AC

CERN -PPE 93-222

su 3406

4

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-PPE/93-222

LAPP-EXP-93.11

9 December 1993

CERN LIBRARIES, GENEVA

P00020952

Influence of the c.m.s. energy-dependent theoretical errors on the measurement of the Z^0 parameters at LEP

M. Martinez*

CERN, PPE Division, CH-1211 Geneva 23, Switzerland
and

B. Pietrzyk

Laboratoire de Physique des Particules LAPP, IN2P3-CNRS,
F-74941 Annecy-le-Vieux, France

Abstract

We have investigated the influence of the c.m.s. energy-dependent theoretical errors on the measurement of the Z^0 parameters at LEP. We have concluded that for experiments measuring the luminosity at small angles these errors will not reduce the high precision of the 1993 measurements. However, in some experimental conditions, these errors may dominate the precision of the measurement of some of the Z^0 parameters, particularly that of Γ_Z . These errors may be reduced by using known theoretical corrections.

(submitted to Physics Letters)

*Present address: Institut de Fisica d'Altes Energies, Buid. C, Univeritat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

Data has been taken at the Large Electron-Positron storage ring LEP since 1989. Each successive year the precision of the Standard Model tests has improved thanks to the increase in the statistical and systematical precision of the measurements and also thanks to improvements of the theoretical predictions [1], [2]. These tests are made by comparing cross-sections and asymmetries with the theoretical predictions for different c.m.s. energies around the Z^0 peak. During 1993 a three-point energy scan has been made at the Z^0 mass M_Z and 2 GeV above and below. Owing to the use of resonant depolarisation [3] for the energy calibration, the c.m.s. energy of LEP should be known to the precision of 2 MeV [4] allowing a substantial improvement in the precision of the determination of the Standard Model parameters, including improvements of the measurement of the Z^0 mass M_Z and width Γ_Z .

In addition ALEPH and OPAL expect a major improvement in 1993 in their luminosity measurements because of the installation of new silicon-tungsten calorimeters [5], [6]. The new ALEPH detector SICAL is already operational since September 1992. The experimental precision of this detector is 0.15% for the 1992 data-taking period and there are even reasonable possibilities of improvement of this precision in 1993.

The Standard Model parameters are extracted from the fit of the Standard Model line shape formulae to the data from the scan. The precision of the determination of the fit parameters depends on the quality of the measurements and also on the precision of the theoretical prediction of the Standard Model line shape formulae. The cross-sections are determined by dividing the number of events of different processes by the luminosity, taking into account the experimental efficiency. Similarly, the luminosity is determined by dividing the number of e^+e^- events scattered at small angles into the "luminosity detector" by the theoretically predicted cross-section, again taking into account the experimental efficiency.

The theoretical errors may be dependent on or independent of the c.m.s. energy. Up to now it has generally been assumed that they are the same for the different c.m.s. energy points of the scan and that the relative errors from point to point are negligible. The purpose of this paper is to investigate the influence of the relative errors in view of the expected improvement in the precision of the 1993 measurements.

The fully correlated theoretical errors, called below the global errors, renormalize the whole line shape and therefore they have little influence on the measurement of the Z mass and width. The relative errors, on the other hand, may shift different points of the line shape differently and therefore strongly influence the measurement. If the peak point is shifted, Γ_Z is changed. If the wing point is shifted, or wing points are shifted in the opposite direction, M_Z is modified.

It is easy to see that some fraction of the theoretical errors may be c.m.s. energy dependent. The theoretical errors concern both the technical and the physical precision. A part of the physical precision is defined as the estimated size of the non-calculated higher order corrections. Usually, the higher order effects are estimated from the calculated fixed order corrections under the assumption that the perturbative expansion behaves reasonably well and therefore, that the size of the calculated

corrections limits the missing ones. This calculated correction may depend on the c.m.s. energy. For example, the photon-Z interference contribution to the luminosity calculation even changes the sign below and above the Z resonance, and the hard photon $O(\alpha)$ correction to the line shape formulae is larger above the Z resonance than below it (more details will be given in the following). Therefore it might be expected that the size of the non-calculated higher order corrections should also have a c.m.s. energy dependence.

In order to test the dependence of the fit parameters on the assumed global and relative errors, we have varied these errors in the line shape fitting programs [7]. In practice, the three 1993 LEP scanning points are considered with their foreseen statistical weights, then the theoretical line shape is computed, afterwards a distortion is applied in the different ways shown in Table 1, and finally the line shape is fitted again to the theoretical expression to get the distortion in the Z parameters. In all cases, if the distortion applied is considered a 1σ uncertainty, then the distortion in the parameters also has the meaning of 1σ systematic uncertainty. This relationship is obvious when just one line shape point is changed. In case (a) in which three points are changed it is still valid because a 100% correlation is assumed for the distortion applied in the three points. In case (c) a 100% anticorrelation is assumed between the two distortions applied.

As a result we have obtained the dependence of different fit parameters, M_Z , Γ_Z , σ_0 , and N_ν , on the global and relative errors. We have checked that the errors of different fit parameters change linearly with the global and relative errors. Therefore in Table 1 we have presented the error in different fit variables only for errors of 0.1%. Note that σ_0 changes less by the distortion of the three points of the scan than by the distortion of one peak point. We have checked that with the pure Breit-Wigner formula the σ_0 variation is the same in both cases. The relatively surprising result in Table 1 is caused by the fact that, with the formula including radiative corrections, σ_{peak} is not equal to σ_0 at the Z^0 peak. The distortion of the peak point changes also the fitted value of Γ_Z , which in turn changes the contribution of the radiative corrections at the peak, and this finally has to be compensated by the additional change of σ_0 .

We will start with a discussion of the theoretical luminosity errors. Very great progress has recently been made in the calculation of the small-angle Bhabha scattering used in the LEP luminosity measurements [8] – [15]. The t -channel photon exchange dominates this process and is responsible for the 99% or more of the cross-section depending on the scattering angle (see Table 1 in Refs. [14] and [15]). The next most important contribution is the interference of the Z in the s channel with the γ in the t channel. This contribution increases the cross-section below the Z^0 mass and decreases it above the Z^0 mass (see Fig. 1).

At present the LEP experiments use the Monte Carlo generator BHLUMI [12] to calculate the dominant photon t -channel contribution and the Monte Carlo program BABAMC [8] to calculate the terms containing the Z^0 exchange at the $O(\alpha)$ [14].

A summary of the theoretical errors on the luminosity calculation in different

Table 1: Error of different fit variables generated by variation of global and relative errors by 0.1% (see text for more details).

Error	Change in			
	$M_Z(\text{MeV})$	$\Gamma_Z(\text{MeV})$	$\sigma_0(\text{pb})$	$N_\nu(\times 10^{-3})$
(a) Global ($\uparrow\uparrow\uparrow$)	0.	0.	42.	7.5
(b) Relative - peak shift($\cdot \uparrow \cdot$)	0.28	2.1	44.	7.9
(c) Relative - wing shift($\downarrow \cdot \uparrow$)	0.8	0.	3.1	0.7
(d) Relative - wing shift($\cdot \cdot \uparrow$)	1.0	1.1	1.8	0.5
(e) Relative - wing shift($\uparrow \cdot \cdot$)	0.7	1.0	4.3	0.9

Table 2: Summary of theoretical errors on the luminosity calculation in different angular regions.

Contribution	$1.61^\circ - 2.8^\circ$	$3.3^\circ - 6.3^\circ$
(1) $\mathcal{O}(\alpha^2)$ LL BHLUMI	0.15	0.15
(2) $\mathcal{O}(\alpha^2)$ SL BHLUMI	0.09	0.09
(3) Z exchange $\mathcal{O}(\alpha)$ BABAMC	0.03	0.03
(4) Z exchange $\mathcal{O}(\alpha^2)$ LL BABAMC	0.015	0.06
(5) Z exchange $\mathcal{O}(\alpha^2)$ SL BABAMC	0.015	0.06
(6) Vacuum polarization	0.05	0.08
Total theoretical error	0.25	0.28

angular regions is presented in Table 2. It is taken from Ref. [15], where the full discussion of different contributions can be found. The contributions (1) and (2) represent the BHLUMI error [12], [15] divided into missing leading-log (LL) $\mathcal{O}(\alpha^2)$ and subleading-log (SL) $\mathcal{O}(\alpha^2)$ contributions. The contributions (3), (4), and (5) represent the technical and physical precision of the Z -term calculation in BABAMC [14], [15] again divided into LL and SL $\mathcal{O}(\alpha^2)$ contributions. The last contribution (6) is the vacuum polarization error [16], [15]. Since the contributions (1) and (2) as well as (4) and (5) can be correlated, they were added linearly to obtain the total theoretical error in Table 2. The other contributions were added quadratically. The two angular regions presented in Table 2 correspond to the angular acceptance of the “old” ALEPH luminosity detector LCAL and the “new” one SICAL, but they can also be treated as the two extreme acceptances of different LEP luminosity detectors.

We have analysed the errors presented in Table 2 in order to divide them into the c.m.s. energy dependent and independent errors. We have concluded that the

Table 3: Leading-log (LL) $O(\alpha^2)$ correction to the $O(\alpha)$ calculation of various parameters in BABAMC with different Z widths, Γ_Z , for different angular acceptances of the luminosity detector and for three c.m.s. energy points of the scan given as the percentage of the total Born cross-section. The right-hand side of the Table shows the change of different fit parameters caused by these corrections in units of MeV for M_Z and Γ_Z , in pb for σ_0 and in 10^{-3} for N_ν .

BABAMC Γ_Z	Angular acceptance	c.m.s. energy			Change of			
		$M_Z - 2$	M_Z	$M_Z + 2$	M_Z	Γ_Z	σ_0	N_ν
2.3098	$1.61^\circ - 2.8^\circ$	+0.002	-0.018	-0.004	0.	+0.35	-8.1	+1.5
2.4870	$1.61^\circ - 2.8^\circ$	+0.001	-0.014	-0.003	0.	+0.27	-6.3	+1.1
2.3098	$3.3^\circ - 6.3^\circ$	+0.011	-0.078	-0.011	0.	+1.6	-35.0	+6.3
2.4870	$3.3^\circ - 6.3^\circ$	+0.002	-0.058	-0.009	+0.1	+1.1	-25.8	+4.6

BHLUMI errors (1) and (2) and also the vacuum polarization error (6) depend on the momentum transfer squared t but not on the c.m.s. energy. So they depend strongly on the geometrical position of the luminosity detector and very little on the c.m.s. energy. Therefore these errors are very strongly correlated for different points of the energy scan and contribute in a negligible way to the relative errors from point to point.

The case of Z-exchange terms calculated in BABAMC is different. The main contribution comes from the interference of the s-channel Z exchange with the t-channel γ exchange and this contribution varies strongly with the c.m.s. energy (Fig. 1 and Table 1 in Refs. [14], [15], respectively).

The LL $O(\alpha^2)$ correction to the $O(\alpha)$ calculation in BABAMC is presented as an error (4) in Table 2, but actually this correction is known. It can be calculated by the semi-analytical program ALIBABA [10]. This correction is presented in Fig. 1 for the default Z width ¹ in BABAMC of 2.3098 GeV and the measured Z width of 2.487 [14]. The largest LL $O(\alpha^2)$ correction to the BABAMC calculation with the measured Z width is given as an error (4) in Table 2. The non-calculated SL $O(\alpha^2)$ correction is conservatively assumed to be equal to the LL $O(\alpha^2)$ correction and is presented as the error (5). The technical precision of the Z-term calculation in BABAMC ((3) in Table 2) was defined as the maximal difference between any component of this calculation in BABAMC and ALIBABA calculated at each c.m.s. energies and was calculated in Refs. [14], [15].

Table 3 shows the LL $O(\alpha^2)$ correction to the $O(\alpha)$ calculation in BABAMC with

¹See Table 8 in Ref. [14] and Table 5 in Ref. [15] where this calculation was made; W. Beenakker, DESY, Hamburg has kindly calculated additional points at $M_Z \pm 2\text{GeV}$ presented in this paper but not in Ref. [14], [15]

different Z widths for different angular acceptances of the luminosity detector and three c.m.s. energy points of the scan. We see that these corrections vary for different points of the scan and therefore give a typical example of the relative errors. The shift of different parameters of the fit caused by this correction is presented on the right-hand side of Table 3. The non-calculated higher order corrections, for example the subleading term $O(\alpha^2)$, can be correlated with the LL correction presented in Table 3. We estimate, therefore, that the errors related to the terms missing in the $O(\alpha)$ calculation in BABAMC are twice the values presented in Table 3. However, if the LL $O(\alpha^2)$ terms are included in the luminosity calculations by the experimental teams, then the numbers presented in Table 3 give the errors related to the missing non-calculated terms above LL $O(\alpha^2)$.

In the extraction of the Z parameters from the line shape study, the other important ingredient is the prediction of the $e^+e^- \rightarrow f\bar{f}$ cross-section. The LEP experiments use “almost model-independent” fitting formulae [7] in order to extract the main Z parameters from the line shape data. In this approach, a Born-like non-radiative cross-section is convoluted with an accurate initial-state radiator function. Explicit final-state QED and QCD corrections are applied where necessary (non-pure Z-exchange terms) and $O(\alpha)$ contributions which do not accommodate to the Born-like parametrization, such as contributions from box diagrams and small imaginary parts of amplitude coming from loop corrections, are added explicitly in the calculation. Therefore, the theoretical uncertainty introduced in the determination of the Z parameters from the hadron line shape data can be classified as follows:

- QED corrections: when dealing with completely inclusive cross-sections (obtained from the extrapolation of the observed cross-section using Monte Carlo event generators) the effect of the QED corrections can be estimated as follows:
 - Final-state corrections: at $O(\alpha)$ these effects are explicitly included in the calculation of the non-pure Z-exchange terms. For the pure Z-exchange term, they are implicit in the definition of the hadronic peak cross-section σ_0^H . We can estimate the missing explicit $O(\alpha^2)$ effects to be $\leq 10^{-5}$.
 - Initial-final state interferences: at $O(\alpha)$ the total correction is of the order of 0.02 % for $|\sqrt{s} - M_Z| \lesssim \Gamma_Z$ but it is already explicitly included in the fitting formulae used by the LEP experiments. Therefore, only uncertainties due to missing higher orders have to be quoted and we can safely estimate them to be $\leq 10^{-5}$.
 - Initial-state corrections: Since complete second-order calculations with soft photon exponentiation are used, only the uncertainty due to missing $O(\alpha^3)$ terms has to be quoted. Owing to the huge size of initial-state radiation corrections, the estimation of the uncertainty for this part deserves a rather long explanation which we will give below.
- Non-radiative corrections: we shall distinguish two sources of uncertainty:

Table 4: Classification of initial-state radiation corrections. The terms in brackets are not included in the YFS formulae discussed in the text.

1			
αL	α		
$\alpha^2 L^2$	$\alpha^2 L$	$[\alpha^2]$	
$\alpha^3 L^3$	$[\alpha^3 L^2]$	$[\alpha^3 L]$	$[\alpha^3]$
$[\dots]$	$[\dots]$	$[\dots]$	$[\dots]$

- Non Born-like contributions: contributions from heavy box diagrams and small imaginary parts of amplitude from loop corrections. They are explicitly included at $O(\alpha)$ and their effect is of the order of 0.02% for $|\sqrt{s} - M_Z| \lesssim \Gamma_Z$. Therefore, only uncertainties due to missing higher orders have to be considered and we have estimated them to be $\leq 10^{-5}$.
- The use of Born-like expressions: one source of uncertainty is related to the approximations used to perform the modelling of the cross-section calculation in terms of a Born-like parametrization. In the process of writing the Z exchange term of the cross-section in terms of Born-like constants (M_Z , Γ_Z , and partial widths) it is necessary to keep only the linear term of the s -dependence in the effective currents and to evaluate the rest at $s = M_Z^2$. The actual size of these approximations and hence their uncertainty will be discussed below.

Concerning the uncertainty due to missing terms in the initial-state radiation (ISR) formulae, a detailed discussion can be found in Ref. [17]. The formulae suggested in these references includes LL up to $O(\alpha^3)$ for hard photon radiation and soft photon and pair production exponentiated in the Yennie-Frautschi-Suura (YFS) [18] formalism. The estimation of uncertainties due to missing higher orders in these papers leads to very small numbers, which we will try to present here for an invariant mass cut matching approximately the LEP experiment analyses, that is $x_{min} \sim 0.04$ (where $\sqrt{s x_{min}}$ is the minimum invariant mass in the hard scattering process). Since we are mainly interested in the estimation of the distortion of the energy dependence which would affect quantities such as M_Z and Γ_Z , we will analyse the theoretical uncertainties as a function of $\sqrt{s}$.

In order to estimate the accuracy of the formulae, it is worth realizing that the missing pieces are (see Table 4):

- In LL, the terms of $O(\alpha^n L^n)$ for $n \geq 4$. We can estimate their size to be conservatively no more than the complete $O(\alpha^3 L^3)$ contribution (see Fig. 2a).

- In SL, the terms of $O(\alpha^n L^m)$ for $n \geq 3$ and $m < n$. In analogy with the previous case, we can estimate their size by taking the complete $O(\alpha^2 L)$ contribution (see Fig. 2a).

Figure 2b shows the result from adding the absolute values of both corrections (to account for the possibility that they are fully correlated) as a function of $\sqrt{s}$ and gives our estimation of the effect of missing QED corrections when using the formulae of Ref. [17].

Nevertheless, given the fact that the formulae from Ref. [17] are not the ones actually applied in packages widely used such as the ZFITTER [7] one, we shall compare the actual predictions using the radiator functions from Ref. [17] with those obtained using the complete second-order exponentiated calculation from Ref. [19]. In Fig. 2c we see that the actual size of the difference between both formulae is much larger than the accuracy that we have estimated for the first one. This apparent contradiction can be traced back to the fact that the YFS approach allows the resummation of the soft photon and lepton pairs to all orders in a systematic way, substantially improving perturbative convergence [20] and accounting naturally for the interplay between hard and soft radiation. Therefore, for the treatment of the soft radiation phenomena, which constitute the bulk of the initial-state corrections, the YFS approach is more precise than that in Ref. [19] and, therefore, the difference in Fig. 2c does not reflect the actual theoretical uncertainty but rather the bias introduced by using simpler approaches than the YFS one.

The energy dependence in Figs. 2b and 2c shows that the distortion is basically constant below the Z peak and changes rapidly above the peak. The physical reason for this is that below the peak soft radiation dominates whereas above the peak hard radiation starts playing a role. Since soft photon radiation is already resummed to all orders and hard photon radiation is not, this is precisely the part of the corrections in which one can expect the bulk of the uncertainty. This physical splitting suggests that effectively the energy-dependent part of the correction is just due to the asymmetry of the correction on the wings of the resonance. Therefore, we shall just take the difference between the correction at the wings as the relative error.

Concerning the bias introduced by the use of fitting formulae, it can easily be computed by comparing the results of complete Standard Model calculations with their Born-like parametrizations. Figure 2d shows the size of the distortion in the hadron line shape introduced by the parametrization used in ZFITTER (a similar curve can be obtained using MIZA). In spite of the fact that this distortion could just be transposed to shifts in the line shape parameters and hence corrected for, this procedure would completely spoil the “almost model-independence” of the approach and hence the nice feature about using parameters to describe the line shape. Therefore, we prefer to consider it as a limitation of the use of the parameter language to describe the line shape information and to take the size of the distortion as an estimate of the theoretical uncertainty inherent in the use of parametrizations.

Table 5 summarizes the contributions of the sources related to the theoretical

Table 5: Estimated uncertainties (in per cent) in the hadron cross-section estimation and the shift of different fit parameters caused by these uncertainties in the same units as in Table 3. In the second part of the table the contributions due to ISR and fitting formulae are added. In the third part of the table the total errors of the fit parameters are presented in the different cases explained in the text.

Source	c.m.s. energy			Change of			
	$M_Z - 2$	M_Z	$M_Z + 2$	M_Z	Γ_Z	σ_0	N_ν
ISR (YFS)	0.	0.	+0.005	+0.1	0.	0.	0.
ISR ($O(\alpha^2)$)	0.	0.	-0.04	+0.4	+0.4	+0.7	-0.2
Fitting formulae	+0.01	+0.02	+0.03	-0.2	0.	-8.9	+1.6
YFS+fitting formulae				-0.1	0.	-8.9	+1.6
$O(\alpha^2)$ +fitting formulae				+0.2	+0.4	-8.2	+1.4
Case 1				0.1	0.54	21.5	3.8
Case 2				0.2	3.6	78.2	14.
Case 3				0.	1.1	34.7	6.2

cross-section expectation we have just discussed and the shift of different fit parameters caused by these uncertainties. In the second part of the table the contributions due to ISR and fitting formulae are added. The sign of the parameter change should not surprise the reader. If in the line shape formulae, for example, the M_Z point value is rising, the fit has to lower the σ_0 value in order to match the value of the experimentally measured point.

Tables 3 and 5 present the changes of different fit parameters caused by the relative theoretical errors in the case of different strategies chosen by experiments. In the third part of Table 5 we present the total experimental error for the two extreme cases. Case 1 represents the experiment measuring the luminosity cross-section in the angular region of $1.61^\circ - 2.8^\circ$, calculating the Z-term correction with the modified Γ_Z BABAMC and using the YFS line shape formulae. Case 2 represents the experiment measuring the luminosity cross-section in the angular region of $3.3^\circ - 6.3^\circ$, calculating the Z-term correction with the default Γ_Z BABAMC and using the $O(\alpha^2)$ line shape formulae. The errors were calculated as twice the entry numbers in Table 3 (to take into account the non-calculated higher order corrections) plus the entry number in Table 5. We have chosen to add errors linearly (including the sign) since all these errors are corrections or are correlated. We see that in both cases the M_Z error is much smaller than the expected 1993 systematical error of 2.5 MeV [2] and also in case 1 the relative theoretical errors will have a negligible influence on the total systematical error. However, in case 2 the Γ_Z error is bigger than the expected 2 MeV systematical error [2]. Also the σ_0 and N_ν errors are approaching the present

systematical errors and will dominate them if the BHLUMI global error should be reduced. In case 2 an improvement can be made by taking advantage of the known theoretical corrections. This can be done by calculating the Z-term contribution with the modified Γ_Z BABAMC, by introducing a LL $O(\alpha^2)$ correction to these terms from ALIBABA and by adding the corrections to the $O(\alpha^2)$ line shape formulae. The error will be then reduced to the values presented as case 3 in Table 5.

In this paper we have discussed the theoretical errors within the Standard Model. Therefore we have not covered here the γZ interference contribution to the line shape. Uncertainties in this contribution can potentially influence the Z^0 mass measurement. It has been shown in Ref. [21] that the variation of the Z^0 mass is not bigger than 1 MeV owing to the variation of the top and Higgs mass in this contribution and also that changing the interference term between zero and twice the Standard Model value shifts the Z^0 mass by ± 3 MeV. Study of the extensions of the Standard Model is beyond the scope of this paper.

To summarize, we have investigated the influence of the c.m.s. energy dependent theoretical errors on the measurement of the Z^0 parameters at LEP. For this purpose we have studied the sensitivity of the Z^0 parameters to an independent shift of scan points of the Z^0 line shape. We have investigated the influence on these parameters of the known theoretical errors and the higher order corrections both for the luminosity calculations and for the line shape formulae. We have concluded that for experiments measuring the luminosity at small angles these errors will not reduce the high precision of the 1993 measurements. However, in some experimental conditions, these errors may dominate the precision of measurement of some Z^0 parameters, particularly the measurement of Γ_Z . These errors may be reduced by using known theoretical corrections.

We would like to thank W. Beenakker, S. Jadach, J. Rander, Z. Wąs and B.F.L. Ward for stimulating discussions on this work.

References

- [1] The LEP Collaborations: ALEPH, DELPHI, L3 and OPAL, Phys. Lett. B276 (1992) 247; preprint CERN-PPE/93-157.
- [2] J. Lefrancois, “Precision tests of the Standard Model”, talk at the Int. Euro-physics Conf. on High Energy Physics, Marseille, 1993.
- [3] A. Blondel, preprint CERN-PPE/93-125, to appear in “Precision Tests of the Standard Electroweak Model”, World Sc. Publ. Co., P. Langacker, editor.
- [4] A. Blondel, in Proc. 3rd Workshop on LEP Performance, Chamonix, 1993, ed. J. Pool, CERN SL/93-19(DI) 1993, p.365.
- [5] J. Rander, ALEPH note 90-008 (ALEPH, CERN); J. Rander and M. Martinez, ALEPH note 90-143 (ALEPH, CERN).
- [6] The OPAL Collaboration, “Proposal for Upgrading the OPAL Luminosity Detector”, LEPC document CERN/LEPC 91-8, LEPC/M-100.
- [7] M. Martinez et al., FORTRAN program MIZA, Z. Phys. C49 (1991)645 ;
D.Bardin et al., FORTRAN program ZFITTER, Comput. Phys. Commun. 59 (1990) 303.
- [8] M. Böhm, A. Denner and W. Hollik, Nucl. Phys. B304 (1988) 687; F.A. Berends, R. Kleiss and W. Hollik, Nucl. Phys. B304 (1988) 712.
- [9] S. Jadach, E. Richter-Wąs, Z. Wąs and B.F.L. Ward, Phys. Lett. B253 (1991) 469.
- [10] W. Beenakker, F.A. Berends and S.C. van der Marck, Nucl. Phys. B349 (1991) 323 and B355 (1991) 281.
- [11] S. Jadach, E. Richter-Wąs, Z. Wąs and B.F.L. Ward, Phys. Lett B260 (1991) 438.
- [12] S. Jadach, E. Richter-Wąs, Z. Wąs and B.F.L. Ward, Phys. Lett. B268 (1991) 253; Comput. Phys. Commun. 70 (1992) 305; S. Jadach and B.F.L. Ward, Phys. Rev. D40 (1989) 3582.
- [13] M. Cacciari, A. Deandrea, G. Montagna, O. Nicrosini and L. Trentadue, Phys. Lett. B271 (1991) 431.
- [14] W. Beenakker and B. Pietrzyk, Phys. Lett. B296(1992)241.
- [15] W. Beenakker and B.Pietrzyk, Phys. Lett. B304(1993)366.

- [16] H. Burkhardt, F. Jegerlehner, G. Penso and C. Verzegnassi, Z. Phys. C43 (1989) 497.
- [17] S. Jadach, M. Skrzypek and M. Martinez, Phys. Lett. B280 (1992) 129 ;
M. Skrzypek, Acta Physica Polonica B23 (1992) 135.
- [18] D.R. Yenni, S.C. Frautschi and H. Suura, Annals of Physics 13 (1961) 379.
- [19] F.A. Berends, W.L. van Neerven and G.J.H. Burgers, Nucl. Phys. B297 (1988) 429 and B304 (1988) 921 (E).
- [20] M. Skrzypek and S. Jadach, Z. Phys. C49 (1991) 577 ;
S. Jadach, M. Skrzypek and B.F.L. Ward, Phys. Lett. B257 (1991) 173.
- [21] M. Grünewald and S. Kirsch, preprint CERN-PPE/93-188;
B. Adeva et al., Z. Phys. C51(1991)179.

Figure 1: Higher order corrections to the terms containing Z-boson exchange calculated in $O(\alpha)$ in BABAMC with different Z widths for two different angular regions studied in this paper. Insets show the contribution of the Z terms calculated in the Born approximation given as the percentage of the total Born cross-section.

Figure 2: Different sources of uncertainties for the $e^+e^- \rightarrow \text{hadrons}$ line shape formulae. (a) ISR corrections from LL $O(\alpha^3)$ and SL $O(\alpha^2)$ terms in the YFS approach. (b) Total estimation of missing higher orders in the YFS approach. (c) Difference between YFS and the complete $O(\alpha^2)$ calculation from Ref. [19]. (d) Differences between the ZFITTER fitting formulae and its full Standard Model prediction.