**GLOBULAR CLUSTER PHOTOMETRY WITH THE HUBBLE SPACE TELESCOPE.
II. U, V, AND I MEASUREMENTS OF M15¹**

Brian Yanny²

Institute for Advanced Study, Princeton, NJ 08540
yanny@guinness.ias.edu

Puragra Guhathakurta²

Princeton University Observatory, Peyton Hall, Princeton NJ, 08544-1001
raja@astro.princeton.edu

John N. Bahcall and Donald P. Schneider

Institute for Advanced Study, Princeton, NJ 08540
jnb@sns.ias.edu, dps@sns.ias.edu

¹ Based on observations with the NASA/ESA Hubble Space Telescope obtained at the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555

² Hubble Fellow

ABSTRACT

Previous ground-based observations have led to proposals that the core of the dense Galactic globular cluster M15 has either collapsed or is the site of a massive black hole. Hubble Space Telescope (HST) observations that were designed to test these proposals are presented for the central 1 arcmin^2 of M15. Planetary Camera (PC) images obtained in the F555W (V) and F785LP (I) filters are used to derive accurate photometry ($1\sigma \sim 0.15 \text{ mag}$) and number counts of stars brighter than the main-sequence turnoff ($V \sim 19$). HST/PC images in the F336W (U) filter, acquired by the Wide Field/Planetary Camera Instrument Definition Team, are also analyzed.

The projected density distribution of resolved stars near the center of M15 is consistent with either a power-law cusp $N(r) \sim r^\alpha$, with $\alpha \sim -0.85 \pm 0.2$, or with a King model with a core of radius $\lesssim 2''$ (best fit $r_{\text{core}} = 1''.3$), when corrected for incompleteness and photometric bias. The data cannot definitively decide between a cusp and a small, flat core because the combination of the extended point-spread function and the high stellar density results in large, model-dependent corrections to the derived star counts. The inferred value of the slope is in excellent agreement with the theoretical value, $\alpha = -0.75$, calculated by Bahcall & Wolf for the distribution of equal-mass stars surrounding a massive black hole. The M15 stellar density profile, and the radial velocity dispersion measurements of Peterson et al., are consistent with that expected for a central black hole of mass $M_{\text{BH}} \sim 2 \times 10^3 (\sigma_v/15 \text{ km s}^{-1})^2 M_\odot$, where σ_v is the line-of-sight velocity dispersion of the stars in the center of the cluster. The observed profile slope is also in agreement with that expected from core collapse without a central black hole. The radial surface-brightness distribution of the residual diffuse light (the light that remains after the removal of the resolved stars) is essentially identical to the distribution derived by Lauer et al. from HST/PC U -band observations, but Monte Carlo simulations demonstrate that the residual light in these images, as determined by current analysis techniques, is not a reliable indicator of the true density distribution.

Photometric measurements in V and I of more than 5×10^3 stars (and in U , V , and I of $\gtrsim 1500$ stars) are used to construct color-magnitude diagrams in the central $1'$ of M15. A list of 14 blue stragglers is presented and a comparison is made with the blue stragglers found in a previous HST Faint Object Camera study.

1. INTRODUCTION

This is the second in a series of papers describing Hubble Space Telescope (HST) observations of the centers of the 19 nearest Galactic globular clusters with $|b| > 15^\circ$. The main aim of this program is to investigate the radial density distribution and stellar populations at the centers of these clusters. In this paper, we extend the techniques used in our study of 47 Tuc (Guhathakurta et al. 1992, hereafter Paper I) in order to analyze data on M15 (NGC 7078). We use Monte Carlo simulations, including a realistic model of the variable HST Point Spread Function (PSF), to evaluate the effects of crowding on star counts and on stellar photometry.

The globular cluster M15 has one of the highest known central stellar densities of Galactic globular clusters; amongst clusters unobscured by heavy Galactic extinction, only M30 and NGC 6244 have comparable projected central densities (Webbink 1985). Within the central $1''$ ($r < 0.06$ pc), the projected density of post-main sequence stars ($V < 19$) is nearly 10^4 stars pc^{-2} ; this corresponds to an estimated volume density of more than 10^5 stars pc^{-3} above turnoff. Phinney & Sigurdsson (1991) estimate a total central mass density (including main sequence stars and heavy remnants, in the form of white dwarfs, neutron stars, etc.) of $10^7 M_\odot \text{pc}^{-3}$. Due to its high central density, M15 has long been considered as a prime candidate for a cluster that has undergone core collapse or that might contain a central black hole (King 1975; Bahcall, Bahcall & Weistrop 1975). In this paper, we make a quantitative determination of the projected density distribution of stars near the cluster center. The distribution is consistent with that expected for a massive central black hole or from core collapse.

The paper is organized as follows: in §2, we describe the acquisition and processing of the HST/Planetary Camera (PC) data; the simulations that are used to estimate the photometric error and incompleteness in the HST data are discussed in §3; a study of the radial density distribution near the center of M15 is presented in §4; in §5, we study the different types of post-main sequence stars as a function of distance from the cluster center; a summary of the principal conclusions is given in §6. The positions, V magnitudes, and $V - I$ and $U - V$ colors for 1186 stars with $V < 18.1$ (that are also detected in I) are presented in Table A1 of the Appendix.

1.1 Radial Density Profile

Djorgovski & King (1984), using ground-based observations, noted that the surface brightness profile near the center of M15 does not show any signs of leveling off as would be expected within the core radius of a “King-profile” distribution (King 1962). They suggested that the cluster appears to fall into the category of “post-core collapse”, with a power-law radial density profile extending in to radii $< 2''$. (Adopting a distance of 11.5 kpc to M15, $1''$ corresponds to 0.06 pc; see Fahlman, Richer & Van den Berg 1985.)

Numerical simulations (*cf.* Heggie 1985) indicate that a core undergoing unchecked collapse will leave a projected stellar density of $N(r) \sim r^\alpha$ with α ranging between -1.2 and -1.4 . The Fokker-Planck calculations of Grabhorn et al. (1992) suggest that the density profile of a cluster with a collapsed core may be quite flat, with a slope shallower than $\alpha = -0.4$. If binary heating halts core collapse (Heggie 1975; Goodman 1989) or if the core undergoes oscillations (Sugimoto & Bettwieser 1983; McMillan 1989; Murphy, Cohn & Hut 1990), one expects a flattening of the density profile near the center, with a core radius of about 0.05 to 0.1 pc.

The steady-state distribution of equal-mass stars around a massive black hole corresponds to a surface density cusp with $\alpha = -0.75$ (Bahcall & Wolf 1976, hereafter BW). This cusp slope can be slightly smaller or larger if the stars in the cusp have different masses (Bahcall & Wolf 1977).

Lauer et al. (1991), using early HST images of M15, reported that the radial surface brightness profile of diffuse residual light in the ultraviolet F336W band (presumably composed of the light of numerous unresolved faint main sequence stars) has a core of radius $2''.2$. Using the same data and realistic Monte Carlo simulations, we demonstrate in §3.3 of this paper that the unresolved light does not reliably trace the underlying density distribution: a true density cusp would look like a flat core in the residual (diffuse) light because of the complications introduced by the extended nature of the HST PSF ($r_{\text{PSF}} \sim 2''.5$). We therefore base our study of the density profile on counts of individual stars.

The ground-based radial velocity data of Peterson, Seitzer & Cudworth (1989) show a strong gradient in the stellar velocity dispersion within the central $6''$ of the cluster. The line-of-sight dispersion, σ_v , in the Peterson et al. data appears to rise in Keplerian fashion ($\sigma_v^2 \sim 1/r$ in the inner few arcminutes) to a central value $> 20 \text{ km s}^{-1}$, consistent with that expected for a black hole of mass $\sim 10^3 M_\odot$ in the center of the cluster (Bahcall & Ostriker 1975; BW). Dubath, Meylan & Mayor (1993), however, have used an independent set of ground-based spectroscopic data and detailed simulations to show that sampling errors make the determination of the central velocity dispersion quite uncertain. Their data do not indicate a steep rise in σ_v towards the center. A post-core collapse cluster without a central black hole is expected to have a velocity dispersion close to the isothermal value (Heggie 1985), with $\sigma_v \lesssim 15 \text{ km s}^{-1}$ in the central $5''$. Further study of the central stellar velocity dispersion is required in order to help distinguish between core-collapse and black-hole models.

1.2 Stellar Types

Ground-based stellar photometry in the outer parts of M15 indicate that the horizontal branch (HB) is at $V_{\text{HB}} = 16.19$, placing the cluster at a distance of 11.5 kpc after correction for an estimated line-of-sight reddening of $E_{B-V} = 0.11$ (Fahlman, Richer & Van den Berg 1985). Early ground-based observations of M15 explored the cluster at radii greater than about $2'$ (Sandage 1970) and showed no clear evidence for blue straggler (BS) stars, although they did reveal a population of stars forming a blue horizontal branch (BHB) extending towards bluer colors and somewhat fainter magnitudes than the HB. A BHB star is thought to be the hot, exposed core of an HB star which has undergone mass loss (see Dupree 1986 for a review). More recent photometry of the outer region of this cluster by Buonanno et al. (1985) confirmed the presence of the BHB but again failed to identify any definite BSs with $V \lesssim 18.5$. Ferraro & Paresce (1993, hereafter FP) have detected several BSs within the central $15''$ of M15 using the HST Faint Object Camera (FOC) and ultraviolet (UV) bandpasses ranging from $1400 \text{ \AA}$ to $3400 \text{ \AA}$. We make a star-by-star comparison of our BS sample with that of FP in §5.2.

Aurière & Cordoni (1981) obtained ground-based images of the central $1'$ of the cluster in $\sim 0''.8$ seeing (full width at half maximum [FWHM]). Even with this excellent resolution, it is not possible to obtain photometry of sufficient accuracy to produce a color-magnitude diagram (CMD) in which the HB and red giant branch (RGB) are clearly distinguishable. Using a fast-guiding camera on the Canada-France-Hawaii Telescope, Stetson (1991) acquired data with $0''.5$ (FWHM) seeing and was able to significantly improve the accuracy of stellar photometry in the central region of M15. Stetson noted that both BHB stars and the brightest stars at the tip of the RGB seem to be underabundant in the central $13''$ of the cluster relative to subgiant stars. In spite of the spherical aberration in the HST primary mirror, the $0''.1$ (FWHM) core of the PC PSF allows stellar photometry of somewhat higher accuracy within $r < 13''$ of the center of M15 compared to that achieved by Stetson. Several

studies (Bailyn et al. 1988; Cederbloom et al. 1992) have noted a color gradient in the overall stellar light in the central portion of M15 in the sense of the colors becoming bluer towards smaller radii. We comment on these findings and compare them to our results in §5.3.

2. OBSERVATIONS AND DATA PROCESSING

2.1 Description of the Data

HST/Planetary Camera observations of the center of M15 were obtained in the F555W and F785LP filters (roughly similar to the Johnson *V* and *I* bandpasses, respectively) on 10 December 1990 and 11 April 1991 (see Griffiths 1989 for the filter response curves and for a description of the instrument). The exposure times were 70 s and 160 s in F555W and 160 s and 180 s in F785LP. In addition, we have reanalyzed four 800 s PC exposures of the cluster taken through the F336W filter (similar to Johnson *U*); these images were obtained by the WF/PC Instrument Definition Team on 19 September 1990 and have previously been analyzed by Lauer et al. (1991).

Each PC image consists of four 800×800 charge-coupled device (CCD) frames with a pixel size of $0''.0439$. Figure 1 shows a (linear) greyscale reproduction of the $8'' \times 8''$ portion of the 70 s F555W exposure centered on M15. The 2σ error circle on the number-weighted center of the stellar distribution (“center of mass” of the cluster) is indicated by the white ring (see §4.1). The four-CCD mosaic of the 70 s F555W exposure is shown in Figure 2, with the greyscale being proportional to the logarithm of the intensity. The full extent of the image is $68'' \times 68''$. There are $\sim 0''.5$ -wide blank strips between the usable field of view of adjacent CCD frames—these are not shown to scale in Figure 2.

Figure 3 shows the sky coverage of all the F555W (*V*), F785LP (*I*), and F336W (*U*) exposures analyzed in this paper. Each four-CCD mosaic image of M15 is represented by a square in Figure 3. The images were taken at a variety of spacecraft roll angles and pointings, but all pointings placed the center of the cluster (indicated by “+” in Fig. 3) in the middle of one of the individual CCDs. There is nearly 100% overlap between the *V* and *I* datasets (*i.e.* a pair of *V* and *I* exposures were acquired for each of two pointings). Each of the dotted squares in Figure 3 corresponds to a pair of *U* exposures, that are separated by roughly $10''$ in the north-south direction. Four *U* exposures overlap in the central $\sim 13''$ of the cluster.

The spherical aberration of the HST optics produces a spatially-varying PSF with a complex shape (see Fig. 1): a sharp core ($\text{FWHM} = 0''.1$) contains $\approx 10\%$ of the light while the rest is distributed in a faint halo extending out to $r \sim 2''.5$ (Burrows et al. 1991). Faint stars that are located within the PSF wings of bright stars are measured reliably only if the model of the PSF accurately subtracts the light in the wings of the bright star. An additional complication is that the 12-bit resolution of the analog-to-digital converters on the HST/PC CCD electronics restricts the dynamic range of the data, causing the cores of the brightest stars to be saturated in *V* and *I* (Lauer 1989).

Some of the images acquired prior to the UV-flooding of the WF/PC in December 1990 suffer from poor flat fielding. The only calibration data available for F336W images obtained during this early phase of HST operation are pre-launch ‘thermal vacuum test’ flats (MacKenty, private communication). The lack of adequate in-flight flat fields significantly limits the extent to which accurate photometry can be obtained from these images. The CMDs shown in this paper specifically exclude all data from F336W CCD#8 and from CCDs #5, #7, and #8 of the 70 s F555W exposure. Further, some of the F336W data near the edges of CCDs #5, #6, and #7 are excluded because of poor flat fielding.

2.2 Data Processing Technique

We have developed a procedure for building an empirical model of the variable PC PSF for each of the four $34'' \times 34''$ CCD frames comprising an image (see Paper I for details). This technique is also capable of recovering the magnitudes of moderately-saturated bright stars to ≈ 0.10 mag. The procedure may be summarized as follows: About 50–100 relatively-isolated, bright but unsaturated stars on each CCD frame are used to build a “core-PSF” template whose shape varies as a function of position in the image plane. The core-PSF is only computed out to a radius of $0''.6$, within which the signal-to-noise ratio (SNR) is high and there is negligible contamination from neighboring stars. Isolated saturated stars then have their central portions restored by fitting the core-PSF model to the unsaturated portions of their profiles. These saturation-corrected stellar images are used to construct a variable “full-PSF” template (out to a radius of $2''.4$) that has a high SNR in both core and wings. This full-PSF template is used to fit stellar images (thereby determining the position and brightness of each star) using the DAOPHOT II software of Stetson (1987, 1992). The PSF construction and fitting process is iterated several times, with the accuracy of the model PSF improving each time as the contaminating neighbors of the bright stars (from which the PSF model is derived) are removed with increasing precision.

While the procedure described in Paper I is sufficient for building an accurate PSF model in areas where the stellar crowding is similar to that in the core of 47 Tuc, the projected central density of M15 is approximately twice as large as that of 47 Tuc and requires additional Monte Carlo simulations. These additional simulations provide estimates of completeness and of photometric accuracy in regions where the surface density of post-main sequence stars ($V < 18$) is $\sim 5\text{--}10 \text{ arcsec}^{-2}$. There are a few minor changes in the data-processing technique, compared to that used in Paper I, most of these specific to the PSF-building method:

- (1) Stars in the central portion of M15 ($r < 4''$) cannot be used to derive a PSF model due to extreme crowding; the PSF model calculated for this region is based on interpolation between stars that are located around the edge of the densest region.
- (2) The outer part of the full-PSF model (beyond a radius of about $2''$) is forced to be a linear function of position, while the inner part is allowed to vary quadratically (as in Paper I), in order to reduce the contaminating effects of poorly-subtracted neighbors on the variable PSF model.
- (3) There are very few stars in the F336W images that are bright enough to have a high SNR in their PSF wings. We are therefore forced to construct a F336W PSF model whose outer part is fixed (*i.e.* constant across a given CCD frame) and whose inner part varies linearly with position.
- (4) In order to keep track of spatial variations in the sky level (*i.e.* the surface brightness of the underlying light in the vicinity of a star; near the cluster center, the background “sky” signal is dominated by the light of unresolved stars), stars within $r \lesssim 20''$ of the cluster center are grouped into regions of size $\sim 4''$ and each group is analyzed separately. If all the stars are analyzed simultaneously, the lower sky value beyond $r \sim 5''$ influences the sky determination in the innermost regions causing it to be underestimated, and this results in photometric bias (see §3.2). Simulated images have been analyzed both with and without the grouping of stars described above, and the photometric bias is about 0.1 mag larger when all the stars are processed together.

Realistic simulations show that this modified processing technique is superior to that used in Paper I, especially when applied to images that are as crowded as those of M15.

2.3 Stellar Photometry

The empirical PSF template derived for each CCD frame is used to make a simultaneous fit to all the resolved stars on that frame, yielding positions and instrumental magnitudes in the process (see Paper I for details). Small photometric zeropoint differences (of up to $\sim \pm 0.2$ mag) between the various CCD frames and exposures are corrected for using a specific reference point in the CMD. The F555W magnitude of the HB and the F555W–F785LP color of the RGB at the level of the HB are forced to be the same across all the CCD frames in the full dataset. We expect this method to be reliable since ground-based observations of M15 show no evidence for any radial variation in the colors of these stellar types (*cf.* Stetson 1991). In regions of the cluster covered by more than one image, the stellar positions derived from the different images are matched by applying linear transformations and the instrumental magnitudes in a given filter are averaged.

As described in Paper I, the F555W and F785LP instrumental magnitudes are converted to the Johnson V and I system (Johnson 1966) using the photometric transformations determined by Harris et al. (1991):

$$(F555W - V) = 0.0768(B - V) - 0.0254(B - V)^2 \quad , \quad (1a)$$

$$(F785LP - I) = -0.0549(V - I) - 0.0343(V - I)^2 \quad . \quad (1b)$$

Equation (1a) may be rewritten in terms of $V - I$ using the $B - V$ vs. $V - I$ relation given by Johnson (1966). Given the F555W and F785LP magnitudes, the above equations are solved by iteration to derive V and I values. The absolute zero point of the HST stellar photometry is set by matching it to ground-based data in the sparse outer parts of the cluster. The HB is normalized to $V_{\text{HB}} = 16.19$ (Fahlman, Richer & Van den Berg 1985) and the I -band zeropoint is calibrated against the data of DaCosta & Armandroff (1990) which shows that $(V - I)_{\text{RGB}} = +1.10$ at $V = 15$. The shape of the bright RGB ($V \lesssim 15$) seen in our HST/PC data is in agreement with that found by DaCosta & Armandroff. The adopted V and I photometric zeropoints are within ± 0.2 mag of those listed in the WF/PC Science Verification Report (Faber 1991). A star-by-star comparison of our V -band data with a ground-based image of the center of M15 taken in $0''.8$ seeing (Aurière & Cordoni 1981) shows good agreement for bright stars ($V \lesssim 16$), indicating that the V photometric zeropoint is accurate. The ground-based magnitudes of fainter stars are systematically too bright due to the effects of crowding.

The HST F336W filter is only a rough approximation to the Johnson U -band, so we do not attempt to convert the F336W magnitudes to the Johnson system. Instead, we set the F336W magnitude zeropoint by comparing the HST/PC photometry with that derived from a spatially-overlapping image of M15, obtained using a Johnson U filter in the CCD camera on the Palomar Observatory 1.5-m telescope (Kirhakos, private communication). The comparison is complicated by severe crowding within the central $1' \times 1'$ overlap area of the datasets given the relatively poor seeing (FWHM $\sim 1''.3$) of the ground-based image. This calibrated F336W magnitude is hereafter loosely referred to as the “ U magnitude” but there may be color-dependent offsets as large as ± 0.2 mag with respect to the true Johnson U magnitude. The HB in the PC data is located at $U_{\text{HB}} \sim 16.2$, in reasonable agreement with (within 0.2 mag of) the measurement by Sandage (1970) in the outer parts of M15. The flat fielding of the F336W data is particularly poor. However, four independent 800 s F336W exposures are available for the center of M15, with $10''$ – $40''$ offsets between them. The errors in the U photometry introduced by inaccurate flat fielding are reduced somewhat by the use of the median of the four measurements wherever possible. The effects of crowding are not as severe

in the U band as they are in V or I because the brightest objects in the field. RGB stars, have relatively little U flux.

More than 10^4 stars are detected in the F555W (V) images in a 1.33 arcmin^2 region; they span the apparent magnitude range $13.0 < \text{F555W} \lesssim 22$. Of the stars detected in F555W, about 5500 stars (53%) are also detected in the I band with magnitudes in the range $11.4 < \text{F785LP} \lesssim 20$. Most of the ~ 5000 stars that are detected in V but not in I are very faint ($\text{F555W} \gtrsim 19$). The UVI overlap covers only 0.43 arcmin^2 . Within this region, a total of 1591 stars are detected in all three bands ($13.8 < U \lesssim 19$).

Table A1 presents positions, V magnitudes, and $V - I$ and $U - V$ colors for the 1186 stars in our dataset with $V < 18.1$ that are also detected in the I band ($I \lesssim 20$). The area covered by Table A1 is 1.33 arcmin^2 . Stars within the UVI -overlap region that are too faint to be detected in U are assigned a “...” in the $U - V$ column of Table A1, while those outside the region covered by the U data are indicated by a long dash. The distribution of V -band stellar apparent magnitudes is discussed in §3.1, and CMDs based on UVI photometry are described in §5.1.

2.4 Astrometry

We use the radio identification of AC 211 (star ID 653 in Table A1) by Kulkarni et al. (1990) and Aurière, LeFevre & Terzan (1984) to fix the absolute astrometric reference frame:

$$\alpha_{2000}(\text{AC 211}) = 21^{\text{h}} 29^{\text{m}} 58^{\text{s}}.26; \quad \delta_{2000}(\text{AC 211}) = +12^{\circ} 10' 02''.90 \quad (2)$$

All the coordinates in Table A1 are relative to AC 211, with both the RA and DEC offsets listed in arcseconds. The overall rotation adopted for the HST/PC image is based on the orientation of the 70 s PC CCD#6 frame, as determined from HST guide star positions (this determination is a part of the standard data pre-processing procedure at Space Telescope Science Institute). Epoch 2000 RA and DEC orientation is used in Table A1. The absolute value of the orientation angle adopted for the HST data agrees well with the orientation of all ground-based images with which we have compared our data.

Positions of stars on all the CCD frames in the four-CCD HST/PC mosaic are related to CCD#6 as follows: The small inter-CCD gaps and rotational offsets for the other three CCDs in the mosaic are determined by comparing the full HST image to a 1 s R -band image of M15 obtained with the Palomar Observatory 5-m telescope. The typical inter-CCD spacing is about 15 pixels ($0''.7$) and is accurate to better than 0.5 pixel ($0''.02$), while the relative rotations range from $0^\circ.2$ to 1° and are accurate to $0^\circ.1$ (see Gould & Yanny 1993). Stellar positions from the other exposures are matched to those on the 70 s F555W image; there is substantial overlap between all the exposures since each exposure images the cluster center. The relative positions of stars on a given CCD frame are accurate to $\lesssim 0''.02$. For two stars on separate CCD frames in the four-CCD mosaic or on different exposures, the relative coordinates are accurate to about $0''.04$. The absolute RA and DEC and the overall orientation of the coordinate system adopted may have slight systematic errors (*e.g.* due to possible offsets between the radio and optical reference coordinate frames). When comparing the HST/PC data with another dataset, it is best to use the positions of a subset of bright isolated stars to determine the relative coordinate transformation between the two datasets.

3. SIMULATIONS

Monte Carlo simulations of HST images are required in order to study: (1) the accuracy with which stellar magnitudes are measured; (2) the fraction of stars that are detected; and (3) the

distortion of the distribution of diffuse (residual) light, that is caused by systematic errors in the photometry and in the subtraction of resolved stars. All of these effects must be studied as a function of the degree of crowding and of stellar brightness. As demonstrated in §3.4 below, systematic effects are especially important for the measurement of unresolved U light near the cluster center. Star counts are best studied in V since more stars are detected in V than in U or I images. However, a correction must be made for the fact that the degree of incompleteness for faint stars varies as a function of distance from the center of M15.

3.1 Simulated Images in V and U

To construct a realistic simulated image, one needs to know: (1) the shape of the stellar luminosity function (LF); (2) the projected density of stars as a function of radius; and (3) the instrumental PSF as a function of position in the focal plane. We have derived each of these quantities from PC images of M15. Independent sets of simulated images are constructed to mimic PC data in the F555W (V) and F336W (U) bands. The V simulations are used to estimate the photometric accuracy for individual stars and the degree of incompleteness, while the U simulations are used to study the diffuse unresolved light.

Figure 4 shows the M15 star counts as a function of their F555W magnitude in four radial bins: $r < 4''$ (solid line), $4'' < r < 9''$ (bold dotted line), $9'' < r < 13''$ (dashed line), and $r > 13''$ (dot-dashed line). The steep fall off in the star counts at faint magnitudes is due to incompleteness in the data caused by crowding, and becomes significantly worse as one approaches the dense cluster center. The bold open circles show the LF used in the F555W simulations. The simulated LF is extrapolated from the observed LF in the outermost annulus by correcting for incompleteness at the faint end; these corrections are based on preliminary simulations appropriate for regions of sparse stellar density and on simulations described in Paper I. The extrapolation only affects the number of faint stars ($V \gtrsim 20$) in the simulated image (these make up the unresolved light in M15); crowding effects, on the other hand, are dominated by the PSF wings of bright stars. We have verified by comparing different simulations that the precise form of extrapolation of the LF is unimportant for the results derived in this paper.

The normalization of the four LFs shown in Figure 4 corresponds to the average projected density of stars within the inner $4''$ of M15—*i.e.* the LFs in the outer three bins have been normalized so that the number of bright stars ($V < 18.5$) matches that in the innermost bin ($r < 4''$). The mean projected stellar densities in the $4'' < r < 9''$ and $9'' < r < 13''$ annuli are 39% and 20%, respectively, of the average density in the central $4''$. The bump in the LF at F555W ~ 16.2 is caused by HB stars.

The LF used in the U simulations is derived from the PC F336W data using a technique similar to that used to estimate the simulated LF in V . The shapes of the V and the U LFs are significantly different for post-main sequence stars since the brightest stars (RGB stars) are redder in $U - V$ than the faint stars (subgiants). The LF rises more steeply with decreasing luminosity at the bright end in the U band than it does in the V band.

Stellar positions for the simulations are drawn at random using one of the following two radial surface density distribution functions: (1) “King model” profile, $N(r) \propto [1 + (r/r_{\text{core}})^2]^{-1}$, with $r_{\text{core}} = 2''$ (King 1962; Lauer et al. 1991); and (2) Power-law central cusp $N(r) \propto r^\alpha$, with $\alpha = -0.75$, as would be expected for a massive black hole (Bahcall & Wolf 1976, 1977). These are hereafter referred to as the ‘King simulation’ and the ‘BW cusp simulation’ and are indicated by the solid curves in Figures 7a and 7b, respectively. Beyond $r = 2''$, where incompleteness is not too severe for

stars with $V < 18$, the two distributions are assigned the same power-law shape—one that matches the observed surface density profile of bright stars in M15. The stellar densities in the inner parts of these simulated images should bracket the range of possible solutions for the true density of M15.

A total of four simulated images are constructed, using the V and the U LFs and the King and the BW cusp radial density profiles. Each image contains 800×800 pixels, the equivalent of a single $34'' \times 34''$ PC CCD frame, with the center of the simulated cluster positioned in the middle of the image. Realistic, spatially-variable PSF templates derived from HST/PC images of M15 in the F555W and the F336W bands are used to create artificial stars in the V and the U simulated images, respectively (see Paper I). Note that there are significant differences in the structure of the U and the V PSFs, especially in the PSF core. Readout noise and Poisson noise are added to the simulated images. Photon counts exceeding the saturation value of about 3×10^4 in the central pixels of bright stars in the V -band simulation ($V \lesssim 13.5$) are truncated to the saturation value in order to represent the saturation observed in the 160 s F555W PC exposure.

The simulated images are processed using exactly the same technique that is used to analyze the HST data. In the process of analyzing the simulations, no use is made of our prior knowledge of the positions and magnitudes of the stars or of the PSF model used to construct the simulated image. Instead, standard DAOPHOT software is used for the preliminary detection of stars and a variable PSF model is reconstructed from the artificial stellar images in the simulation (as described in §2.2 and in Paper I).

In addition to the four simulated images described above, tests have been carried out using special-purpose simulations with the aim of answering specific questions relating to the accuracy of stellar photometry. These tests are described in the following section (§3.2).

3.2 Accuracy of V Photometry

The PSF model that is reconstructed from the simulated F555W image is used to make a simultaneous fit to the positions and magnitudes of all the detected stars, and this yields an ‘output list’. Every star in this list is matched to the nearest star in the ‘input list’ (the list that is used to construct the simulated image). Only input stars whose positions are within 1 pixel ($0''.04$) of the output star and whose magnitudes are within 3.5 mag of the output magnitude are considered to be eligible for matching; the limit on the magnitude difference avoids spurious matches between bright, resolved stars and the numerous faint stars ($V \gtrsim 21$) in the simulation. The input and output stellar magnitudes are then compared in order to determine the accuracy of the V photometry as a function of the brightness of the star and of the distance from the cluster center (crowding).

Figure 5 shows the 1σ photometric error estimates derived from the V simulations as a function of input F555W magnitude. Figures 5a (top panel) and 5b (middle panel) are for the King and the BW cusp simulations, respectively. Figure 5c (bottom panel) shows stars in the sparser regions of the F555W King simulation (beyond $11''$ from the center). The 1σ rms errors increase from ~ 0.05 mag at the bright end to ~ 0.3 mag at F555W = 19. For stars fainter than about $V = 16$, the errors are somewhat larger in the central portions of BW cusp simulation than they are in the King simulation; this is due to the higher degree of crowding within the central $2''$ of the BW cusp simulation. Photometry of faint stars is more accurate in the less-dense outer regions ($1\sigma \sim 0.10$ – 0.15 mag) than it is near the center.

A systematic positive offset of 0.1–0.2 mag in the mean difference between the input and output magnitudes is apparent in Figure 5 for stars with $V \gtrsim 16$ within the inner $20''$. The photometric bias (systematic overestimation of the stellar brightness) increases with decreasing distance from the

cluster center, exceeding 0.2 mag for stars in the crowded central portions ($r < 4''$), and is somewhat worse in the F555W BW cusp simulation than in the King simulation due to the higher density of stars in the inner portions of the former. We have undertaken the following tests to characterize the nature of this bias:

- (1) In order to check whether the photometric bias is related to errors in the PSF reconstruction, we have analyzed the F555W King and the BW cusp simulations using the same PSF model that was used to construct these simulated images. The bias is reduced by about 30% (to ~ 0.1 mag) when a “perfect” PSF template is used in the fitting procedure.
- (2) We have constructed an F555W simulated image containing a uniform stellar distribution with a density comparable to that in the inner part of M15 ($r \lesssim 4''$). This image has been analyzed in the usual way and the bias in stellar photometry in this uniform density simulation is comparable to that in the central $4''$ of the F555W King and BW cusp simulations (especially when a reconstructed PSF template is used in the fitting procedure). This shows that the steep gradient in the surface brightness of the background light near the center of the F555W King and the BW cusp simulations is not solely responsible for the photometric bias.
- (3) We have carried out a detailed comparison between a selected set of stars in the output list and their counterparts in the input list. Whenever an input star has one or more faint neighbors with a separation $\lesssim 2$ pixels ($= 0''.1 = \text{FWHM of PSF core}$), the brightness of the star is overestimated by the PSF-fitting software. For example, a star with $V_{\text{input}} = 16.0$ and a close faint neighbor with $V_{\text{input}} = 17.7$ are fit as a single star with $V_{\text{output}} = 15.8$. Since the degree of crowding increases towards the cluster center, the amount of contamination from neighboring stars and the resulting photometric bias are worse. Inaccuracies in the modeling of the sharp undersampled PSF core (specifically, the fact that the reconstructed PSF core tends to be broader than the true PSF core) cause the amount of contamination from neighbors to be greater when the reconstructed PSF template is used in the PSF-fitting procedure, compared to when the “perfect” PSF template is used.

The I -band photometric error for faint subgiant stars is slightly larger than the V errors (see Paper I), while that in the U band (due to crowding effects alone) is expected to be somewhat smaller. This is due to the fact that the brightness ratio between the red stars at the bright end of the stellar LF (tip of the RGB) and the subgiants is largest in the I band (so that the crowding effects of the overlapping PSF wings of bright stars are worst) and smallest in U . The errors in the M15 U data, however, are dominated by systematic errors in flat-fielding and are typically 0.15–0.3 mag. An estimate of the overall photometric errors in UVI can also be made by examining the width of various features in the CMDs derived later in this paper (see Figs. 12 and 14 and §5.1).

3.3 Correction of V -Magnitude-Limited Star Counts

The process of matching the input star list for a simulation with the list of stars recovered by the PSF-fitting procedure (output list) indicates the fraction of the input stars that have been detected. This fraction decreases towards fainter input magnitudes and in regions of higher stellar density. However, a somewhat different quantity C must be derived from the simulation in order to correct the M15 star counts:

$$C = \frac{N(V_{\text{output}} < V_{\text{lim}})}{N(V_{\text{input}} < V_{\text{lim}})}, \quad (3)$$

where C is the ratio of the number of stars with output magnitude (V_{output}) brighter than some limiting magnitude (V_{lim}) to the number of stars with input magnitude (V_{input}) brighter than V_{lim} .

While the non-detection of stars by the PSF-fitting software tends to lower this ratio compared to unity, the photometric bias discussed in the previous section ($V_{\text{output}} < V_{\text{input}}$ for faint stars) tends to increase this ratio. The reason for the increase is that stars whose input magnitudes are fainter than the cutoff V_{lim} are included in the numerator of the above ratio when $V_{\text{output}} < V_{\text{lim}}$. The amount by which the photometric bias increases C depends on the size of the bias and on the steepness of the stellar LF at V_{lim} . Even in the absence of photometric bias, the steeply rising faint end of the LF implies that photometric errors cause more stars near the magnitude limit to be scattered into the allowed magnitude range than out of it.

The star-count correction factor C is plotted as a function of radius (and therefore of stellar density) in Figure 6 for the F555W King simulation (upper panel) and for the F555W BW cusp simulation (lower panel). The open circles show ratios of star counts using $V_{\text{lim}} = 18$ binned in radial annuli of width $1''$, and the error bars represent 1σ Poisson fluctuations in $N(V_{\text{output}} < V_{\text{lim}})$. The choice of $V_{\text{lim}} = 18$ avoids faint stars for which incompleteness is severe and yet includes enough stars to permit an accurate determination of the stellar density profile.

Near the center of each simulated image, the effect of incompleteness dominates over the effect of photometric bias, so that the value of C is smallest in the innermost bin. The increase in $C(r)$ from the center out to $r \sim 5''$ is due to the fact that the degree of completeness increases with distance from the crowded central region while the effect of photometric bias is virtually unchanged. At larger radii ($r \gtrsim 15''$), practically 100% of the stars are detected and the photometric bias/error is smaller; this causes $C(r)$ to approach unity. The values of C derived from the simulations (open circles in Fig. 6) are noisy due to the small number of stars in each radial bin ($N \lesssim 50$). In order to reduce the effect of these fluctuations, we have fit smooth functions (double exponentials) to $C(r)$. These analytic fits are shown by solid lines in Figure 6, while the dotted line in the lower panel shows an extreme $C(r)$ that is consistent with the results of the BW cusp simulation. The differences between the various curves are a measure of the uncertainties in the star-count correction factor within the innermost $2''$. Corrections to the M15 star counts using these smooth approximations to $C(r)$ are given in §4.2 and in Figure 9 below.

Figures 7a and 7b show the projected density profile of all stars with $V < 18$ that are detected in the F555W King and the BW cusp simulations, respectively (open circles, with 1σ Poisson error bars). The solid line in each panel indicates the underlying distribution from which the input stars have been drawn—i.e. it is the appropriately-normalized analytic radial distribution function used to construct the simulated image (§3.1). The stars detected in the King and the BW simulations are corrected for incompleteness and photometric bias using the smooth approximations to $C(r)$ shown by the solid lines in Figure 6. These corrected star counts are indicated by crosses in Figure 7 and lie closer to the input distribution function (solid line) than the raw counts (open circles), thus providing a consistency check on the correction procedure.

3.4 Residual Unresolved Light in U-Band Simulation

The U simulations are used solely for the purpose of understanding the properties of the unresolved light in the M15 U image (that remains after the removal of the resolved stars). These simulations provides a calibration for the study of Lauer et al. (1991), who analyzed the residual light in the same F336W (U) images of M15 and concluded that there is a flattening of the density profile interior to $r \sim 2''$. The U band is better suited to the study of diffuse residual light than the V band since the brightest stars in the cluster (at the tip of the RGB) are relatively red in color. Even in the U band, however, our technique, which is essentially identical to that used by Lauer

et al., is unable to remove the light from resolved stars with sufficient accuracy. The inaccuracies in the subtraction cause a distortion of the brightness profile of the diffuse light (as shown below).

The F336W (U) King and the BW cusp simulated images are analyzed using the PSF-fitting software described in §2.2. The reconstructed PSF template and the best-fit magnitudes and positions of the detected stars are used to subtract the light from each detected star. The diffuse light that remains consists primarily of stars that are too faint to be detected individually ($U \gtrsim 19$). Figure 8 shows the measured radial surface brightness profile of the unresolved light in the F336W BW cusp simulation (open circles). The error bars show the 1σ probable error in the mean surface brightness in each annulus, based on the observed pixel-to-pixel intensity variation alone, but the actual measurement error is dominated by systematic effects that are much larger than this. The solid line in Figure 8 indicates the power-law radial distribution function used to construct the simulated image (representing the deconvolved brightness profile) while the dashed line shows the result of convolving it with the HST PSF. The latter is the residual diffuse brightness profile expected if the light of all bright stars (down to some magnitude limit) were to be perfectly subtracted.

The cusp in the input brightness profile is “flattened” by the PSF-fitting/subtraction software. One can ask the question: “Where are the photon counts represented by the gap between the flattened output profile in Figure 8 (open circles) and the steeper input curve (dashed line)?” The answer lies in the fact that resolved stars with $16 < U < 18$ are found to be systematically too bright (as in the V band; see §3.2 and Fig. 5) so that some of the “background” unresolved light is mistakenly removed in the process of subtracting these stars. The mean photometric bias is ~ 0.1 mag in the innermost $4''$ of the F336W BW cusp simulation. The bias is caused by contamination of resolved stars by faint neighbors: for example, about one-fourth of all stars with $17 < U_{\text{input}} < 18$ within $r < 4''$ of the center of the simulated cluster have one or more contaminating neighbors with $18 \lesssim U_{\text{input}} \lesssim 20.5$ within $0''.1$. The output magnitude of each of these stars correspond to the sum of the brightnesses of the star and its faint neighbors. These faint stars are contain too little light to be detected individually, but those in close proximity to a bright star have their light removed when one attempts to subtract the light of the bright star. Since the central portion of the cluster is the most crowded, the bias in stellar photometry and the consequent oversubtraction is most severe in this region; the net effect is an apparent flattening of the residual light profile.

The dotted line in Figure 8 shows the brightness profile of the diffuse light after the bright resolved stars are subtracted using their correct magnitudes (*i.e.* input magnitudes, rather than the biased output magnitudes derived by PSF fitting) and the reconstructed PSF template. This profile is in good agreement with the (PSF-convolved) input profile (dashed line in Fig. 8), demonstrating that the distortion of the unresolved light is a result of bias in stellar photometry and is not caused by errors in the modeling of the PSF. In particular, the cusp at the center of the BW cusp simulation is found to have $U_{\text{output}} \sim 16$, but it is actually composed of the light of several stars within $r < 0''.1$ of the center with $U_{\text{input}} \gtrsim 17.5$.

Thus, our F336W simulations show that the unresolved U light does not provide a direct measure of the stellar density distribution inside $r \lesssim 2''$ for the range of profile shapes described in §3.1. The radial brightness profile of the residual diffuse light in the BW cusp simulation is seen to have a spurious central flat portion with a core radius of about $2''$, while the residual light in the $r_{\text{core}} = 2''$ King simulation is also suppressed near the center and has a best-fit core radius of about $3''$ (without deconvolution).

4. DENSITY DISTRIBUTION

The principal goal of this paper is to determine accurately the radial density profile near the center of M15. The overall light profile is dominated by a few stars at the bright end of the LF and consequently suffers from large statistical fluctuations in their number and distribution. We therefore use the projected star density (number of resolved stars brighter than some magnitude limit) as our best estimate of the true radial mass density profile in M15. In §4.1, we describe the determination of the position of the center of the cluster. We analyze the V -band star counts in §4.2 and estimate the black hole mass that could explain the counts. We discuss the radial distribution of the U residual light in §4.3.

4.1 Cluster Center

The center of the stellar distribution in M15 is determined from a maximum-likelihood (ML) fit (Sarazin 1980) to the (x, y) positions of the stars modeled with a King profile with a small core, $r_{\text{core}} = 2''$. Each resolved star, no matter how bright, is given equal weight. Unlike the center of the light distribution, the number-weighted ML center is not unduly influenced by the light of a few bright RGB stars. The ML center is insensitive to the details of the density profile used to fit the star counts: by varying r_{core} between $1''$ and $4''$ and by using different magnitude-limited star lists, we estimate the rms error in the center to be $0''.2$. The number-weighted centroid of the (x, y) positions of all stars detected in V within $r < 13''$ is only 9 pixels ($0''.4$) away from the ML center. As a check of the method, the center of the F555W BW cusp simulation is also determined using the ML technique; the difference between the measured and true centers is 3 pixels ($0''.13$).

The assumed cluster center influences the inferred (azimuthally-averaged) density in the inner regions: deviations of the adopted center from the true center cause the measured density distribution to appear flatter than the true distribution near the center. If the ML center we use is significantly in error, then the true density distribution near the center is steeper than we have inferred. Based on the astrometry described in §2.4 (using the radio identification of the star AC 211), our best estimate of the number-weighted center of the stellar distribution in M15 is:

$$\alpha_{2000}(\text{center}) = 21^{\text{h}} 29^{\text{m}} 58^{\text{s}}.28 \pm 0^{\text{s}}.013; \quad \delta_{2000}(\text{center}) = +12^{\circ} 10' 00''.75 \pm 0''.2 \quad (4)$$

Figure 1 shows the 2σ error circle ($r = 0''.4$) around the ML solution for the center of the cluster. There are four stars with $V < 18$ (three of these are within $0''.1$ of one another; see §5.4), and one with $V \sim 18.5$, within the error circle. In addition, there are three stars with $U \sim 18$ also within the circle which are not detected in V or I .

4.2 Distribution of Stellar Surface Density

Since a larger number of stars are detected in the V band than in U or in I , the radial profile is best determined from V -band stellar counts. Tests made using the F555W simulations (see §3.3 and Figs. 6 and 7) suggest that using all stars with $V < 18$ to determine the radial density profile provides a good compromise between avoiding large incompleteness corrections in the star counts and reducing the statistical fluctuations due to the small number of stars in the innermost radial bins.

Figure 9 presents the projected number density of detected stars ($F555W < 18.0$) as a function of angular distance from the center of M15 with error bars representing the square root of the number of stars in each radial bin ($\Delta r = 1''$). The open circles in Figure 9 show the data uncorrected for incompleteness or photometric bias. Comparing King profiles with a range of core radii to the

uncorrected cumulative radial distribution of stars with $V < 18$ near the cluster center yields a best-fit value of $r_{\text{core}} = 1''.3$.

The M15 star counts have been corrected using analytic fits to $C(r)$, the star-count correction factor derived from the F555W simulations (as described in §3.3; see Fig. 6). Independent sets of corrected counts, based on the results of the King ($\times$) and the BW cusp ($+$) simulations, are shown in Figure 9. Note that the correction factors derived from the two simulations are different. The open squares in the four innermost bins in Figure 9 represent an extreme correction to the counts based on the BW cusp simulation (dotted line in Fig. 6b). In all cases however, the corrected counts rise more steeply towards the center than the uncorrected counts since the value of $C(r)$ increases from the center out to $r \sim 5''$ (see Fig. 6).

The different sets of corrected star counts shown in Figure 9, based on the King and the BW cusp simulations, should bracket the actual density profile of M15. The solid lines in Figure 9 represent a King profile with $r_{\text{core}} = 2''$ and a BW cusp profile with a power-law index of -0.75 —the same radial distribution functions used in the construction of the F555W simulations (§3.1). The corrected M15 star counts are consistent with having a central region of constant density extending out to:

$$r_{\text{core}} \lesssim 2'' \quad , \quad (5)$$

(best-fit core radius $\lesssim 1''.3$), or with a power-law density profile $N(r) \sim r^\alpha$ with an index:

$$\alpha = -0.85 \pm 0.20 \quad . \quad (6)$$

Accurate determination of the shape of the stellar density profile of M15 is limited by: (1) Poisson errors and by (2) uncertainties in the correction factor $C(r)$ for $r < 2''$.

In order to verify that the V -band star counts are not significantly affected by the overlapping PSF wings of a few bright RGB stars near the center, the radial density profile has also been derived from counts in the U band where the RGB light is not as dominant. The U -band star counts are fit by essentially the same power-law slope and King core radius as are the V -band counts, but have larger Poisson error bars.

If a massive black hole is present in the inner part of M15, dynamical friction would cause the massive object to settle into the center of the cluster within a short timescale. Excursions of the massive black hole from the center are predicted to be small (BW, Eq. 102) and its expected mean angular separation from the cluster center is:

$$\langle r \rangle = 0''.01 (r_{\text{core}}/1'')(4 \times 10^3 M_\odot / M_{\text{BH}}) \quad , \quad (7)$$

where r_{core} is the core radius that the density distribution would have in the absence of the black hole and is expected to be smaller than $4''$ (see below), while M_{BH} is the black hole mass. Thus any massive black hole near the center of M15 should lie well within the error circle for the cluster center (white circle in Fig. 1).

A black hole significantly affects stellar dynamics within a “gravitational radius” $r_g = \frac{GM_{\text{BH}}}{\sigma_v^2 D}$, where σ_v is the rms dispersion in the line-of-sight velocity at the center of the cluster, and D is the distance to the cluster. For M15, r_g is $0''.35$ for a black hole of mass $10^3 M_\odot$ if we adopt $\sigma_v = 15 \text{ km s}^{-1}$. The Bahcall-Wolf formula describing the modification of an underlying projected stellar density profile in the presence of a central massive object is:

$$N(< r) = N_{\text{King}}(< r) [1 + 2.6 (r/r_{\text{core}})(r/r_g)^{-1.75}] \quad , \quad (8)$$

where $N(< r)$ is the modified cumulative radial distribution of stars as a function of projected distance from the cluster center and N_{King} is a standard King profile (King 1962) with a core radius r_{core} .

The ground-based radial velocity measurements of Peterson, Seitzer & Cudworth (1989) are consistent with a central black hole of mass $M_{\text{BH}} = 1 \times 10^3 M_{\odot}$. If M_{BH} were larger than $3 \times 10^3 M_{\odot}$, the line-of-sight velocity dispersion within the inner $1''$ of M15 would be significantly larger than the 25 km s^{-1} observed by Peterson et al.

As discussed below, the simple King (1962, Eqs. 18–20) fitting formula is an inadequate description of the M15 star counts. The stellar density profile in the range $2'' < r < 20''$ is approximately a power-law, and it may be the result of a variety of processes, possibly including core collapse. In order to obtain a range of estimated black hole masses, we adopt a double King profile (linear combination of two King profiles with different core radii and central surface densities) as the appropriate “background” density distribution upon which to “add” a black hole.

Figure 10 presents a range of illustrative fits to the data. As in Figure 9, the open circles show the uncorrected M15 star counts, while the $\times$ ’s and $+$ ’s show the corrected counts with $C(r)$ derived from the F555W King and the BW cusp simulations, respectively. Various pairs of King models with $r_{\text{core}} = 1'', 2'', 4'', 8'',$ and $16''$ have been added in order to represent the underlying density distribution. For each of the four cases shown in Figures 10a–10d, the ratio of central densities in the two King models comprising the double King profile is adjusted to approximate best the M15 star count data between $r \gtrsim 1''$ and $r = 20''$. The sum of the two King profiles, with zero black hole mass, is shown by the lowest curve in each panel. Double King models with $r_{\text{core}} = [2'', 16'']$ (Fig. 10a) and with $r_{\text{core}} = [1'', 8'']$ (Fig. 10b) fit the corrected stellar distribution in M15 if the cluster is assumed to contain also a central black hole of mass $\gtrsim 1 \times 10^3 M_{\odot}$. Figure 10c shows that, if the smaller core radius of the double King profile is $4''$ or larger, the black hole mass required to fit the observed star counts is too large ($\sim 10^4 M_{\odot}$) to be consistent with the measured velocity dispersion (Peterson et al. 1989). Furthermore, the larger core radius of the double King model must be larger than $4''$ in order to fit the stellar density beyond $r \sim 5''$ (Fig. 10d).

If the steep, cusp-like stellar density profile of M15 is due to a black hole in the center of the cluster, then the above tests using the sum of two King profiles (and the upper limit on M_{BH} imposed by the Peterson et al. radial velocity data) imply that the black hole mass must be:

$$1 \times 10^3 \left(\frac{\sigma_v}{15 \text{ km s}^{-1}} \right)^2 M_{\odot} \lesssim M_{\text{BH}} \lesssim 3 \times 10^3 \left(\frac{\sigma_v}{15 \text{ km s}^{-1}} \right)^2 M_{\odot} \quad . \quad (9)$$

The range in allowed masses is a result of not being able to disentangle the background density profile from the portion of the density profile affected by the possible presence of a black hole. With star counts alone, it is not possible to distinguish statistically between an approximately isothermal collapsed core ($-0.4 \lesssim \alpha \lesssim -1.2$) with no black hole and a cusp of equal-mass stars dominated by a central black hole ($\alpha = -0.75$). It is only when one assumes the presence of a black hole and that the background density distribution of stars is roughly described by a composite King profile (with an inner core size ranging from $1''$ – $4''$) that the above mass limits on the black hole follow.

4.3 Diffuse U Light

We have analyzed the radial distribution of the residual diffuse light in the F336W (U) image of M15 that remains after the subtraction of all bright, resolved stars ($U \lesssim 19$). The diffuse brightness in the U -band is plotted versus the logarithm of the radius in Figure 11 (open circles) and it is in good agreement with that measured by Lauer et al. (1991) (dashed line). We have analyzed the same

images as Lauer et al. and have used similar stellar photometry and PSF-subtraction techniques (both based on DAOPHOT). The agreement between the two sets of diffuse light measurements is therefore expected. The Lauer et al. profile has been shifted downwards (fainter) by 0.1 mag in order to normalize it to our measurements. This small difference in the overall normalizations could be due to the fact that we have removed somewhat fainter stars than Lauer et al. did, or could reflect minor differences in the photometric zeropoints or in the empirical PSF templates used.

While the residual diffuse light is mostly produced by faint main sequence stars, imperfect photometry of a bright star can cause an excess or a deficiency in the diffuse light within $r \sim r_{\text{PSF}} \sim 2''.5$ of the bright star depending upon the sign of the photometric error. The bias in the photometry of resolved stars located near the cluster center leads to a systematic oversubtraction of the diffuse light in the central regions when these stars are removed (see §3.2 and §3.4 for details). This effect is best illustrated in Figure 8 of §3.4, where we have analyzed the diffuse light in the F336W BW cusp simulation. Although the stars in this simulation are distributed as a power-law function of radius, with $\alpha = -0.75$, the residual diffuse light recovered by the PSF fitting/subtraction software flattens within the innermost $2''$ (solid line in Fig. 11), over a region whose size is comparable to the extent of the PSF wings. Thus, the diffuse residual light does not directly trace the underlying stellar distribution in the spherically-aberrated HST images of M15. It is difficult to distinguish between a power-law density profile and one with a $2''$ core on the basis on the unresolved cluster light.

Correction of HST optics (WF/PC2) should remove the large wings of the instrumental PSF. The contamination of the unresolved cluster light by the PSF wings of bright stars near the crowded center of M15 is expected to be greatly reduced, and this will enable a more accurate measurement of the diffuse light. The reduction of the PSF wings will also enable the detection of fainter stars, particularly in the central region, thereby reducing the Poisson errors in the star counts. WF/PC2 images of M15 will allow a determination of the radial profile into $r \sim 0''.4$.

5. STELLAR POPULATIONS AND RADIAL SEGREGATION

Color-magnitude diagrams (CMDs) of post-main sequence stars in M15, as a function of radius (and therefore of density), provide information on various stellar populations, such as red giant branch, horizontal branch, and blue straggler stars, and enable a comparison of the radial distributions of different stellar types. In this section, we describe CMDs based on *UVI* data (§5.1) and present a study of blue straggler stars in the dense inner portions of M15 (§5.2). Radial segregation between stellar types is discussed in §5.3. The object AC 214, located near the center of the cluster, has unusual morphology and colors and is investigated in §5.4.

5.1 Color-Magnitude Diagrams

Figure 12 displays a V versus $V - I$ CMD of ~ 5000 stars located within $\sim 1'$ of the center of M15 over an area of 1.33 arcmin^2 . The horizontal branch (HB) at $V \sim 16$ is tightly defined and well separated from the red giant branch (RGB), to be expected for a cluster with a metallicity as low as that of M15. The blue horizontal branch (BHB) stars form a diffuse extension to the blue side of and fainter than the HB in the color-magnitude plane. While some of the scatter in the BHB distribution is probably intrinsic (*cf.* Stetson 1991), some of it is undoubtedly caused by the increase in I -band photometric errors towards fainter I magnitudes. The bold open squares in Figure 12 mark the blue straggler (BS) candidates found by Ferraro & Paresce (1993, FP) in an

ultraviolet-band study of M15 using the HST/FOC. The BHB sequence extends into the region of the CMD where one expects to find BS stars ($V \sim 18$, $V - I \sim 0.2$), making it difficult to distinguish between these two types of stars on the basis of V and I photometry alone.

A restricted (V , $V - I$) CMD for a “clean” sample of 830 stars within $r < 13''$ of the center of M15 is shown in Figure 13. Only those stars are included for which there are two independent magnitude measurements on different images that agree to within 0.5 mag in each of the V and I bands. The V and I magnitudes used in the CMD in Figure 13 each represent the mean of a pair of measurements. This $r < 13''$ clean CMD compares favorably, in terms of photometric accuracy, to the (V , $B - R$) CMD obtained by Stetson (1991) for the same region of M15 using $0''.5$ seeing ground-based images. A clear separation is seen between the HB, the subgiants, and the BHB/BS stars in Figure 13. Of all the stars with $V < 18$ ($V < 19$) detected in both F555W and F785LP images within $r < 13''$ of the cluster center, 83% (73%) appear in Figure 13. The degree of completeness for faint stars ($V \gtrsim 19$) is much lower in this inner, clean CMD than in the unrestricted CMD (Fig. 12). The increased crowding in the inner region results in: (1) a smaller fraction of faint stars being detected by the analysis software, and (2) larger photometric errors for faint stars, causing them to fail the criterion placed on the accuracy of the pair of measurements. Of the eight FP BS candidates shown in Figure 12 (open bold squares), six are seen in the restricted CMD (Fig. 13), implying that the difference between the pair of measurements for each of these stars (ΔV , ΔI) is less than 0.5 mag.

Figure 14 shows a (V , $U - I$) CMD for the $\gtrsim 1500$ stars detected in all three filters distributed within an area of 0.43 arcmin^2 and extending out to $50''$ from the center of M15. The (V , $U - I$) CMD offers two advantages over the (V , $V - I$) CMD: (1) the longer color baseline results in a clearer separation between various stellar types, amplifying small differences in stellar effective temperatures, and (2) mutually independent quantities are plotted along the abscissa and ordinate (any photometric error in V results in correlated errors in the V versus $V - I$ CMD). The requirement that a star be detected in each of U , V , and I results in a lower degree of completeness in the (V , $U - I$) CMD than in the (V , $V - I$) CMD. The (V , $U - I$) CMD is reproduced in Figure 15 with different symbols indicating the different stellar types: bright red giant branch stars (triangles), horizontal branch stars (+), blue horizontal branch stars ($\times$), and blue straggler stars (open squares). No strict defining criteria have been used in the labeling of stellar types in Figure 15; the task is made relatively straightforward by the fact that the different types are fairly tightly localized and well separated from one another in the (V , $U - I$) CMD, with the possible exception of BSs.

The bright star above and to the left of the bright RGB stars, with $V = 13.11$ and $U - I = 1.46$ (encircled dot in Fig. 15), is believed to be a foreground star. There is an *a priori* probability of $\sim 10\%$ of finding a Galactic “field” star with $V < 15$ within the 1 arcmin^2 M15 image (Ratnatunga & Bahcall 1985). The dashed box in Figure 15 ($16.5 < V < 17.7$, $+0.85 < U - I < +2.0$) encloses a set of faint RGB stars whose radial distribution is used as the standard against which to compare the distributions of other stellar types (§5.3). These faint RGB stars are hereafter referred to as the ‘control’ sample.

The spread in the distribution of any given stellar type in the (V , $V - I$) and (V , $U - I$) CMDs can be used to place an upper limit on the scatter introduced by photometric error. For example, the width of the RGB increases from $\Delta(V - I) = 0.2$ at $V = 14$ to $\Delta(V - I) \gtrsim 0.3$ at $V = 18$, while the width of the HB is about $\Delta V = 0.3$. The quoted widths correspond to roughly 2σ . If the intrinsic widths of the RGB and HB are assumed to be negligible, this implies that the 1σ photometric error in the V band increases from $\lesssim 0.1$ mag at $V = 14$ to $0.15\text{--}0.20$ mag at

$V = 18$. These error estimates represent averages over the inner 1 arcmin^2 of M15. A comparison with Figure 5 shows that the error estimates derived from the F555W simulations are realistic. The photometric uncertainty increases as the degree of crowding increases towards the cluster center (see §3.2 and Fig. 5). Near the center of M15, the V -band photometric error is $\sigma \gtrsim 0.2 \text{ mag}$ for stars with $V \lesssim 18$, so that stellar photometry is less accurate than the 0.1 mag photometry obtained for stars brighter than the main sequence turnoff in the less dense cluster 47 Tuc (Paper I).

5.2 Blue Stragglers

Ground-based observations of the outer parts of M15 ($r > 2'$) have not made a convincing case for the presence of blue straggler stars (Buonanno et al. 1985; Sandage 1970). Recent HST/FOC images of the inner $22'' \times 22''$ region of M15 obtained by Ferraro & Paresce (1993) have revealed nine BS candidates. The FP BSs were selected on the basis of a far-ultraviolet CMD (F220 mag vs. F220–F342 color), where the F220 filter is roughly centered on $2200 \text{ \AA}$ and the F342 filter is similar to the WF/PC F336W filter. We have identified 14 BS candidates in the central $20''$ of M15 on the basis of their position in the $(V, U - I)$ CMD (brighter than the main sequence turnoff, bluer and brighter than the subgiants).

Table 1 shows the FOC UV magnitudes and colors as well as the PC magnitudes and colors of all the BS candidates identified by FP and by this paper. We divide the BS candidates listed in the table into those detected by both the FOC and the PC, those detected by the FOC alone in the region of overlap, those detected by the PC alone in the region of overlap, and those detected by the PC outside the region of overlap. Eight out of the nine candidate BS stars selected by FP on the basis of their UV colors are detected in each of the U , V , and I bands in the PC dataset and lie in the BS region of our $(V, U - I)$ CMD (filled squares in Fig. 15). Figure 16 shows the positions on the sky of the FP BS candidates (filled triangles) and of the BS candidates identified in our study (open circles). The solid outline marks the region of intersection of the PC U -, V -, and I -band coverage, showing where BSs could have been detected, while the dotted square shows the area covered by the HST/FOC image used by FP.

Of the eight BS candidates in common between FP and our PC dataset, five are well separated from the subgiant branch, the BHB, and the HB regions in the $(V, U - I)$ CMD of Figure 15 (star IDs 612, 799, 891, 1009, and 1043 in Table A1). Another star (ID 580) is at $V = 16.93$, $U - I = 0.23$, and is below the HB and might be a BHB star. Of the nine FP BS candidates, the reddest (F220 – F342 = 0.9, star ID 649) is also the FP BS located closest to the cluster center ($r \sim 0''.4$). This star is bluer than, but close to, the faint RGB of the $(V, U - I)$ CMD (Fig. 15). The photometry for this star is uncertain due to the presence of several bright neighbors. Star ID 922 is identified as a BS by FP, and lies close to the blue edge of the UVI subgiant branch.

The bluest FP BS (F220 – F342 = -0.4 , FP ID 83) is only detected in our U -band image, not in the V or I datasets. This star is well off to the blue side of the BS region in the (F220, F220–F342) CMD.

One star defined as a BS in the $(V, U - I)$ CMD (ID 985) is detected by FP but they do not label it as a UV BS star (FP ID 261). It lies just to the red and is fainter than the other nine FP BSs in the far-ultraviolet CMD. Three more BS candidates detected by us lie within $1''$ of the center of the cluster (star IDs 642, 644, and 676) and were not detected by FP. While two of these three stars also lie in the BS region of the $(V, V - I)$ CMD (Fig. 12), one of them, star ID 644, is in the subgiant branch of the $V, V - I$ CMD. This apparent discrepancy may be a result of photometric error in the crowded center, and makes the identification of star ID 644 as a BS less certain.

We also detect two BSs (star IDs 866 and 878) that lie outside the area covered by the FOC image. These additional BSs are listed at the end of Table 1 and have similar colors to the BSs that are in common between the FOC and PC datasets.

The 14 stars we have defined to be BSs lie in the region of the $(V, U - I)$ CMD that is $\gtrsim 1$ mag brighter than the main sequence turnoff, but somewhat redder than the zero-age main sequence (ZAMS). Based on the colors expected for the theoretical ZAMS (Green et al. 1987), the naive interpretation of the BSs as single stars with no peculiarities suggests that they have masses at least as large as $1.2 M_{\odot}$. For comparison, the turnoff mass in M15 is about $0.8 M_{\odot}$. The colors and luminosities of the BSs are consistent with their being contact binaries (*cf.* Mateo et al. 1990) or the product of stellar collisions.

Most of the BSs we detect, as well as those identified by FP, appear to lie on a projected east-west line. A similar alignment has been observed for the six bright BS stars in the center of the nearby cluster NGC 6397 (Aurière, Ortolani & Lauzeral 1990). These configurations are not physically significant since the stars change their positions on time scales short ($< 10^5$ yr) compared to the lifetime of the cluster or of the individual stars ($> 10^8$ yr).

5.3 Radial Segregation by Stellar Type

In this section, we compare the radial distribution of the different kinds of post-main sequence stars within $r < 1'$ of the center of M15. Several authors have reported that the central $6''$ of M15 is bluer than the region further out (Bailyn et al. 1988; Cederbloom et al. 1992). Following Stetson (1991), we investigate whether this color change is due to a depletion of red stars, an excess of blue stars, or some combination of the two. We present evidence that the brightest RGB stars are depleted relative to other stellar types near the cluster center ($r < 6''$). Tidal interactions between stars are thought to be responsible for the destruction of RGB envelopes in the dense central region (Piotto et al. 1988; Djorgovski et al. 1991), biasing the radial distribution of such stars relative to the underlying density distribution.

Figure 17 shows the $(V, U - I)$ CMD in four concentric annular bins. Since the overall stellar density in the cluster is a steep function of radius, we choose the limiting radii of the bins so that the number of ‘control’ stars (as defined in §5.1 and Fig. 15) is equal in each of the four annular regions. Stetson (1991) noted that the stars at the tip of the RGB ($V < 14$) appeared to be depleted for $r < 13''$. Figure 17 confirms this observation; few very bright ($V < 13.5$) stars are present in the central bin ($r < 5''.6$).

The ‘control’ stars, against which we compare the radial distribution of various stellar types, are faint RGB stars. They are selected using the criteria: $16.5 < V < 17.7$ and $+0.85 < U - I < +2.0$. The faint V magnitude limit of 17.7 used to define the control stars is comparable to the faintest BS and BHB stars (see Fig. 15), and is bright enough to ensure that the control stars are not affected by differential incompleteness in the inner parts of the cluster. Since a control star has roughly the same mass as an HB, BHB, or a bright RGB star, the effects of mass segregation are not expected to differentiate between these stellar types and the control population.

In order to determine whether the color trend in the center of M15 is mainly due to a deficiency of bright RGB stars or to an excess of HB stars, we list in Table 2 the number of stars in the four radial bins as a function of stellar type. The numbers in parentheses in Table 2 indicate the fraction of stars of a given type that lie within a specific radial bin (0.25 implies no radial segregation), along with 1σ Poisson errors. Presented in this manner, it is apparent that the bright RGB stars are depleted in the central $6''$; the density of HB stars, if anything, shows a marginal decrease near the

cluster center.

The statistical significance of the bright RGB depletion may be calculated (in a binning-independent fashion) by applying a Kolmogorov-Smirnov (K-S) test. The cumulative radial distributions of the various types of stars are compared to test if they are consistent with being drawn from the same parent distribution. The cumulative radial distributions of the various types of stars in the central 40'' of M15 are plotted in Figure 18. The bright RGB stars appear significantly depleted in the inner 6'' of the cluster, while the HB stars are not strongly depleted. A K-S test shows that the bright RGB depletion is significant at the 98% confidence level. The BS stars appear strongly concentrated in the center, but given the small number of stars and the limited U -band areal coverage (see Fig. 16), the depletion is significant only at the 2σ level.

5.4 AC 214

The object AC 214 (see Fig. 1) is interesting for several reasons. A close inspection of the F555W or the F336W image shows that what has been previously referred to as AC 214 in the literature (Aurière & Cordoni 1981) is actually the superposition of at least three stars (star IDs 695, 699, and 691 in Table A1). The resolution of AC 214 into its three components is even more striking in the HST/FOC UV image of M15 obtained by De Marchi & Paresce (1994), available from the HST archive. The chance for an overlap of three $V < 17$ stars within 0''.1 is less than 1%, if the surface density of stars is assumed to be constant within $r < 1''$ of the cluster center.

The center of M15, as determined from the maximum-likelihood fit (§4.1), is 0''.36 W and 0''.16 S from the brightest component of AC 214 (star ID 695). It is plausible that the multiple object AC 214 is the location of the central cusp in M15. Our “cusp simulations” of clusters, where stars are drawn from a power-law radial density distribution, $N(r) \propto r^\alpha$, generally show a bright central “image” that looks roughly stellar. This central image corresponds to $V \sim 16 \pm 0.5$ for $\alpha = -0.75$. For a steeper cusp, say $\alpha = -1.0$, the brightness of the central image increases to $V \sim 15$. The two brightest components of the AC 214 triplet have $V \sim 15.5$. Figure 14 shows the locations of these three stars in the (V , $U - I$) CMD (bold open squares) along with the color and magnitude that would result if the three components were treated as a single star (large open circle).

A black hole of mass $10^3 M_\odot$ in the center of M15 would produce a line-of-sight velocity dispersion of $\sim 30 \text{ km s}^{-1}$ within the innermost 0''.1 $\sim 1000 \text{ AU}$ (corresponding to the separation between the components of AC 214). A ground-based spectrum of AC 214 has been obtained by Murphy et al. (1991), who were unaware of the multiple nature of this object, but the data are inconclusive with regard to the velocity dispersion among the components. If, on the other hand, AC 214 represents the collapsed core of M15 (without a massive central object), there is no reason to expect an unusually large velocity dispersion among its three components.

AC 214 does not appear to be a strong X-ray source. The nearest X-ray source to the cluster center is AC 211, located about 2''.5 from the stellar center of mass and from AC 214.

6. SUMMARY

The principal results obtained in this paper are summarized below:

1. HST/PC images of M15 in the F555W (V) and F785LP (I) filters are used to derive accurate photometry ($1\sigma = 0.15 \text{ mag}$ for $V < 18$) of the $\sim 10,000$ stars to $V \sim 20$ (1 mag below the main sequence turnoff) that are within $r \lesssim 45''$ of the cluster center. F336W (U) band images yield slightly less accurate magnitudes ($1\sigma = 0.2 \text{ mag}$) due to the lack of U -band flat fields taken

in orbit. Photometric and positional data for 1186 stars brighter than $V = 18.1$ are given in Table A1 of the Appendix.

2. The radial surface density profile of post-main sequence stars within the central $5''$ of the cluster is consistent with a power-law cusp of slope $\alpha = -0.85 \pm 0.2$. After correcting for incompleteness and photometric bias (see §3.3 and Fig. 6), the best-fit slope is slightly steeper. The data are well fit by a cusp with a black hole of mass: $1 \times 10^3 M_{\odot} \lesssim M_{\text{BH}} \lesssim 3 \times 10^3 M_{\odot}$ or by a collapsed-core model without a black hole. The density distribution is also consistent with a core of radius less than $2''$, the best-fit core radius being $1''.3$. Further observations are required to decide which of these descriptions (cusp or small core) is correct. According to our Monte Carlo simulations, corrected optics for the WF/PC2 will allow a determination of the radial profile into $r \sim 0''.4$.
3. Realistic simulations of HST/PC images are used to determine the level of precision of the photometry and to evaluate corrections for incompleteness in the star counts. The simulations also show that the brightness of the U -band diffuse light is not a reliable probe of the star distribution for $r < 2''$ in M15. We derive a radial surface-brightness profile for the U -band diffuse light that is essentially identical to that of Lauer et al. (1991), but the simulations indicate that it is not possible to use the diffuse light to distinguish between a flat core and a central cusp. The large PSF of the existing HST optics results in a flat core in the diffuse light even in simulations in which the input radial distribution of stars follows a power-law cusp (see Fig. 8).
4. Monte Carlo simulations of a cluster with a power-law cusp suggest that, on average, the superposition of several stars should appear as a single bright central object with $V = 16 \pm 0.5$. The object AC 214, located within 2σ of the maximum-likelihood cluster center, consists of at least three stars separated by $\sim 0''.1$ with the two brightest components having $V = 15.5$. AC 214 is a candidate for the cusp in M15. Based on the observed stellar density in M15 and assuming a constant surface density within $r < 1''$, the chance of random superposition of three $V < 17$ stars within $0''.1$ is less than 1%.
5. Fourteen blue straggler candidates are identified in the center of the cluster ($r < 30''$; see Table 1) on the basis of a V vs. $U - I$ color-magnitude diagram. There is excellent agreement between our blue straggler candidates and those identified by Ferraro & Paresce (1993) using far-ultraviolet HST/FOC images. There is marginal evidence for an increase in the blue straggler fraction towards the cluster center.
6. Earlier reports of gradients in the stellar population near the center of M15 (Stetson 1991) are confirmed. The bluer colors in the central portion of the cluster ($r < 6''$) are caused by a depletion of bright red giant branch stars in this region, rather than by an excess of BHB or HB stars.

APPENDIX: TABLE OF STARS

Table A1 lists the positions, magnitudes and colors of all 1186 stars with $V < 18.1$ that are also detected in the I -band image. A long dash in the $U - V$ column indicates that the star lies outside the region covered by the U image, while an ellipsis indicates that the star is too faint to be detected in the U band. The positions are in equinox 2000 coordinates relative to the star AC 211 (star ID 653).

Computer-readable copies of the photometric and positional data may be obtained by contacting B.Y. or P.G..

We thank S. Kirhakos for obtaining several ground-based images of M15 which allowed the photometric zero points to be determined for the *U*-band data. We are grateful to P. Stetson for providing DAOPHOT II software and to C. Sarazin for providing a copy of his maximum-likelihood programs. We are grateful to T. Lauer for a number of valuable suggestions about the data analysis. We thank J. MacKenty for information on WF/PC flat fields. We acknowledge useful discussions with C. Bailyn, H. Cohn, S. Faber, P. Hut, I. King, M. Schwarzschild, and L. A. Willson. Support for this work was provided by NASA through grant number NAG5-1618 and grants HF-1013.01-90A (B.Y./Hubble fellowship) and HF-1033.01-92A (P.G./Hubble fellowship) from the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., under NASA contract NAS5-26555.

REFERENCES

- Aurière, M., & Cordoni, J.-P. 1981, *A&ASup*, 46, 347 (AC)
- Aurière, M., LeFevre, O., & Terzan, A. 1984, *A&A*, 138, 415
- Aurière, M., Ortolani, C., & Lauzeral, S. 1990, *Nature*, 344, 638
- Bahcall, J. N., Bahcall, N. A., & Weistrop, D. 1975, *Astrophys. Lett.*, 16, 159
- Bahcall, J. N., & Ostriker, J. P. 1975, *Nature*, 256, 23
- Bahcall, J. N., & Wolf, R. A. 1976, *ApJ*, 209, 214 (BW)
- Bahcall, J. N., & Wolf, R. A. 1977, *ApJ*, 216, 883
- Bailyn, C. D., Grindlay, J. E., Cohn, H., & Lugger, P. M. 1988, *ApJ*, 331, 303
- Buonanno, R., Corsi, C. E., & Fusi Pecci, F. 1985, *A&A*, 145, 97
- Burrows, C. J., Holtzman, J. A., Faber, S. M., Bely, P. Y., Hasan, H., Lynds, C. R., & Schroeder, D. 1991, *ApJL*, 369, L21
- Cederbloom, S. E., Moss, M. J., Cohn, H. N., Lugger, P. M., Bailyn, C. D., Grindlay J. E., & McClure, R. D. 1992, *AJ*, 103, 480
- DaCosta, G. S., & Armandroff, T. E. 1990, *AJ*, 100, 162
- De Marchi, G., & Paresce, F. 1994, *ApJ* (20 Feb)
- Djorgovski, S., & King, I. R. 1984, *ApJ*, 277, L49
- Djorgovski, S., Piotto, G., Phinney, E. S., & Chernoff, D. F. 1991, *ApJL*, 372, L41
- Dubath, P., Meylan, G., & Mayor, M. 1993, *ApJ*, in press
- Dupree, A. K. 1986, *ARAA*, 24, 377
- Faber, S. M. (editor) 1991, "Final Orbital/Science Verification Report", Space Telescope Science Institute Publication
- Fahlman, G. G., Richer, H. B., & Van den Berg D. A. 1985, *ApJSup*, 58, 225
- Ferraro, F. R., & Paresce, F. 1993, *AJ*, 106, 154 (FP)
- Goodman, J. 1989, in "Dynamics of Dense Stellar Systems", ed. D. Merrit (New York: Cambridge Univ. Press), p 183
- Gould, A. P., & Yanny, B. 1993, *PASP*, in press
- Grabhorn, R. P., Cohn, H. N., Lugger, P. M., & Murphy, B. W. 1992, *ApJ*, 392, 86
- Green, E. M., Demarque, P., & King, C. R. 1987, in "The Revised Yale Isochrones and Luminosity Functions" (New Haven: Yale Univ. Observatory Publication), p 80
- Griffiths, R. 1989, "Wide Field and Planetary Camera Instrument Handbook", Space Telescope Science Institute Publication
- Guhathakurta, P., Yanny, B., Schneider, D. P., & Bahcall, J. N. 1992, *AJ*, 104, 1790 (Paper I)
- Harris, H. C., Baum, W. A., Hunter, D. A., & Kreidl, T. J. 1991, *AJ*, 101, 677

- Heggie, D. C. 1975, MNRAS, 173, 729
- Heggie, D. C. 1985, in "Dynamics of Star Clusters", Proc. IAU Symp. 113, eds. J. Goodman & P. Hut, p 139
- Johnson, H. L. 1966, ARAA, 4, 193
- King, I. R. 1962, AJ, 67, 471
- King, I. R. 1975, in "Dynamics of Stellar Systems", Proc. IAU Symp. 69, ed. A. Hayli (Dordrecht: Reidel), p 99
- Kulkarni, S. R., Goss, W. M., Wolszczan, A., & Middleditch, J. M. 1990, ApJL, 363, L5
- Lauer, T. R. 1989, PASP, 101, 445
- Lauer, T. R., et al. 1991, ApJL, 369, L45
- Mateo, M., Harris, H. C., Nemec, J., & Olszewski, E. W. 1990, AJ, 100, 469
- McMillan, S. L. W. 1989, in "Dynamics of Dense Stellar Systems", ed. D. Merrit (New York: Cambridge Univ. Press), p 207
- Murphy, B., Cohn, H., & Hut, P. 1990, MNRAS, 245, 335
- Murphy, B., Rutten, R. G. M., Callanan, P. J., Seitzer, P., Charles, P. A., Cohn, H. N., & Lugger, P. M. 1991, Nature, 351, 130
- Peterson, R. C., Seitzer, P., & Cudworth, K. M. 1989, ApJ, 347, 251
- Phinney, E. S., & Sigurdsson, S. 1991, Nature, 349, 220
- Piotto, G., King, I. R., & Djorgovski, S. 1988, AJ, 96, 1918
- Ratnatunga, K. U., & Bahcall, J. N. 1985, ApJSup, 59, 63
- Sandage, A. R. 1970, ApJ, 162, 841
- Sarazin, C. L. 1980, ApJ, 236, 75
- Stetson, P. B. 1987, PASP, 99, 191
- Stetson, P. B. 1991, in "Precision Photometry", eds. A. G. Davis Philip, A. R. Upgren, & K. A. Janes (Schenectady: L. Davis Press), p 69
- Stetson, P. B. 1992, in "Astronomical Data Analysis Software", ed. D. M. Worrall, C. Biemesderfer, & J. Barnes (ASP Conf. series, no. 25), p 297
- Sugimoto, D., & Bettwieser, E. 1983, MNRAS, 204, 19p
- Webbink, R. F. 1985, in "Dynamics of Star Clusters", Proc. IAU Symp. 113, eds. J. Goodman & P. Hut, p 541

Table 1.
Blue Stragglers in M15

Table A1 ID	FP ID	F220-F342	F220	F342	U	V	I	U - I	Comments
<i>BSs detected by PC and FOC in overlap</i>									
651	122	0.9	18.4	17.5	16.86	16.95	16.17	0.68	$r < 1''$; reddest FP BS; near faint RGB in <i>UVI</i> CMD
612	109	0.4	18.1	17.7	17.29	17.45	17.13	0.16	
799	196	0.6	18.3	17.7	17.37	17.81	17.28	0.09	
922	238	0.6	18.5	17.9	17.69	17.83	17.26	0.43	$r \sim 2''$; on edge of <i>UVI</i> subgiant region
891	222	0.5	18.3	17.8	17.50	17.73	17.56	-0.06	
580	81	0.5	18.0	17.5	17.17	16.93	16.94	0.23	below HB, possibly BHB
1009	273	0.4	18.1	17.7	17.67	17.87	17.75	-0.07	
1043	280	0.6	18.3	17.7	17.68	17.87	17.67	0.01	
<i>BSs detected by FOC alone in overlap</i>									
...	83	-0.4	18.1	18.5	18.19	...	...	...	bluest FP BS; no V or I detection
<i>BS detected by PC alone in overlap</i>									
985	261	1.1	18.8	17.7	17.59	17.76	17.45	0.14	near FP BS region
642	...	...	...	...	17.59	17.77	17.12	0.47	$r < 1''$; on edge of <i>UVI</i> subgiant region
644	...	...	...	...	16.81	17.59	16.42	0.39	$r < 1''$; near <i>UVI</i> subgiant region
676	...	...	...	...	17.46	17.80	17.44	0.02	$r < 1''$
<i>BSs detected by PC outside overlap</i>									
866	...	...	...	...	17.42	17.80	17.51	-0.09	
878	...	...	...	...	17.87	17.72	17.55	0.32	near <i>UVI</i> subgiant region

Table 2.

Counts^a of various stellar types as a function of distance from the center of M15

Stellar Type	$[0'' < r < 5.6'']$	$[5.6'' < r < 10.1'']$	$[10.1'' < r < 15.9'']$	$[r > 15.9'']$
Control stars ^b	63 (0.25±0.03)	63 (0.25±0.03)	63 (0.25±0.03)	65 (0.26±0.03)
Bright RGB	25 (0.14±0.03) ^c	43 (0.23±0.04)	62 (0.34±0.04)	54 (0.29±0.04)
HB	18 (0.17±0.04)	32 (0.30±0.05)	25 (0.24±0.05)	30 (0.29±0.05)
BS	7 (0.5±0.19)	4 (0.28±0.14)	2 (0.14±0.10)	1 (0.07±0.07)
BHB	12 (0.18±0.05)	19 (0.28±0.07)	14 (0.21±0.06)	22 (0.33±0.07)

^a The figures in parentheses indicate the fraction of the total number stars of a given type that are contained in a particular radial bin, along with 1σ Poisson errors.

^b ‘Control stars’ are defined to be faint RGB stars in the range $16.5 < V < 17.7$ and $+0.85 < U - I < +2.0$. The limiting radii of the bins are selected so that the number of control stars in each bin is approximately the same.

^c Statistically-significant deficit at the $\geq 3\sigma$ level.

TABLE A1. Stellar Photometry and Astrometry for M 15 ($V < 18.1$)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
1	-47.24	-9.03	17.93	1.09	—	61	-34.28	-1.09	15.41	1.04	0.62	121	-26.11	-15.77	17.72	0.47	—
2	-46.73	-38.55	17.23	1.16	—	62	-34.17	-3.83	16.16	0.31	0.07	122	-26.07	-6.56	17.58	1.02	0.34
3	-46.38	-10.22	17.35	1.00	—	63	-34.16	-6.97	17.51	1.02	...	123	-26.01	-32.11	16.76	1.08	—
4	-45.70	-49.12	16.39	1.26	—	64	-34.12	-46.66	17.69	1.05	—	124	-26.00	-36.05	17.69	1.16	—
5	-44.96	-18.82	15.66	1.14	—	65	-33.58	-46.77	16.24	0.28	—	125	-25.92	3.92	17.22	1.20	-0.08
6	-44.87	-48.64	14.49	1.08	—	66	-33.32	-3.27	16.28	0.34	0.16	126	-25.78	6.89	17.72	-0.11	-1.38
7	-44.79	-3.29	16.48	0.17	—	67	-33.22	0.01	16.54	1.06	0.20	127	-25.65	-9.80	17.53	1.05	0.53
8	-43.84	4.40	17.75	1.14	...	68	-33.15	-29.70	16.81	0.22	—	128	-25.50	-39.63	17.08	1.07	—
9	-43.63	-16.98	16.30	0.28	—	69	-32.89	-32.79	17.91	0.05	—	129	-25.46	-13.41	16.92	1.06	1.09
10	-43.54	0.50	17.38	1.39	0.19	70	-32.81	-2.79	14.83	1.06	1.07	130	-25.05	2.87	17.03	1.01	0.23
11	-42.88	-12.94	16.21	1.07	—	71	-32.79	7.11	15.93	0.12	0.21	131	-24.96	-35.08	17.56	1.08	—
12	-42.70	-49.20	16.42	1.11	—	72	-32.76	-7.52	17.44	1.06	0.29	132	-24.92	-37.14	16.64	1.05	—
13	-42.19	-45.54	15.16	1.19	—	73	-32.52	-9.88	15.94	0.49	—	133	-24.74	-11.00	15.98	0.81	0.39
14	-41.90	8.37	17.47	1.08	0.17	74	-32.48	-4.21	17.46	1.13	0.05	134	-24.74	-10.66	17.82	1.09	0.18
15	-41.78	-28.69	16.58	0.76	—	75	-32.40	-15.35	17.68	1.02	—	135	-24.60	-25.68	15.99	1.16	—
16	-41.78	-17.53	17.20	1.04	—	76	-32.28	-17.54	15.23	0.97	—	136	-24.54	-11.24	17.40	1.09	0.43
17	-41.34	-42.29	17.21	1.11	—	77	-32.15	6.16	18.03	0.01	-1.51	137	-24.45	-17.56	17.77	1.09	—
18	-41.03	-20.06	15.96	1.00	—	78	-32.14	-44.04	14.67	1.08	—	138	-24.40	-16.14	13.01	1.38	—
19	-40.99	-8.21	16.14	1.02	0.77	79	-32.13	-53.91	18.02	1.08	—	139	-24.14	-7.06	16.07	0.55	-0.08
20	-40.92	-28.85	17.07	1.09	—	80	-32.05	-1.99	18.06	1.14	0.09	140	-24.08	-33.30	17.80	1.04	—
21	-40.89	-13.55	15.87	0.84	—	81	-31.65	3.74	16.95	0.13	-1.08	141	-24.05	8.99	15.79	0.99	0.42
22	-40.59	-24.09	16.17	0.40	—	82	-31.56	-31.77	15.82	1.17	—	142	-24.04	0.67	17.72	1.08	0.08
23	-40.58	-55.82	14.71	1.09	—	83	-31.44	-24.94	15.93	0.55	—	143	-23.96	-7.64	17.28	1.09	0.29
24	-40.53	-11.80	17.87	1.03	—	84	-31.22	-53.78	16.75	1.02	—	144	-23.95	9.36	17.91	1.04	0.15
25	-40.50	-5.83	17.43	1.04	0.34	85	-31.05	-39.62	17.92	1.03	—	145	-23.86	-28.16	15.80	1.14	—
26	-40.35	-15.32	17.05	1.01	—	86	-31.04	-35.29	14.28	1.16	—	146	-23.70	4.88	15.61	0.96	0.35
27	-40.33	-55.92	16.25	0.90	—	87	-30.81	-24.73	17.07	1.11	—	147	-23.59	-18.79	16.97	1.11	—
28	-40.15	-0.23	17.13	1.08	0.23	88	-30.67	-54.17	17.47	1.02	—	148	-23.59	-2.00	17.41	0.89	0.20
29	-39.98	-37.30	16.38	0.39	—	89	-30.55	7.91	17.42	1.06	-0.03	149	-23.50	-16.91	17.34	1.28	—
30	-39.86	-54.70	16.07	1.17	—	90	-30.05	-5.05	16.10	0.34	0.11	150	-23.45	-19.01	14.56	1.02	—
31	-38.64	-42.23	16.46	0.25	—	91	-30.02	6.98	17.85	1.01	0.03	151	-23.39	-2.18	16.24	0.37	0.29
32	-38.62	-2.10	17.08	0.13	-1.01	92	-30.00	-46.29	17.38	1.20	—	152	-23.23	-13.83	16.50	1.04	—
33	-38.58	-23.88	16.16	0.24	—	93	-29.83	-8.11	13.66	1.19	1.77	153	-23.06	-4.16	17.48	1.02	0.49
34	-38.25	-10.97	16.36	1.00	0.66	94	-29.48	2.78	17.38	0.11	-1.19	154	-22.83	-26.64	14.25	1.14	—
35	-37.86	-3.67	14.86	1.01	0.73	95	-29.34	-49.40	17.23	1.01	—	155	-22.67	3.21	17.65	0.95	0.09
36	-37.84	-23.91	16.03	0.88	—	96	-29.32	9.50	18.06	0.98	0.10	156	-22.64	-30.27	17.86	1.08	—
37	-37.75	-23.52	17.54	0.60	—	97	-29.16	-2.65	16.90	1.08	0.36	157	-22.50	-22.20	14.78	1.08	—
38	-37.70	-25.57	15.62	0.41	—	98	-29.11	9.94	16.25	0.69	-0.17	158	-22.49	-14.32	17.17	0.18	—
39	-37.31	-5.18	17.79	1.08	0.03	99	-28.66	-33.40	15.22	1.11	—	159	-22.38	3.70	16.39	0.16	-0.30
40	-37.19	9.44	16.20	0.30	-0.05	100	-28.64	-10.64	14.60	1.07	1.12	160	-22.21	-5.02	16.33	0.68	0.15
41	-37.07	-18.53	17.89	0.90	—	101	-28.52	-11.98	17.37	1.06	—	161	-21.91	-29.42	16.37	1.11	—
42	-36.81	5.85	16.83	1.10	0.25	102	-28.51	-19.27	15.96	0.73	—	162	-21.88	-14.90	17.44	1.06	—
43	-36.80	-28.68	17.39	1.16	—	103	-28.47	-51.09	15.74	0.02	—	163	-21.82	2.36	18.09	0.89	-0.19
44	-36.35	-8.83	14.36	0.99	1.13	104	-28.32	-0.96	16.12	0.31	0.00	164	-21.77	-10.88	16.18	0.25	—
45	-36.31	-10.83	17.88	1.08	0.25	105	-28.18	-48.35	16.15	0.20	—	165	-21.44	1.98	16.23	0.59	0.06
46	-36.30	6.25	16.02	0.51	0.16	106	-27.98	7.53	16.21	0.18	-0.15	166	-21.28	-51.92	14.08	1.06	—
47	-36.09	-15.10	14.38	1.01	—	107	-27.84	-6.26	17.67	1.14	0.08	167	-20.92	6.50	17.94	0.95	-0.02
48	-36.00	2.32	15.10	1.08	0.74	108	-27.70	-5.29	17.56	1.08	0.19	168	-20.91	-13.17	17.83	0.22	—
49	-35.67	-29.99	14.18	1.21	—	109	-27.46	-2.82	14.15	1.16	1.57	169	-20.91	-9.61	18.09	1.14	—
50	-35.59	-22.82	15.16	1.05	—	110	-27.37	-18.67	17.33	1.04	—	170	-20.85	-0.49	16.26	0.16	0.03
51	-35.49	-10.50	16.80	1.02	0.47	111	-27.22	-41.83	16.79	0.12	—	171	-20.51	-17.55	16.07	0.36	—
52	-35.48	0.16	16.14	1.06	0.39	112	-27.03	-0.17	16.98	1.05	0.24	172	-20.46	-9.54	15.56	1.06	—
53	-35.41	-52.64	18.02	1.08	—	113	-26.86	-3.21	17.56	1.22	0.41	173	-20.13	-16.55	16.95	0.21	—
54	-35.29	-12.38	16.23	0.55	0.00	114	-26.75	-54.86	15.07	1.01	—	174	-20.13	-17.08	17.57	1.01	—
55	-35.10	-33.17	17.76	1.14	—	115	-26.60	-54.72	16.12	1.20	—	175	-20.12	-4.43	17.94	1.11	-0.04
56	-35.01	-31.48	17.42	1.10	—	116	-26.51	-35.00	15.72	1.08	—	176	-20.04	-30.86	17.88	1.04	—
57	-34.80	-8.68	17.54	1.04	0.09	117	-26.41	3.93	14.05	1.15	1.38	177	-19.96	-53.20	17.27	1.07	—
58	-34.76	-11.35	17.83	1.03	0.04	118	-26.39	1.37	17.27	1.03	0.15	178	-19.93	-49.00	17.28	0.96	—
59	-34.72	-19.13	15.71	0.34	—	119	-26.31	-26.57	16.78	1.14	—	179	-19.88	-48.02	17.56	0.94	—
60	-34.68	-2.99	16.94	1.11	0.33	120	-26.16	-20.93	15.74	1.05	—	180	-19.86	-13.40	17.23	1.04	—

^a Positions relative to the star AC 211 (star ID #653), which has $\alpha_{2000} = 21^{\text{h}}29^{\text{m}}58.^{\text{s}}26$; $\delta_{2000} = 12^{\circ}10'02''9$

“—” indicates the star is located outside the area covered by the U image; “...” indicates the star is not detected in U .

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
181	-19.79	2.24	17.96	1.07	0.27	241	-15.17	-5.09	17.99	0.98	0.71	301	-10.74	-10.42	17.87	-0.19	-1.15
182	-19.78	-20.51	16.13	1.10	—	242	-15.00	-13.33	15.97	1.06	—	302	-10.72	-30.22	16.11	0.41	—
183	-19.65	4.88	18.02	0.76	-0.22	243	-14.96	-6.47	17.45	0.83	0.73	303	-10.71	10.37	17.95	1.09	1.03
184	-19.43	-3.55	17.83	-0.22	-1.43	244	-14.96	4.12	16.15	1.14	0.54	304	-10.66	-5.62	16.33	1.51	0.29
185	-19.39	4.03	16.75	1.01	0.21	245	-14.95	4.62	15.59	1.14	0.70	305	-10.66	10.29	18.00	1.26	0.04
186	-19.34	1.07	15.99	0.31	0.26	246	-14.90	-5.73	17.06	1.04	0.54	306	-10.63	7.28	16.22	0.81	-0.17
187	-19.25	10.73	15.92	0.17	0.18	247	-14.90	-15.79	17.63	0.98	—	307	-10.62	-18.92	12.99	1.35	—
188	-19.19	-25.02	13.27	1.44	—	248	-14.89	-6.40	17.53	0.90	0.65	308	-10.57	-8.17	16.17	0.44	0.17
189	-19.16	1.86	15.49	1.17	0.62	249	-14.78	6.75	17.85	1.05	0.39	309	-10.49	-7.47	13.39	1.29	2.34
190	-19.12	-31.30	17.57	0.05	—	250	-14.72	-12.39	16.77	-0.09	—	310	-10.46	11.43	17.87	-0.04	-1.21
191	-18.87	5.63	16.09	1.01	0.32	251	-14.58	-40.61	16.25	0.30	—	311	-10.44	-15.93	18.08	0.90	0.56
192	-18.86	-26.58	17.09	0.44	—	252	-14.55	-36.04	17.49	0.89	—	312	-10.35	0.34	17.97	0.65	0.40
193	-18.83	6.61	14.36	0.99	1.00	253	-14.52	9.69	17.79	0.99	...	313	-10.33	-9.20	17.68	1.01	0.09
194	-18.80	-9.61	15.59	1.12	—	254	-14.45	9.63	17.94	1.02	0.45	314	-10.28	-2.22	16.25	1.09	0.51
195	-18.77	-17.43	16.55	1.06	—	255	-14.12	-44.34	17.86	0.95	—	315	-10.15	-11.49	17.96	0.92	0.49
196	-18.43	-11.38	16.94	0.97	—	256	-13.87	-3.84	16.82	1.01	0.58	316	-10.12	1.22	18.07	1.04	0.25
197	-18.29	1.97	16.21	0.35	0.04	257	-13.83	1.79	15.16	1.18	1.02	317	-10.06	-1.41	16.35	0.32	-0.03
198	-18.29	2.97	16.65	1.09	0.31	258	-13.51	-6.01	15.57	1.09	0.85	318	-10.05	-17.67	16.07	0.64	0.37
199	-18.28	-7.95	14.57	1.23	—	259	-13.49	-13.35	17.12	1.05	—	319	-10.02	1.79	15.31	0.36	0.15
200	-18.17	-20.64	17.81	1.10	—	260	-13.44	2.71	16.46	1.12	0.53	320	-10.01	-4.01	17.51	1.03	0.31
201	-18.05	-17.24	17.24	0.11	—	261	-13.43	-8.61	13.92	1.31	1.79	321	-10.01	-11.36	17.88	0.06	-0.97
202	-17.63	-14.69	15.47	1.08	—	262	-13.38	-5.07	17.88	0.90	0.44	322	-10.01	-32.55	17.66	0.98	—
203	-17.62	-4.52	17.81	0.82	0.57	263	-13.38	-12.86	17.04	1.01	—	323	-9.97	-9.92	17.96	0.94	0.27
204	-17.61	-7.29	16.05	1.04	0.71	264	-13.37	-15.63	16.04	0.29	—	324	-9.97	-15.88	17.03	0.99	—
205	-17.59	-20.06	15.18	1.04	—	265	-13.33	-10.84	16.95	1.07	0.71	325	-9.91	10.76	16.47	1.14	1.63
206	-17.53	-17.31	16.24	1.02	—	266	-13.19	-29.89	15.91	0.35	—	326	-9.89	-4.37	17.99	0.89	0.45
207	-17.43	1.73	13.61	1.41	1.67	267	-13.18	-19.37	16.13	0.85	—	327	-9.86	10.69	16.53	1.25	0.38
208	-17.40	-45.07	13.44	1.44	—	268	-13.02	-11.23	17.65	0.98	0.71	328	-9.82	-14.46	15.96	1.01	1.93
209	-17.32	-14.39	17.22	1.08	—	269	-13.00	-12.61	17.45	1.12	0.60	329	-9.80	-14.36	15.79	0.96	1.83
210	-17.30	-0.09	17.87	0.61	0.30	270	-12.84	-48.68	15.96	1.20	—	330	-9.77	5.34	15.94	1.22	0.69
211	-17.25	6.77	15.40	1.05	0.74	271	-12.71	-24.38	17.93	0.89	—	331	-9.75	-2.99	15.55	1.12	0.62
212	-17.12	-21.36	16.43	1.10	—	272	-12.65	0.14	16.50	1.09	0.64	332	-9.70	11.92	16.28	0.88	0.09
213	-17.12	5.37	17.93	0.87	0.24	273	-12.57	9.47	17.98	-0.06	-1.27	333	-9.47	8.42	15.88	1.19	0.66
214	-17.08	0.60	15.95	1.13	0.45	274	-12.55	-13.45	17.08	0.87	0.80	334	-9.45	-5.84	17.06	0.97	0.49
215	-16.97	5.55	17.74	0.91	0.14	275	-12.54	-30.04	17.43	0.95	—	335	-9.45	-34.47	17.76	1.18	—
216	-16.86	-21.74	16.72	1.04	—	276	-12.47	3.33	18.03	0.10	-1.28	336	-9.44	4.78	17.74	0.30	-1.15
217	-16.68	-50.57	16.22	0.23	—	277	-12.40	-10.04	16.46	1.14	0.49	337	-9.40	-17.89	17.58	1.09	1.08
218	-16.66	6.66	17.37	0.95	0.22	278	-12.39	-7.66	16.90	1.01	0.52	338	-9.37	-4.65	16.92	0.01	-0.67
219	-16.62	-13.60	17.09	1.00	—	279	-12.27	-26.16	16.70	0.95	—	339	-9.37	-2.22	14.66	1.13	1.08
220	-16.55	-10.93	17.21	0.92	—	280	-12.16	1.75	17.59	1.08	0.31	340	-9.31	-3.77	16.14	0.39	0.19
221	-16.35	-48.84	17.40	0.95	—	281	-12.11	-31.38	16.13	0.26	—	341	-9.23	10.85	15.00	1.24	2.15
222	-16.32	-0.69	17.53	1.03	0.26	282	-12.10	-48.18	17.20	1.08	—	342	-9.20	-19.03	16.23	0.28	-0.10
223	-16.23	-2.20	15.30	1.18	0.86	283	-12.09	-18.74	13.18	1.32	—	343	-9.10	-1.31	16.15	0.38	0.15
224	-16.22	1.61	18.04	0.76	0.22	284	-12.07	-4.38	17.57	1.01	0.36	344	-9.07	-22.86	16.81	0.66	—
225	-16.22	1.92	16.49	1.08	0.55	285	-12.07	-24.60	17.61	0.86	—	345	-8.96	-4.59	16.69	0.10	-0.31
226	-16.19	2.65	15.87	1.10	0.50	286	-11.95	-11.88	14.98	1.12	1.12	346	-8.84	-34.06	16.13	1.19	—
227	-16.05	7.05	17.98	0.95	-0.01	287	-11.89	-11.85	14.98	1.04	1.12	347	-8.84	7.17	15.91	1.17	0.64
228	-16.00	8.63	17.19	0.05	-0.94	288	-11.79	1.01	17.60	1.01	0.22	348	-8.79	-1.50	18.03	0.76	-0.22
229	-15.98	-27.27	17.35	1.06	—	289	-11.78	-6.39	17.11	1.01	0.46	349	-8.72	8.51	17.76	1.17	0.33
230	-15.93	-4.88	17.89	0.87	...	290	-11.73	-10.41	17.02	1.01	0.41	350	-8.66	-2.92	15.79	0.58	0.01
231	-15.81	-38.24	17.52	0.99	—	291	-11.59	-25.33	18.03	0.96	—	351	-8.64	-0.43	15.11	1.12	0.77
232	-15.70	7.76	17.54	0.00	-0.94	292	-11.55	-47.65	17.67	1.08	—	352	-8.63	2.86	15.85	1.14	0.50
233	-15.67	7.10	17.38	0.99	0.54	293	-11.48	-7.26	15.68	1.10	0.81	353	-8.60	-2.87	16.13	1.15	-0.20
234	-15.58	-1.12	17.45	-0.03	-0.92	294	-11.38	-48.36	15.71	1.17	—	354	-8.59	-11.01	17.83	0.48	0.10
235	-15.48	-0.08	17.72	1.08	0.33	295	-11.37	-5.25	17.07	1.03	0.43	355	-8.52	-18.59	17.62	1.24	0.38
236	-15.39	-5.83	18.05	0.82	0.52	296	-11.33	-14.20	16.85	0.86	0.78	356	-8.45	-12.19	17.96	0.90	0.33
237	-15.36	0.08	17.73	1.05	0.41	297	-11.25	-0.02	15.32	1.12	0.93	357	-8.40	2.50	17.48	1.03	0.16
238	-15.32	-11.01	16.12	0.20	0.25	298	-11.08	9.11	16.36	1.18	1.12	358	-8.35	-13.43	16.40	1.03	0.45
239	-15.27	-3.14	17.66	1.08	0.26	299	-11.07	-4.39	16.63	1.02	0.55	359	-8.34	12.12	16.39	1.28	0.51
240	-15.23	-1.13	16.87	0.12	-0.77	300	-10.97	-39.52	17.73	0.96	—	360	-8.28	4.11	16.06	1.14	0.64

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
361	-8.27	-13.58	17.48	0.17	-1.01	421	-6.61	-51.32	16.09	1.15	—	481	-4.44	-19.32	17.83	0.68	—
362	-8.27	1.64	17.09	0.03	-0.67	422	-6.61	2.77	17.33	1.01	0.22	482	-4.43	-11.36	18.09	1.10	0.23
363	-8.26	-35.92	17.43	1.01	—	423	-6.59	5.60	15.60	1.15	0.72	483	-4.41	-19.21	17.74	0.76	0.25
364	-8.25	1.80	18.00	0.96	0.15	424	-6.57	-2.64	18.08	0.97	0.25	484	-4.38	-5.87	15.93	0.35	0.18
365	-8.24	-19.86	16.81	1.01	0.69	425	-6.50	-3.56	16.13	0.44	0.25	485	-4.38	11.84	16.48	0.32	-0.25
366	-8.22	8.90	16.30	0.34	0.14	426	-6.47	-9.30	16.24	0.35	0.13	486	-4.35	12.29	14.11	1.21	1.42
367	-8.21	-15.57	17.01	0.79	—	427	-6.45	-12.90	15.42	0.93	0.95	487	-4.29	1.26	17.74	1.17	0.09
368	-8.18	-10.19	17.64	0.96	0.47	428	-6.33	-50.55	17.19	0.14	—	488	-4.29	13.81	15.63	1.22	0.44
369	-8.17	-4.75	17.85	1.03	0.13	429	-6.32	-14.70	16.05	0.23	0.08	489	-4.17	-2.83	17.24	0.95	0.31
370	-8.16	-15.50	16.90	0.88	0.49	430	-6.29	1.62	17.80	1.12	0.11	490	-4.16	-0.73	17.25	1.12	-0.03
371	-8.13	-1.49	16.78	0.18	-0.48	431	-6.21	-2.91	17.11	1.08	0.08	491	-4.16	-11.85	18.03	0.67	0.36
372	-8.02	-0.95	17.58	1.12	0.28	432	-6.19	-4.18	16.59	1.06	0.28	492	-4.14	-6.25	17.14	0.88	0.52
373	-7.98	4.54	17.53	1.00	0.08	433	-6.18	-5.69	17.81	1.00	0.38	493	-4.13	-2.40	17.85	0.85	0.38
374	-7.94	-17.84	14.24	0.95	—	434	-6.17	-10.03	17.82	1.02	0.34	494	-4.12	-7.19	17.77	0.87	0.24
375	-7.90	-17.74	14.14	1.01	1.57	435	-6.16	-5.17	17.83	0.16	-1.26	495	-4.11	7.93	15.32	1.04	0.38
376	-7.88	-34.82	13.40	1.41	—	436	-6.14	-9.56	13.47	1.26	1.90	496	-3.99	-5.35	17.27	0.77	0.37
377	-7.85	-39.79	16.09	0.32	—	437	-6.14	-38.93	16.52	1.16	—	497	-3.98	-24.10	17.33	1.00	—
378	-7.83	-13.30	17.37	1.03	0.28	438	-6.12	-39.21	15.12	1.15	—	498	-3.98	-45.01	16.37	1.11	—
379	-7.80	-28.52	16.78	0.14	—	439	-6.08	12.19	17.93	1.40	0.38	499	-3.90	-0.61	16.16	0.38	-0.03
380	-7.74	11.34	17.30	1.52	-0.10	440	-6.02	-3.29	15.19	1.14	0.77	500	-3.88	-13.89	15.23	0.96	0.98
381	-7.72	1.61	16.09	0.41	0.09	441	-5.87	1.32	16.04	0.62	0.14	501	-3.86	-4.55	17.02	1.00	0.09
382	-7.69	-25.53	17.93	0.92	—	442	-5.80	-41.29	17.70	1.09	—	502	-3.82	-4.19	17.96	0.95	-0.10
383	-7.67	-27.30	16.58	0.21	—	443	-5.75	6.04	18.07	1.04	0.01	503	-3.82	0.45	17.90	-0.20	-1.43
384	-7.66	-46.11	14.93	1.14	—	444	-5.71	13.86	13.45	1.32	1.93	504	-3.79	7.15	17.83	1.05	0.22
385	-7.64	-10.16	16.27	0.97	0.78	445	-5.68	-8.35	17.12	1.01	0.23	505	-3.76	6.44	17.85	1.07	0.06
386	-7.60	4.71	16.17	1.10	0.62	446	-5.67	-0.98	16.18	0.42	0.05	506	-3.75	5.71	17.85	0.97	0.04
387	-7.57	8.49	14.54	1.23	1.22	447	-5.65	-0.71	17.37	0.93	0.12	507	-3.73	-0.86	16.90	0.96	0.16
388	-7.56	-19.20	17.65	0.63	—	448	-5.51	4.29	16.11	0.00	0.13	508	-3.62	-2.95	17.54	1.00	0.27
389	-7.52	-14.12	17.96	0.92	0.37	449	-5.48	-1.12	16.57	1.05	0.34	509	-3.62	-6.40	16.12	0.65	0.07
390	-7.51	-19.08	17.64	0.90	0.57	450	-5.43	-7.88	17.12	0.88	0.40	510	-3.61	-8.15	17.67	0.92	0.18
391	-7.46	-27.04	17.09	1.08	—	451	-5.32	0.85	17.03	1.07	0.09	511	-3.52	-14.39	15.83	0.37	-0.01
392	-7.35	-27.51	15.55	1.14	—	452	-5.31	-6.41	15.94	0.38	0.13	512	-3.51	1.93	17.52	1.22	0.20
393	-7.32	-0.08	15.87	0.61	0.11	453	-5.22	-4.29	13.87	1.19	1.65	513	-3.49	-42.12	15.11	1.13	—
394	-7.24	-5.38	17.85	1.01	0.38	454	-5.20	4.62	17.91	0.87	0.24	514	-3.48	2.33	17.86	1.02	0.12
395	-7.23	-31.36	17.51	1.02	—	455	-5.18	-16.77	17.82	0.67	—	515	-3.47	-18.33	17.72	0.80	—
396	-7.22	-13.98	15.78	0.66	0.30	456	-5.17	-8.17	17.74	0.91	0.39	516	-3.45	14.18	14.45	1.19	1.26
397	-7.15	-1.14	16.24	0.37	-0.02	457	-5.15	-3.15	17.75	0.89	0.14	517	-3.44	-18.23	17.62	0.91	0.33
398	-7.12	-9.38	17.47	-0.04	-1.13	458	-5.15	-16.69	17.73	0.86	0.28	518	-3.42	0.72	16.88	0.95	0.23
399	-7.05	-0.25	14.87	1.20	0.75	459	-5.13	-16.16	17.14	-0.14	—	519	-3.40	-3.04	17.84	0.95	0.40
400	-7.04	3.98	17.43	1.04	0.14	460	-5.10	-16.09	16.99	-0.11	-0.83	520	-3.34	-1.55	17.84	0.82	0.14
401	-7.02	-2.22	16.21	0.73	0.05	461	-5.09	-24.47	16.21	1.07	—	521	-3.28	10.65	16.05	0.48	-0.05
402	-6.99	4.56	15.06	1.25	0.78	462	-5.04	-29.30	17.25	1.06	—	522	-3.25	-4.12	15.22	1.01	0.60
403	-6.95	0.89	17.98	0.92	-0.09	463	-5.02	5.75	16.22	0.28	0.05	523	-3.25	-5.38	14.11	1.12	1.35
404	-6.92	7.37	18.04	0.96	0.06	464	-4.91	-17.50	14.18	0.78	—	524	-3.24	3.47	17.75	0.92	-0.09
405	-6.91	-10.71	15.74	0.54	0.90	465	-4.90	-12.99	16.11	0.17	0.32	525	-3.21	-1.23	14.14	1.07	1.29
406	-6.89	-3.17	17.93	0.97	0.30	466	-4.88	-5.76	15.82	0.98	0.18	526	-3.18	-3.87	16.10	1.09	0.55
407	-6.88	-38.06	17.12	1.12	—	467	-4.87	-17.42	14.01	0.99	1.39	527	-3.18	-11.79	17.37	0.91	0.47
408	-6.86	-2.34	17.29	0.00	-1.19	468	-4.86	-18.62	15.61	0.67	—	528	-3.17	4.74	17.53	1.01	0.24
409	-6.82	-29.89	17.51	1.01	—	469	-4.83	-18.52	15.57	1.01	0.76	529	-3.08	13.73	16.43	0.52	-0.06
410	-6.81	-25.82	14.93	1.00	—	470	-4.79	2.30	18.01	0.34	0.33	530	-3.04	0.72	16.96	1.08	0.15
411	-6.81	-53.75	18.04	1.14	—	471	-4.71	-6.08	13.77	1.12	1.47	531	-2.99	-6.85	17.91	-0.37	-1.25
412	-6.77	-19.10	16.16	-0.04	—	472	-4.71	-6.95	17.86	0.95	0.09	532	-2.95	2.02	17.58	0.91	0.04
413	-6.74	6.84	16.23	1.08	0.42	473	-4.71	-38.38	15.01	1.07	—	533	-2.92	-14.74	17.84	1.31	0.18
414	-6.73	12.48	16.83	1.33	0.43	474	-4.70	8.18	14.98	1.16	0.86	534	-2.88	-3.29	16.13	0.37	0.11
415	-6.72	-18.99	15.98	0.22	0.38	475	-4.67	2.95	13.41	1.30	1.95	535	-2.87	-2.82	17.44	1.02	0.30
416	-6.71	-41.19	16.11	0.46	—	476	-4.61	0.15	17.68	1.07	0.05	536	-2.84	-1.68	14.08	1.21	1.31
417	-6.67	-56.02	17.27	1.32	—	477	-4.58	-5.34	17.53	0.99	0.23	537	-2.84	-8.56	16.95	0.89	0.39
418	-6.66	-5.35	17.91	1.01	0.26	478	-4.52	2.56	15.82	1.21	0.50	538	-2.81	-6.72	17.52	0.99	0.13
419	-6.65	-6.13	17.59	0.06	-1.13	479	-4.48	-4.84	14.15	1.14	1.23	539	-2.81	4.92	16.41	1.04	0.22
420	-6.64	-25.56	16.15	1.11	—	480	-4.44	-3.75	16.01	0.61	0.37	540	-2.78	-6.31	17.60	0.96	0.23

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
541	-2.76	-6.03	15.83	1.06	0.53	601	-1.05	-18.78	17.46	0.68	—	661	0.16	3.53	17.21	1.03	-0.03
542	-2.72	-6.46	17.80	0.90	0.52	602	-1.04	-5.91	17.60	0.93	0.00	662	0.18	-28.95	15.89	0.50	—
543	-2.66	-2.12	18.04	1.28	0.43	603	-1.04	-34.66	14.18	1.22	—	663	0.19	-1.18	17.69	0.77	-0.06
544	-2.66	-5.33	17.66	1.03	0.28	604	-1.03	-18.68	17.34	0.75	0.49	664	0.19	-2.29	17.75	0.80	0.10
545	-2.64	-15.88	13.01	1.12	2.63	605	-0.98	1.39	16.08	0.44	-0.03	665	0.19	-5.86	17.99	0.88	0.01
546	-2.57	5.65	16.60	1.07	0.20	606	-0.97	13.65	14.57	1.21	1.04	666	0.20	0.85	17.02	0.86	0.22
547	-2.55	-7.34	16.48	0.88	0.39	607	-0.95	-2.09	17.49	0.93	-0.01	667	0.20	-5.00	16.62	0.96	0.19
548	-2.55	-8.26	17.86	0.80	0.25	608	-0.91	-2.67	17.70	1.17	-0.04	668	0.25	-1.75	16.83	0.12	-0.37
549	-2.55	-25.79	17.55	1.07	—	609	-0.89	-3.18	15.04	0.98	0.56	669	0.26	1.01	17.67	0.82	0.21
550	-2.45	1.65	16.56	1.01	0.20	610	-0.84	-3.40	16.11	1.02	0.31	670	0.30	-37.01	15.66	1.11	—
551	-2.44	4.49	17.84	0.89	-0.03	611	-0.84	-12.91	15.59	0.95	0.80	671	0.31	-4.06	17.88	0.70	0.13
552	-2.43	-24.02	17.18	0.90	—	612	-0.77	-0.57	17.45	0.32	-0.16	672	0.31	2.18	17.62	1.04	-0.05
553	-2.40	-15.53	17.04	1.07	...	613	-0.76	-23.88	15.07	1.15	—	673	0.32	-29.51	17.96	1.08	—
554	-2.32	8.65	15.25	1.04	0.55	614	-0.74	-39.18	17.60	1.13	—	674	0.33	5.54	16.74	1.02	0.06
555	-2.30	-14.93	17.82	1.05	0.09	615	-0.72	13.29	15.42	1.25	0.69	675	0.34	-4.65	17.13	1.01	0.10
556	-2.24	-41.89	17.39	0.14	—	616	-0.68	-40.44	17.73	0.11	—	676	0.37	-2.70	17.80	0.36	-0.34
557	-2.20	4.22	16.13	0.76	-0.32	617	-0.67	2.53	16.29	0.61	-0.14	677	0.40	0.65	17.91	0.88	-0.12
558	-2.19	-3.40	18.10	0.90	-0.06	618	-0.62	-9.07	16.22	1.00	0.51	678	0.42	-1.50	17.48	0.95	-0.10
559	-2.18	5.06	15.77	0.90	-0.06	619	-0.57	-2.02	17.64	0.39	-1.35	679	0.42	15.76	17.00	1.22	0.26
560	-2.10	-0.13	16.68	1.02	0.34	620	-0.57	-12.04	17.87	0.74	0.17	680	0.44	-0.26	18.00	0.97	-0.09
561	-2.08	-8.61	17.61	0.88	0.47	621	-0.50	-2.98	17.65	0.91	0.26	681	0.44	-9.82	17.48	1.00	0.32
562	-2.07	4.77	15.59	1.08	0.46	622	-0.50	12.21	15.69	1.28	0.39	682	0.46	1.66	17.55	0.96	-0.09
563	-2.06	-5.05	14.29	1.11	1.17	623	-0.49	-49.84	13.72	1.38	—	683	0.46	-16.71	12.90	1.24	2.53
564	-2.06	-20.34	18.00	1.01	0.34	624	-0.48	-7.62	18.07	0.99	-0.05	684	0.49	-6.86	17.06	0.94	0.38
565	-2.05	-2.26	16.27	0.23	-0.11	625	-0.47	0.15	17.96	0.78	0.18	685	0.56	-4.91	17.28	0.94	0.17
566	-2.02	-14.59	16.22	1.01	0.48	626	-0.46	-0.19	17.75	0.95	-0.18	686	0.57	3.14	17.57	0.97	-0.01
567	-2.02	-23.96	16.16	0.27	—	627	-0.46	5.96	14.91	1.03	0.55	687	0.60	3.77	17.72	0.12	-1.54
568	-1.99	-2.53	16.06	0.82	0.02	628	-0.45	-19.30	17.03	-0.22	—	688	0.60	-5.11	17.60	0.82	0.35
569	-1.91	-0.17	17.12	0.82	0.15	629	-0.44	-19.20	16.87	-0.23	-0.75	689	0.61	8.42	17.66	1.01	0.01
570	-1.88	1.79	17.63	1.01	0.06	630	-0.43	-0.31	18.03	1.21	-0.20	690	0.63	-6.99	17.54	0.94	0.36
571	-1.83	-26.31	18.09	0.03	—	631	-0.43	-4.27	17.15	-0.11	-0.95	691	0.64	-2.03	16.67	0.23	-0.95
572	-1.78	2.81	17.98	0.14	-1.53	632	-0.40	-3.86	17.85	0.97	-0.07	692	0.66	0.01	17.91	0.85	-0.37
573	-1.74	6.79	16.21	0.29	-0.09	633	-0.40	3.31	17.81	0.95	0.06	693	0.66	-15.91	17.80	1.17	0.17
574	-1.71	-12.40	17.70	0.96	0.39	634	-0.39	-7.00	17.37	0.92	0.26	694	0.67	-11.60	15.83	0.76	0.31
575	-1.68	-4.11	15.49	0.92	0.41	635	-0.39	-12.15	17.11	0.99	0.21	695	0.70	-1.98	15.45	1.31	0.30
576	-1.66	-3.56	17.31	0.92	0.30	636	-0.38	2.64	16.15	0.29	-0.14	696	0.70	-5.68	15.89	0.31	0.16
577	-1.64	4.25	16.44	1.09	0.09	637	-0.37	-0.96	17.58	1.14	-0.22	697	0.72	-1.77	15.41	1.02	0.58
578	-1.61	-9.10	18.07	0.82	-0.27	638	-0.36	-9.55	16.44	0.99	0.31	698	0.72	4.16	15.50	1.04	0.53
579	-1.59	0.92	17.77	0.92	0.00	639	-0.33	-2.76	17.12	0.20	-1.05	699	0.75	-2.06	15.48	0.11	0.83
580	-1.58	-7.10	16.93	-0.01	0.24	640	-0.29	-15.60	15.55	0.96	0.64	700	0.78	-6.00	17.88	0.17	...
581	-1.58	-8.79	18.00	0.75	0.28	641	-0.27	2.28	17.69	0.96	0.23	701	0.80	6.41	16.39	1.07	0.24
582	-1.46	-18.33	15.31	0.75	—	642	-0.25	-1.64	17.77	0.65	-0.18	702	0.81	-6.15	15.84	0.98	0.45
583	-1.45	11.61	17.49	1.19	-0.03	643	-0.24	-50.92	17.59	1.11	—	703	0.81	-8.12	17.80	0.19	-1.12
584	-1.44	-18.24	15.04	0.91	1.10	644	-0.22	-1.91	17.59	1.17	-0.78	704	0.82	-0.42	17.94	0.85	-0.09
585	-1.42	-5.48	16.78	0.18	-0.24	645	-0.18	-12.11	17.40	-0.15	-0.97	705	0.86	-52.57	16.19	1.14	—
586	-1.40	-4.69	16.66	1.01	0.26	646	-0.15	-1.36	14.74	1.13	0.61	706	0.93	-1.36	16.20	0.23	-0.21
587	-1.31	-8.41	17.22	0.98	0.29	647	-0.14	-3.23	17.66	0.82	-0.12	707	0.97	9.82	16.10	0.82	-0.32
588	-1.29	-17.34	17.52	0.84	0.39	648	-0.13	-2.51	13.65	1.21	1.75	708	1.01	-2.68	17.22	0.81	-0.11
589	-1.25	-53.68	18.01	0.07	—	649	-0.09	-7.27	17.09	0.95	0.51	709	1.03	9.17	17.30	0.00	-1.37
590	-1.23	-40.29	15.78	1.11	—	650	-0.08	-50.64	15.73	1.17	—	710	1.05	-3.20	18.06	0.82	0.32
591	-1.21	-1.39	17.10	0.97	-0.05	651	-0.04	-1.73	16.95	0.77	-0.09	711	1.05	-3.36	16.26	0.27	-0.09
592	-1.18	-2.20	17.62	1.14	0.02	652	-0.03	-33.33	16.40	0.33	—	712	1.07	-0.86	16.23	0.83	-0.08
593	-1.18	1.16	17.99	0.91	-0.01	653	0.00	0.00	16.44	0.26	-1.82	713	1.07	-1.60	18.00	0.64	0.16
594	-1.18	-20.16	17.91	0.92	0.17	654	0.07	-1.86	17.08	0.80	0.03	714	1.10	-0.62	17.31	0.78	0.13
595	-1.16	-2.68	18.04	0.87	-0.32	655	0.08	-26.30	16.60	1.11	—	715	1.10	-3.59	16.48	0.96	0.27
596	-1.16	-9.97	16.54	0.02	-0.36	656	0.09	-18.81	15.42	0.70	—	716	1.10	1.32	16.24	0.36	-0.16
597	-1.16	7.15	17.95	0.97	0.02	657	0.10	-18.72	15.22	0.92	1.03	717	1.13	-7.10	17.87	0.90	0.32
598	-1.12	-4.72	18.05	0.68	0.42	658	0.13	2.31	18.08	0.94	-0.04	718	1.14	0.68	17.89	0.91	0.07
599	-1.08	-2.05	16.67	1.05	0.31	659	0.15	-4.23	17.92	0.91	-0.06	719	1.25	-0.85	16.75	0.87	0.16
600	-1.08	6.91	17.57	1.07	-0.07	660	0.16	-0.36	17.46	0.94	-0.09	720	1.25	-10.85	15.65	1.00	0.57

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
721	1.27	-8.95	16.50	1.01	0.54	781	2.90	2.49	17.95	1.04	-0.05	841	4.89	8.92	16.21	0.38	-0.15
722	1.28	7.16	16.13	0.45	-0.27	782	2.93	-2.87	15.78	1.01	0.38	842	4.94	-15.41	15.86	0.94	1.06
723	1.32	0.89	17.99	0.93	0.05	783	2.94	6.69	17.58	1.04	0.04	843	4.98	5.31	17.17	0.89	-0.04
724	1.41	-1.22	16.29	0.20	-0.25	784	2.97	-13.43	16.17	0.66	-0.17	844	5.04	-5.75	14.99	1.10	0.96
725	1.44	7.17	17.79	1.03	-0.07	785	3.02	-0.38	16.97	1.01	0.01	845	5.05	-8.58	17.99	1.05	0.06
726	1.47	-3.61	17.17	-0.11	-1.08	786	3.05	-6.17	17.83	0.96	0.25	846	5.06	-21.41	15.68	1.03	1.00
727	1.48	3.33	16.16	0.26	-0.09	787	3.07	1.56	16.91	0.89	0.27	847	5.13	0.34	17.80	1.16	0.14
728	1.48	6.17	15.78	0.99	0.44	788	3.07	-3.17	17.63	0.69	0.29	848	5.13	-17.84	17.79	0.99	0.26
729	1.48	8.41	15.96	0.47	0.01	789	3.08	-3.05	16.75	0.95	0.23	849	5.15	-2.42	17.71	0.91	0.24
730	1.51	7.64	16.09	0.43	-0.13	790	3.08	-13.42	17.13	0.76	0.94	850	5.17	-6.12	17.70	0.69	0.04
731	1.52	-8.13	18.05	0.90	0.10	791	3.12	-4.41	16.03	0.60	0.19	851	5.18	14.22	17.73	1.11	-0.02
732	1.53	14.25	17.91	1.26	0.10	792	3.21	-1.63	15.38	0.89	0.27	852	5.26	-10.66	17.17	0.94	0.38
733	1.54	-0.14	15.02	0.85	0.53	793	3.21	3.65	16.08	0.41	-0.08	853	5.32	-6.75	17.18	1.11	0.36
734	1.54	-36.13	16.77	1.14	—	794	3.22	1.73	17.37	0.83	0.01	854	5.33	-5.02	17.57	0.91	0.13
735	1.63	0.25	17.70	1.04	-0.05	795	3.22	-41.39	13.08	1.52	—	855	5.33	-22.58	15.81	1.07	—
736	1.64	3.97	17.92	0.84	-0.00	796	3.23	17.93	18.07	1.11	-0.67	856	5.34	4.11	17.94	1.14	-0.18
737	1.66	-31.13	15.97	0.54	—	797	3.27	-45.63	14.53	1.08	—	857	5.37	-0.01	17.81	1.01	-0.09
738	1.67	5.30	16.20	0.73	0.53	798	3.35	-3.77	17.68	0.88	0.02	858	5.37	-3.07	17.95	0.93	-0.14
739	1.77	-17.65	17.76	1.17	...	799	3.36	-0.44	17.81	0.53	-0.44	859	5.38	-4.61	17.44	0.90	0.13
740	1.77	-17.74	18.09	0.65	...	800	3.39	-1.39	18.02	1.01	0.07	860	5.38	-11.80	17.00	0.97	0.64
741	1.77	-19.24	16.23	0.02	—	801	3.44	9.20	17.86	0.95	-0.20	861	5.38	-13.29	18.01	0.79	0.13
742	1.78	-13.91	17.50	0.95	0.53	802	3.51	-29.85	16.29	0.43	—	862	5.41	14.64	16.28	0.41	-0.16
743	1.78	-19.15	16.00	0.15	0.33	803	3.52	-10.39	16.57	-0.03	-0.33	863	5.43	-8.55	17.76	1.01	0.17
744	1.83	-8.34	17.02	0.93	0.24	804	3.55	-2.05	17.12	1.02	0.15	864	5.46	-4.06	16.88	1.00	0.23
745	1.84	-0.45	16.91	0.91	0.06	805	3.55	-28.43	16.23	0.45	—	865	5.53	9.73	16.88	1.13	0.14
746	1.87	0.54	17.79	0.99	-0.08	806	3.57	4.13	17.20	1.02	-0.21	866	5.58	10.94	17.80	0.29	-0.38
747	1.90	0.10	15.25	0.30	0.10	807	3.58	-3.99	15.83	0.96	0.38	867	5.58	13.75	16.39	0.35	-0.24
748	1.94	-2.55	16.67	1.07	0.21	808	3.58	-9.87	18.06	0.89	...	868	5.59	-6.01	15.54	1.15	0.26
749	1.95	-2.98	17.90	0.93	-0.40	809	3.59	-1.59	17.89	1.07	-0.35	869	5.65	-8.79	17.80	0.99	-0.06
750	1.95	2.64	16.18	0.37	-0.15	810	3.70	-1.47	17.22	1.00	0.05	870	5.66	-6.38	16.27	1.15	0.37
751	1.99	-33.79	17.29	1.11	—	811	3.70	-3.00	18.06	0.97	-0.25	871	5.68	-0.66	13.91	1.17	1.49
752	2.03	-5.14	17.33	0.82	-0.03	812	3.75	-10.91	15.54	0.99	0.91	872	5.70	3.98	16.53	1.02	0.23
753	2.08	-2.23	13.41	1.14	1.86	813	3.85	-8.97	18.10	0.92	0.28	873	5.73	14.00	16.51	0.40	-0.44
754	2.08	-34.15	17.29	1.13	—	814	3.87	-51.32	15.74	1.14	—	874	5.75	-1.93	15.94	1.05	0.39
755	2.09	-1.01	16.77	0.87	0.11	815	3.90	-7.89	17.73	0.89	0.49	875	5.76	-5.85	13.79	0.96	1.59
756	2.11	-9.85	15.62	1.01	0.74	816	3.91	-4.40	17.83	0.98	-0.31	876	5.76	-34.52	16.43	0.65	—
757	2.13	3.61	17.16	1.08	-0.20	817	3.92	-2.11	15.07	0.48	0.54	877	5.76	16.33	17.74	1.10	0.14
758	2.15	12.17	15.79	1.39	0.28	818	3.97	-6.03	14.84	1.03	1.44	878	5.80	-20.51	17.72	0.17	0.15
759	2.22	6.24	15.69	1.08	0.51	819	3.99	-6.32	17.41	1.14	0.25	879	5.81	11.16	17.01	1.11	0.07
760	2.30	-27.86	17.28	1.10	—	820	4.10	6.77	15.15	0.99	0.42	880	5.95	2.79	15.57	1.04	0.49
761	2.30	-33.19	17.13	1.14	—	821	4.15	-3.42	16.56	1.01	0.15	881	5.99	-6.05	17.43	1.62	-0.36
762	2.38	-5.45	17.51	1.02	0.00	822	4.19	1.29	17.61	1.26	-0.07	882	5.99	-15.59	17.83	0.93	0.07
763	2.38	-13.55	16.04	0.96	0.92	823	4.20	-0.41	16.03	0.83	-0.04	883	6.01	1.46	16.40	1.05	0.18
764	2.41	-3.07	16.44	0.99	0.22	824	4.23	-3.29	16.15	0.35	-0.02	884	6.06	10.29	16.29	0.39	-0.15
765	2.42	6.09	16.37	0.21	-0.35	825	4.32	-18.84	15.83	1.01	0.77	885	6.10	-4.00	17.74	0.86	0.04
766	2.51	3.70	17.84	0.94	...	826	4.35	5.23	13.57	1.28	1.54	886	6.11	-4.70	16.43	1.06	0.28
767	2.53	3.60	16.72	1.02	-0.06	827	4.35	-11.62	16.08	0.06	0.39	887	6.12	4.48	16.92	1.13	-0.07
768	2.53	7.38	17.48	1.05	0.03	828	4.37	-0.09	13.39	1.26	1.69	888	6.12	-46.72	16.26	0.41	—
769	2.64	-3.18	17.97	0.93	-0.23	829	4.45	-3.31	16.81	0.93	0.25	889	6.17	-10.55	17.76	0.95	0.26
770	2.64	11.85	17.38	1.18	0.18	830	4.55	-7.02	17.31	0.37	-1.04	890	6.19	-15.85	17.46	0.95	0.34
771	2.66	-5.92	17.58	0.92	0.31	831	4.57	-0.85	17.94	0.73	-0.08	891	6.20	-2.46	17.73	0.17	-0.23
772	2.69	2.37	15.47	1.01	0.58	832	4.63	6.76	17.61	1.20	0.12	892	6.22	2.65	16.87	1.02	0.18
773	2.71	-12.34	15.81	0.97	0.86	833	4.63	-4.30	16.11	0.24	0.00	893	6.22	4.58	17.51	0.84	0.08
774	2.77	-13.68	16.02	0.96	0.93	834	4.67	-6.43	13.90	1.14	1.43	894	6.29	-6.69	18.02	1.09	-0.25
775	2.79	-1.19	16.79	0.08	-0.83	835	4.68	10.81	16.58	1.14	0.24	895	6.30	2.32	17.76	0.99	0.12
776	2.81	-10.39	17.71	0.95	0.55	836	4.70	-37.72	17.98	1.10	—	896	6.30	7.49	16.23	0.31	-0.17
777	2.83	-0.25	16.67	0.95	0.17	837	4.75	4.89	16.63	1.04	0.12	897	6.35	5.38	17.64	1.00	0.02
778	2.84	-13.89	17.60	0.92	0.34	838	4.76	6.49	15.17	1.01	0.42	898	6.39	-0.20	16.79	1.08	0.08
779	2.86	-1.95	16.72	1.05	0.17	839	4.83	15.13	15.19	1.20	0.77	899	6.43	-8.49	14.77	1.01	0.92
780	2.90	1.31	16.23	0.31	-0.12	840	4.86	-5.41	17.47	0.97	0.07	900	6.47	-16.35	15.46	1.00	0.75

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$	ID	$\Delta\alpha$	$\Delta\delta$	V	$V-I$	$U-V$
901	6.48	-41.30	16.46	0.86	—	961	8.11	-8.82	18.09	0.97	0.10	1021	10.85	-40.35	18.08	1.11	—
902	6.48	19.66	16.46	0.02	-0.54	962	8.21	11.88	17.06	-0.01	-1.52	1022	10.92	1.96	17.94	0.92	0.07
903	6.51	2.71	17.27	0.86	-0.01	963	8.22	-2.51	17.07	0.97	0.15	1023	10.97	1.73	17.16	1.04	0.07
904	6.52	-9.83	16.68	0.98	0.61	964	8.24	-34.98	18.07	1.22	—	1024	10.98	-5.81	14.69	1.14	1.08
905	6.54	2.91	17.36	0.96	0.20	965	8.29	-0.77	15.82	1.12	0.38	1025	10.98	-12.81	14.05	1.13	1.89
906	6.54	-3.92	16.98	0.93	0.06	966	8.31	-19.89	17.23	1.01	0.60	1026	10.99	-3.23	16.92	1.01	0.28
907	6.56	-18.38	18.01	0.96	0.31	967	8.37	-52.29	14.59	1.20	—	1027	11.02	2.72	17.54	0.87	0.19
908	6.60	-26.04	17.40	1.04	—	968	8.41	0.27	17.43	1.09	0.17	1028	11.06	-7.27	17.27	0.99	0.47
909	6.60	-44.70	15.76	1.13	—	969	8.42	3.99	17.14	0.95	0.04	1029	11.11	7.03	17.67	1.00	0.17
910	6.62	15.32	14.55	1.23	1.03	970	8.43	-0.07	16.57	1.01	0.13	1030	11.17	-0.21	18.09	0.96	-0.07
911	6.65	0.62	16.00	0.64	-0.03	971	8.45	1.08	17.18	0.89	0.05	1031	11.18	-30.02	17.90	1.14	—
912	6.65	-2.65	15.97	0.67	0.03	972	8.47	5.04	17.93	0.97	0.08	1032	11.36	2.58	15.72	1.06	0.46
913	6.67	-15.96	17.87	0.92	0.35	973	8.48	16.89	17.84	1.18	...	1033	11.36	-8.46	17.48	1.09	0.34
914	6.76	3.12	18.00	-0.19	-1.70	974	8.53	-23.62	17.86	1.07	—	1034	11.39	3.47	17.85	0.91	-0.12
915	6.79	6.81	17.72	1.14	-0.34	975	8.67	-3.95	17.51	0.97	0.11	1035	11.51	6.92	16.73	1.10	0.18
916	6.80	-23.73	17.97	1.02	—	976	8.72	-5.17	15.25	1.04	0.79	1036	11.51	8.41	16.38	1.13	0.37
917	6.82	19.33	18.02	0.72	-0.29	977	8.77	-4.45	16.21	0.25	0.01	1037	11.59	-4.73	17.75	1.03	...
918	6.85	-34.43	13.65	1.34	—	978	8.80	-2.88	17.98	0.82	-0.07	1038	11.59	-8.57	16.05	0.85	0.21
919	6.86	-19.65	17.35	1.02	0.41	979	8.81	15.79	17.79	1.05	-0.03	1039	11.60	-4.82	17.38	-0.11	-1.08
920	6.87	-11.64	16.74	-0.05	-0.35	980	8.86	3.80	15.60	1.03	0.54	1040	11.63	-2.05	17.45	0.97	0.32
921	6.97	-34.98	17.00	1.42	—	981	8.87	-7.41	16.09	0.46	0.13	1041	11.65	-18.59	17.15	1.04	0.71
922	7.02	-1.32	17.83	0.57	-0.14	982	9.05	7.13	17.78	0.98	-0.15	1042	11.65	-27.63	16.41	0.33	—
923	7.08	-1.18	17.33	0.90	0.18	983	9.13	-0.75	16.73	0.95	0.21	1043	11.66	-3.65	17.87	0.20	-0.19
924	7.12	-6.46	13.25	1.21	2.10	984	9.24	3.90	17.34	1.11	-0.03	1044	11.66	-23.85	16.31	1.13	—
925	7.17	-12.19	16.83	1.03	0.75	985	9.26	-2.28	17.76	0.31	-0.17	1045	11.77	-9.88	13.35	1.15	2.27
926	7.19	2.83	17.33	1.01	-0.09	986	9.31	0.43	16.26	0.65	-0.26	1046	11.78	12.88	14.68	1.23	1.04
927	7.23	5.52	16.24	0.25	-0.19	987	9.31	-33.87	17.35	1.18	—	1047	11.79	-1.81	15.41	1.06	0.75
928	7.27	-49.07	15.75	1.12	—	988	9.32	3.18	17.61	0.95	-0.04	1048	11.81	-3.15	18.03	0.95	0.19
929	7.28	2.90	17.15	0.86	0.15	989	9.32	8.62	17.45	1.05	0.03	1049	11.92	-46.96	15.42	1.15	—
930	7.31	-7.30	18.04	0.62	0.21	990	9.38	-1.62	15.55	1.09	0.52	1050	11.93	-52.05	16.13	1.09	—
931	7.35	19.37	17.99	0.84	-0.35	991	9.39	-25.47	16.35	0.72	—	1051	11.99	-0.84	17.69	1.01	0.12
932	7.37	-3.55	17.03	0.95	-0.01	992	9.62	-0.54	16.26	1.07	0.29	1052	12.04	6.60	18.04	1.04	0.17
933	7.38	0.87	14.91	1.09	0.71	993	9.66	-3.83	17.12	-0.02	-1.10	1053	12.05	-4.03	17.20	1.00	0.42
934	7.40	7.55	18.01	1.00	-0.16	994	9.73	-2.82	16.85	1.01	0.16	1054	12.06	2.80	15.74	1.10	0.49
935	7.41	-9.69	17.42	1.01	0.27	995	9.77	-5.93	17.63	0.97	0.38	1055	12.15	10.78	17.68	0.23	-1.24
936	7.43	-29.45	17.76	1.12	—	996	9.86	-12.74	16.61	1.00	0.73	1056	12.15	-36.42	17.23	1.09	—
937	7.44	-0.63	17.75	1.08	-0.08	997	9.90	-17.33	16.71	0.99	0.57	1057	12.33	4.28	13.11	0.86	0.60
938	7.48	-10.12	17.06	0.96	0.44	998	9.92	-0.82	15.53	1.11	0.58	1058	12.35	5.48	16.81	1.05	0.01
939	7.48	-18.39	17.69	1.01	0.56	999	9.97	-1.75	18.04	0.84	0.08	1059	12.35	5.17	17.29	1.08	-0.26
940	7.55	-8.31	17.79	0.96	0.22	1000	10.01	13.35	17.31	1.12	0.02	1060	12.35	12.64	14.75	1.21	0.84
941	7.60	1.94	14.64	1.08	0.71	1001	10.05	-5.28	14.62	1.14	1.02	1061	12.44	-16.37	17.18	1.08	0.44
942	7.62	-13.90	16.79	0.95	0.70	1002	10.07	-6.21	13.62	1.26	1.76	1062	12.62	-0.07	17.29	0.95	0.20
943	7.70	-14.89	15.90	1.01	1.01	1003	10.08	9.66	15.90	0.87	0.08	1063	12.71	-7.61	17.54	1.05	0.51
944	7.82	6.81	16.74	1.06	0.12	1004	10.18	-25.30	17.07	1.08	—	1064	12.75	-5.16	17.73	0.99	0.29
945	7.82	-8.86	15.67	1.03	0.72	1005	10.19	12.10	17.95	1.08	-0.07	1065	12.79	-55.57	17.33	1.20	—
946	7.83	-6.19	17.81	0.86	0.31	1006	10.19	-51.04	17.79	1.05	—	1066	12.83	7.00	17.48	1.02	0.71
947	7.85	18.88	17.31	0.77	0.23	1007	10.22	-11.13	15.67	1.04	1.10	1067	12.84	-42.16	17.18	1.10	—
948	7.86	7.50	17.92	1.08	-0.10	1008	10.27	-14.42	17.78	0.87	0.16	1068	13.11	-28.97	16.76	1.15	—
949	7.86	-32.09	15.71	1.20	—	1009	10.29	-3.53	17.87	0.13	-0.20	1069	13.20	4.62	17.44	0.95	-0.29
950	7.92	3.99	17.55	1.04	-0.06	1010	10.31	16.03	13.52	1.21	1.73	1070	13.28	6.17	15.92	1.07	0.54
951	7.93	-7.04	14.23	1.17	1.22	1011	10.33	7.44	17.94	1.02	-0.04	1071	13.28	5.57	17.94	0.91	-0.06
952	7.95	14.50	13.46	1.31	1.79	1012	10.34	-7.80	17.34	1.02	0.26	1072	13.29	-10.81	16.26	0.82	0.45
953	7.96	-34.45	17.39	1.10	—	1013	10.40	-6.89	16.42	1.10	0.50	1073	13.36	1.55	16.22	0.26	0.07
954	7.96	-46.56	17.77	0.08	—	1014	10.51	1.92	17.95	0.94	-0.15	1074	13.44	-8.23	15.30	1.06	0.95
955	7.99	-0.85	17.17	0.98	0.16	1015	10.60	1.28	16.12	0.36	-0.07	1075	13.46	-23.82	17.76	1.05	—
956	7.99	-3.38	15.66	1.02	0.59	1016	10.66	-9.00	13.89	1.24	2.01	1076	13.57	-26.88	16.76	1.14	—
957	8.02	16.87	15.91	0.70	0.12	1017	10.70	-2.47	18.09	1.03	-0.05	1077	13.58	-43.76	15.49	1.18	—
958	8.04	-16.20	15.35	1.01	0.96	1018	10.72	6.07	14.93	1.14	0.81	1078	13.62	-15.20	16.10	0.37	0.35
959	8.06	-6.41	17.49	1.40	0.05	1019	10.72	8.03	16.65	1.12	0.17	1079	13.66	-7.96	18.04	0.47	0.75
960	8.09	2.79	16.86	1.01	0.16	1020	10.72	13.79	17.70	1.09	0.23	1080	13.67	-15.00	17.20	1.01	0.59

TABLE A1. (continued)

ID	$\Delta\alpha$	$\Delta\delta$	V	V-I	U-V	ID	$\Delta\alpha$	$\Delta\delta$	V	V-I	U-V	ID	$\Delta\alpha$	$\Delta\delta$	V	V-I	U-V
1081	13.70	-26.15	16.94	1.10	—	1117	15.38	6.30	17.29	1.08	0.30	1153	17.75	-1.67	17.96	0.93	0.40
1082	13.70	-27.45	17.79	1.06	—	1118	15.39	0.15	13.53	1.28	2.07	1154	17.79	-6.59	16.29	0.14	0.15
1083	13.73	-18.71	17.88	1.16	-0.15	1119	15.42	-18.96	15.50	1.01	0.55	1155	17.91	-12.62	16.52	1.07	0.66
1084	13.78	-39.00	15.98	0.61	—	1120	15.53	-0.64	15.86	1.08	0.82	1156	18.07	-37.48	16.50	1.10	—
1085	13.83	-49.46	15.26	1.17	—	1121	15.58	-11.10	18.01	0.98	0.35	1157	18.08	-14.89	15.50	1.21	0.92
1086	13.93	-35.98	16.60	1.14	—	1122	15.65	-21.00	15.34	1.19	0.95	1158	18.12	3.92	17.78	-0.01	—
1087	14.00	-1.80	16.21	0.34	0.21	1123	15.66	-12.49	17.85	0.19	-0.00	1159	18.24	-26.36	16.55	0.23	—
1088	14.05	2.33	16.93	1.07	0.30	1124	15.76	8.08	17.00	1.27	—	1160	18.25	-3.13	17.05	0.84	0.59
1089	14.05	-34.19	16.29	0.57	—	1125	15.79	-29.60	17.69	1.11	—	1161	18.33	4.02	17.85	1.06	—
1090	14.15	-11.03	17.58	0.89	...	1126	15.93	9.41	17.24	0.20	—	1162	18.37	-18.34	16.79	0.26	-0.41
1091	14.15	-51.42	13.21	1.56	—	1127	16.03	-43.06	17.36	1.06	—	1163	18.43	-10.17	16.45	0.90	0.66
1092	14.17	-1.75	15.98	0.42	0.41	1128	16.08	-14.63	18.04	1.06	0.46	1164	18.46	-0.98	14.43	1.07	1.78
1093	14.19	-26.58	17.69	0.11	—	1129	16.09	-11.20	17.06	0.95	0.48	1165	18.47	-10.72	16.56	0.09	-0.08
1094	14.21	9.58	17.32	0.08	—	1130	16.11	9.65	18.02	1.02	—	1166	18.48	-14.23	17.56	1.08	0.55
1095	14.34	-49.68	17.93	1.07	—	1131	16.12	-15.58	16.23	1.10	0.77	1167	18.67	-4.64	14.12	1.05	1.68
1096	14.35	-16.44	17.52	1.08	...	1132	16.13	8.44	14.47	1.26	—	1168	19.03	-29.94	15.58	0.19	—
1097	14.49	0.32	16.31	0.75	-0.42	1133	16.31	-1.81	17.66	1.14	0.04	1169	19.09	-2.96	17.15	0.10	-0.48
1098	14.49	-6.48	17.53	0.86	0.16	1134	16.46	-9.67	16.70	0.97	...	1170	19.10	-39.84	16.37	1.13	—
1099	14.57	-3.45	15.40	0.98	0.63	1135	16.49	-4.72	17.65	1.00	...	1171	19.13	-37.88	17.97	0.08	—
1100	14.57	-12.07	16.77	1.01	0.57	1136	16.54	-28.53	15.64	1.21	—	1172	19.18	-38.62	17.93	0.97	—
1101	14.59	6.29	17.14	1.03	0.23	1137	16.65	-20.10	17.89	1.06	0.34	1173	19.19	-18.49	18.10	0.27	-0.11
1102	14.66	-14.79	15.06	1.13	1.14	1138	16.66	4.93	16.31	0.24	0.14	1174	19.24	-19.94	17.90	1.11	0.25
1103	14.76	-18.99	17.16	1.13	0.52	1139	16.76	2.68	15.59	1.11	0.91	1175	19.30	-4.32	17.76	0.87	0.50
1104	14.80	5.49	17.77	1.03	0.10	1140	16.81	-19.16	15.18	0.99	0.59	1176	19.38	-11.66	17.98	0.92	0.19
1105	14.84	3.27	18.04	1.03	0.23	1141	16.92	4.62	17.62	0.92	0.76	1177	19.38	-50.23	15.43	1.14	—
1106	14.91	-10.54	17.68	1.01	0.43	1142	16.93	11.55	17.60	1.11	—	1178	19.39	-2.53	18.07	0.06	0.02
1107	15.02	-16.50	16.92	0.14	-0.76	1143	16.97	-2.60	17.91	1.05	0.52	1179	19.60	-48.93	17.34	1.13	—
1108	15.04	-23.88	16.60	1.13	—	1144	16.97	-14.39	16.39	0.80	-0.15	1180	19.98	-52.66	18.02	1.02	—
1109	15.08	5.84	15.62	0.42	0.95	1145	17.07	-7.45	15.91	0.88	0.88	1181	20.00	-25.66	17.62	1.10	—
1110	15.08	-10.24	15.21	1.03	1.10	1146	17.22	2.63	17.89	0.99	0.38	1182	20.42	-3.48	17.61	0.55	0.41
1111	15.08	-30.24	15.47	1.15	—	1147	17.29	-39.68	16.13	0.65	—	1183	20.46	-39.08	15.76	0.23	—
1112	15.10	-3.04	16.59	1.03	0.57	1148	17.38	-11.45	16.51	1.08	0.58	1184	21.12	-8.42	18.08	0.51	0.37
1113	15.11	0.18	16.14	0.37	0.38	1149	17.40	-3.70	16.92	1.01	0.67	1185	21.17	-3.40	15.29	0.65	1.06
1114	15.29	-6.37	15.79	0.50	0.65	1150	17.60	-9.93	18.01	0.92	0.06	1186	23.15	-6.93	16.04	0.31	1.13
1115	15.32	-2.14	16.06	0.52	0.25	1151	17.69	-13.20	16.14	0.78	0.37						
1116	15.35	-49.91	16.49	0.83	—	1152	17.70	-26.06	17.64	1.11	—						

FIGURE CAPTIONS

1. Halftone reproductions of the $8'' \times 8''$ section of the 70 s F555W (V) HST/PC image centered on M15, with the greyscale linearly proportional to image intensity. The white ring marks the $0''.4$ (9 pixel) radius 2σ error circle around the maximum-likelihood estimate of the center of mass of the cluster, giving equal weight to each resolved star. AC 211 is a known X-ray source and AC 214 is resolved in our images into at least three stars within $0''.1$ of one another. Note, the sharp core of the HST point-spread function ($\text{FWHM} \sim 0''.1$) and complicated pattern of the extended PSF wings. The orientation of this image is such that the vertical ($+y$) axis is 2° west of north.
2. The full four-CCD mosaic of the 70 s F555W (V) PC image of M15, with the greyscale proportional to the logarithm of the intensity. The orientation of this image is the same as Figure 1. The CCD images have been trimmed to remove most of the unexposed area of the PC. The gaps between adjacent CCDs shown in this figure are approximate, and do not accurately represent the slight rotations (up to 1°) and gaps ($0''.7$) that are present between the four CCDs.
3. Areal coverage of the U -, V -, and I -band HST/PC exposures analyzed in this paper. Each solid square represents a pair of exposures through the F555W (V) and F785LP (I) filters, each exposure being a 2×2 mosaic of CCD frames. Each dotted square represents a pair of 800 s F336W (U) exposures separated by about $10''$ in the north-south direction. The “+” sign marks the center of M15. The exposures were obtained at a variety of spacecraft roll angles, but the cluster center was positioned at the center of one of the CCD frames in each exposure.
4. The distribution of stellar apparent F555W (similar to V) magnitudes in M15 in four radial bins: $r < 4''$ (solid line); $4'' < r < 9''$ (bold dotted line); $9'' < r < 13''$ (dashed line); and $r > 13''$ (dot-dashed line). The turnover at the faint end of the stellar luminosity function is due to incompleteness, which sets in at increasingly bright magnitudes as one approaches the cluster center where the degree of crowding is the greatest. The LFs in the three outermost bins have been scaled to match that in the innermost ($r < 4''$) radial bin. The bold open circles represent the stellar LF derived from the the four distributions (by correcting the outermost LF for incompleteness beyond $V \gtrsim 20$). This extrapolated LF is used in the simulations to represent both bright and faint (unresolved) stars.
5. (a) Photometric errors (1σ) for the $34'' \times 34''$ F555W (V) King simulation, as a function of stellar apparent magnitude, obtained by comparing the input magnitude to the output magnitude (*i.e.* that derived from PSF fitting). The errors increase from about $\lesssim 0.1$ mag for $V < 16$ to about 0.3 mag at the faint end ($V \gtrsim 19$). There is a systematic positive bias (overestimation of output brightness) of about 0.10–0.15 mag for stars with $V \gtrsim 16$ caused by contamination from faint neighbor stars (see §3.2 for a detailed discussion).
 (b) Same as (a) for the F555W BW cusp simulation. The rms photometric errors and the magnitude bias are somewhat worse for the BW cusp simulation than it is for the King simulation due to the higher degree of crowding in the central $2''$ of the BW cusp simulation.
 (c) Same as (a) for the sparse outer parts of the F555W King simulation ($r > 13''$). The photometric errors and magnitude bias are smaller in the outer portions of the image than it is in the crowded central region.
6. (a) Star-count correction factors, $C(r)$, as a function of radius (and hence of stellar density), derived from the F555W (V) King simulation (as defined by Eq. 3 in §3.3). The error bars represent Poisson fluctuations in the number of stars with $V < 18$ detected in a given $1''$ -wide

annulus. The smooth solid curve is an analytic approximation to the data points that is used to correct the M15 star counts. While incompleteness tends to lower the value of C (as seen in the dense central region), the systematic magnitude bias and photometric errors for faint stars tend to raise it above unity beyond $r \sim 3''$. At large radii, C approaches unity since the completeness is essentially 100% and the photometric bias and errors are negligible.

(b) Same as (a) for the F555W BW cusp simulation. The higher density of stars in the inner $2''$ of this simulation, compared to the King simulation, causes the incompleteness to be worse, lowering the central value of C . The dotted line is an extremely steep $C(r)$, but one that is consistent with the data points derived from the simulation.

7. (a) The projected density of stars with $F555W < 18$ vs. radius in the F555W (V) King simulation. The open circles indicate the number of stars in the output list, binned in $1''$ -wide annuli, and Poisson error bars. The crosses show the star counts after correcting by the factor $C(r)$ derived from the King simulation (smooth solid curve in Fig. 6a). The thin solid line shows the input radial distribution function from which the simulated stars have been drawn: $N(r) \propto [1 + (r/r_{\text{core}})^2]^{-1}$, with $r_{\text{core}} = 2''$, for $r < 2''$ and a power-law beyond. The fact that the corrected counts (crosses) lie closer to the input distribution than the uncorrected counts (open circles) provides a consistency check on the star-count correction factors derived from the simulation.
 (b) Same as (a) for the F555W BW cusp simulation. The corrections applied to the raw counts have been derived from the BW cusp simulation itself (smooth solid curve in Fig. 6b). The input distribution function is a power-law— $N(r) \propto r^\alpha$, with $\alpha = -0.75$ —interior to $2''$, but is forced to be identical to the radial distribution function of the King simulation at larger radii.
8. The diffuse residual light in the F336W (U) BW cusp simulation, after the detection and subtraction of the images of all bright stars (open circles). The error bars show the 1σ probable error in the mean brightness within each 4 pixel-wide ($0''.18$) annulus, based on the measured pixel-to-pixel intensity variation. The solid line shows the $\alpha = -0.75$ power-law input distribution function, while the dashed curve is a result of convolving the input distribution with the HST point-spread function. The dotted curve is the result of subtracting the light of bright stars using their correct (*i.e.* input) magnitudes. Even though this simulation was constructed to have a central power-law cusp, the diffuse residual brightness profile (open circles) has a flat inner portion with $r_{\text{core}} \sim 2''$. The bias in the measurement of the magnitudes of resolved stars in the dense central region leads to a systematic oversubtraction of the light (see §3.4 for details).
9. The surface density of resolved stars with $V < 18$ in M15 (open circles) with 1σ Poisson error bars. The radial bins are $1''$ in width. The “ $\times$ ” and “+” symbols show the result of applying the star-count correction $C(r)$ derived from the F555W King and BW cusp simulations, respectively (smooth solid curves in Fig. 6). The open squares in the four innermost bins show the effect of the extreme correction based on the BW cusp simulation (dotted line in Fig. 6). The differences between the various corrections are a measure of the uncertainty in $C(r)$ within $r < 2''$. In all cases, however, the corrected counts rise more steeply towards the center than the raw counts. The solid curves show a King model density distribution, with $r_{\text{core}} = 2''$, and a power-law distribution, with $\alpha = -0.75$, that are consistent with the M15 star counts. The best-fit power-law index for the M15 star count distribution is -0.85 ± 0.2 and the best-fit core radius is $1''.3$.

10. (a) The M15 $V < 18$ raw and corrected star counts (same as in Fig. 9) compared to the sum of two King distribution functions, with $r_{\text{core}} = [2'', 16'']$, with a 10:1 ratio of central surface densities (lower solid curve). Also, shown are the distributions in the presence of a massive central black hole (see Eq. 8 of §4.2) of mass $M_{\text{BH}} = 1.5 \times 10^3 M_{\odot}$ (middle solid curve) and $M_{\text{BH}} = 3 \times 10^3 M_{\odot}$ (top solid curve).
 (b) Same as (a) with a double King distribution characterized by $r_{\text{core}} = [1'', 8'']$ normalized 4:1, and black hole masses of $0 M_{\odot}$, $10^3 M_{\odot}$, and $2 \times 10^3 M_{\odot}$.
 (c) Same as (a) with a double King distribution with $r_{\text{core}} = [4'', 8'']$, and black hole masses of $0 M_{\odot}$ and $10^4 M_{\odot}$. If the inner King core radius exceeds $\sim 3''$, the black hole mass required to fit the star-count distribution is so large that it is conflict with the radial velocity measurements of Peterson et al. (1989).
 (d) Same as (a) with a double King distribution with $r_{\text{core}} = [2'', 4'']$, and black hole masses of $0 M_{\odot}$ and $2 \times 10^3 M_{\odot}$. The larger of the two King core radii must be $\gtrsim 8''$ in order to fit the M15 star count data in at $r \sim 10''$.
11. The surface brightness profile of the residual unresolved light in the F336W (U) image of M15, after all bright stars have been subtracted from the image (open circles). The dashed line shows the same quantity as measured by Lauer et al. (1991). The error bars on both sets of measurements (1σ probable error in the mean brightness, derived from pixel-to-pixel fluctuations) are comparable to the size of the open circles. The two measurements are in good agreement; this is only to be expected since our analysis technique is similar to that of Lauer et al., and the same data were analyzed in both cases. The solid curve shows the residual light in the U -band BW cusp simulation (same as the open circles in Fig. 8), and it is similar to the M15 diffuse light profile. The large wings of the HST point-spread function make the diffuse light an unreliable tracer of the underlying stellar density distribution.
12. A V versus $V - I$ color-magnitude diagram for stars in the central $r \lesssim 1'$ of M15. This diagram shows the ~ 5000 stars which are detected in both F555W (V) and F785LP (I) bands. The bold open squares indicate the blue straggler candidates identified by Ferraro & Paresce (1993) on the basis of far-ultraviolet colors. The large spread in $V - I$ color at the base of the red giant branch is mostly a result of photometric error caused by crowding in the dense inner regions of the cluster. These errors, and the presence of a prominent blue horizontal branch (extension of stars bluer than and fainter than horizontal branch stars), make the identification of blue stragglers difficult.
13. A restricted (V , $V - I$) color-magnitude diagram for 830 selected stars in the central $13''$ of M15. Only those stars are included that are detected in both V exposures and in both I exposures, and have magnitude differences less than 0.5 mag (in both bands) between the two measurements.
14. A V versus $U - I$ color-magnitude diagram for the inner portion of M15, using the largest available color baseline, $U - I$; the ordinate and abscissa of this diagram are independent. All stars with VUI magnitudes are plotted. The bold open squares mark the three components of what has been previously known as AC 214. The bold open circle indicates the combined magnitude and color of the three component stars.
15. The same (V , $U - I$) color-magnitude diagram as in Figure 14 with stars encoded by type. Selected subgroups of stars are indicated with different symbols: red giant branch (RGB), (red) horizontal branch (HB), blue horizontal branch (BHB), and blue stragglers (BS). Ferraro &

Paresce (1993) blue stragglers are marked with filled boxes. BSs identified on the basis of our *UVI* CMD are marked with open boxes. A set of faint RGB stars are denoted ‘control stars’ (dashed rectangle); these stars are used as the standard against which we compare the radial distribution of stars of various stellar types. A probable foreground star is marked with a large circle.

16. The positions of the blue straggler stars in M15. The area where our *U*, *V*, and *I* HST/PC images overlaps (where a BS could be detected) is shown by the solid outline. BS detections are indicated by open circles. The area covered in Ferraro & Paresce’s UV study is shown by the dashed square. FP BS candidates are indicated by filled triangles. There are three blue stragglers detected very near the center of the cluster (+).
17. A set of (*V*, *U* − *I*) color-magnitude diagrams in four radial annuli centered on M15. The limiting radii of the four annuli are chosen so that each annulus contains the same number of selected faint RGB stars ($16.5 \leq V \leq 17.7$ and $+0.85 \leq U - I \leq +2.0$) that serve as a control sample. Note that the *total* number of stars in each panel increases radially outwards due to the increased probability of detection of faint stars ($V \sim 19$) in the sparse outer parts of the cluster. The bright RGB stars appear to be depleted in the innermost annulus.
18. Cumulative radial distributions of stars of selected types in the central 40'' of M15. The different stellar types are identified on the basis of the (*V*, *U* − *I*) color-magnitude diagram. The bright RGB stars are significantly depleted with respect to the faint RGB and HB stars. The BS stars show a marginally-significant excess near the cluster center.

Fig 1

Fig 2

Fig 4

F555W mag (Input)

Fig 5

Fig 6

Fig 7

Fig 8

Fig 9

Fig 10

Fig 11

Fig 12

Fig 13

Fig 14

Fig 15

Fig 16

Fig 18

Fig 17