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Radiative Corrections to Neutral Current Processes^{*}

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^{*} to appear in "Precision Tests of the Standard Model", edited by P. Langacker

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CHAPTER 2d
RADIATIVE CORRECTIONS TO NEUTRAL CURRENT PROCESSES*

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1. Introduction

The standard $SU(3)_c \times SU(2)_L \times U(1)_Y$ model of strong and electroweak interactions has been extremely successful. It correctly predicted the existence and observed properties of weak neutral currents as well as the $W^\pm$ and Z bosons.¹ Furthermore, because that gauge theory is renormalizable, its predictions can be scrutinized at the quantum loop level by high precision measurements. A deviation from expectations would signal the presence of “new physics”. Remarkably, at this time there are no solid experimental results which cannot be accommodated by the standard model at the 1 or 2 sigma level. That accomplishment is very impressive when one considers the wealth of experimental data which must be confronted. Nevertheless, we do anticipate the emergence of “new physics” as higher energy and precision are probed. That conviction stems from a general dissatisfaction with electroweak symmetry breaking and mass generation via the simple Higgs mechanism. Although that mechanism can accommodate all known particle masses and mixing (even CP violation), it does not explain their origin. One hopes that a truly fundamental theory would have no free parameters and would elucidate the origin of mass either through additional symmetry, new dynamics, or by some as yet “unknown”. Uncovering those missing ingredients is the goal of high energy physics.

This chapter presents a general overview of electroweak radiative corrections and their implications for the standard model. As illustrative examples, the focus will be on low energy weak neutral current processes, but the discussion will be somewhat general.

Radiative corrections depend to some extent on the as yet unknown top quark and Higgs scalar masses. Global comparisons between theory and all high precision experiments have thereby been used to infer (for the top quark pole mass)²

$$m_t(pole) = 143 \pm 26 \text{ GeV} \quad (1.1)$$

for an assumed central value of $m_H \simeq 200 \text{ GeV}$ and the range $57 \text{ GeV} < m_H < 800 \text{ GeV}$. Fortunately (or unfortunately), the dependence on the Higgs mass, m_H , is small. When top is discovered and m_t measured, the comparison with Eq. (1.1) will be a beautiful test of the standard model at the loop level. The present experimental bound³

$$m_t > 108 \text{ GeV} \quad (\text{CDF}) \quad (1.2)$$

suggests that Eq. (1.1) is likely to be on the mark.

After m_t is directly measured, one can go back and use precision measurements to differentiate light and heavy Higgs scenarios or search for hints of “new physics”. Those pursuits will compare experiments at the few tenths of a percent level and will be limited to some extent by theoretical uncertainties. For that reason, our discussion will often emphasize underlying theory uncertainties from hadronic loops and higher order effects.

Precision tests of the standard model not only probe for signals of “new physics”, but also provide important model building constraints. Already, they suggest significant constraints on Z' bosons,⁴ heavy fermions, technicolor,⁵ etc. Should a “new physics” signal be directly uncovered at the next generation of high energy colliders, precision measurements carried out at low energies will be invaluable for deciphering it.

The contents of this overview are organized as follows. In section 2, a particular electroweak renormalization scheme is described. That prescription is based on the physical observables α , G_μ , m_Z , m_W , fermion masses, and m_H (as well as quark mixing angles which will not be discussed). Those quantities are not completely independent. They are connected by the lowest order natural relation

$$\frac{e_0^2}{g_{2_0}^2} = 1 - (m_W^0/m_Z^0)^2 \quad (1.3)$$

which is maintained for renormalized physical observables, up to finite calculable corrections.⁶ Those radiative corrections exhibit a sensitive dependence on m_t and thus play an important role in constraining its value.⁷ It is also useful to introduce the bare weak mixing angle, θ_W^0 , which depends on the other parameters via

$$\sin^2 \theta_W^0 = \frac{e_0^2}{g_{2_0}^2} = 1 - (m_W^0/m_Z^0)^2. \quad (1.4)$$

A specific renormalized quantity

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} \equiv \frac{e^2(m_Z)_{\overline{MS}}}{g_2^2(m_Z)_{\overline{MS}}} \quad (1.5)$$

defined by $\overline{MS}$ (modified minimal subtraction) is advocated here.^{8,9} That definition is particularly convenient for discussing weak neutral current processes and testing grand unified theories.

It can be perturbatively related to experimental observables or more physical definitions of $\sin^2 \theta_W$ such as^{8,10,11}

$$\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2 \quad (1.6)$$

$$\sin^2 \theta_W(0) \equiv \pi\alpha/\sqrt{2}m_W^2G_\mu \quad (1.7)$$

$$\sin^2 \bar{\theta}_W \text{ LEP Asymmetry Definition} \quad (1.8)$$

using the natural relation in Eq. (1.4) and calculated loop corrections. Section 3 illustrates the form of one loop radiative corrections to several weak neutral current processes. Those include atomic parity violation, polarized electron scattering asymmetries, and neutrino-electron scattering. Modulo some QED effects such

as bremsstrahlung and Coulombic interactions, it turns out that radiative corrections to each of those reactions can be parametrized by two (process dependent) quantities ρ and κ each of which is $1 + \mathcal{O}(\alpha)$ terms. In section 4, two examples of “new physics” are considered. Z' bosons motivated by E_6 grand unification and the Peskin-Takeuchi⁵ S , T , and U parametrization of gauge boson loop corrections are discussed. Their effects on various observables are illustrated. Finally, in section 5, conclusions and an outlook on the future is given.

2. Renormalization Prescription

The basic renormalized parameters of the standard model can be categorized as masses, couplings, and mixing angles. Masses are traditionally defined¹² as the real part of the propagator pole $m^2 = \text{Re}s_0$. That definition is manifestly gauge independent and directly related to physical observables. However, in the case of the Z resonance, a slightly different definition, m_Z , has been employed in Breit-Wigner fits¹²

$$m_Z^2 = m_Z^2(\text{pole}) + \Gamma_Z^2. \quad (2.1)$$

Those definitions differ by about 34 MeV which is much larger than the experimental uncertainty

$$m_Z = 91.187 \pm 0.007 \text{ GeV}. \quad (2.2)$$

Similarly, in the case of the $W^\pm$ bosons, a renormalized W mass, m_W , is more appropriate for low energy studies than the pole mass; so here

$$m_W^2 \equiv m_W^2(\text{pole}) + \Gamma_W^2 \quad (2.3)$$

will be employed. That W mass definition differs from the pole by about 25 MeV, while ± 40 MeV is the goal of future experiments. At hadron colliders, m_W will be normalized using $m_Z = 91.187$ GeV; so, m_W as defined in Eq. (2.3) is quite natural. At $e^+e^- \rightarrow W^+W^-$ facilities, such as LEP II, the W mass extraction will be set by definition and inclusion of the relevant radiative corrections.

In the case of quark masses, running $\overline{MS}$ defined masses are sometimes more convenient than physical masses, e.g. renormalization group studies. The distinction for light quarks will not concern us. However, the top quark mass definition should be properly specified in any discussion of precision electroweak studies, particularly if two loop QCD corrections are included. The top quark pole and $\overline{MS}$ masses are related by¹³

$$m_t(\text{pole}) = m_t(m_t)_{\overline{MS}} \left(1 + \frac{4}{3} \frac{\alpha_s(m_t)}{\pi} + 10.9 \left(\frac{\alpha_s(m_t)}{\pi} \right)^2 + \dots \right) \quad (2.4)$$

In addition, the leading $\mathcal{O}(G_\mu m_t^2)$ corrections are (for $m_H \simeq 200$ GeV) approximately $-5/4 \frac{G_\mu m_t^2}{8\sqrt{2}\pi^2} \simeq -0.3\%$ and will be neglected. For $\alpha_s(m_t) \simeq 0.10-0.11$, the QCD corrections give rise to a 5-6% difference, which can be sizable ~ 8 GeV for $m_t \simeq 150$ GeV. In our discussion, we employ $m_t \equiv m_t(m_t)_{\overline{MS}}$ in all one loop radiative corrections. As we shall illustrate, that prescription has the advantage of not inducing large $\mathcal{O}(\alpha\alpha_s m_t^2/m_W^2)$ two loop effects. Hence, one can largely ignore two loops, modulo leading log summations which are controlled by the renormalization group. Eq. (2.4) can be used to translate to the top pole mass definition.

2.1 α , G_μ , m_Z , and m_W

In addition to fermion masses and mixing angles, m_H and three other independent parameters are needed to specify the standard electroweak model. The three parameters can be any combination of α , G_μ , m_Z , and m_W . Since the first three are already precisely measured, they are generally used as input to predict m_W . We shall follow that approach; however, for some discussions it can be cumbersome or misleading. The introduction of an auxiliary mixing angle via $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ often facilitates a simple closed form presentation of radiative corrections.

The usual fine structure constant, α , is defined in terms of the physical electric charge

$$\alpha \equiv \frac{e^2}{4\pi} = Z_3 \frac{e_0^2}{4\pi} \mu^{n-4} \quad (2.5)$$

where μ is the 't Hooft mass unit¹⁴ introduced to keep α dimensionless in n dimensions. The Z_3 photonic wavefunction renormalization is conventionally defined using the vacuum polarization function $\Pi_{\gamma\gamma}(q^2)$

$$Z_3^{-1} = 1 + \Pi_{\gamma\gamma}(0) \quad (2.6)$$

$$\begin{aligned} \Pi_{\gamma\gamma}(q^2) = & -\frac{e_0^2}{6\pi^2} \left\{ \sum_f Q_f^2 \left(\frac{1}{n-4} + \frac{\gamma}{2} - \ln\sqrt{4\pi} + 3 \int_0^1 dx (x-x^2) \ln(m_f^2 + q^2(x^2-x)) \right) \right. \\ & \left. - \frac{21}{4} \left(\frac{1}{n-4} + \frac{\gamma}{2} - \ln\sqrt{4\pi} - \frac{1}{21} + 3 \int_0^1 dx (x-x^2) \ln(m_W^2 + q^2(x^2-x)) \right) \right\} \quad (2.7) \end{aligned}$$

with the sum taken over all charged fermions. Of course, at $q^2 = 0$, one must actually employ non-perturbative techniques or real data to obtain the hadronic contribution.

Experimentally, one knows α very precisely¹⁵

$$\alpha^{-1} = 137.0359895 (61) \quad (2.8)$$

That atomic physics parameter is, however, not directly applicable for weak interaction studies. Instead, one can employ $\overline{MS}$ renormalization of $\Pi_{\gamma\gamma}(q^2)$ by subtracting only $(\frac{1}{n-4} + \frac{\gamma}{2} - \ell n \sqrt{4\pi})$ terms, to define $\alpha(\mu)_{\overline{MS}}$. Using a dispersion relation to evaluate the hadronic contributions, one finds

$$\alpha^{-1}(m_Z)_{\overline{MS}} = 127.9 \pm 0.1 \quad (2.9)$$

where the error stems from uncertainties in $e^+e^- \rightarrow \text{hadrons}$ data.¹⁶ Even though α has essentially no uncertainty, the error in Eq. (2.9) in extrapolating to short-distances represents a limitation for precision tests of the standard model. It can be reduced by better measurements of low energy $e^+e^- \rightarrow \text{hadrons}$ or eliminated by employing some short-distance (precision) parameter in place of α . For perturbative expansions with $g_2^2/4\pi$ we will use $\alpha(m_Z)_{\overline{MS}}/\sin^2 \theta_W(m_Z)_{\overline{MS}}$.

The Fermi constant, G_μ , is obtained from the muon lifetime via the defining equation

$$\begin{aligned} \tau_\mu^{-1} = \Gamma(\mu \rightarrow \text{all}) &= \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right) \left(1 + \frac{3}{5} \frac{m_\mu^2}{m_W^2}\right) \left[1 + \frac{\alpha(m_\mu)}{2\pi} \left(\frac{25}{4} - \pi^2\right)\right] \\ f(x) &= 1 - 8x + 8x^3 - x^4 - 12x^2 \ell n x \\ \alpha^{-1}(m_\mu) &\simeq \alpha^{-1} - \frac{2}{3\pi} \ell n \left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} = 136 \end{aligned} \quad (2.10)$$

Employing $\tau_\mu = 2.197035 \pm 0.000040 \times 10^{-6} \text{s}$ then gives

$$G_\mu = 1.16639(2) \times 10^{-5} \text{ GeV}^{-2} \quad (2.11)$$

with essentially no theoretical uncertainty. Most of the radiative corrections to muon decay, including m_t dependent loops, have been absorbed into G_μ . Those effects will reappear when we examine the natural relations in Eqs. (1.3) & (1.4) or use G_μ to normalize weak neutral current amplitudes.

The Z boson mass in Eq. (2.2) has been precisely measured at LEP. The experimental uncertainty is insignificant, but the specific definition of m_Z is important for comparison with m_W and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ for the purpose of extracting m_t .

Given the value of α , G_μ , and m_Z , the $W^\pm$ mass is determined by using Eq. (1.3) in conjunction with the one loop corrections to $\Pi_{\gamma\gamma}(0)$, muon decay, and the Z and $W^\pm$ masses. Those loop effects can be collected into a quantity traditionally called Δr and used in the relation^{10,17}

$$1 - m_W^2/m_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu m_W^2(1 - \Delta r)} \quad (2.12)$$

where

$$\Delta r = \frac{2\delta e}{e} - \frac{2\delta g_2}{g_2} - \cot^2 \theta_W \left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} \right). \quad (2.13)$$

Normalizing at $m_t = 140$ GeV and $m_H = 200$ GeV, one finds^{18,19}

$$\Delta r = 0.0469 \pm 0.0011 \quad (2.14)$$

which corresponds to the central prediction $m_W = 80.18$ GeV. The dependence of Δr on m_t is illustrated in table 1 and the implication for m_W is shown in table 2. In the asymptotic limit of large m_t and m_H , the contribution to Δr is given by δ

$$\delta = \frac{\alpha(m_Z)_{\overline{MS}}}{\pi \sin^2 \theta_W(m_Z)_{\overline{MS}}} \left[\frac{-3}{16} \frac{m_t^2}{m_W^2} \cot^2 \theta_W(m_Z)_{\overline{MS}} + \frac{11}{48} \ln \left(\frac{m_H^2}{m_Z^2} \right) \right] \quad (2.15)$$

Because they contribute with opposite sign, large m_t effects can be offset by very large m_H . So, if m_t turns out to be much larger than the 143 GeV central value in Eq. (1.1), it could signal a very large m_H . The exact dependence of Δr on m_t and m_H can be found in ref. 17.

Table 1 Standard model predictions for Δr , $\Delta r(m_Z)_{\overline{MS}}$, and $\Delta \hat{r}$ as functions of $m_t(m_t)_{\overline{MS}}$ for $m_H \simeq 200$ GeV. The uncertainty comes from hadronic vacuum polarization and 2 loop effects. These values incorporate the updates of QCD and top threshold effects given by Fanchiotti, Kniehl and Sirlin.²⁰

$m_t(m_t)_{\overline{MS}}$ (GeV)	Δr ± 0.0011	$\Delta r(m_Z)_{\overline{MS}}$ ± 0.0011	$\Delta \hat{r}$ ± 0.0011
100	0.0596	0.0693	0.0664
110	0.0565	0.0694	0.0656
120	0.0534	0.0698	0.0649
130	0.0503	0.0700	0.0642
140	0.0469	0.0702	0.0633
150	0.0435	0.0704	0.0625
160	0.0398	0.0705	0.0616
170	0.0360	0.0706	0.0605
180	0.0319	0.0707	0.0594
190	0.0277	0.0708	0.0585
200	0.0232	0.0709	0.0574
210	0.0186	0.0710	0.0562
220	0.0137	0.0711	0.0551
230	0.0086	0.0712	0.0539
240	0.0033	0.0712	0.0525
250	-0.0021	0.0712	0.0510

Table 2 Standard model predictions for m_W and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ as functions of $m_t(m_t)_{\overline{MS}}$. those predictions are based on $m_Z = 91.187$ GeV, $\alpha = 1/137.036$, $G_\mu = 1.16639 \times 10^{-5}$ GeV $^{-2}$, and the assumption $m_H \simeq 200$ GeV. The quoted uncertainties correspond to hadronic vacuum polarization and two loop effects.

$m_t(m_t)_{\overline{MS}}$ (GeV)	$m_t(pole)$ (GeV)	m_W (GeV) ± 0.02	$\sin^2 \theta_W(m_Z)_{\overline{MS}}$ ± 0.0004
100	106.2	79.96	0.2336
110	116.7	80.01	0.2333
120	127.2	80.07	0.2331
130	137.7	80.12	0.2328
140	148.2	80.18	0.2325
150	158.7	80.24	0.2322
160	169.2	80.30	0.2319
170	179.7	80.37	0.2315
180	190.2	80.44	0.2311
190	200.6	80.51	0.2308
200	211.1	80.58	0.2304
210	221.6	80.65	0.2300
220	232.0	80.73	0.2296
230	242.5	80.81	0.2292
240	252.9	80.89	0.2287
250	263.5	80.97	0.2282

2.2 $\sin^2 \theta_W(m_Z)_{\overline{MS}}$

A variety of renormalized weak mixing angles have been advocated in the literature. Some physical definitions are given in Eqs. (1.6)–(1.8). Two of those quantities are related through Δr

$$\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2 = \sin^2 \theta_W(0)/(1 - \Delta r) \quad (2.16)$$

Each of those definitions has its own drawback. The quantity $\sin^2 \theta_W(0) \equiv \pi\alpha/\sqrt{2}m_W^2 G_\mu$ is the ratio of a long-distance parameter, α , defined at zero momentum transfer and short-distance quantities G_μ and m_W^2 . It is, therefore, not a good expansion parameter for weak neutral current amplitudes. When it is employed, spurious vacuum polarization corrections of $\mathcal{O}(7\%)$ are induced⁸ (approximately the difference between α and $\alpha(m_Z)_{\overline{MS}}$). Since one strives to employ Born level parameters which do not introduce significant higher order effects, $\sin^2 \theta_W(0)$ is rarely used except in discussions of Δr .

The mass definition $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2$ in Eq. (1.6) does not induce large distance vacuum polarization effects. It does, however, introduce weak isospin breaking loop effects. An important example is the top-bottom mass difference. When neutral current amplitudes are expressed in terms of $\sin^2 \theta_W$, radiative corrections of

$\mathcal{O}(\alpha m_t^2/m_W^2)$ are induced. Such an effect can be misleading since it gives the appearance of m_t sensitivity, e.g. in Z decay asymmetries, when none intrinsically exists. For that reason, $\sin^2 \theta_W$ has fallen out of favor as m_t has been growing. An exception is the use of $\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2$ in the ratio of deep-inelastic neutrino neutral to charged current scattering $R_\nu \equiv \sigma_{NC}(\nu_\mu N \rightarrow \nu_\mu X)/\sigma_{CC}(\nu_\mu N \rightarrow \mu X)$. That ratio has an intrinsic dependence on $\alpha m_t^2/m_W^2$ which cancels with the $\alpha m_t^2/m_W^2$ dependence induced by the use of $\sin^2 \theta_W$ in the neutral current amplitude. Therefore, to a very good approximation, R_ν measures $\sin^2 \theta_W$ independent of m_t and when used in conjunction with m_Z can be viewed as an indirect determination of m_W .²⁰

An alternative weak mixing angle has been employed by LEP workers.¹¹ They define an effective $\sin^2 \bar{\theta}_W$ at the Z pole via the ratio of vector and axial vector couplings. In principle, that definition should allow a direct determination of $\sin^2 \bar{\theta}_W$ via forward-backward lepton or tau polarization asymmetry measurements at the Z . Unfortunately, there are still radiative corrections that must be applied to the asymmetries; so, the definition of $\sin^2 \bar{\theta}_W$ is somewhat buried in computer codes. Fortunately, $\sin^2 \bar{\theta}_W$ is numerically very close to the $\overline{MS}$ definition (to be discussed below)

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} \simeq \sin^2 \bar{\theta}_W + 0.0002 \quad (2.17)$$

so, the two can be used interchangeably for now. When the errors on $\sin^2 \bar{\theta}_W$ are reduced to the level of ± 0.0005 , the approximate relationship in Eq. (2.17) should be scrutinized and refined. (Eq. (2.17) is an estimate and should not be taken too literally.)

Rather than employing any of the physical weak mixing angles described above, it is more convenient to define an unphysical angle via modified minimal subtraction ($\overline{MS}$)

$$\sin^2 \theta_W(\mu)_{\overline{MS}} \equiv e^2(\mu)_{\overline{MS}}/g_2^2(\mu)_{\overline{MS}} \quad (2.18)$$

Each of the couplings $e^2(\mu)_{\overline{MS}}$ and $g_2^2(\mu)_{\overline{MS}}$ is individually defined by $\overline{MS}$ with μ the mass scale of dimensional regularization. When μ is greater than m_t , the $\overline{MS}$ prescription is very simple. It corresponds to subtracting all $(\frac{1}{n-4} + \frac{\gamma}{2} - \ell n \sqrt{4\pi})$ loop corrections. For the standard model with 3 generations and one Higgs doublet, that translates to⁸

$$\sin^2 \theta_W(\mu)_{\overline{MS}} = \sin^2 \theta_W^0 \left[1 + \frac{e_0^2 \mu^{n-4}}{8\pi^2} \left(\frac{11}{3} + \frac{19}{6\sin^2 \theta_W^0} \right) \left(\frac{1}{n-4} + \frac{\gamma}{2} - \ell n \sqrt{4\pi} \right) + \dots \right] \quad (2.19)$$

where $\dots$ indicates higher order terms. Because of its simple definition, $\sin^2 \theta_W(\mu)_{\overline{MS}}$ is very convenient for grand unified theories (GUTS). For example, in leading log approximation, the minimal SU(5) model predicts⁸ ($\sin^2 \theta_W^0 = 3/8$)

$$\sin^2 \theta_W(\mu)_{\overline{MS}} = \frac{3}{8} \left[1 - \frac{109}{18} \frac{\alpha(\mu)_{\overline{MS}}}{\pi} \ell n \left(\frac{m_X}{\mu} \right) \right], \mu > m_t \quad (2.20)$$

where m_X is the heavy X boson mass, $\mathcal{O}(10^{15} \text{ GeV})$.

For most low-energy processes as well as measurements at the Z pole, it is convenient to keep μ fixed at $\mu = m_Z$. That requirement causes a slight complication because $m_t > m_Z$ (from CDF bounds). To deal with that issue is a bit subtle since $g_2(\mu)_{\overline{MS}}$ can be defined by charged or neutral current couplings and top affects them differently. It is convenient to subtract an extra^{21,22}

$$\frac{\alpha}{\pi} \left(\frac{1}{3} - \frac{8}{9} \sin^2 \theta_W \right) \ell n \left(\frac{m_t}{\mu} \right) \quad (2.21)$$

from Eq. (2.19) for $\mu < m_t$. That additional finite subtraction keeps $\gamma - Z$ mixing independent of $\ell n m_t$ in neutral current amplitudes and allows $\sin^2 \theta_W(\mu)_{\overline{MS}}$ to be continuous at $\mu = m_t$. Numerically, the additional subtraction is very small, ~ 0.0001 at $\mu = m_Z$, but its inclusion is important for strict adherence to the $\overline{MS}$ formalism.

Predictions for $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ can be obtained from α , G_μ , and m_Z using the relation

$$\sin^2 2\theta_W(m_Z)_{\overline{MS}} = \frac{4\pi\alpha}{\sqrt{2}G_\mu m_Z^2 (1 - \Delta\hat{r})} \quad (2.22)$$

where the radiative correction $\Delta\hat{r}$ is given in table 1 as a function of m_t . Note that $\Delta\hat{r}$ is much less sensitive to m_t than Δr . From the values of $\Delta\hat{r}$, the predictions for $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ in table 2 follow. For GUT extrapolations

$$\sin^2 \theta_W(m_t)_{\overline{MS}} = \sin^2 \theta_W(m_Z)_{\overline{MS}} + \frac{\alpha(m_Z)_{\overline{MS}}}{2\pi} \left[\frac{23}{6} + \frac{17}{9} \sin^2 \theta_W \right] \ell n \left(\frac{m_t}{m_Z} \right) \quad (2.23)$$

In section 3, we will employ $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ to parametrize weak neutral current amplitudes. In terms of that quantity, the electroweak radiative corrections will be simplified.

2.3 Top Quark Mass

Electroweak observables are sensitive to the top quark mass via loop corrections to the W and Z self-energies. As illustrated in table 2, when α , G_μ , and m_Z are given, m_W and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ become sensitive determinators of m_t . Those two latter quantities are related by

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2 \theta_W(m_Z)_{\overline{MS}} (1 - \Delta r(m_Z)_{\overline{MS}})} \quad (2.24)$$

where the loop corrections $\Delta r(m_Z)_{\overline{MS}}$ are given in table 1. Note that they are very insensitive to the value of m_t . From (2.24), one can see that the combination $m_W^2 \sin^2 \theta_W(m_Z)_{\overline{MS}}$ is rather insensitive to m_t . Therefore, a $\pm 0.05\%$ determination

of m_W (i.e. ± 40 MeV) is equal to a ± 0.0002 measurement of $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ for the purpose of determining m_t .

At present, direct measurements of m_W give¹⁵

$$m_W = 80.22 \pm 0.26 \text{ GeV} \quad (2.25)$$

which suggests (from table 2) $m_t(\text{pole}) \simeq 152$ GeV. Indirect determinations of m_W via R_ν imply^{2,23}

$$m_W = 80.00 \pm 0.13 \pm 0.27 \text{ GeV} \quad (R_\nu \text{ \& } m_Z) \quad (2.26)$$

which corresponds to a lower m_t .

Measurements of $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ at LEP and SLAC via decay asymmetries give²

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2326 \pm 0.0011 \quad (\text{LEP Average}) \quad (2.27)$$

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2372 \pm 0.0056 \quad (\text{SLD}) \quad (2.28)$$

From table 2, we see that those results suggest $m_t(\text{pole}) \simeq 141$ GeV which is quite consistent with the m_W implications. A more powerful LEP constraint on m_t follows from measurements of the Z partial and total decay widths. Taken together with the above results, one finds globally

$$\begin{aligned} m_t(m_t)_{\overline{MS}} &= 135 \pm 26 \text{ GeV} \\ m_t(\text{pole}) &= 143 \pm 26 \text{ GeV} \end{aligned} \quad (\text{Global Fit}) \quad (2.29)$$

where the central values correspond to $m_H \simeq 200$ GeV and the errors allow for $57 \text{ GeV} \leq m_H \leq 800 \text{ GeV}$.

Given the good agreement between theory and experiment, we will assume $m_t(m_t)_{\overline{MS}} \simeq 140$ GeV, $m_H \simeq 200$ GeV, and $\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2325$ as central values in discussions of radiative corrections which follow. (Preliminary updates from LEP suggest that increases in Eq. (2.29) are imminent.) For the examples we have chosen as illustrations, the sensitivity to shifts in m_t and m_H is very small; so, they serve primarily as probes of “new physics” rather than refinements of the above constraints.

3. Radiative Corrections and Neutral Currents

Electroweak radiative corrections have been calculated at the one loop level for a variety of weak neutral current processes. Here, I will discuss three relevant examples which are particularly easy to illustrate, atomic parity violation, polarized electron scattering and neutrino-electron scattering. In all cases, the renormalization prescription entails choosing G_μ and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ as the lowest order expansion parameters and explicitly computing the one loop quantum corrections. As

we shall see, the effect of those corrections can be (almost entirely) parameterized by the replacements $G_\mu \rightarrow \rho G_\mu$ and $\sin^2 \theta_W(m_Z)_{\overline{MS}} \rightarrow \kappa \sin^2 \theta_W(m_Z)_{\overline{MS}}$ where ρ and κ are both $1 + \mathcal{O}(\alpha)$ process dependent quantities. In addition to ρ and κ , there are generally bremsstrahlung effects which require a careful study of detector characteristics and experimental cuts. We will not discuss bremsstrahlung in any detail, but when necessary present radiative inclusive cross-sections.

3.1 Atomic Parity Violation

The interference of weak neutral current and electromagnetic amplitudes gives rise to parity violation in all physical systems. At low energies, such effects are generally much too small to observe. However, under special circumstances they may be significantly enhanced. For example, Bouchiat and Bouchiat²⁴ pointed out that parity violating effects in heavy atoms are enhanced by roughly $Z^3 \sim 10^5 - 10^6$ relative to hydrogen. To date, atomic parity violation has been observed in bismuth, thallium, cesium, and lead.²⁵ Those experiments confirm standard model expectations and constrain “new physics” appendages. In the case of cesium, progress has been particularly impressive. An experiment in Boulder, Colorado has measured atomic parity violation with $\pm 2\%$ precision.²⁶ Concurrently, the atomic theory²⁷ uncertainty has been lowered to about $\pm 1\%$. Hopefully, both errors will be further reduced in the near future. Anticipating a $\pm 0.5\%$ total error, implies a sensitive test of the standard model at the level of its radiative corrections and probe of “new physics” such as Z' bosons²⁸ and technicolor loop effects.²¹

In addition, the possibility of scrutinizing atomic theory and even eliminating most of its uncertainty by comparing parity violation in chains of isotopes has been advocated.²⁹ Given the impressive history of atomic physics experiments, one might envision a time when those presently difficult measurements become routine laboratory procedures and reach $\pm 0.1\%$ accuracy. Given such possibilities, a thorough understanding of electroweak radiative corrections is required. Here, we review the status of such calculations.^{30,31,32,33}

The low energy electron-quark parity violating Hamiltonian (due to Z exchange) is conventionally parametrized by³⁰

$$\begin{aligned}
 H_{PV} = \frac{G_\mu}{\sqrt{2}} & \left(C_{1u} \bar{e} \gamma_\mu \gamma_5 e \bar{u} \gamma^\mu u + C_{2u} \bar{e} \gamma_\mu e \bar{u} \gamma^\mu \gamma_5 u \right. \\
 & + C_{1d} \bar{e} \gamma_\mu \gamma_5 e \bar{d} \gamma^\mu d + C_{2d} \bar{e} \gamma_\mu e \bar{d} \gamma^\mu \gamma_5 d \\
 & + C_{1s} \bar{e} \gamma_\mu \gamma_5 e \bar{s} \gamma^\mu s + C_{2s} \bar{e} \gamma_\mu e \bar{s} \gamma^\mu \gamma_5 s \\
 & \left. + \dots \right) \quad (3.1)
 \end{aligned}$$

where $\dots$ represent heavy quark terms $q = c, b, t$. Following our renormalization prescription, G_μ has been used to normalize the effective interaction and the C 's are given by (in lowest order)

$$\begin{aligned}
C_{1u} &= \frac{1}{2} \left(1 - \frac{8}{3} \sin^2 \theta_W(m_Z)_{\overline{MS}} \right) \\
C_{1d} = C_{1s} &= -\frac{1}{2} \left(1 - \frac{4}{3} \sin^2 \theta_W(m_Z)_{\overline{MS}} \right) \\
C_{2u} &= \frac{1}{2} (1 - 4 \sin^2 \theta_W(m_Z)_{\overline{MS}}) \\
C_{2d} = C_{2s} &= -\frac{1}{2} (1 - 4 \sin^2 \theta_W(m_Z)_{\overline{MS}}). \tag{3.2}
\end{aligned}$$

The C_{2q} terms are suppressed by $1 - 4 \sin^2 \theta_W(m_Z)_{\overline{MS}} \simeq 0.07$, due to the electron's neutral current vector coupling, and thus naturally small. That suppression is fortunate, because those amplitudes undergo strong interaction renormalizations at low energies and are therefore less clean theoretically than the C_{1q} which are not renormalized by strong interactions³¹ (due to CVC) at $q^2 = 0$. In fact, for heavy atoms, the C_{1q} are of prime importance because they add coherently over all quarks in the nucleus and parity violating effects are proportional to the so-called weak charge Q_W

$$Q_W(Z, A) \equiv 2[(A + Z)C_{1u} + (2A - Z)C_{1d}] \tag{3.3}$$

Compared to Q_W , C_{2q} , C_{1s} and electron-electron parity violating effects in heavy atoms are quite negligible²⁷ (e.g. $< 0.1\%$ for cesium); hence, we ignore them in this discussion. More important are nuclear anapole moment effects which can be several percent relative to Q_W and are not reliably computable.²⁵ Fortunately, those contributions to atomic parity violating transitions can be eliminated by averaging transitions involving pairs of hyperfine lines.²⁶

To obtain the one loop radiative corrections to Q_W , we proceed in steps. At the quark level, one combines the radiative corrections to muon decay and the electron-quark neutral current amplitudes (at $q^2 = 0$) to give (perturbatively)³¹

$$\begin{aligned}
C_{1u} &= \frac{1}{2} \rho'_{PV} \left(1 - \frac{8}{3} \kappa'_{PV}(0) \sin^2 \theta_W(m_Z)_{\overline{MS}} \right) \\
C_{1d} &= -\frac{1}{2} \rho'_{PV} \left(1 - \frac{4}{3} \kappa'_{PV}(0) \sin^2 \theta_W(m_Z)_{\overline{MS}} \right) \tag{3.4}
\end{aligned}$$

where

$$\begin{aligned}
\rho'_{PV} &= 1 - \frac{\alpha}{2\pi} + \frac{\alpha(m_Z)_{\overline{MS}}}{2\pi} \left\{ \frac{3}{8s^2} \frac{m_t^2}{m_W^2} + \frac{3}{8s^4} \ell n c^2 - \frac{7}{8s^2} + \frac{3\xi}{8s^2} \left(\frac{\ell n(c^2/\xi)}{c^2 - \xi} + \frac{1}{c^2} \frac{\ell n \xi}{1 - \xi} \right) \right. \\
&\quad \left. - \frac{1}{s^2} - 4(1 - 4s^2) \left[\ell n \left(\frac{m_Z^2}{M^2} \right) + \frac{3}{2} \right] - \frac{9}{16s^2 c^2} \left(1 - \frac{16}{9} s^2 \right) (1 + (1 - 4s^2)^2) \right\} \tag{3.5a}
\end{aligned}$$

$$\begin{aligned}
\kappa_{PV}(0) = 1 - \frac{\alpha}{2\pi s^2} & \left\{ \frac{1}{3} \sum_i (T_{3i} Q_i - 2s^2 Q_i^2) \ell n \left(\frac{m_i^2}{m_Z^2} \right) - \left(\frac{7}{2} c^2 + \frac{1}{12} \right) \ell n c^2 + \frac{7}{9} - \frac{s^2}{3} \right. \\
& - \frac{1}{6} (1 - 4s^2) \left(\ell n \left(\frac{m_Z^2}{m_e^2} \right) + \frac{1}{6} \right) \left. \right\} - \frac{\alpha(m_Z)_{\overline{MS}}}{2\pi s^2} \left\{ \frac{9 - 8s^2}{8s^2} \right. \\
& + \left(\frac{9}{4} - 4s^2 \right) (1 - 4s^2) \left(\ell n \left(\frac{m_Z^2}{M^2} \right) + \frac{3}{2} \right) + \frac{9}{16s^2 c^2} \left(\frac{1}{2} - 2s^2 + \frac{16}{9} s^4 \right) (1 + (1 - 4s^2)^2) \left. \right\}
\end{aligned} \tag{3.5b}$$

In those expressions $s^2 \equiv \sin^2 \theta_W(m_Z)_{\overline{MS}}$, $c^2 \equiv \cos^2 \theta_W(m_Z)_{\overline{MS}}$, $\xi = m_H^2/m_Z^2$, and M is a hadronic cutoff $\simeq 0.7 \sim 1$ GeV used in γZ box diagrams. The M dependence signals hadronic uncertainties which we subsequently address at the nucleon level.³² Fortunately, that uncertainty is suppressed by a $1 - 4s^2$ factor and rendered negligible.

In the perturbative expressions of Eq. (3.5), we have judiciously employed $\alpha = 1/137$ and $\alpha(m_Z)_{\overline{MS}} = 1/127.9$ for different contributions. (The difference is of two loop order.) The first coupling is employed for QED loops and low momentum photon exchange (e.g. γZ mixing) while the latter is more appropriate for short-distance electroweak loops such as box diagrams. Our choice reduces two loop uncertainties since it incorporates most leading logs.

The corrections in Eq. (3.5a) have the following origins.³¹ The first term is due to a QED renormalization of the electron's axial-vector current at $q^2 = 0$. That term as well as some other low energy photonic effects should actually be recalculated for heavy atoms using bound state electron propagators.³⁴ However, those modifications are expected to be small; so, we neglect them. The next four terms in ρ'_{PV} result from self-energy and vertex corrections to the neutral current amplitudes¹¹ and muon decay corrections (via G_μ normalization). The dependence on m_t and m_H reflects a difference in Z and W self-energies. It is similar to the dependence exhibited in Eq. (2.15) from the comparison of m_W and m_Z . The $\mathcal{O}(\alpha m_t^2/m_W^2)$ term is potentially important for large m_t ; so, it is of interest to also examine two loop corrections of that form. Employing $m_t \equiv m_t(m_t)_{\overline{MS}}$ and assuming $m_H \simeq 1.4m_t$, one finds (roughly)

$$m_t^2 \rightarrow m_t^2 \left(1 - \frac{2\alpha_s(m_t)}{3\pi} \left(\frac{\pi^2}{3} - 3 \right) - 0.4 \frac{\alpha(m_Z)_{\overline{MS}}}{\pi s^2} \frac{m_t^2}{m_W^2} + \dots \right) \tag{3.6}$$

where $\dots$ represent non-enhanced two loop effects. As previously mentioned, the QCD correction is very small¹⁹ because an $\overline{MS}$ definition of m_t is employed. Effects that grow as m_t^4 are also fairly insignificant if m_t is not too large. Since other uncalculated two loop corrections are potentially just as large as those in Eq. (3.6), we ignore all such effects and instead assign an uncertainty to ρ'_{PV} which reflects their neglect. The last three terms in Eq. (3.5a) come from WW , γZ and ZZ box diagrams.

The corrections to $\kappa'_{PV}(0)$ in Eq. (3.5b) also exhibit hadronic sensitivities. The sum in that expression is over all fermions with mass $< m_Z$ and $Q_i =$ electric

charge, $T_{3i} = \pm 1/2$ (a color factor of 3 must be included for quarks). That term and its neighbor (from $W^\pm$ loops) are induced by γZ vacuum polarization mixing. Such mixing involves quarks at $q^2 = 0$ and hence cannot be reliably obtained from the naive perturbative result in Eq. (3.5b). Instead, a dispersion relation must be used in conjunction with $e^+e^- \rightarrow \text{hadrons}$ data to properly determine the quark contribution.³² We subsequently give the results of such an analysis. The remaining corrections to κ'_{PV} are from the electron charge radius and box diagrams.

We now focus on hadronic effects and uncertainties in Eq. (3.5) when applied to heavy atoms. The following discussion summarizes and updates the original analysis of ref. 32 where details can be found.

The hadronic vacuum polarization corrections to γZ mixing have been extracted from $e^+e^- \rightarrow \text{hadrons}$ data most recently by Jegerlehner.¹⁶ Employing his results and the analysis of ref. 32, one finds (for $s^2 = 0.2325$)

$$\frac{1}{3} \sum_{\text{quarks}} (T_{3i} Q_i - 2s^2 Q_i^2) \ell n \left(\frac{m_i^2}{m_Z^2} \right) \rightarrow -6.85 \pm 0.50 \quad (3.7)$$

The uncertainty in that contribution will turn out to be the dominant theoretical uncertainty (modulo atomic theory). In principle, it could be reduced somewhat by better $e^+e^- \rightarrow \text{hadrons}$ data.

The non-perturbative result in Eq. (3.7) can be mimicked by employing effective quark masses¹⁶ $m_u \simeq 62$ MeV, $m_d \simeq 83$ MeV, $m_s \simeq 215$ MeV, $m_c \simeq 1.5$ GeV, $m_b \simeq 4.5$ GeV and multiplying by the QCD correction $(1 + \alpha_s/\pi) \simeq 1.042$. We subsequently use those masses to estimate changes in $\kappa'_{PV}(q^2)$ for $q^2 \neq 0$.

A second hadronic uncertainty enters ρ'_{PV} and κ'_{PV} via low frequency contributions to γZ box diagrams. Those effects were studied in ref. 32 for nucleons using a form factor analysis. There, it was shown that the electron-nucleon parity violating Hamiltonian was given by

$$H_{PV} = \frac{G_\mu}{\sqrt{2}} \bar{e} \gamma_\mu \gamma_5 e (C_{1p} \bar{p} \gamma^\mu p + C_{1n} \bar{n} \gamma^\mu n) + \dots \quad (3.8)$$

where

$$\begin{aligned} C_{1p} &= \frac{1}{2} \rho'_{PV} (1 - 4\kappa'_{PV}(0) \sin^2 \theta_W(m_Z) \overline{MS}) \\ C_{1n} &= -\frac{1}{2} \rho'_{PV} - 0.3 \frac{\alpha(m_Z) \overline{MS}}{\pi} (1 - 4s^2) \end{aligned} \quad (3.9)$$

with ρ'_{PV} and $\kappa'_{PV}(0)$ as given previously, but the γZ box contribution replaced by

$$\ell n \left(\frac{m_Z^2}{M^2} \right) + \frac{3}{2} \rightarrow 11.0 \pm 1.0 \quad (3.10)$$

(That value is slightly updated compared with ref. 32.) Here, we again have some hadronic uncertainty. Fortunately, it is suppressed by $1-4s^2$ in Eq. (3.5). That suppression factor will be maintained in going from nucleons to heavy nuclei and thus render additional two nucleon correlation effects (of the type discussed in ref. 35) small and presumably unimportant.

In terms of C_{1p} and C_{1n} , the weak charge is given by

$$Q_W(A, Z) = 2(A - Z)C_{1n} + 2ZC_{1p} \quad (3.11)$$

From the above formulas, one finds (for $m_t = 140$ GeV, $m_H = 200$ GeV, $\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2325$)

$$\rho'_{PV} = 0.9857 \pm 0.0004 \pm 0.0004 \quad (3.12a)$$

$$\kappa'_{PV}(0) = 1.0029 \pm 0.0025 \pm 0.0005 \quad (3.12b)$$

where the first error comes from 1 loop hadronic uncertainties and the second corresponds to (uncalculated) higher order effects. From those results, one finds

$$C_{1p} = 0.03317 \pm 0.00013 \quad (3.13a)$$

$$C_{1n} = -0.49290 \pm 0.00028 \quad (3.13b)$$

$$Q_W(Z, A) = (0.9858 \pm 0.0006)(Z - A) + (0.06634 \pm 0.0024)Z \quad (3.13c)$$

In the case of $^{133}_{55}\text{Cs}$ where the most precise measurements have been carried out

$$Q_W(^{133}_{55}\text{Cs})^{\text{theory}} = -73.24 \pm 0.14 \quad (3.14)$$

That prediction is extremely insensitive to the values of m_t and m_H employed.^{21,36} Changing those parameters modifies ρ'_{PV} via Eq. (3.5a) but that effect is almost completely canceled by the change in $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ (see table 2). Therefore, Eq. (3.14) represents a testable prediction with very little uncertainty. It is to be compared with the experimental result²⁶

$$Q_W(^{133}_{55}\text{Cs})^{\text{exp.}} = -71.04 \pm 1.58 \pm 0.88 \quad (3.15)$$

where the errors are experimental and atomic theory.²⁷ The agreement is quite good and must be viewed as a success for the standard model. As the errors in Eq. (3.15) are further reduced, we should see an interesting confrontation with theory at the level of electroweak radiative corrections. One can even probe for “new physics”. Two examples, Z' bosons and technicolor models will be subsequently discussed.

Rather than comparing Q_W with a $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ derived from α , G_μ , and m_Z , one could employ alternative strategies. For example, a $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ directly measured in Z decay asymmetries or derived from α , G_μ , and m_W (see Eq. (2.24)) could be used in the Q_W predictions. In that case, $Q_W(^{133}_{55}\text{Cs})$ will exhibit the

dependence on m_t and m_H given by the ρ'_{PV} in Eq. (3.5a). To be competitive with other loop constraints on m_t and m_H , one must aim for a $\pm 0.2\%$ measurement of $Q_W(^{133}_{55}\text{Cs})$. Such an advance represents a significant challenge for experiment and atomic theory.

As mentioned before, one might envision measuring $Q_W(Z, A)$ for a chain of isotopes and comparing those values to eliminate most atomic theory uncertainties.²⁹ (Neutron distributions may cause variations between isotopes.³⁷) The ρ'_{PV} dependence also cancels in such a comparison; so, it can be considered merely as an alternate way to measure $\sin^2 \theta_W(m_Z)_{\overline{MS}}$. To be competitive with Z asymmetry measurements, ratios of weak charges for widely separated isotopes would need to be measured to $\pm 0.1\%$ precision.

3.2 Polarized Electron Scattering

Low energy polarized electron scattering experiments also probe the weak neutral current and extensions of the standard model. By measuring the parity violating asymmetry

$$A_{pol} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \quad (3.16)$$

for right and left-handed electron scattering, one detects the interference between γ and Z exchange amplitudes. Various asymmetry experiments have already been carried out or are planned. They range from the classic SLAC deep-inelastic e - D experiment³⁸ to lower energy measurements at BATES³⁹ and CEBAF.

One possibility suggested by G. Feinberg⁴⁰ is particularly simple. It involves the elastic scattering of polarized electrons on a spinless isoscalar target such as ^{12}C or ^4He . For those cases, axial-vector hadronic matrix elements are negligible and the vector current matrix elements are parametrized by a single form factor which is identical for the electromagnetic and weak neutral current amplitudes. (For now, we neglect strange quark radii⁴¹ and nuclear isospin impurity effects⁴² which can modify the asymmetry. Such subtleties will be subsequently addressed.) Therefore, hadronic matrix element uncertainties cancel in the A_{pol} ratio and the predicted lowest order asymmetry depends on $C_{1u} + C_{1d}$ (or $C_{1p} + C_{1n}$). One finds (in lowest order)

$$A_{pol} = \frac{1}{2} \frac{G_\mu q^2}{\sqrt{2}\pi\alpha} (C_{1p} + C_{1n}) = -\frac{G_\mu q^2}{\sqrt{2}\pi\alpha} \sin^2 \theta_W(m_Z)_{\overline{MS}} \quad (0^+ \rightarrow 0^+) \quad (3.17)$$

where q is the momentum transfer. An experiment³⁹ at BATES on ^{12}C gave $\sin^2 \theta_W(m_Z) = 0.200 \pm 0.051$ where the error is mainly statistical. One could envision a reduction in the experimental uncertainty to $\pm 1\%$ by longer runs. At that level, eC is competitive with atomic parity violation experiments; so, it becomes

important to assess all theoretical uncertainties and include electroweak radiative corrections.

Discussion of the radiative corrections is facilitated by considering the quantity

$$\frac{A_{pol}}{\left(-\frac{G_\mu}{\sqrt{2}\pi} \frac{q^2}{\alpha}\right)} = \sin^2 \theta_W(m_Z)_{\overline{MS}} \quad (3.18)$$

Modulo nuclear isospin impurity effects (which are $< 1\%$ for ^4He and ^{12}C and should be corrected separately⁴²), in the limit $q^2 \rightarrow 0$, radiative corrections to that ratio have the same form as in the atomic parity violation discussion. They modify the r.h.s. of Eq. (3.18) to³²

$$-\frac{1}{2}(C_{1p} + C_{1n}) = \rho'_{PV} \kappa'_{PV}(0) \sin^2 \theta_W(m_Z)_{\overline{MS}} + 0.15 \frac{\alpha(m_Z)_{\overline{MS}}}{\pi} (1 - 4s^2) \quad (3.19)$$

where ρ'_{PV} and $\kappa'_{PV}(0)$ were previously given in Eqs. (3.5) & (3.12). Numerically, one finds the prediction

$$\frac{A_{pol}}{\left(-\frac{G_\mu}{\sqrt{2}\pi} \frac{q^2}{\alpha}\right)} \xrightarrow{q^2 \rightarrow 0} 0.2299 \pm 0.0003 \quad (0^+ \rightarrow 0^+) \quad (3.20)$$

which is applicable to ^4He and ^{12}C (after correcting for nuclear isospin impurities).

Of course, real scattering asymmetry measurements must be made at $q^2 \neq 0$. Varying q^2 will modify the asymmetry prediction in Eq. (3.20) due to changes in the radiative corrections and the presence of nuclear strange quark charge radii effects. (Nuclear isospin impurity effects will also exhibit some q^2 dependence. We continue to assume that they are treated separately or are measured experimentally under the phenomenological prescription we subsequently give.)

Collecting all q^2 dependent modifications of Eq. (3.20) into a correction factor $1 + R(q^2)$, one expects

$$\frac{A_{pol}}{\left(-\frac{G_\mu}{\sqrt{2}\pi} \frac{q^2}{\alpha}\right)} = 0.2299(1 + R(q^2)) \quad (3.21)$$

$$R(0) = 0$$

where $R(q^2)$ will exhibit some process dependence. For low q^2 , one expects contributions to $R(q^2)$ to be small. QED corrections from bremsstrahlung and low momentum photon exchange largely factorize and cancel in the asymmetry ratio. Residual effects can be computed and should not introduce an error larger than $\pm 0.5\%$ in $R(q^2)$. In the case of ρ'_{PV} and κ'_{PV} , they are slightly modified for $q^2 \neq 0$ primarily from changes in γZ mixing and γZ box diagrams. For low q^2 , those effects are small and tend to cancel.³² In the case of the BATES kinematics, $|q^2| \simeq 3 \times 10^4$

MeV², we expect $R(q^2)$ to be modified by about $0.001 \sim 0.002$ due to changes in ρ'_{p1} and κ'_{p1} .

More controversial is the effect of strange quarks on $R(q^2)$. For $q^2 \neq 0$, the strange radius of the nucleus can give rise to a shift in the asymmetry. For example, Musolf and Donnelly⁴³ found that strange quarks can give a q^2/m_N^2 contribution to $R(q^2)$ for small $|q^2|$ when Jaffe's estimate⁴¹ of the strange radius is employed. That corresponds to about a 3% reduction in the asymmetry for BATES' kinematics. Given the uncertainty in such a prediction (the strange radius has not been experimentally observed), it represents an obstacle for interpreting precision measurements of the asymmetry at the $\pm 1\%$ level. One could overcome that uncertainty somewhat by first measuring the strange radius of a nucleon. Such experiments are planned.⁴³ Alternatively, we propose to overcome this problem by a direct experimental approach to all $R(q^2)$ effects. If one can measure the polarization asymmetry with high experimental precision for several values of q^2 , then the $R(q^2)$ dependence should be ascertained and one can extrapolate to $q^2 = 0$ for comparison with the standard model prediction in Eq. (3.20). With such a procedure, it seems feasible that a confrontation between experiment and theory can be made at the $\pm 1\%$ level. We shall assume such a determination is possible in our later discussion of "new physics" sensitivity.

3.3 Neutrino-Electron Scattering

High statistics neutrino scattering experiments have been carried out for deep-inelastic $\bar{\nu}_\mu N$, elastic $\bar{\nu}_\mu p$, and $\bar{\nu}_\mu e$. Here, we discuss $\bar{\nu}_\mu e$ scattering, including effects of electroweak radiative corrections. The results are taken from Sarantakos, Sirlin, and Marciano.⁴⁴

Consider the differential cross-sections for $\nu_\mu e \rightarrow \nu_\mu e$ and $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$. Neglecting Z propagator effects, the lowest order cross-sections are given by⁴⁴

$$\begin{aligned} \frac{d\sigma(\nu_\mu e \rightarrow \nu_\mu e)}{dz} &= \frac{2G_\mu^2 p_1 \cdot p_2}{\pi} \left\{ \varepsilon_-^2 + \varepsilon_+^2 (1-z)^2 - \varepsilon_- \varepsilon_+ \frac{m_e^2}{p_1 \cdot p_2} z \right\} \\ \frac{d\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)}{dz} &= \frac{2G_\mu^2 p_1 \cdot p_2}{\pi} \left\{ \varepsilon_+^2 + \varepsilon_-^2 (1-z)^2 - \varepsilon_- \varepsilon_+ \frac{m_e^2}{p_1 \cdot p_2} z \right\} \\ \varepsilon_- &= \frac{1}{2}(1 - 2\sin^2 \theta_W(m_Z)_{\overline{MS}}) \\ \varepsilon_+ &= -\sin^2 \theta_W(m_Z)_{\overline{MS}} \end{aligned} \quad (3.22)$$

where p_1 and p_2 are the initial momenta of the neutrino and electron, and

$$z = \frac{-q^2}{2p_1 \cdot p_2} = \frac{E'_e - m_e}{E_\nu}, \quad 0 \leq z < \left(1 + \frac{m_e}{2E_\nu}\right)^{-1} \quad (3.23)$$

where E'_e and E_ν are the energies of the final state electron and initial state neutrino in the lab (electron rest) frame.

Consistent with our renormalization scheme, we have normalized the lowest order cross sections in terms of G_μ and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$. The $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$ cross-sections are obtained from Eq. (3.22) via the replacement

$$\varepsilon_- \rightarrow -\frac{1}{2}(1 + 2\sin^2 \theta_W(m_Z)_{\overline{MS}}) \quad (3.24)$$

where charged current amplitudes reduce ε_- by -1 due to destructive interference with the weak neutral current.

Incorporating electroweak radiative corrections is straightforward. Modulo QED correction, the electroweak loop corrections lead to the replacements

$$\begin{aligned} G_\mu &\rightarrow \rho_{NC}^{(\nu_\mu e)} G_\mu \\ \sin^2 \theta_W(m_Z)_{\overline{MS}} &\rightarrow \kappa^{(\nu_\mu e)}(q^2) \sin^2 \theta_W(m_Z)_{\overline{MS}} \end{aligned} \quad (3.25)$$

where⁴⁴

$$\begin{aligned} \rho_{NC}^{(\nu_\mu e)} &= 1 + \frac{\alpha(m_Z)_{\overline{MS}}}{2\pi} \left\{ \frac{3}{8s^2} \frac{m_t^2}{m_W^2} + \frac{3}{8s^4} \ell n c^2 - \frac{7}{8s^2} + \frac{3\xi}{8s^2} \left(\frac{\ell n(c^2/\xi)}{c^2 - \xi} + \frac{1}{c^2} \frac{\ell n \xi}{1 - \xi} \right) \right. \\ &\quad \left. + \frac{1}{c^2 s^2} \left(\frac{19}{8} - \frac{7}{2} s^2 + 3s^4 \right) \right\} \end{aligned} \quad (3.26a)$$

$$\begin{aligned} \kappa^{(\nu_\mu e)}(q^2) &= 1 - \frac{\alpha}{2\pi s^2} \left\{ 2 \sum_i (T_{3i} Q_i - 2s^2 Q_i^2) J_i(q^2) - \left(\frac{7}{2} c^2 + \frac{1}{12} \right) \ell n c^2 - 2J_\mu(q^2) \right. \\ &\quad \left. + \frac{1}{3} \ell n c^2 + \frac{1}{3} c^2 + \frac{1}{2} \right\} - \frac{\alpha(m_Z)_{\overline{MS}}}{2\pi s^2} \left\{ \frac{1}{c^2} \left(\frac{19}{8} - \frac{17}{4} s^2 + 3s^4 \right) \right\} \end{aligned} \quad (3.26b)$$

$$J_i(q^2) = \int_0^1 dx \, x(1-x) \ell n \left(\frac{m_i^2 - q^2 x(1-x)}{m_Z^2} \right)$$

Notice the similarity with Eq. (3.5). Many of the corrections are universal, having been introduced via G_μ normalization or from γZ mixing. The sum in Eq. (3.26b) is over all charged fermions (with a color factor of 3 for quarks). For low q^2 , quark loops must be evaluated using a dispersion relation in conjunction with $e^+e^- \rightarrow$ hadrons data. Alternatively, one can employ the effective quark masses given in section 3.1. Using Eq. (3.7) and the formulas given above with $m_t = 140$ GeV, $m_H = 200$ GeV, and $s^2 = 0.2325$, one finds at $q^2 = 0$

$$\begin{aligned} \rho_{NC}^{(\nu_\mu e)} &= 1.0104 \\ \kappa^{(\nu_\mu e)}(0) &= 0.9966 \pm 0.0025 \end{aligned} \quad (3.27)$$

where the uncertainty comes from hadronic vacuum polarization effects in γZ mixing.

For $q^2 \neq 0$, $\kappa^{(\nu_\mu e)}(q^2)$ is somewhat modified due to the q^2 dependence of the $J_i(q^2)$ terms in Eq. (3.26b). Those effects are, however, very small for two reasons. The electron vacuum polarization contribution to γZ mixing, which is the most q^2 dependent (at low q^2) is suppressed by $(1-4s^2)$. The light quark mass contributions (u, d, s) to γZ mixing are to a large extent canceled by the neutrino charge radius (the J_μ term) and that cancellation carries over to the q^2 dependence.⁴⁴ Therefore, the error in neglecting the q^2 dependence of $\kappa^{(\nu_\mu e)}(q^2)$ for moderate E_ν , 1–100 GeV, is insignificant compared to the error in Eq. (3.27). For very high energy neutrinos, the shift in κ can be obtained from the formulas in ref. 44. For most applications, however, employing $\kappa^{(\nu_\mu e)}(0)$ is perfectly adequate.

Turning to the QED corrections, we consider the extreme relativistic electron limit such that terms proportional to $m_e^2/p_1 \cdot p_2$ and m_e^2/q^2 can be dropped. Integrating over all bremsstrahlung, one finds that the scattering cross sections are shifted by⁴⁴

$$\begin{aligned} \frac{\Delta\sigma(\nu_\mu e \rightarrow \nu_\mu e(\gamma))}{\sigma(\nu_\mu e \rightarrow \nu_\mu e)} &= \frac{\alpha}{\pi} \left\{ \varepsilon_-^2 F_- + \frac{1}{3} \varepsilon_+^2 F_+ \right\} \\ \frac{\Delta\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e(\gamma))}{\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)} &= \frac{\alpha}{\pi} \left\{ \varepsilon_+^2 F_- + \frac{1}{3} \varepsilon_-^2 F_+ \right\} \end{aligned} \quad (3.28)$$

where

$$F_+ = F_- + 1 = -\frac{2}{3} \ell n \left(\frac{2p_1 \cdot p_2}{m_e^2} \right) - \frac{1}{6} \left(\pi^2 - \frac{43}{4} \right) \quad (3.29)$$

Corrections to the spectrum are given in ref. 44.

In total, we find that $\bar{\nu}e$ cross-sections are predicted up to about the $\pm 0.3\%$ level with the dominant error coming from hadronic vacuum polarization in γZ mixing. As in the case of atomic parity violation, those uncertainties could be reduced by better measurements of $e^+e^- \rightarrow$ hadrons. At present, however, that uncertainty is negligible compared to experimental errors.

To reduce systematic errors, high precision experiments actually measure ratios such as $R \equiv \sigma(\nu_\mu e)/\sigma(\bar{\nu}_\mu e)$ which is predicted (modulo very small q^2 and $p_1 \cdot p_2$ dependent $\mathcal{O}(\alpha)$ effects) to be

$$R = \frac{3 \left\{ 1 - 4\kappa^{(\nu_\mu e)}(0) \sin^2 \theta_W(m_Z) \overline{MS} + \frac{16}{3} (\kappa^{(\nu_\mu e)}(0) \sin^2 \theta_W(m_Z) \overline{MS})^2 \right\}}{\left\{ 1 - 4\kappa^{(\nu_\mu e)}(0) \sin^2 \theta_W(m_Z) \overline{MS} + 16 (\kappa^{(\nu_\mu e)}(0) \sin^2 \theta_W(m_Z) \overline{MS})^2 \right\}} \quad (3.30)$$

The quantity $R' \equiv \sigma(\nu_\mu e)/[\sigma(\nu_e e) + \sigma(\bar{\nu}_\mu e)]$ has also been discussed⁴⁵ as a vehicle for precision studies.

At present, the best measurement of R comes from the CHARM II collaboration.⁴⁶ Using their results, one finds

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2328 \pm 0.0092 \pm 0.0006 \quad (\text{CHARM II}) \quad (3.31)$$

where the first error is experimental and the second comes from the uncertainty in $\kappa^{(\nu_\mu e)}(0)$. The central value is very close to the world average of 0.2325 ± 0.0006 ; but, the experimental error is not competitive with more precise measurements of Z decay asymmetries which presently give $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ to about ± 0.0011 and ultimately may reduce the error to ± 0.0004 . Nevertheless, precision measurements of $\nu_\mu e$ and $\nu_e e$ scattering can be used to search for “new physics”. In the next section, we discuss the sensitivities of atomic parity violation, polarized electron-Carbon, and $\nu_e e$ scattering to Z' bosons and technicolor effects and show how improvements in those low energy experiments could probe “new physics” up to about the TeV scale.

4. New Physics Signatures

Precision electroweak measurements test the standard model at the level of its radiative corrections and probe for “new physics”. The “new physics” may arise at the tree level, as in the case of extra Z' bosons or be induced via quantum loop effects, e.g. technicolor scenarios. Whatever, the source, we already know that existing experiments allow at most from a few tenths of a percent to perhaps a few percent deviations from the standard model. Therefore, clear discovery of some “new physics” is unlikely to come simply from improved precision in a single experimental measurement. Instead, one will check various scenarios by global fits to all data. Also, should some new phenomenon be directly observed at high energies, low energy constraints will allow us to sort out its properties.

To illustrate the utility of low energy electroweak measurements, we consider here two possible scenarios: 1) The effect of Z' bosons on low energy neutral current phenomena and 2) Modifications of low energy observables due to heavy loop effects, e.g. technicolor models. Our discussion for both cases will be somewhat general; however, we will focus primarily on atomic parity violation, polarized e -C scattering, and $\nu_\mu e$ scattering, since they were extensively reviewed in section 3. For concreteness, we will assume a future set of experiments in those areas will lead to combined theoretical and experimental errors

$$\begin{aligned} \frac{\Delta Q_W}{Q_W}({}^{133}_{55}\text{Cs}) &= \pm 0.5\% \\ \Delta A(eC)_{pol}/A_{pol} &= \pm 1\% \\ \Delta R/R &= \pm 1\% \end{aligned} \quad (4.1)$$

Those hopes are to be compared with the existing best experiments which give $\Delta Q_W(Cs)/Q_W(Cs) = \pm 2.5\%$, $\Delta A/A = \pm 25\%$, and $\Delta R/R = \pm 7\%$. Achievement of those goals is most likely for atomic parity violation but the other two improvements also seem possible if dedicated efforts are made. The values in Eq. (4.1) were chosen because, as we shall see, they probe new physics at roughly the same level.

4.1 Z' Bosons

Extra neutral gauge bosons (generically called Z' bosons) can be easily appended to the standard model via additional $U'(1)$ gauge symmetries. They arise quite naturally in grand unified theories (GUTS) and some superstring models. For example, the $SO(10)$ model has one such additional boson which is denoted by Z_χ while E_6 has Z_χ as well as a second flavor diagonal neutral boson, Z_ψ .⁴⁷

The discovery of additional Z' bosons would have profound implications. Besides signaling new forces in nature, their specific properties could point to a particular GUT, Left-Right model, compositeness, etc.

If such bosons exist, they may eventually be produced and studied at high energy colliders. For example, the Fermilab tevatron $p\bar{p}$ collider CDF group has given the bounds (at 95% CL)¹⁵

$$\begin{aligned} m_{Z_\chi} &> 340 \text{ GeV} \\ m_{Z_\psi} &> 320 \text{ GeV} \end{aligned} \quad (4.2)$$

We anticipate those searches to continue up to about 500 GeV as the Fermilab luminosity and energy (1.8 TeV $\rightarrow$ 2.0 TeV) increase. At the SSC, the potential for finding Z' bosons should extend to the 5–10 TeV region depending on their specific properties.

To complement Z' production searches, it is useful to look for indirect hints of a Z' 's existence via low energy phenomenology. Here, we will give a general discussion and then focus on atomic parity violation, polarized e - C scattering and R as specific probes of Z' bosons.

We consider the appendage of N weak neutral gauge bosons $Z_i, i = 1, 2, \dots$ to the standard model. The fermionic sector of the electroweak neutral current interaction Lagrangian is given by⁴⁸

$$\mathcal{L}_{int} = -eA^\mu J_\mu^{em} - \frac{g_2}{\cos \theta_W} Z^\mu J_\mu^{NC} - \sqrt{\frac{3}{8}} g_2 \tan \theta_W \sum_i \sqrt{\lambda_i} Z_i^\mu J_\mu^i \quad (4.3)$$

where

$$J_\mu^{em} = \sum_f Q_f (\bar{f}_R \gamma_\mu f_R + \bar{f}_L \gamma_\mu f_L) \quad (4.4a)$$

$$J_\mu^{NC} = \sum_f (-\sin^2 \theta_W Q_f \bar{f}_R \gamma_\mu f_R + (T_{3f} - Q_f \sin^2 \theta_W) \bar{f}_L \gamma_\mu f_L) \quad (4.4b)$$

$$J_\mu^i = \sum_f (Q_{f_R}^i \bar{f}_R \gamma_\mu f_R + Q_{f_L}^i \bar{f}_L \gamma_\mu f_L) \quad (4.4c)$$

with Q_f the fermion electric charge ($Q_e = -1$), T_{3f} = weak isospin. The λ_i in Eq. (4.3) are arbitrary but the normalization is chosen such that $\lambda_i \simeq 1$ in many GUTS. In fact, for E_6 models

$$3Q_{e_R}^X = 3Q_{u_R}^X = -Q_{d_R}^X = +1$$

$$Q_{e_L}^X = Q_{\nu_{eL}}^X = -3Q_{u_L}^X = -3Q_{d_L}^X = +1 \quad (4.5a)$$

$$Q_{e_R}^\psi = Q_{u_R}^\psi = Q_{d_R}^\psi = -\sqrt{5/27}$$

$$Q_{e_L}^\psi = Q_{\nu_L}^\psi = Q_{u_L}^\psi = Q_{d_L}^\psi = \sqrt{5/27} \quad (4.5b)$$

In general one would expect all of the Z_i to mix with one another as well as the ordinary Z . However, we know from LEP physics that the Z couplings exhibit little if any mixing. Therefore, we do not consider ordinary Z mixing in this discussion. For E_6 -inspired models, the combinations

$$Z_\beta = Z_\chi \cos \beta + Z_\psi \sin \beta$$

$$Z'_\beta = -Z_\chi \sin \beta + Z_\psi \cos \beta$$

$$-\pi/2 \leq \beta \leq \pi/2 \quad (4.6)$$

will be taken as mass eigenstates with $m_{Z_\beta} < m_{Z'_\beta}$.

Given the above interactions, one can easily work out the modifications of weak neutral current amplitudes due to Z' bosons. Here we give the results for $Q_W(Cs)$, $A_{pol}(eC)$, and $\sigma(\nu_\mu e)$ in the case of E_6 inspired models with $\lambda_\chi = \lambda_\psi = 1$ and the couplings in Eq. (4.5). One finds⁴⁹

$$Q_W(^{133}_{55}\text{Cs}) = -73.24 \pm 0.14 + 65.41 \left\{ \frac{m_Z^2}{m_{Z_\beta}^2} \left(\cos^2 \beta + \sqrt{\frac{5}{3}} \sin \beta \cos \beta \right) \right.$$

$$\left. + \frac{m_Z^2}{m_{Z'_\beta}^2} \left(\sin^2 \beta - \sqrt{\frac{5}{3}} \sin \beta \cos \beta \right) \right\} \quad (4.7)$$

$$\frac{A_{pol}(eC)}{\left(\frac{-G_F}{\sqrt{2}\pi} \frac{q^2}{\alpha} \right)} = (0.2299 \pm 0.0003) \left(1 + R(q^2) - \frac{m_Z^2}{m_{Z_\beta}^2} \left(\cos^2 \beta + \sqrt{\frac{5}{3}} \sin \beta \cos \beta \right) \right.$$

$$\left. - \frac{m_Z^2}{m_{Z'_\beta}^2} \left(\sin^2 \beta - \sqrt{\frac{5}{3}} \sin \beta \cos \beta \right) \right) \quad (4.8)$$

while for $\nu_\mu e$ scattering one finds the ε_- and ε_+ in Eq. (3.22) are changed to⁵⁰

$$\begin{aligned}\varepsilon_- &= \frac{1}{2}(1 - 2s^2) - \frac{3}{4}s^2 \left[\frac{m_Z^2}{m_{Z_\beta}^2} \left(\cos \beta + \sqrt{\frac{5}{27}} \sin \beta \right)^2 + \frac{m_Z^2}{m_{Z'_\beta}^2} \left(-\sin \beta + \sqrt{\frac{5}{27}} \cos \beta \right)^2 \right] \\ \varepsilon_+ &= -s^2 - \frac{3}{4}s^2 \left[\frac{m_Z^2}{m_{Z_\beta}^2} \left(\cos \beta + \sqrt{\frac{5}{27}} \sin \beta \right) \left(\frac{1}{3} \cos \beta - \sqrt{\frac{5}{27}} \sin \beta \right) \right. \\ &\quad \left. + \frac{m_Z^2}{m_{Z'_\beta}^2} \left(\sin \beta - \sqrt{\frac{5}{27}} \cos \beta \right) \left(\frac{1}{3} \sin \beta + \sqrt{\frac{5}{27}} \cos \beta \right) \right] \quad (4.9)\end{aligned}$$

The first two quantities are most sensitive for $\beta = 0$, the SO(10) model where $Z_\beta = Z_\chi$; so, we focus on that scenario. In that case, the above expressions become (ignoring Z_ν) (including radiative corrections)

$$\begin{aligned}Q_W(^{133}_{55}\text{Cs}) &= -73.24 \pm 0.14 + 65.4 \frac{m_Z^2}{m_{Z_\chi}^2} \\ \frac{A_{pol}}{\left(\frac{-G_\mu}{\sqrt{2}\pi} \frac{q^2}{\alpha} \right)} &= (0.2299 \pm 0.0003) \left(1 + R(q^2) - \frac{m_Z^2}{m_{Z_\chi}^2} \right) \\ \varepsilon_- &= 0.2683 \left(1 - 0.65 \frac{m_Z^2}{m_{Z_\chi}^2} \right) \\ \varepsilon_+ &= -0.2317 \left(1 + 0.25 \frac{m_Z^2}{m_{Z_\chi}^2} \right) \quad (4.10)\end{aligned}$$

One sees that a $\pm 0.5\%$ measurement of $Q_W(Cs)$ probes M_{Z_χ} at the 1.2 TeV level. Indeed, if we were to attribute the central values spread in Eqs. (3.14) and (3.15), i.e. $\Delta Q_W = 2.2$ to a Z_χ , it would correspond to $m_\chi \simeq 500$ GeV. Amazingly, such a massive Z_χ would not have been detected in any other precision measurement or collider experiment (see Eq. (4.2)). A new measurement of $Q_W(Cs)$ to ± 0.35 would see a 500 GeV Z_χ at the 6σ level. Also, for other cesium isotopes, one finds

$$Q_W(^{133+X}_{55}\text{Cs}) = -73.24 - 0.986X + (65.4 + 0.62X) \frac{m_Z^2}{m_{Z_\chi}^2} \quad (4.11)$$

Measuring Q_W for chains of Cs isotopes and other atoms could reveal the couplings of an extra Z' .

To probe the $m_{Z_\chi} \simeq 1$ TeV level in polarized eC scattering requires a measurement of A_{pol} to $\pm 0.8\%$ which is hard but not impossible. At issue is the control of $R(q^2)$ in Eq. (4.10) at that level. If a deviation from the standard model is found in

atomic parity violation, low energy polarized electron scattering experiments may provide our best means of confirmation.

In the case of $\bar{\nu}_\mu e$ scattering, one finds from Eq. (4.10) that a 1 TeV Z_χ can be observed (at the 1σ) level) by a $\pm 0.8\%$ measurement of $\sigma(\nu_\mu e)$ or a similar measurement of R (since $\sigma(\bar{\nu}_\mu e)$ is rather insensitive to a Z_χ). Alternatively, $R' \equiv \sigma(\nu_\mu e)/[\sigma(\nu_e e) + \sigma(\bar{\nu}_\mu e)]$ is somewhat more sensitive to Z_χ

$$R' = R'_{\text{standard model}} \left(1 - 1.3 \frac{m_Z^2}{m_{Z_\chi}^2} \right) \quad (4.12)$$

A 1% measurement of R' would probe the TeV scale.

4.2 S , T , and U

A nice formalism for studying heavy physics effects on gauge boson self-energies was introduced by Peskin and Takeuchi,⁵ the S , T and U parametrization. Here, we describe the implementation of that formalism and current experimental constraints on those parameters.

Heavy new fermions, such as technifermions, a heavy fourth generation, etc. enter low-energy phenomenology through gauge boson self-energies. In the electroweak sector those include the vacuum polarization functions $\Pi_{\gamma\gamma}(q^2)$, $\Pi_{\gamma Z}(q^2)$, $\Pi_{ZZ}(q^2)$, and $\Pi_{WW}(q^2)$. Using $\overline{MS}$ subtraction removes heavy particle contributions to the first two by absorbing such effects into the definitions $\alpha(m_Z)_{\overline{MS}}$ and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$. There are, however, residual effects in Π_{ZZ} and Π_{WW} that remain after renormalization and are observable as corrections to the natural relationships in Eq. (1.4).

The heavy new fermion loop effects can be parametrized by three observables S_W , S_Z and T defined by²¹

$$\left. \frac{\Pi_{WW}^{new}(m_W^2) - \Pi_{WW}^{new}(0)}{m_W^2} \right|_{\overline{MS}} = \frac{\alpha(m_Z)_{\overline{MS}}}{4\sin^2 \theta_W(m_Z)_{\overline{MS}}} S_W = Z_W^{new} - 1 \quad (4.13)$$

$$\left. \frac{\Pi_{ZZ}^{new}(m_Z^2) - \Pi_{ZZ}^{new}(0)}{m_Z^2} \right|_{\overline{MS}} = \frac{\alpha(m_Z)_{\overline{MS}}}{\sin^2 2\theta_W(m_Z)_{\overline{MS}}} S_Z = Z_Z^{new} - 1 \quad (4.14)$$

$$\frac{\Pi_{WW}^{new}(0)}{m_W^2} - \frac{\Pi_{ZZ}^{new}(0)}{m_Z^2} = \alpha(m_Z)_{\overline{MS}} T = \rho(0)^{new} - 1 \quad (4.15)$$

where “new” means only heavy new particle loops are included. $|_{\overline{MS}}$ means a modified minimal subtraction is applied, and $\alpha(m_Z)_{\overline{MS}}$ has been factored out. In terms of the Peskin-Takeuchi parameters

$$S = S_Z$$

$$\begin{aligned}
T &= T \\
U &= S_W - S_Z
\end{aligned}
\tag{4.16}$$

The T and U correspond to isospin violating effects, while S is isospin conserving.

Even without new physics, S_W , S_Z , and T provide a convenient means for approximating deviations from our assumed $m_t = 140$ GeV and $m_H = 200$ GeV values. Deviations give rise to^{22,51}

$$\begin{aligned}
S_W &\simeq \frac{1}{6\pi} \ln \left(\frac{m_H}{200 \text{ GeV}} \right) + \frac{2}{3\pi} \ln \left(\frac{m_t}{140 \text{ GeV}} \right) \\
S_Z &\simeq \frac{1}{6\pi} \ln \left(\frac{m_H}{200 \text{ GeV}} \right) - \frac{1}{3\pi} \ln \left(\frac{m_t}{140 \text{ GeV}} \right) \\
T &\simeq \frac{3}{16\pi s^2} \left(\frac{m_t^2 - (140 \text{ GeV})^2}{m_W^2} \right) - \frac{3}{8\pi c^2} \ln \left(\frac{m_H}{200 \text{ GeV}} \right)
\end{aligned}
\tag{4.17}$$

Note that $U = \frac{1}{\pi} \ln(m_t/140 \text{ GeV})$ is much smaller than T and can be generally ignored.

S_W , S_Z , and T are measured in the following way. Employing α , G_μ , m_Z , $m_t = 140$ GeV, $m_H = 200$ GeV (along with light fermion masses and mixing) completely specifies the standard model, modulo “new physics” or deviations in m_t or m_H . If such effects are present, they lead to shifts in the radiative corrections previously discussed (see section 2)²¹

$$\begin{aligned}
\Delta r &= 0.0469 \pm 0.0011 - 0.0195 S_W + 0.0361 S_Z - 0.0258 T \\
\Delta r(m_Z)_{\overline{MS}} &= 0.0702 \pm 0.0011 + 0.0084 S_W \\
\Delta \hat{r} &= 0.0633 \pm 0.0011 + 0.011 S_Z - 0.00782 T
\end{aligned}
\tag{4.18}$$

as well as a change in the ρ parameters of section 3

$$\rho^{\text{new}}(0) = 1 + 0.00782 T \tag{4.19}$$

used in all weak neutral current amplitudes.

From the changes given above, one finds the new predictions

$$\sin^2 \theta_W(m_Z)_{\overline{MS}} = 0.2325 \pm 0.0004 + 0.00365 S_Z - 0.00261 T \tag{4.20a}$$

$$m_W = 80.18 \pm 0.02 + 0.45 T - 0.63 S_Z + 0.34 S_W \text{ GeV} \tag{4.20b}$$

$$Q_W(^{133}_{55}\text{Cs}) = -73.24 \pm 0.14 - 0.8 S_Z - 0.005 T \tag{4.20c}$$

$$\frac{A_{pol}(eC)}{\left(\frac{-G_\mu}{\sqrt{2}\pi} q^2/\alpha\right)} = 0.2299(1 + R(q^2) + 0.016S_Z - 0.003T) \quad (4.20d)$$

$$R = \frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)} = 1 - 0.029S_Z + 0.021T \quad (4.20e)$$

$$\Gamma_Z/\Gamma_Z^{\text{Standard Model}} = 1 - 0.0038S_Z + 0.0105T \quad (4.20f)$$

$$\Gamma(Z \rightarrow e^+e^-)/\Gamma_{e^+e^-}^{S.M.} = 1 - 0.0021S_Z + 0.0093T \quad (4.20g)$$

From those expressions, we see that most precision experiments are sensitive to a linear combination of S_Z and T . The exception is m_W which depends on both S_W and S_Z . In fact, one can isolate S_W by comparing m_W and $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ (measured via a Z decay asymmetry)²¹

$$S_W = 118 \left\{ 2 \left(\frac{m_W - 80.18 \text{ GeV}}{80.18 \text{ GeV}} \right) + \frac{\sin^2 \theta_W(m_Z)_{\overline{MS}} - 0.2325}{0.2325} \right\} \quad (4.21)$$

In addition, one notes that $Q_W(Cs)$ and $A_{pol}(eC)$ are rather insensitive to T and thus provide direct measurements of S_Z .²¹

By comparing experiments with the theoretical predictions, including its S & T dependence, one obtains constraints on those parameters. Various existing constraints along with anticipated future advances are illustrated in table 3. Taking $S_W = S_Z = S$, one finds²

$$S = -0.05 \pm 0.26 + 1.15T \quad (4.22)$$

The data is very consistent with S & $T \simeq 0$, but could accommodate an $S \simeq 1$ if T is also $\mathcal{O}(1)$.

Why is S interesting? Each new heavy chiral $SU(2)_L$ doublet contributes $1/6\pi$ to S ,⁵² so, a signal for heavy fermions could be $S > 0$. Technicolor models have many heavy doublets. Indeed, one expects (including enhancements)

$$S \simeq (0.05 - 0.10)N_T N_D + 0.12 \quad (\text{technicolor}) \quad (4.23)$$

where N_T is the number of technicolors and N_D is the number of flavor doublets. (The $+0.12$ comes from a heavy effective Higgs in such a scenario.) For a generic one generation model with $N_D = 4$ and $N_T = 4$, one expects⁵ $S \simeq 0.9 \sim 1.7$. So far, there is no evidence for $S > 0$, but $S \sim 1$ is not completely ruled out.

In the future, individual experiments are expected to reach a sensitivity of $0.1 \sim 0.5$ for S and T . At that level, technicolor should unveil itself if it is real. We note that supersymmetric theories do not introduce additional chiral fermions and thus naturally give very small S and T . So, as long as their values remain close to zero, it bodes well for supersymmetry enthusiasts.

Table 3 Present experimental constraints on S_W , S_Z , and T along with anticipated future sensitivities. Central values of $m_t = 140$ GeV and $m_H = 200$ GeV are assumed.

Experiment	Current Constraint	Future
$m_W = 80.22 \pm 0.26$ GeV	$T - 1.4S_Z + 0.76S_W = 0.09 \pm 0.60$	± 0.09
LEP Asymmetries	$S_Z - 0.69T = 0.03 \pm 0.32$	± 0.15
$(\sin^2 \theta_W(m_Z))_{\overline{MS}} = 0.2326 \pm 0.0011$		
A_{LR} (SLD)	$S_Z - 0.69T = 1.33 \pm 1.5$	± 0.10
$Q_W(^{133}\text{Cs}) = -71.04 \pm 1.81$	$S_Z - 0.005T = -2.7 \pm 2.0 \pm 1.1$	± 0.5
$\Gamma_Z = 2492 \pm 7$ MeV	$T - 0.36S_Z = 0.27 \pm 0.27$	± 0.27
$\Gamma_{t+l^-} = 83.33 \pm 0.29$ MeV	$T - 0.23S_Z = -0.35 \pm 0.37$	± 0.37
$R = \frac{\sigma(\nu_\mu e)}{\sigma(\bar{\nu}_\mu e)}$	$S_Z - 0.69T = 0.08 \pm 2.5$	?
$R_\nu \equiv \sigma(\nu_\mu N)_{NC}/\sigma(\nu_\mu N)_{CC}$	$T - 0.4S_Z = -0.22 \pm 0.67$	± 0.24
$R_{\bar{\nu}} \equiv \sigma(\bar{\nu}_\mu N)_{NC}/\sigma(\bar{\nu}_\mu N)_{CC}$	$T - 0.028S_Z = 1.4 \pm 1.3$	± 0.65

5. Outlook

The discovery of weak neutral currents marked a tremendous success for the $SU(2)_L \times U(1)$ model. Now, that theory has been tested at the level of its quantum corrections via 1% (or better) measurements. So far, no hints of “new physics” have emerged, but m_t has been constrained to 143 ± 26 GeV. Confronting that prediction with a direct determination of m_t , after its discovery, will be very exciting. An intriguing outcome would be a very heavy m_t , well beyond the loop constraint. Indeed, the heavier top turns out to be, the more interesting it becomes.

After m_t is known, precision measurements may provide some constraint on m_H . At least the light, $m_H \simeq 100$ GeV, and heavy $m_H \simeq 800$ GeV, cases may be differentiated. Confirming the Higgs’ existence, will require direct discovery at high energies and exploration of its properties. That occurrence must wait for the next generation of colliders.

To the dismay of technicolor advocates, there is no evidence for $S > 0$; however, $S \sim \mathcal{O}(1)$ is not ruled out. The next generation of precision measurements will probe S down to about ± 0.2 . At that level, technicolor or some other heavy fermion effect might well emerge. In addition, we saw that Cs atomic parity violation is already sensitive to $m_{Z_\chi} \simeq 500$ GeV and is capable of reaching to ~ 1 TeV. Similar sensitivities are possible with polarized eC and $\nu_\mu e$ scattering experiments; but reaching that level will not be easy. Nevertheless, such pursuits are worth the effort. Those low energy measurements complement precision determinations of m_W , $\sin^2 \theta_W(m_Z)_{\overline{MS}}$ etc., searches for rare or forbidden processes, and high energy collider explorations. Collectively, they test the standard model and provide

guidance for our scientific imaginations, as we strive to decipher and appreciate the laws of Nature.

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