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Universal Parametrization for Quark and Lepton Substructure

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Abstract

A universal parametrization for possible quark and lepton substructure is advocated in terms of quark and lepton form factors. It is emphasized that the lower bounds on compositeness scale, Λ_c , to be determined experimentally strongly depend on their definitions in composite models. From the recent HERA data, it is estimated to be $\Lambda_c > 50\text{GeV}$, 0.4TeV and 10TeV , depending on the parametrizations with a single-pole form factor, a contact interaction and a logarithmic form factor, respectively.

After the detailed theoretical investigations of composite models of quarks and leptons for the last two decades [1], the possible substructure of quarks and leptons has recently become a serious subject for experimental searches. In early 1970's, several authors suggested that deviations from Bjorken-scaling behavior for deep inelastic ep scattering should appear due to the substructure of constituents, the quarks [2]. Chanowitz and Drell [3] further proposed to analyze not only the electromagnetic form factors and structure functions of the proton but also the cross sections for $e^+ + e^- \rightarrow \text{hadrons}$ and $p + p \rightarrow l^+ + l^- + \text{hadrons}$ in terms of the form factors of quarks,

$$\mathcal{F}_q(q^2) \equiv (1 - q^2/M^2)^{-1} \cong 1 + q^2/M^2 \quad \text{for} \quad |q^2|/M^2 \ll 1 \quad (1)$$

where q^2 is the momentum transfer squared and M is a constant parameter for the new mass scale. Indeed, immediately after their suggestion, a clear breakdown of Bjorken scaling was observed in deep inelastic μp scattering. However, it has been successfully explained not as an effect of the substructure of quarks but as an effect of the asymptotically free behavior of quantum chromodynamics (QCD). In 1983, Eichten, Lane and Peskin [4] suggested that the unknown strong binding forces induce, by constituent exchange, new flavor-diagonal helicity-conserving contact interactions of composite quarks and leptons (ψ) of the form,

$$\mathcal{L}_{\psi\psi} = \frac{g^2}{2\Lambda^2} \left[\eta_{LL} \bar{\psi}_L \gamma_\mu \psi_L \bar{\psi}_L \gamma^\mu \psi_L + \eta_{RR} \bar{\psi}_R \gamma_\mu \psi_R \bar{\psi}_R \gamma^\mu \psi_R + 2\eta_{RL} \bar{\psi}_R \gamma_\mu \psi_R \bar{\psi}_L \gamma^\mu \psi_L \right] \quad (2)$$

where they define the compositeness energy scale Λ such that $g^2/4\pi = 1$ and the largest $|\eta_{ij}| = 1$ ($i, j = L, R$). They further argued that this effect overwhelms the contribution of the form factors of the constituents since $\mathcal{L}_{\psi\psi}$ produces interference terms in the cross section of order $(4\pi\alpha_\psi/q^2)^{-1}(g^2/\Lambda^2) = q^2/\alpha_\psi\Lambda^2$ relative to the standard-model contribution, where α_ψ is a gauge coupling constant in the standard model of $SU(3) \times SU(2) \times U(1)$. After their suggestion, most of theoretical predictions and experimental analyses have been made in terms of their parametrizations [5, 6].

The purpose of this letter is to point out that the lower bounds on the compositeness scale to be determined experimentally strongly depend on their definitions in the composite models. From the recent HERA ep scattering data [7], the lower bounds are estimated to be $M > 50\text{GeV}$, $\Lambda > 0.4\text{TeV}$, and $\Lambda_c > 10\text{TeV}$, depending on the parametrizations with a single-pole form factor, a contact interaction and a logarithmic form factor, respectively, where Λ_c will be defined in the explanation of the “logarithmic form factor” below.

Our universal way to parametrize the possible effects of quark and lepton substructure is the following: Let $d\sigma(ab \rightarrow c)_0$ be the theoretical cross section for a certain scattering process of $a + b \rightarrow c$ calculated in the standard model of $SU(3) \times SU(2) \times U(1)$ without the substructure of quarks and leptons but with all the higher-order QCD and other radiative corrections, if any. Define $d\sigma(ab \rightarrow c)$ as the corrected theoretical differential cross section by inserting the quark or lepton form factor, $\mathcal{F}_q(q^2)$ or $\mathcal{F}_l(q^2)$ ($q = u, d, c, s, t, b$ and $l = e, \mu, \tau$) in every vertex of the gauge boson (γ or Z) and quarks or leptons. Furthermore, assume an approximate universality of quark form factors, i. e. $\mathcal{F}_u(q^2) \cong \mathcal{F}_d(q^2) \cong \dots$. In general, there may exist two or more form factors for a vertex [8]. Also assume an approximate universality for them, if necessary. Then, the corrected theoretical differential cross sections for almost all the experimentally relevant scattering processes are factorized by the quark and lepton form factors as

$$d\sigma(e^+e^- \rightarrow l^+l^-) = |\mathcal{F}_e(s)\mathcal{F}_l(s)|^2 d\sigma(e^+e^- \rightarrow l^+l^-)_0 \quad \text{for } l = \mu, \tau, \quad (3)$$

$$d\sigma(e^+e^- \rightarrow \text{hadrons}) = |\mathcal{F}_e(s)\mathcal{F}_q(s)|^2 d\sigma(e^+e^- \rightarrow \text{hadrons})_0, \quad (4)$$

$$d\sigma(ep \rightarrow ep) = |\mathcal{F}_e(q^2)\mathcal{F}_q(q^2)|^2 d\sigma(ep \rightarrow ep)_0, \quad (5)$$

$$d\sigma(ep \rightarrow eX) = |\mathcal{F}_e(q^2)\mathcal{F}_q(q^2)|^2 d\sigma(ep \rightarrow eX)_0, \quad (6)$$

$$d\sigma(p\bar{p} \rightarrow l^+l^-X) = |\mathcal{F}_l(M_{l^+l^-}^2)\mathcal{F}_q(M_{l^+l^-}^2)|^2 d\sigma(p\bar{p} \rightarrow l^+l^-X)_0 \quad \text{for } l = e, \mu, \tau \quad (7)$$

where s is the total c. m. energy squared and $M_{l^+l^-}^2$ is the invariant mass squared of l^+ and l^- . For some limited kinematical regions, a similar factorization also holds

approximately for two other experimentally relevant processes as

$$d\sigma(e^+e^- \rightarrow e^+e^-) \cong |\mathcal{F}_e(q^2)|^4 d\sigma(e^+e^- \rightarrow e^+e^-)_0 \quad \text{for } |q^2| \ll s, \quad (8)$$

$$d\sigma(p\bar{p} \rightarrow jjX) \cong |\mathcal{F}_q(\hat{t})|^4 d\sigma(p\bar{p} \rightarrow jjX)_0 \quad \text{for } |\hat{t}| \ll \hat{s} \quad (9)$$

where j denotes a jet, and $\hat{s}$ and $\hat{t}$ are, respectively, the total c. m. energy squared and momentum transfer squared of the hard scattering parton quarks decaying into jets. In some models, the form factors may not be so universal and the cross sections may not strictly be factorized. Even in such cases, however, the expressions (3)–(9) are still useful because, as far as $|q^2| \ll M^2$, the cross sections take the forms of (3)–(9) with the effective form factors which are weighted combinations of each form factor. Note also that, as far as $|q^2| \ll \Lambda^2$, the effects of the contact interaction parametrization [4] are equivalent to those of the form factor,

$$\mathcal{F}_q(q^2) = 1 - \frac{q^2}{2\alpha_q\Lambda^2}. \quad (10)$$

Therefore, by comparing the experimental data with the corrected theoretical differential cross sections (3)–(9), we can extract valuable information on the possible effects of quark and lepton substructure as deviations of the quark and lepton form factors from unity.

Our parametrization of quark and lepton substructure is similar but different from that of Chanowitz and Drell [3] in the following two ways: Not only the possible substructure of quarks but also that of leptons is taken into account, and the strong interaction effect of gluons is not taken as a substructure effect but as a radiative correction to be included in the zeroth-order theoretical differential cross section, $d\sigma(ab \rightarrow c)_0$. On the other hand, the application of the Eichten, Lane and Peskin parametrization [4] is sometimes limited as follows: For example, consider the minimal composite model [1], which consists of an iso-doublet of subquarks, w_1 and w_2 (the charges $+1/2$ and $-1/2$), and a color-quartet of subquarks, C_0 and C_i ($i=1,2,3$; the charges $-1/2$ and $+1/6$). The electron is a composite of w_2 and C_0

and the $u_i(d_i)$ quark is a composite of w_i (w_j) and C_i . Then, although e and d_i may have a contact interaction due to an exchange of the w_j subquark, e and u_i have no such interaction since they have no common subquark. Therefore, processes such as $e p$ scattering cannot be parametrized universally in the original spirit of Eichten, Lane and Peskin. Even if a process may have a contact interaction, in some cases the contact interaction should not be taken as a new interaction to be added to the familiar interactions in the standard model, but it should be absorbed into one of the latter interactions as a possible effect of the quark and lepton form factors. This is particularly the case if not only quarks and leptons but also the gauge bosons are taken as composites as in our unified composite model [9]. Note that their parametrization can be taken effectively as a special case of our parametrizations where the quark or lepton form factor is approximated as (10) for $|q^2| \ll \Lambda_c^2$. The apparent large discrepancy between the experimental bound on M in the single-pole parametrization (1) and that on Λ in the contact term parametrization (2) stems from the extra factor α_y^{-1} in (10). Note also that for $q^2 < 0$ the parametrization (2) definitely produces a constructive (or destructive if negative sign in front of q^2) interference with the standard-model contribution while ours may produce either a constructive interference or a destructive one, depending on the q^2 behavior of the quark or lepton form factor.

For $|q^2| \ll \Lambda_c^2$ ($\Lambda_c = M$ or Λ), both of the parametrizations depend linearly on q^2/Λ_c^2 . In general, however, the logarithmic term of $\ln(|q^2|/m^2)/\ln(\Lambda_c^2/m^2)$, if any, dominates over the linear terms, where m is some small mass scale in the theory. In fact, such a logarithmic term is expected in the models where the subconstituents are light and confined. For definiteness, suppose that the quarks and leptons are made of a fermionic subquark w and a bosonic subquark C . Then, the interaction of a composite quark (or lepton) with a gauge boson is given by the subquark loop diagrams illustrated in Fig. 1, where the blob indicates a vertex function of the composite quark (or lepton) and subquarks. Although we do not know the precise form

of the vertex function, we expect that it should decrease rapidly above the compositeness scale Λ_c . Thus, we approximate it by inserting a constant vertex to the blobs and by cutting off the momenta at Λ_c . In practice, we use the gauge invariant Pauli-Villars regulators. The form factor $\mathcal{F}(q^2)$ is extracted by estimating the q^2 dependence on the mass shell of the composite quark (or lepton) and normalizing it at $q^2 = 0$, where q is the momentum of the gauge boson. This procedure to estimate the form factor can be demonstrated in a dynamical model by the following steps:

- (i) starting with the Nambu-Jona-Lasinio type Lagrangian for the subquarks with the interaction term of the form $(\bar{w}C^\dagger)(Cw)$ [10],
- (ii) writing down the equivalent Lagrangian with the auxiliary field for the composite quark or lepton, and
- (iii) following the ordinary way of perturbative calculation.

After a lengthy calculation, we obtain

$$Q_f \mathcal{F}(q^2) = Q_w \mathcal{F}_w(q^2) + Q_C \mathcal{F}_C(q^2) \quad (11)$$

with

$$\mathcal{F}_w(q^2) = N \int_0^1 dx \int_0^{1-x} dy \left[\ln(\Lambda_c^2/L_w) + K_w/L_w - 1 \right] \quad (12)$$

$$\mathcal{F}_C(q^2) = N \int_0^1 dx \int_0^{1-x} dy \left[\ln(\Lambda_c^2/L_C) + K_C/L_C \right] \quad (13)$$

$$K_w = (xm_f + m_w)^2 + y(1-x-y)q^2 \quad (14)$$

$$K_C = 2x[(1-x)m_f + m_w]m_f \quad (15)$$

$$L_w = xm_w^2 + (1-x)m_w^2 - x(1-x)m_f^2 - y(1-x-y)q^2 \quad (16)$$

$$L_C = xm_w^2 + (1-x)m_w^2 - x(1-x)m_f^2 - y(1-x-y)q^2 \quad (17)$$

for $m_w^2, m_C^2, m_f^2, |q^2| \ll \Lambda_c^2$, where m_w, m_C and m_f are the masses of the subquarks w, C and the quark (or lepton) f , respectively; Q_w, Q_C and $Q_f (\equiv Q_w + Q_C)$ are the charges (with respect to the relevant gauge symmetry) of w, C and f , respectively, and N is the normalization factor such that $\mathcal{F}_w(0) = \mathcal{F}_C(0) = 1$. Note that N is common to $\mathcal{F}_w(q^2)$ and $\mathcal{F}_C(q^2)$ due to charge conservation. In the following, we

put $m_w = m_C \equiv m$ and assume $m_f^2 \ll m^2, |q^2|$, for simplicity. In the region of $|q^2| \ll m^2$, the form factor behaves linearly in q^2 as

$$\mathcal{F}(q^2) = 1 + \frac{(3Q_w + Q_C)q^2}{12Q_fm^2 \ln(\Lambda_c^2/m^2)}. \quad (18)$$

On the other hand, if $m^2 \ll |q^2|$, the logarithmic behavior of

$$\mathcal{F}(q^2) = \frac{\ln(\Lambda_c^2/|q^2|)}{\ln(\Lambda_c^2/m^2)} = 1 - \frac{\ln(|q^2|/m^2)}{\ln(\Lambda_c^2/m^2)} \quad (19)$$

dominates over the linear one. In this case, the assumed approximate universality of $\mathcal{F}_u(q^2) \cong \mathcal{F}_d(q^2) \cong \dots$ holds so that the factorizations in Eqs. (3)–(9) may be realized. The logarithmic behavior implies that we can probe very high compositeness scale by relatively low-energy experiments. The expressions (12)–(17) indicate that the logarithmic behavior exists only if all the masses of the composite and the constituents are much smaller than $\sqrt{|q^2|}$ and Λ_c . In this case, the same diagram causes a non-negligible anomalous magnetic moment. Therefore, we can apply it only to the form factors of quarks but not to those of leptons, whose anomalous moments are known to be very small. Note also that we do not expect such logarithmic behavior for form factors of hadrons, nuclei, atoms and molecules because the composites are as heavy as or heavier than its compositeness scale.

Now we apply the parametrizations to the recent HERA ep scattering data [7] for a illustration. Here we do not take into account the effects of Z boson exchange since neither do the authors of Ref. [7]. For simplicity, we take into account only the effect of quark substructure by neglecting that of lepton substructure. Then, the factorization in Eq. (6) effectively indicates that the structure function $F_2(x, q^2)$ is factorized as

$$F_2(x, q^2) = [\mathcal{F}_q(q^2)]^2 F_2(x, q^2)_0 \quad (20)$$

where x is the Bjorken scaling variable and $F_2(x, q^2)_0$ is the standard model prediction for the structure function. Let us first use the MRS D'_- prediction [11] for

$F_2(x, q^2)_0$ and evaluate the lower bounds on the parameters M , Λ and Λ_c in the parametrizations with the single-pole form factor (1), the contact interaction (2) and the logarithmic form factor (19), respectively. Then, a detailed χ^2 analysis results in $M > 40 \text{ GeV}$, $\Lambda > 0.7 \text{ TeV}$ and $\Lambda_c > 1100 \text{ TeV}$ for $m = 1 \text{ GeV}$ (95% C.L.). In Fig. 2, the predictions for the form factors with the compositeness scale at these lower bounds are plotted against $|q^2|$ together with the experimental values derived from the HERA data.

The analysis in the last paragraph is somewhat too crude because the QCD prediction for $F_1(x, q^2)_0$ involves the following ambiguity: QCD renders a definite prediction for the q^2 dependence of $F_2(x, q^2)_0$, but not for its x -dependence. To eliminate the ambiguity, we logarithmically differentiate (20) as

$$q^2 \frac{\partial}{\partial q^2} \log F_2(x, q^2) = 2q^2 \frac{\partial}{\partial q^2} \log \mathcal{F}_q(q^2) + q^2 \frac{\partial}{\partial q^2} \log F_2(x, q^2)_0. \quad (21)$$

The QCD prediction for the second term in the right hand side of (21) is less ambiguous. Furthermore, for the logarithmic form factor (19), the first term in the right hand side of (21) becomes $2/\ln(|q^2|/\Lambda_c^2)$, which is independent of the subquark mass m . We determine the left hand side of (21) from the experimental data and evaluate the lower bounds on the parameters M , Λ and Λ_c by a detailed χ^2 analysis. We obtain the following similar but somewhat different values: $M > 50 \text{ GeV}$, $\Lambda > 0.4 \text{ TeV}$ and $\Lambda_c > 10 \text{ TeV}$ (95% C.L.). In Fig. 3, the predictions on $q^2(\partial/\partial q^2) \log F_2(x, q^2)$ of the logarithmic form factor (19) are plotted against x together with the experimental values derived from the HERA data. We thus conclude that the lower bounds strongly depend on the definitions of the compositeness scale. We hope that the universal parametrization and the systematic analysis which we have discussed in this letter will be applied to the future experiments to search for substructure of quarks and leptons.

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Figure Captions

1. The subquark diagrams which give rise to the quark or lepton form factor. w and C denote the subquarks, and q , l , γ and Z denote the quarks, lepton, photon and Z -boson, respectively.
2. The q^2 -dependence of the form factors. The solid, dashed and dotted curves show, respectively, the single-pole form factor with $M = 40\text{GeV}$, the contact-interaction equivalent one with $\Lambda = 0.7\text{TeV}$ and the logarithmic one with $\Lambda_c = 1100\text{TeV}$ and $m = 1\text{GeV}$. The points with error bars are the experimental values derived from the HERA data.
3. The x -dependence of $q^2(\partial/\partial q^2)\log F_2(x, q^2)$. The solid line indicates the QCD prediction in the range of q^2 of the HERA data. The dotted, dashed and long-dashed lines show the predictions by the logarithmic form factor with QCD corrections for $\Lambda_c = 1\text{TeV}$, 10TeV and 100TeV , respectively. The points with error bars are the experimental values derived from the HERA data.

Fig. 1

Fig. 2

Fig. 3