SHEAR FIELDS AND THE EVOLUTION OF GALACTIC-SCALE DENSITY PEAKS

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ABSTRACT

We present preliminary results of our investigation into the influence of shear fields on the evolution of galactic scale fluctuations in a primordial Gaussian random density field. Specifically, we study how the matter associated with a galaxy-scale peak evolves, to determine whether the shear can affect the peak’s ability to form a virialized structure. We find that the evolution of the mass distribution in the immediate surroundings of \( \geq 1\sigma \) initial density peaks is sensitive to the nature and magnitude of the shear. Its final fate and configuration spans a plethora of possibilities, ranging from accretion onto the peak to complete disruption. On the other hand, the mass defining the peak itself always tends to form a virialized object, though a given galaxy-scale peak need not necessarily form only one halo. Furthermore, galaxy-size halos need not necessarily be associated with initial galaxy-scale density peaks. Under certain conditions, the shear field is capable of breaking up a single primordial peak into two (and perhaps, more) distinct halos, or of promoting the growth of smaller-scale peaks into galaxy-size halos.

Subject headings: Cosmology: theory – large-scale structure of the Universe – Galaxies: formation

1. Introduction

It is generally believed that structure formation in the universe is the result of initially small Gaussian random density fluctuations having been gravitationally amplified, with objects such as galaxies having evolved from peaks in the initial field. The statistical properties of peaks in a Gaussian random field have been extensively studied (cf. Peacock & Heavens 1985, Bardeen et al. 1986). A major drawback in linking the results of such studies to objects like galaxies is that
there is no clear association between features in the initial conditions and eventual halos of a given mass. Consequently, halos of a characteristic mass scale are generally identified, in a one-to-one mapping, with peaks in the initial fluctuation field that is artificially filtered on the same scale. This ignores the fact that actual collapse occurs where density and velocity waves interfere constructively (Bond & Meyers 1993). Apart from questions regarding the validity of the identification procedure, smoothing itself can lead to overinterpretations. For example, the mean density profile of a peak can be more extended than the filter profile (Bardeen et al. 1986) and hence, the actual mass associated with a peak can be considerably larger than the mass scale of the filter. Also, the filtering masks the complicated process by which structures form.

An important physical effect that gets relegated into the background by the one-to-one peak-halo identification scheme is the influence of shear on structure formation. Shear on an evolving density fluctuation can be due to its own intrinsic asphericity (internal shear) or can be induced by the surrounding mass distribution (external shear). Studies of collapsing homogeneous ellipsoids have shown that internal shear can alter the collapse history of structures (Icke 1973; White & Silk 1979). Studies also show that large-scale structures can induce significant tidal forces in their vicinities (e.g. the Local Supercluster, see Lilje, Yahil & Jones 1986) and that the resulting torques are capable of imparting angular momentum to the protohalos (Efstathiou & Jones 1979; Barnes & Efstathiou 1987) as well as, of influencing the spatial and the kinematical structure of the virialized halos (Binney & Silk 1979; Dubinski 1992). Hoffman (1986, 1989) was amongst the first to recognize the importance of external shear on structure formation itself, noting that shear speeds up collapse. Bertshinger & Jain (1993) and Bond & Myers (1993) have recently elaborated on various aspects of this. In particular, Bond & Myers (1993) find that tidal fields play an important role in the formation of low-mass objects.

At this point, a clear distinction ought to be made between a collapse and virialization. Collapse refers to a state where the local density approaches infinity. A positive energy structure that has evolved into a thin expanding rod can be thought of as having collapsed, but not virialized. Collapse and virialization are synonymous only in the case of spherically symmetric infall, although in the
case of bound structures one will follow the other fairly rapidly. In this study, we are interested in the effect of cosmological shear fields on the formation of virialized objects. Specifically, we generate a series of initial density field realizations with a galaxy-scale peak at a designated location, where we vary the height of the peak, its peculiar velocity, as well as the magnitude and the orientation of the shear field acting upon it, and follow the evolution of the associated matter distribution to see whether the shear can cause the peak to spawn many smaller clumps instead of forming a single halo. Such phenomena may explain the results of Katz et al. (1993), who found that some of the virialized objects in their N-body simulation did not correspond to primordial density peaks. Also, we explore the impact of shear on the fate of the matter distribution immediately surrounding the density peaks. The degree to which this matter accretes onto the halo determines its density distribution and final mass.

In this letter, we discuss the preliminary results of our study, concentrating on four cases. In section 2, we describe our technique for generating a constrained random initial density field. In section 3, we present our preliminary results, and in section 4, we attempt to draw some conclusions.

2. Initial Conditions

In order to generate constrained density fields (see Bertschinger 1987), we use the prescription of Hoffman & Ribak (1991). They showed that in the case where the constraints are linear functionals of the field, the problem of generating a constrained random field has a simple and elegant solution. The details of the method is best presented within the context of the problem at hand (see Van de Weygaert & Bertschinger 1993 for a more comprehensive discussion). Basically, we are interested in studying the evolution of the matter distribution associated with and in the neighbourhood of a galaxy-size peak in the initial density field. Apart from being able to determine the location, the scale and the height of the peak, we also wish to sculpt the total matter distribution in order to subject the peak to a desired amount of net gravitational and tidal forces. As a first step, we need to define the term “galaxy-scale peak”. We identify such a peak as a peak in the density field that has been smoothed by a Gaussian with a characteristic scale $R_G = 0.585h^{-1} \text{Mpc}$. (The enclosed mass is comparable to that of an $L_*$-ish galaxy, $\approx 10^{12}M_\odot$.)
In addition to its scale and location, a peak in the smooth density field is characterized by 18 constraints. We need to specify its height and ensure that the 3 first derivatives of the smooth density field vanish at its summit. The 6 second-order derivatives of the density field are set by specifying the compactness, the axis ratios, and the orientation of the peak. These 10 constraints together determine the density distribution in the immediate vicinity of the peak. The specification of the velocity field around the peak introduces 8 additional constraints: The 3 components of the smoothed peculiar velocity field at the location of the peak and the 5 independent components of the traceless shear tensor, which when linearly extrapolated to the present are given by

\[ \nu_G(r_{pk}) = \frac{H_0}{4\pi} \int d\mathbf{y} \int d\mathbf{x} \delta(\mathbf{x}) W_G(\mathbf{x}, \mathbf{y}) \frac{(\mathbf{y} - r_{pk})}{|y - r_{pk}|}, \]

\[ \sigma_{G,ij}(r_{pk}) = \frac{1}{2} \left\{ \frac{\partial \nu_{G,i}}{\partial r_j} + \frac{\partial \nu_{G,j}}{\partial r_i} \right\} - \frac{1}{3} \left( \nabla \cdot \mathbf{v}_G \right) \delta_{ij} \bigg|_{r=r_{pk}}, \]

where \( \delta(\mathbf{x}) \) is the unsmoothed density fluctuation field linearly extrapolated to the present, \( W_G(\mathbf{x}, \mathbf{y}) \) is the Gaussian smoothing function, \( H_0 \) is the Hubble constant and \( F(\Omega_0) \approx \Omega_0^{0.6} \).

Let us denote the constraints as a set of linear functionals \( C_i[\delta] \) on the constrained field \( \delta \) with the value \( c_i \) at the location of the peak, \( \{ C_i[\delta; r_{pk}] = c_i; \ i = 1, \ldots, M \} \). These linear constraints can be cast in the form of a convolution between \( \delta(\mathbf{x}) \) and some kernel \( H_i(\mathbf{x}; r_{pk}) \),

\[ C_i[\delta; r_{pk}] = \int d\mathbf{x} f(\mathbf{x}) H_i(\mathbf{x}, r_{pk}) = \int \frac{dk}{(2\pi)^3} \mathcal{H}(\mathbf{k}) \mathcal{H}_i(\mathbf{k}) = c_i, \]

where \( \mathcal{H}(\mathbf{k}) \) and \( \mathcal{H}_i(\mathbf{k}) \) are the Fourier transforms of \( \delta(\mathbf{x}) \) and \( H_i(\mathbf{x}; r_{pk}) \), respectively. The kernels for the peculiar velocity and the shear constraints, for example, are

\[ \mathcal{H}_v(\mathbf{k}) = -iH_0F(\Omega_0) \frac{k}{k^2} \tilde{W}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}_{pk}}, \]

\[ \mathcal{H}_{\sigma,ij}(\mathbf{k}) = -H_0F(\Omega_0) \left( \frac{k_i k_j}{k^2} - \frac{1}{3} \delta_{ij} \right) \tilde{W}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}_{pk}}, \]

where \( \tilde{W}(\mathbf{k}) = \exp(-k^2 \sigma^2/2) \) is the Fourier transform of the Gaussian filter. Note that the position of the peak enters into the kernels via the phase \( \mathbf{k} \cdot \mathbf{r}_{pk} \). Finally, the constrained fluctuation field \( \tilde{\delta}(\mathbf{r}) \) can be constructed according to (see Van de Weygaert & Bertschinger 1993)

\[ \tilde{\delta}(\mathbf{r}) = \int \frac{dk}{(2\pi)^3} \left[ \hat{\delta}(\mathbf{k}) + P(k) \mathcal{H}_i(\mathbf{k}) \delta_{ij}^{-1} (c_j - \hat{c}_j) \right] e^{-i\mathbf{k} \cdot \mathbf{r}}, \]
where $\tilde{\delta}(x)$ is an unconstrained realization of the fluctuation field with the same power spectrum $P(k)$ as that characterizing the constrained field $\delta(x)$, $\tilde{r}_j$ is the value of the linear functional $C_j$ for the field $\tilde{\delta}(x)$, and the random Gaussian variate $\hat{\delta}(k)$ is its Fourier transform. Also,

$$\xi_{ij} = \langle C_i C_j \rangle = \int \frac{dk}{(2\pi)^3} \hat{H}_i(k) \hat{H}_j(k) P(k).$$

(5)

3. Simulation and Results

For our simulations, we assume an $\Omega = 1$ universe, a Hubble constant of $H_0 = 50 \text{ km/s/Mpc}$ ($h = 0.5$), and adopt the cold dark matter power spectrum of Davis et al. (1985) — normalized such that $\sigma(8h^{-1}\text{Mpc}) = 1.0$ at $a = 1$, the present epoch (Davis & Peebles 1983). The fluctuation field is sampled onto a $64^3$ grid in a periodic box of length $25h^{-1}\text{Mpc}$, transformed into positions and growing mode velocities of $64^3$ particles, and evolved using the P$^3$M N-body code (with the force softening distance, $\epsilon = 0.019h^{-1}\text{Mpc}$) of Bertschinger & Gelb (1991). The different simulation configurations are listed in Table 1 — the numerical values listed are linear extrapolations at $a = 1$.

Our tailor-made galaxy-scale peaks either are at rest or are streaming at 1000 km/s, a velocity that is approximately equal to the rms velocity ($\approx 884 \text{ km/s}$) of galaxy-scale peaks. The value of the expansion scalar, $\nabla \cdot \mathbf{v}_G$, associated with the peak is $-197.5 \text{ km/s/Mpc}$ for $1\sigma$ peaks and twice that for $2\sigma$ peaks. The shear tensor at the location of the peak is orientated so that the off-diagonal terms are zero. For most configurations, the diagonal term with the largest magnitude is positive (dilation) while the other two are equal and negative (contraction). In Runs 7–9, the situation is reversed. Since we wish to induce non-negligible shear, the magnitude of the largest element is chosen to be 100–150 km/s/Mpc on the scale of $0.585h^{-1}\text{Mpc}$, 1.5–2 times the dispersion ($\approx 70 \text{ km/s/Mpc}$) for the diagonal shear components (Bond 1987).

In the cases where the gravitational and the tidal forces on the peak are primarily due to one large mass concentration (as in Runs 1 & 2), the evolution of the core-envelope material (we define the core as that material which is initially within a comoving radius of $0.78h^{-1}\text{Mpc}$ of the density peak and defines the peak, and we define the envelope as that material which is within $1.56h^{-1}\text{Mpc}$ from the peak and constitutes its immediate surroundings) is not interesting as it ends up
merging into the massive halo. The rest of the simulations, however, clearly illustrate the impact of shear on the fate of the envelope material. In the absence of shear, the envelope would accrete onto the density peak. Shear complicates the nature of secondary infall and hence, affects both the final mass and the density profiles of the halos (Gunn & Gott 1972; Hoffman & Shaham 1985; Bond & Myers 1993). An analytical study by Zaroubi & Hoffman (1993) also suggests that the evolution of the envelope is sensitive to tidal influences. This effect of shear on the envelope material is best seen in Runs 3 & 4.

The first row in Figure 1 shows the matter distribution in Run 3, at $a = 0.5$. The shear on the $1\sigma$ peak is induced by a set of mass concentrations aligned along a filamentary structure. The left panel presents the particle distribution in a $3h^{-1}$ Mpc slice centred on the segment of the filament where the matter associated with the initial peak is located. The core material is unaffected by shear and forms a small halo (circled). In contrast, as the peak streams towards the filament, the increasing shear stretches the envelope and eventually tears most away (see centre panel). Note that two clumps that are massive enough to be galaxies have condensed out from the debris of the disrupted envelope. Identifying the initial density peaks with the particles closest to the positions of their summit, we find that the above two “galaxies” do not correspond to any initial $\geq 1\sigma$ galaxy-scale peaks. The location of all such peaks within a thicker $5h^{-1}$ Mpc slice are plotted in the right panel. This results conflicts with the underlying assumption of the peak-halo identification scheme that galaxies only form around galaxy-scale density peaks.

In Run 4, the $2\sigma$ galaxy-size peak starts is initially at rest and embedded in a filament. The second row in Figure 1 displays the matter distribution at $a = 0.9$. The particle distribution in a $2h^{-1}$ Mpc slice through the simulation volume is presented in the left panel. The centre panel offers a detailed view of the region enclosed by the box, where the matter associated with our peak ends up. The distribution of the core-envelope material (right panel) shows that the core is in the process of merging with a larger halo. It was not significantly affected by the shear. However, a good fraction of the envelope has been stripped away and captured by other halos.

In hindsight, the profound influence of shear on the envelope material is not surprising. But,
can shear affect the core? To explore this, we imposed a considerable shear ($|\sigma_{xx}| = 350 \text{ km/s/Mpc}$) on a $1\sigma$ galaxy-scale peak that is initially at rest (Run 6). The particle distribution at $a = 0.8$ in a $1.24 h^{-1}$ Mpc slice through the entire simulation volume is shown in the left panel of the third row in Figure 1. The centre panel shows a detailed, $3h^{-1}$ Mpc thick slice of the region enclosed by the box. The circled mass concentrations are the fragments of the core of the original density peak. The two core pieces, shown in isolation in the right panel, collapse to form two distinct halos. This is an example of a case where a single initial peak forms more than one halo. Only one of the two halos contains the peak particle associated with the original density peak; the other halo contains no peak particle corresponding to an initial $\geq 1\sigma$ galaxy-scale peak and would have been missed under the peak-halo identification scheme. The arrows in the centre panel locate all the peak particles in a slice that extends beyond the particle slice by $1h^{-1}$ Mpc on either side.

The shear configurations discussed thus far involve dilation in one direction and contraction in the other two. In Run 8, the situation is reversed and the $1\sigma$ galaxy-scale under consideration is initially embedded in a plane. It is also given an initial velocity perpendicular to the plane. The left panel of the last row in Figure 1 shows the particle distribution at $a = 0.8$ in a $1h^{-1}$ Mpc thick slice centred on the plane. The envelope of the peak is widely dispersed over this plane but more interestingly, the core region fragments and forms two distinct halos, although the shear is not unusually large. A close-up of the region enclosed by the box is shown in the centre and the right panels. The projected velocity vectors (centre panel) reveal that the two halos are more or less virialized. As the arrow in the right panel indicates, only one of the halos contains a peak particle associated with an initial $\geq 1\sigma$ galaxy-scale peak. We note that the evolution of a $2\sigma$ peak under the same conditions is markedly different (Run 7). Although the peak initially starts to dissipate, mergers with nearby small clumps reverses the trend and gives rise to a group-size halo containing most of the core material as well as about half of the original envelope.

4. Conclusions

Although preliminary, our study into the role of shear on the formation of galaxy-scale halos has produced some tantalizing results. We find that the tidal forces induced by large-scale inhomoge-
Geneities can significantly affect the matter distribution in and around a primordial density peak, with the extent of the influence depending on the size of the density fluctuation, on the strength and the orientation of the shear field, and on whether its source is localized or distributed:

[1] When the shear is primarily caused by a single mass concentration, the galaxy-scale peak ends up merging with this object (e.g. Runs 1 & 2).

[2] When the source of the shear is distributed (e.g. a filament or a wall), the core of the peak tends to evolve undisturbed, while the envelope is deformed and even dispersed (e.g. Runs 3–5). The determination of halo properties, like its final mass, based on the spherically symmetric secondary infall model are likely to be in error.

[3] When the tidal forces are sufficiently strong, and favourably oriented with respect to the peak’s mass distribution and peculiar velocity, the peak can be torn into at least two distinct pieces, with each eventually forming a galaxy-scale halo (e.g. Runs 6 & 8). In this case, it is quite likely that not all of the halos will contain peak particles demarcating initial galaxy-scale density peaks. These halos are potential sites of galaxy formation though they would not have been flagged as such under the peak-halo identification scheme. We note, however, that we have not found a case of an initial galaxy-scale peak that did not form a halo, contrary to the findings of Katz et al. (1993).

[4] In regions of high shear, galaxy-size halos can spring up from primordial density peaks on scales smaller than $0.6h^{-1}$ Mpc, the commonly adopted “galaxy-scale” (e.g. Run 3). The convergence of matter streams due to shear boosts the rate of accretion onto the small peaks.

The frequency of occurrence of the above effects, particularly [3] and [4], is currently under study. If generic, they suggest that identifying galaxy-scale peaks in the initial field as sites of galaxy formation, in a one-to-one mapping, is perhaps an inaccurate oversimplification.

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References
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All numerical values are linear extrapolations evaluated at the present epoch.

†Initial conditions for Runs 4, 5 & 6 generated using same random number seed

‡Initial conditions for Runs 7, 8 & 9 generated using same random number seed
**Figure Caption**

**FIGURE 1** First row: The matter distribution at $a = 0.5$ in Run 3. The left panel shows the particle distribution in a $3 h^{-1}$ Mpc slice centred on the core (circled) formed by our initial galaxy-scale peak, while the distribution of the core-envelope particles are plotted in the centre panel. The right panel shows the location of all peak particles associated with initial $\geq 1\sigma$ galaxy-scale peaks in a thicker slice. Second row: The matter distribution at $a = 0.9$ in Run 4. The left panel illustrates the particle distribution in a $2 h^{-1}$ Mpc thick slice though the simulation volume. The centre and the right panels offer a more detailed view of the region enclosed by the box, with the right panel showing the distribution of the core-envelope particles. Third row: The matter distribution at $a = 0.8$ in Run 6. The particle distribution in a $1.24 h^{-1}$ Mpc slice is presented in the left panel. The centre panel shows the particle distribution in a $3 h^{-1}$ Mpc slice of the region defined by the box. The two core fragments are circled, and the corresponding particles are also plotted in the right panel. The arrows in the centre panel mark the location of all peak particles associated with initial $\geq 1\sigma$ galaxy-scale peaks in a yet thicker slice. Fourth row: The matter distribution at $a = 0.8$ in Run 8. The left panel illustrates the particle distribution in a $1 h^{-1}$ Mpc thick slice. The centre and right panels show a close-up of the region enclosed by the box. The projected velocity vectors of the particles are plotted in the centre panel and the core particles are plotted in the right panel. The arrow marks the location of the only peak particle associated with initial $\geq 1\sigma$ galaxy-scale peaks in the vicinity.