi_a \\ & + i f_{\rho\pi\pi} \varphi_a^+ \rho_b^\mu \partial_\mu \varphi_c \epsilon_{abc} + \text{h.c.}, \end{aligned} \quad (3.1)$$

where $\rho_{b,\mu}$, ω_μ , ϕ_μ and φ_a represent ρ , ω , ϕ and π fields, respectively. The f 's are dimensionless coupling constants. The pion mass difference is neglected ($m_\pi = 0.138 \text{ GeV}/c^2$).

The interaction with the electromagnetic field is introduced by the Vector Meson Dominance model Lagrangian

$$\mathcal{L}_{\text{elmg}} = -\frac{eg_{\rho\gamma}}{2} m_\rho^2 A^\mu \rho_\mu^0 - \frac{eg_{\omega\gamma}}{2} m_\omega^2 A^\mu \omega_\mu - \frac{eg_{\phi\gamma}}{2} m_\phi^2 A^\mu \phi_\mu. \quad (3.2)$$

We use the notation of [26], Eq. (2.4). Above Lagrangians were successfully used already in the sixties to describe the decays of mesons. Let us only mention the recent discovery of the decay mode $\rho^0 \rightarrow \pi^+ \pi^- \gamma$ [27] with the branching ratio in nice agreement with what was calculated [28] more than 20 years before the experiment.

Our approach to the vector meson sector is simple. We ignore the fact that the $\rho^0 - \omega - \phi$ mixing may influence various processes in a different way. All vector meson states we deal with are assumed to be “physical”. This simplification is to a good level of accuracy justified by the experimental data on $e^+ e^- \rightarrow \pi^+ \pi^- \pi^0$, which does not allow, even in conjunction with $\pi\gamma$ and $K_L^0 K_S^0$ final states, to distinguish significantly among various scenarios [29].

The generic diagram for the process $\pi^+ \pi^- \pi^0 \rightarrow \gamma^*$ (Fig. 2) represents three diagrams which differ by the charge of the virtual ρ meson. All internal momenta are time-like, so the mass m_V ($V = \rho, \omega$, and ϕ) in vector meson propagators is replaced by a complex quantity $m_V - i\Gamma_V/2$. The final expression for the matrix element of $\pi^+ \pi^- \pi^0 \rightarrow \gamma^*$ is

$$\begin{aligned}
|\overline{\mathcal{M}_{123 \rightarrow \gamma^*}}|^2 &= \frac{\pi\alpha}{m_\pi^2} \left[s_{12}s_{13}s_{23} - m_\pi^2 (M^2 - m_\pi^2)^2 \right] \left| \sum_{i < j} \frac{1}{s_{ij} - (m_\rho - i\Gamma_\rho/2)^2} \right|^2 \\
&\times \left| \sum_{V=\omega, \phi} \frac{m_V^2 G_V}{M^2 - (m_V - i\Gamma_V/2)^2} \right|^2
\end{aligned} \tag{3.3}$$

with $s_{ij} = (p_i + p_j)^2$ and $G_V = f_{\rho\pi\pi} f_{V\rho\pi} g_{V\gamma}$. The matrix elements for the processes $\pi^+\pi^- \rightarrow \gamma^*\pi^0$, $\pi^+\pi^0 \rightarrow \gamma^*\pi^+$ and $\pi^-\pi^0 \rightarrow \gamma^*\pi^-$ (they are all equal) can be obtained from (3.3) by crossing symmetry and making the ρ -propagators for space-like virtual momenta (t and u channels) real.

The two coupling constant products G_ω and G_ϕ which enter Eq. 3.3 were determined in the following way (see the Appendix B for formulae):

- The experimental information [30] on the widths of decays $\omega \rightarrow 3\pi$ and $\phi \rightarrow 3\pi$ with Eqs. (B2) and (B3) were used to get $(f_{\omega\rho\pi} f_{\rho\pi\pi})^2 = 156.6 \pm 2.1$ and $(f_{\phi\rho\pi} f_{\rho\pi\pi})^2 = 0.498 \pm 0.036$.
- The direct electromagnetic decays $\omega \rightarrow e^+e^-$ and $\phi \rightarrow e^+e^-$ were utilized to fix the values of $g_{\omega\gamma}^2 = 0.0138 \pm 0.0005$ and $g_{\phi\gamma}^2 = 0.0241 \pm 0.0009$ by means of Eq. (B4).
- Combining the above numbers we arrived at $G_\omega^2 \equiv (f_{\rho\pi\pi} f_{\omega\rho\pi} g_{\omega\gamma})^2 = 2.16 \pm 0.08$ and $G_\phi^2 \equiv (f_{\rho\pi\pi} f_{\phi\rho\pi} g_{\phi\gamma})^2 = 0.012 \pm 0.001$.
- To determine the relative sign of G_ω and G_ϕ we calculated the decay width (B6) of the radiative decay $\rho \rightarrow \pi\gamma$ (in our simple approach it is charge blind). The squares of necessary coupling constant products $f_{\omega\rho\pi} g_{\omega\gamma}$ and $f_{\phi\rho\pi} g_{\phi\gamma}$ were obtained by dividing G_ω^2 and G_ϕ^2 by $f_{\rho\pi\pi}^2 = 36.6 \pm 0.3$ (B7). They are $(5.90 \pm 0.22) \times 10^{-2}$ and $(3.82 \pm 0.32) \times 10^{-4}$, respectively. Under the assumption that the relative sign of the coupling constant products is positive (negative) we got $\Gamma_{\rho \rightarrow \pi\gamma} = (112 \pm 4)$ keV $[(82.9 \pm 3.6) \text{ keV}]$. The negative relative sign fits better the experimental values ($\Gamma_{\rho^\pm \rightarrow \pi^\pm \gamma} = 120 \pm 30$ keV, $\Gamma_{\rho^0 \rightarrow \pi^0 \gamma} = 68 \pm 8$) keV. This is in agreement with what is expected from the SU(3) symmetry [26]. The remaining discrepancy offers an estimate of the systematic error of our approach less than 10%.
- To make one more check of the soundness of our approach represented by the Lagrangians (3.1, 3.2) we calculated $r = f_{\omega\rho\pi}^2 / f_{\phi\rho\pi}^2$ from the experimental value of $\Gamma_{\omega \rightarrow 3\pi} / \Gamma_{\phi \rightarrow 3\pi}$ (B5) on the one hand and from that of $\Gamma_{\omega \rightarrow \pi^0 \gamma} / \Gamma_{\phi \rightarrow \pi^0 \gamma}$ (B2, B3) on the other. We got $r = 314 \pm 23$ in former case and $r = 286 \pm 34$ in the latter.

Now we are ready to calculate the dilepton yield from the subprocess $\pi^+\pi^-\pi^0 \rightarrow l^+l^-$. We evaluate the integral (2.8) by Monte Carlo method, mapping first the integration region into the unit six-dimensional cube by means of the following substitutions ($i = 1, 2, 3$):

$$\begin{aligned}
\xi_i &= \exp\left(\frac{m_\pi - E_i}{T'}\right), \\
\xi_4 &= \frac{1}{2}(1 - \cos\theta_{13}),
\end{aligned}$$

$$\begin{aligned}\xi_5 &= \frac{1}{2}(1 - \cos \theta_{23}), \\ \xi_6 &= \varphi - 2\pi).\end{aligned}\tag{3.4}$$

The parameter $T' > T$, where T is the actual temperature of the hadron gas, was introduced in order to lower the Monte Carlo statistical errors in the high dilepton mass region. The results obtained with different values of T' are, of course, mutually compatible. We finally used $T' = 3T$. With matrix element (3.3) it would perhaps be possible to perform at least some of the integrations analytically. We did not do it in order not to lose the great advantage of the Monte Carlo method – a possibility to evaluate simply any differential distribution by binning the integrand according to values of respective variable(s) [31]. We calculated dR/dM^2 for three different temperatures: $T = 100, 150$, and 200 MeV.

B. Reference processes

In order to assess the importance of the processes considered in this paper we have to compare them to some “standards”. As the first standard we have chosen the process $\pi^+\pi^- \rightarrow l^+l^-$ with a simple ρ pole parametrization [18] for the pion form factor. The two-pion annihilation with this or similar one-pole parametrizations has been used very often as the only reaction producing dileptons in the hadron gas. On the other hand, in Ref. [15] all possible pairwise reactions among the members of the pseudoscalar and vector nonets ($P + P \rightarrow l^+l^-$, $V + V \rightarrow l^+l^-$, $V + P \rightarrow l^+l^-$) were considered. The total yield from those processes will be our second standard.

In any calculation of dilepton yield from hadronic reactions or decays the issue of formfactors is extremely important. Let us only note that in the case of pion annihilation the pion formfactor [32] that describes correctly the e^+e^- annihilation data also in the region of higher ρ recurrences [33] gives dilepton yield at $M = 2.5$ GeV/c² 6.5 times higher than a single pole approximation [18].

In [15] the formfactors [32] based on experimental data and formulae with correct analytic behavior were used wherever possible. In this work the same choice of formfactors is made except for the reaction $\pi + \rho \rightarrow \mu^+\mu^-$, where the kaon isoscalar factor used in [15] is replaced by a linear combination of ω and ϕ poles in order to be consistent with the formalism used for the three-pion reactions. It leads to the cross section

$$\sigma_{\pi\rho \rightarrow l^+l^-}(s) = \frac{\alpha\pi^2}{3m_\pi^2} |\vec{p}_\pi^*| \sqrt{s} \left| \sum_{V=\omega,\phi} \frac{m_V^2 f_{V\rho\pi} g_{V\gamma}}{s - (m_V - i\Gamma_V/2)^2} \right|^2 \mathcal{T}(s).\tag{3.5}$$

In this reaction, and also in the $K\bar{K}$ annihilation, the ϕ resonance region lies very near to threshold. It makes the Monte Carlo calculations a rather delicate matter. We therefore used an alternative method. It corresponds in spirit to the approach used in [18] but allows different masses of incoming particles and relaxes the Boltzmann occupation number approximation. To derive the relevant formula we start with (2.17). We are now considering processes in which two incoming hadrons convert to an l^+l^- pair. Their invariant energy is therefore equal to the dilepton mass and we can write in a free gas approximation

$$\frac{dR_{12 \rightarrow l+l^-}}{dM^2} = \frac{\mathcal{N}}{2} \lambda^{1/2}(M^2, m_1^2, m_2^2) \sigma_{12 \rightarrow l+l^-}(M^2) \int \prod_{i=1,2} \frac{d^3 p_i}{(2\pi)^3 E_i} f_i(E_i) \delta[M^2 - (p_1 + p_2)^2].$$

For the momentum-space integral we use what is derived in Appendix A. We arrive at

$$\frac{dR_{12 \rightarrow l+l^-}}{dM^2} = \frac{\mathcal{N}}{2(2\pi)^4} \frac{T}{M} \lambda(M^2, m_1^2, m_2^2) \tilde{K}(M, m_1, m_2, T) \sigma_{12 \rightarrow l+l^-}(M^2). \quad (3.6)$$

In the Boltzmann occupation numbers approximation and with equal masses of incoming particles the formula (3.6) becomes identical with Eq. (17) in [18]. The numerical analysis showed that the quantum statistics effects are important at low dilepton masses. Let us quote as an example that for the $\pi^+\pi^-$ annihilation at $T = 150$ MeV they increase the dilepton yield by a factor of 1.62, 1.19, and 1.01 at $M = 0.3, 0.7$, and 1.7 GeV/ c^2 , respectively.

On the basis of Eq. (3.6) we recalculated the differential rates in dilepton mass squared for all processes considered in [15]. The deviations from Monte Carlo results [15] were largest for the $K\bar{K}$ annihilation in the ϕ region (up to 30%). They were smaller in the $\pi\rho$ case and negligible for the other reactions.

C. Results

We compare the dilepton mass spectrum from the 3-pion processes with those from other reactions at temperatures $T = 100, 150$ and 200 MeV. The qualitative features are similar at all temperatures. We therefore detail only the results for $T = 150$ MeV.

In Fig. 3 the differential rate in M^2 for dimuons from the $\pi^+\pi^-\pi^0$ annihilation (solid curve) and $\pi\pi \rightarrow \mu^+\mu^-\pi$ reaction (dashed) are compared with the “standard” two-pion annihilation in a single pole approximation (dotted). At low masses ($M \lesssim 0.7$ GeV/ c^2) the reaction with a pion in final state is more efficient than the 3-pion annihilation, at high masses ($M \gtrsim 1.1$ GeV/ c^2) they reverse the roles. This can be easily understood. In the former reaction only two pions bring energy, which is then shared by dilepton and outgoing pion. In the 3-pion annihilation the energy income is larger and the whole invariant energy transforms into dilepton mass. This also makes it superior to 2-pion annihilation at high masses. The crossover is located at $M \approx 1.4$ GeV/ c^2 not only for $T = 150$ MeV but for all three temperatures. The most interesting feature of Fig. 3 is the presence of sharp ω and ϕ peaks, which are absent in the 2-pion annihilation.

The contribution of the process $\pi\pi \rightarrow l^+l^-\pi$ to the peaks rises faster with the temperature than that of the 3-pion annihilation (see last two columns of Table I for a detailed study of the ω peak). The reason lies again in former reaction being more energy deficient than the latter.

The differential rates from both 3-pion processes are summed and displayed in Fig. 4 by the solid curve. Besides the 2-pion annihilation (same as in Fig. 3, dotted) we show also (dashed) the sum of differential rates from all reactions considered in Ref. [15]: $P+P, P+V, V+V \rightarrow \mu^+\mu^-$ [P (V) denotes the pseudoscalar (vector) nonet]. In comparison to the latter, the three-pion processes give negligible contribution everywhere despite the ω peak (where no other reaction contributes) and partly also the ϕ peak.

The complete differential rate (sum of all pairwise reactions considered in [15] and the 3-pion processes) for three different temperatures is depicted by solid curves in Fig. 5, again accompanied by a simple $\pi^+\pi^-$ annihilation (dotted). The comparison clearly shows how insufficient it is to consider only $\pi^+\pi^-$ annihilation when calculating dilepton production rate from a hadron gas in the continuum region above the ϕ peak. But this is exactly the mass region that was suggested as convenient for thermal dilepton observation long time ago [2]. Fig. 5 also shows that the ω and partly also ϕ peak become more pronounced with rising temperature. This is due to three-pion processes, as we have already discussed.

IV. CALIBRATING THE HADRONIC FIREBALL CLOCK AND THERMOMETER

A method has recently been proposed [34] for measuring hadronic fireball lifetimes by comparing the dilepton yield under the ρ peak to the mean multiplicity of produced ρ mesons (as seen *via* their $\pi\pi$ decay mode). In a less explicit way the main idea appeared also in [35]. The method is based on the fact that the mean number of direct decays $\rho^0 \rightarrow l^+l^-$ that occur during the lifetime τ of a thermalized hadronic fireball is proportional to τ . The ω and ϕ peaks are less convenient because longer lifetimes are required to enhance the dilepton yield well above the level expected from the ω and ϕ mean multiplicities and their dilepton branching ratios.

The inclusion of vector meson lifetime dilatation (not considered in [34]) makes the method dependent on the temperature history of fireball. The multiplicative correction factor for dilepton yield is $\langle m_V/E_V \rangle$. For ρ mesons and $T = 100, 150$ and 200 MeV it has the value 0.833, 0.767 and 0.710, respectively [36].

In [34] it has been assumed that the direct decays are the only sources of dileptons that are peaked in the corresponding mass regions.

Recently, several reactions have been shown [15] to contribute to the ϕ peak. They are: $K^+K^- \rightarrow l^+l^-$, $K^0\bar{K}^0 \rightarrow l^+l^-$, and $\pi\rho \rightarrow l^+l^-$. None of the reactions with participation of only two hadrons can contribute to the ω peak. The reason is either isospin and G-parity conservation ($\pi^+\pi^- \rightarrow l^+l^-$) or higher threshold (reaction listed above, $\eta\omega \rightarrow l^+l^-$, $K^*\bar{K}^* \rightarrow l^+l^-$, and so on).

In this work we have learned that the three-pion reactions ($\pi^+\pi^-\pi^0 \rightarrow l^+l^-$, $\pi\pi \rightarrow l^+l^-\pi$) contribute to the ϕ and ω peaks. The latter contribution is especially important, because it is the only additional source of resonance dileptons in that region. We will show that a strong dependence of the dilepton yield from the 3-pion processes on temperature makes the ω peak a sensitive tool for measuring the (average) fireball temperature.

To simplify the terminology we will denote as “trivial” the dileptons coming from the direct decays of free-streaming vector mesons after their freeze-out. Because the dilepton branching ratios $B_{V \rightarrow l^+l^-}$ are small, the expected mean number of dileptons $\langle N_t \rangle$ originating from the decays $V \rightarrow l^+l^-$ can be calculated as the mean number of V 's (as seen *via* their hadronic decay modes) multiplied by $B_{V \rightarrow l^+l^-}$. On the theoretical side, we can introduce $\langle n_t \rangle = d\langle N_t \rangle/d^3x$ as the mean number of dileptons per unit volume of the fireball that released those V 's after breaking up. In the free gas approximation we have

$$\langle n_t \rangle = 3B_{V \rightarrow e^+e^-} \int \frac{d^3p}{(2\pi)^3} f(E_V) . \quad (4.1)$$

The “additional” dileptons are produced during the lifetime of a hadronic fireball. Their presence violates the relation between the mean number of dileptons in peak and the mean number of V ’s. If $R(t, \vec{x})$ is the dilepton production rate in the considered mass range then the mean number of additional dileptons creating during the lifetime of the fireball is

$$\langle N_a \rangle = \int_0^\tau dt \int_{\Omega(t)} d^3x R(t, \vec{x}). \quad (4.2)$$

In principle, one can imagine two sources of additional dileptons: (i) direct decays of resonances; (ii) reactions which are peaked in corresponding dilepton mass regions. There is a strong argument (at least in the case of $\rho^0 \rightarrow l^+l^-$ versus $\pi^+\pi^- \rightarrow l^+l^-$) that these two sources are equivalent and correspond not to two distinct real processes but to two alternative ways of describing the same phenomenon [17]. The coincidence of dilepton rates from direct decays and those from $1+2 \rightarrow e^+e^-$ reactions in the ρ and ϕ peak regions seems to support this view. But the three-pion processes represent definitely an independent source of dileptons. The corresponding rates have a steeper temperature dependence than the rates from direct decays and in the ω peak the former exceeds the latter substantially (≈ 14 times at $T = 200$ MeV).

The rate of additional dileptons from direct decays is given in the free gas approximation by

$$R_{V \rightarrow l^+l^-} = 3\Gamma_{V \rightarrow l^+l^-} \int \frac{d^3p_V}{(2\pi)^3} \frac{m_V}{E_V} f(E_V) . \quad (4.3)$$

The contribution to the additional dileptons rate from a specific reaction can be calculated either by Monte Carlo method [Eq. (2.8) or (2.15)] with rejection of “events” with dilepton masses outside the considered peak or by numerical integration of differential rate (3.6).

The number of additional dileptons depends on the space-time-temperature evolution of the hadronic fireball. Not to burden the basic principle with technical details we will illustrate it using the simplest possible scenario, in which the temperature T and volume V of a free-gas fireball do not vary during its lifetime τ .

The results obtained in this simple model are presented in Table II. The $\langle n_t \rangle$ in its third column denotes the mean number of trivial dielectrons that are produced by decays of vector mesons emitted at freeze-out from a unit volume of an equilibrium hadron gas, calculated using Eq. (4.1). The next three columns contain the slope of the lifetime dependence of the additional-to-trivial dielectron ratio, which is independent of the fireball volume. First of them concerns the additional dielectrons from direct decays inside the fireball, second those originating from various reactions. To avoid a possible double counting we quote only sums of those two contributions to the ω peak, where it is clear that they represent independent sources. But the presence of an independent three-pion component can be noticed also in the ϕ peak. It manifests itself by the $\langle N_a \rangle / \langle N_t \rangle$ slope from reactions not decreasing with temperature, in contrary to that from direct decays.

The most salient feature of Table II is the copious production of dielectrons in the ω mass region by three-pion reactions. For a fireball with lifetime $\tau = 10$ fm/c and temperature

$T = 200$ MeV the yield of dielectrons is enhanced by a factor of 4.3 above the level expected on the basis of the number of produced ω mesons. At this temperature the number of dielectrons in ρ and ω peaks should be roughly equal, their ratio (but not the individual magnitudes) almost independent of the fireball lifetime. But the enhancement of the ω peak drops sharply with decreasing temperature, whereas that of the ρ slightly rises. The analysis of the enhancement (if observed) of the ρ and ω peaks in dilepton mass spectrum thus offers a unique opportunity to measure simultaneously the lifetime and temperature of hadronic fireballs. To avoid any confusion, let us stress that we mean the enhancement in the number of dileptons relative to what is expected on the basis of direct dilepton decay branching ratios and multiplicities of vector mesons, as seen in their hadronic decay channels.

In the case of the broad ρ^0 peak the separation of “resonance” and “background” is a little ambiguous. In theoretical calculations we can subtract a pure QED $\pi^+\pi^-$ annihilation rate from the vector meson dominance formula with a simple ρ pole. The estimates presented in Table II were obtained in that way.

If enhancement of the resonance dilepton production is observed experimentally for the ρ and ω peaks separately, the mean temperature and lifetime of hadronic fireballs can be determined. The ω/ρ ratio in additional dilepton yields depends only on the fireball temperature. The latter can thus be extracted by comparison with expectations like those presented in the second column of Table III (where only additional dileptons from direct decays were included in the ρ peak). Once the mean temperature is known, the mean lifetime can be estimated by dividing the measured additional-to-trivial ratio, in either peak, by the corresponding slope from Table II.

Separating the ρ and ω peaks in experiment is a difficult task. If only unseparated data is available, neither of the two fireball parameters can be determined on the basis of it. This is in contrary to what was proposed in [34], where the temperature dependence of additional-to-trivial slopes in lifetime was neglected. The mean fireball temperature must be first estimated from other phenomenon, for example from transverse spectra of dileptons [37]. Alternatively, one may assume that it does not differ very much from the freeze-out temperature, which is related to transverse momentum (or transverse mass) spectra of hadrons.

Let us finally note that the 3-pion processes manifest themselves by the additional-to-trivial ratio in the $\rho - \omega$ region that rises with temperature. If only additional dileptons from direct decays were considered (as in Table II), the trend would be just opposite not only for the corporate peaks, but also for all individual ones. The sharply changing ω/ρ ratio for additional dileptons (the 2nd column in Table III) is again caused by the 3-pion processes.

V. CONCLUSIONS

It is obvious that the three-pion processes play a more important role in producing dileptons from dense hadronic systems than it has been anticipated so far. Comparing to $\pi^+\pi^-$ annihilation, which has frequently been considered as the only source of dileptons from hadronic gas, the yield from the three-pion annihilation decreases less slowly with increasing dilepton mass. The crossover is located at approximately $1.4 \text{ GeV}/c^2$. Beyond it the 3-pion annihilation takes over above the 2-pion one. But the presence of other 2-hadron reactions

[15] diminishes the role of three-pion processes in the continuum mass region even at the highest dilepton masses considered in our calculations (3 GeV/c²).

Three-pion processes play a unique and exceptional role in amplifying the ω peak in dilepton mass spectrum, where none of other processes can contribute. It can provide a signature of the presence of hadronic fireballs. Because the processes with three pions have a different temperature dependence than the two-body processes, simultaneous measurement of ρ and ω enhancements can determine both mean lifetime and mean temperature of hadronic fireballs. Three-pion processes also amplify the ϕ peak in dilepton spectrum, but only marginally.

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APPENDIX A: *S*-CONSTRAINED MOMENTUM-SPACE INTEGRALS FOR BOSONS AND FERMIONS

Let us calculate the following integral

$$\Phi(s) = \int \prod_{i=1,2} \frac{d^3 p_i f_i(E_i)}{(2\pi)^3 E_i} \delta[s - (p_1 + p_2)^2], \quad (\text{A1})$$

where f_i 's denote mean occupation numbers for particles 1 and 2. The spherical symmetry in momentum space allows us to perform three of the integrations immediately. The δ function is used to remove the integral over $\cos \theta_{12}$. We thus get

$$\Phi(s) = \frac{1}{(2\pi)^4} \int_{m_1}^{\infty} dE_1 f_1(E_1) \int_{E_2^-}^{E_2^+} dE_2 f_2(E_2),$$

where

$$E_2^{\pm} = \frac{1}{2m_1^2} \left[E_1 (s - m_1^2 - m_2^2) \pm p_1 \lambda^{1/2}(s, m_1^2, m_2^2) \right].$$

The triangle function λ has been defined previously (2.12). Integration over E_2 can be done analytically for both Bose-Einstein and Fermi-Dirac functions, leaving us with the final result in a form of integral which must be evaluated numerically. In order to be able to study the Boltzmann limit (high $\sqrt{s}/T$) easily, we will rather perform first the substitutions $u = E_1 + E_2$ and $v = E_1 - E_2$, which lead to

$$\int_{m_1}^{\infty} dE_1 \int_{E_2^-}^{E_2^+} dE_2 \cdots = \frac{1}{2} \int_{\sqrt{s}}^{\infty} du \int_{v_-}^{v_+} dv \cdots$$

The integration boundaries are given by

$$v_{\pm} = \frac{1}{s} \left[(m_1^2 - m_2^2) u \pm \lambda^{1/2}(s, m_1^2, m_2^2) \sqrt{u^2 - s} \right]. \quad (\text{A2})$$

The integral over v can be expressed in terms of elementary functions. A little algebra leads to

$$\Phi(s) = \frac{T}{(2\pi)^4} \sqrt{\frac{\lambda(s, m_1^2, m_2^2)}{s}} \tilde{K}(\sqrt{s}, m_1, m_2, T). \quad (\text{A3})$$

The function $\tilde{K}$ assumes different forms for different statistics. In the case that both particles are bosons, it is given by

$$\begin{aligned} \tilde{K}_{BB}(\sqrt{s}, m_1, m_2, T) &= \sqrt{\frac{s}{\lambda(s, m_1^2, m_2^2)}} \int_{\sqrt{s}}^{\infty} \frac{du}{\exp(u/T) - 1} \\ &\times \ln \left[\sinh \frac{u + v_+}{4T} \sinh \frac{u - v_-}{4T} \left(\sinh \frac{u + v_-}{4T} \sinh \frac{u - v_+}{4T} \right)^{-1} \right]. \end{aligned} \quad (\text{A4})$$

Distribution of various factors between the above function and the rest of the formula (A3) may seem a little unnatural. The reason for it becomes clear if we consider the high $\sqrt{s}/T$ limit. Then we can neglect the 1 in the denominator of the first fraction in (A4) and replace the hyperbolic sinuses by exponentials. Argument of the logarithm thus becomes $\exp[(v_+ - v_-)/2T]$. Using Eq. (A2) we conclude that for $\sqrt{s}/T \rightarrow \infty$

$$\tilde{K}_{BB}(\sqrt{s}, m_1, m_2, T) \rightarrow \frac{T}{\sqrt{s}} \int_{\sqrt{s}/T}^{\infty} \exp(-y) \sqrt{y^2 - \frac{s}{T^2}} dy \equiv K_1\left(\frac{\sqrt{s}}{T}\right), \quad (\text{A5})$$

where K_1 is the modified Bessel function. It says that to account for quantum statistics effects in some formulae which were derived (*e.g.* in [12,18]) assuming classical Boltzmann statistics one has to replace the modified Bessel function K_1 by the corresponding function $\tilde{K}$. If both particles are fermions, then

$$\begin{aligned} \tilde{K}_{FF}(\sqrt{s}, m_1, m_2, T) &= \sqrt{\frac{s}{\lambda(s, m_1^2, m_2^2)}} \int_{\sqrt{s}}^{\infty} \frac{du}{\exp(u/T) + 1} \\ &\times \ln \left[\cosh \frac{u + v_+}{4T} \cosh \frac{u - v_-}{4T} \left(\cosh \frac{u + v_-}{4T} \cosh \frac{u - v_+}{4T} \right)^{-1} \right]. \end{aligned} \quad (\text{A6})$$

If particle 1 is fermion and particle 2 boson we get

$$\begin{aligned} \tilde{K}_{FB}(\sqrt{s}, m_1, m_2, T) &= \sqrt{\frac{s}{\lambda(s, m_1^2, m_2^2)}} \int_{\sqrt{s}}^{\infty} \frac{du}{\exp(u/T) + 1} \\ &\times \ln \left[\cosh \frac{u + v_+}{4T} \sinh \frac{u - v_-}{4T} \left(\cosh \frac{u + v_-}{4T} \sinh \frac{u - v_+}{4T} \right)^{-1} \right]. \end{aligned} \quad (\text{A7})$$

The high $\sqrt{s}/T$ limit of $\tilde{K}_{FF}$ and $\tilde{K}_{FB}$ is obviously $K_1(\sqrt{s}/T)$ again. Eqs. (A4) and (A6) do not change if $v_+ \leftrightarrow -v_-$. This is a consequence of the $m_1 \leftrightarrow m_2$ symmetry [see Eq. (A2)]. From the practical point of view, the integrals that enters our formulae for $\tilde{K}$'s can safely be handled, after the substitution $x = (u - \sqrt{s})/T$ by the 8-point Gauss quadrature with the weight $\sqrt{x} \exp(-x)$ [38]. The Gauss-Laguerre quadrature, although better known, is inconvenient here because of the $\sqrt{x}$ -like behavior of the integrand.

Generalization to the case of nonzero chemical potentials can simply be done by the following substitutions:

$$\begin{aligned} u &\rightarrow u + \mu_1 + \mu_2, \\ u + v_{\pm} &\rightarrow u + v_{\pm} + 2\mu_1, \\ u - v_{\pm} &\rightarrow u - v_{\pm} + 2\mu_2 \end{aligned}$$

in equations (A4), (A6), and (A7).

APPENDIX B: SOME DECAYS OF VECTOR MESONS

From the general expression for the three particle decay width of an unpolarized vector meson with mass m_V

$$\Gamma_{V \rightarrow 123} = \frac{1}{2m_V} \int (2\pi)^4 \delta(p_V - p_1 - p_2 - p_3) |\overline{\mathcal{M}_{V \rightarrow 123}}|^2 \prod_{i=1}^3 \frac{d^3 p_i}{2E_i (2\pi)^3} \quad (\text{B1})$$

we get after a little manipulation

$$\begin{aligned} \Gamma_{V \rightarrow \pi^+ \pi^- \pi^0} &= \frac{1}{3(4\pi)^3 m_V} \int_{m_\pi}^{m_V - 2m_\pi} dE_1 \int_{m_\pi}^{m_V - m_\pi - E_1} dE_2 \\ &\quad \Theta(1 - |C(E_1, E_2)|) \sum_{\lambda} |\mathcal{M}_{V \rightarrow \pi^+ \pi^- \pi^0}|^2. \end{aligned} \quad (\text{B2})$$

The argument of the step function contains the expression

$$C(E_1, E_2) = \frac{m_V^2 + m_\pi^2 + 2E_1 E_2 - 2m_V (E_1 + E_2)}{2\sqrt{E_1^2 - m_\pi^2} \sqrt{E_2^2 - m_\pi^2}},$$

which represents the value of cosine of the angle between momenta $\vec{p}_1$ and $\vec{p}_2$ in the V meson rest frame which satisfies the overall energy-momentum conservation. The sum over the vector meson spin states of the matrix elements squared comes out from the Lagrangian (3.1) as

$$\begin{aligned} \sum_{\lambda} |\mathcal{M}_{V \rightarrow \pi^+ \pi^- \pi^0}|^2 &= \left(\frac{f_{V\rho\pi} f_{\rho\pi\pi}}{m_\pi} \right)^2 \left[s_{12} s_{13} s_{23} - m_\pi^2 (m_V^2 - m_\pi^2)^2 \right] \\ &\quad \times \left| \sum_{i < j} \frac{1}{s_{ij} - (m_\rho - i\Gamma_\rho/2)^2} \right|^2. \end{aligned} \quad (\text{B3})$$

The kinematical invariants s_{ij} are simply related to energies in the parent meson rest frame:

$$\begin{aligned}s_{12} &= m_\pi^2 - m_V^2 + 2m_V(E_1 + E_2), \\ s_{13} &= m_\pi^2 + m_V^2 - 2m_VE_2, \\ s_{23} &= m_\pi^2 + m_V^2 - 2m_VE_1.\end{aligned}$$

The direct dilepton decay width of $V = \rho^0, \omega, \phi$ is equal

$$\Gamma_{V \rightarrow l^+ l^-} = \frac{\pi \alpha^2}{3} m_V g_{V\gamma}^2 \left(1 + \frac{2m_l^2}{m_V^2}\right) \left(1 - \frac{4m_l^2}{m_V^2}\right)^{1/2}. \quad (\text{B4})$$

The radiative decay of $V = \omega, \phi$ is described by

$$\Gamma_{V \rightarrow P + \gamma} = \frac{\alpha}{96m_\pi^2} (f_{V\rho P} g_{\rho\gamma})^2 \left(\frac{m_V^2 - m_P^2}{m_V}\right)^3. \quad (\text{B5})$$

In the ρ case both ω and ϕ are allowed in the intermediate state and thus

$$\Gamma_{\rho \rightarrow P + \gamma} = \frac{\alpha}{96m_\pi^2} (f_{\omega\rho P} g_{\omega\gamma} + f_{\phi\rho P} g_{\phi\gamma})^2 \left(\frac{m_\rho^2 - m_P^2}{m_\rho}\right)^3. \quad (\text{B6})$$

Finally, the coupling constant between ρ and π fields can be extracted from the relation

$$\Gamma_{\rho \rightarrow \pi\pi} = \frac{f_{\rho\pi\pi}^2}{48\pi m_\rho^2} (m_\rho^2 - 4m_\pi^2)^{3/2}. \quad (\text{B7})$$

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TABLES

TABLE I. Dilepton rates within the ω peak in a free hadron gas

T (MeV)	R (c/fm ⁴)		
	$V \rightarrow e^+e^-$	$\pi^+\pi^-\pi^0 \rightarrow e^+e^-$	$\pi\pi \rightarrow e^+e^-\pi$
100	6.95×10^{-10}	7.82×10^{-10}	1.87×10^{-10}
150	1.77×10^{-8}	2.77×10^{-8}	4.28×10^{-8}
200	1.03×10^{-7}	2.17×10^{-7}	1.12×10^{-6}

TABLE II. Trivial and additional dileptons from a hadronic fireball

peak	T (MeV)	$\langle n_t \rangle$ (fm ⁻³)	$\tau^{-1}\langle N_a \rangle / \langle N_t \rangle$ (c/fm)			contributing reactions
			$V \rightarrow e^+e^-$	reactions	sum	
ρ^0	100	1.37×10^{-8}	0.640	0.767		
	150	3.63×10^{-7}	0.589	0.595		$\pi^+\pi^- \rightarrow e^+e^-$
	200	2.23×10^{-6}	0.545	0.560		
ω	100	1.95×10^{-8}	0.036	0.050	0.086	$\pi^+\pi^-\pi^0 \rightarrow e^+e^-$,
	150	5.38×10^{-7}	0.033	0.132	0.165	$\pi^+\pi^- \rightarrow e^+e^-\pi^0$,
	200	3.37×10^{-6}	0.031	0.400	0.431	$\pi^\pm\pi^0 \rightarrow e^+e^-\pi^\pm$.
ϕ	100	1.12×10^{-8}	0.020	0.018		$K\bar{K}, \pi\rho \rightarrow e^+e^-$,
	150	6.65×10^{-7}	0.018	0.018		$\pi^+\pi^-\pi^0 \rightarrow e^+e^-$,
	200	6.06×10^{-6}	0.017	0.020		$\pi\pi \rightarrow e^+e^-\pi$.

TABLE III. Additional dileptons within ρ and ω peaks

T (MeV)	additional	additional/trivial
	ω/ρ	in $\rho + \omega$
100	0.191	0.315
150	0.415	0.336
200	1.195	0.476

FIG. 1. Structure of the matrix element of the process $1 + 2 + 3 \rightarrow l^+ + l^-$ in one-photon approximation.

FIG. 2. Feynman diagram for massive photon production from 3-pion initial state.

FIG. 3. Differential rate in M^2 as a function of dimuon mass M for $\pi^+\pi^-\pi^0 \rightarrow \mu^+\mu^-$ (full), $\pi^+\pi^- \rightarrow \mu^+\mu^-\pi$ (dashed) and $\pi^+\pi^- \rightarrow \mu^+\mu^-$ in one-pole approximation (dotted) in a hadron gas at $T = 150$ MeV.

FIG. 4. Comparison of summed differential rate for three-pion processes (full) with the sum of $P + P$, $P + V$, $V + V \rightarrow \mu^+\mu^-$ reaction rates (dashed) and with the rate for $\pi^+\pi^- \rightarrow \mu^+\mu^-$ in one-pole approximation (dotted) in a hadron gas at $T = 150$ MeV.

FIG. 5. Total differential rate of dimuon production (full) compared to $\pi^+\pi^- \rightarrow \mu^+\mu^-$ in one-pole approximation (dotted) at three different temperatures of hadron gas.