Electroweak Symmetry Breaking and Bottom-Top Yukawa Unification

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Abstract

The condition of unification of gauge couplings in the minimal supersymmetric standard model provides successful predictions for the weak mixing angle as a function of the strong gauge coupling and the supersymmetric threshold scale. In addition, in some scenarios, e.g. in the minimal SO(10) model, the tau lepton and the bottom and top quark Yukawa couplings unify at the grand unification scale. The condition of Yukawa unification leads naturally to large values of $\tan \beta$, implying a proper top quark-bottom quark mass hierarchy. In this work, we investigate the feasibility of unification of the Yukawa couplings, in the framework of the minimal supersymmetric standard model with (assumed) universal mass parameters at the unification scale and with radiative breaking of the electroweak symmetry. We show that strong correlations between the parameters $\mu_3$, $M_{1/2}$ and $\delta = B_0 - (6r/7)A_0$ appear within this scheme, where $r$ is the ratio of the top quark Yukawa coupling to its infrared fixed point value. These correlations have relevant implications for the sparticle spectrum, which presents several characteristic features. In addition, we show that due to large corrections to the running bottom quark mass induced through the supersymmetry breaking sector of the theory, the predicted top quark
mass and $\tan\beta$ values are significantly lower than those previously estimated in the literature.

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1 Introduction

The minimal supersymmetric standard model provides a well motivated and predictive extension of the successful standard model of the strong and electroweak interactions. The condition of unification of couplings is implicit within this scheme and the predictions for the weak mixing angle are in good agreement with the values measured by the most recent measurements at LEP [1]–[3]. In addition to the gauge coupling unification condition, relations between the values of the Yukawa couplings of the quarks and leptons of the third generation appear in the minimal supersymmetric grand unification scheme. In particular, in the minimal SU(5) model the unification of bottom and tau Yukawa couplings is obtained. The bottom–tau Yukawa unification condition leads to predictions for the top quark mass as a function of the running bottom quark mass, the strong gauge coupling value and the value of \( \tan \beta \), the ratio of vacuum expectation values [4]–[6].

Recently, it has been observed that for the phenomenologically allowed values of the bottom quark mass and moderate values of \( \tan \beta < 10 \), large values of the top quark Yukawa coupling are needed in order to contravene the strong gauge coupling renormalization of the bottom Yukawa coupling [7]–[9]. In general, for large enough values of the top quark Yukawa coupling at the grand unification scale, the low energy Yukawa coupling is strongly focussed to a quasi infrared fixed point [10]–[11]. In the minimal supersymmetric standard model, the quasi infrared fixed point predictions for the physical top quark mass \( M_t \) are given by \( M_t \sim A \sin \beta \), with \( A \simeq 190 - 210 \) GeV for the strong gauge coupling \( \alpha_3(M_Z) = 0.11 - 0.13 \). It has been recently shown that for the values of the strong gauge coupling consistent with the condition of gauge coupling unification, with reasonable threshold corrections at the grand unification and supersymmetry breaking scales, the top quark mass should be within 10% of its quasi infrared fixed point values if the condition of bottom–tau Yukawa unification is required [12].

A more predictive scheme is obtained in the framework of the minimal SO(10) unification. In this case top–bottom quark Yukawa unification is also required, implying that, for a given value of the bottom quark mass and the strong gauge coupling value [13], not only the top quark mass but also the value of \( \tan \beta \) may be determined. Remarkably, large values of \( \tan \beta \geq 40 \) are obtained in this case, leading to a proper bottom–top mass hierarchy [14]–[16]. For these large values of \( \tan \beta \), the bottom quark Yukawa coupling itself plays a relevant role in the running of the top quark Yukawa coupling, as well as in the running of the ratio of the bottom to tau Yukawa couplings. This leads to a somewhat weaker convergence of the top quark Yukawa coupling to its infrared fixed point value,
together with a slight modification of the infrared fixed point expression [8].

Moreover, it has been recently observed that for these large values of tan $\beta$, potentially large corrections to the running bottom quark mass may be induced through the supersymmetry breaking sector of the theory [17] – [18]. Although in the exact supersymmetric theory the bottom quark and tau lepton only couple to one of the Higgs fields $H_1$, a coupling of these fermions to the Higgs field $H_2$ is induced at the one loop level in the presence of soft supersymmetry breaking terms. These corrections are decisive in obtaining the predictions for the top quark mass. Indeed, for the characteristic values of tan $\beta$ arising if the top-bottom Yukawa unification is required, tan $\beta = \mathcal{O}(50)$, the bottom mass corrections would be very large, unless the supersymmetric mass parameter $\mu$ and the gluino mass are much lower than the characteristic squark masses. This hierarchy of masses may be achieved by imposing certain symmetries in the theory. These symmetries may, however, be in conflict with the radiative breaking of the electroweak symmetry, particularly in simple supersymmetry breaking scenarios.

The question of radiative electroweak symmetry breaking with large tan $\beta$ appears hence, as an independent issue, which has been investigated in minimal supersymmetric models with universal soft supersymmetry breaking parameters at the grand unification scale with encouraging results [15], [19], [20], [21]. However, not enough attention was paid either to the full consistency with the requirement of the unification of the gauge and Yukawa couplings, nor to a systematical identification of the complete parameter space at the GUT scale, which gives electroweak symmetry breaking with large tan $\beta$. Recently, we presented an investigation of the properties of the radiative electroweak symmetry breaking solutions for small and moderate values of tan $\beta$ and a top quark Yukawa coupling taking values close to its infrared fixed point solution, as required by bottom–tau Yukawa coupling unification [22]. We obtained quite remarkable correlations between different supersymmetric mass parameters, as well as an effective reduction of the number of free independent parameters at the grand unification scale. It is the purpose of this work to perform a similar analysis for the large tan $\beta$ regime.

We use the recently developed bottom-up approach to radiative electroweak symmetry breaking, [21], which is particularly suitable for a systematic search for large tan $\beta$ solutions, and possibly to identify the symmetries underlying those solutions. In our calculation we use the two loop renormalization group evolution of gauge and Yukawa couplings, while the Higgs and supersymmetric mass parameters are evolved at the one loop level. The leading supersymmetric threshold corrections to the Higgs quartic couplings
and to all supersymmetric mass parameters are included in the analysis. We proceed by fixing the experimentally known values of $M_Z$, $\alpha(M_Z)$, $\sin^2 \theta_W(M_Z)$ and $M_t$ (with their corresponding uncertainties). After choosing a set of values for $M_t$ and $\tan \beta$, the unification condition of the three Yukawa couplings fixes their running in the range from $M_Z$ to $M_{GUT}$. Next, the search for electroweak symmetry breaking solutions is performed by scanning over the CP odd Higgs mass and the low energy stop mass parameters. For each solution the one-loop correction to the running bottom mass at $M_Z$ is calculated and finally the pole bottom mass is obtained. The predictions for the top quark mass and $\tan \beta$ is the collection of those values of $M_t$ and $\tan \beta$ for which there are solutions with the pole bottom mass within the experimentally acceptable range. A more detailed explanation of this procedure will be given below.

We will show that under the requirement of top-bottom-tau Yukawa unification, the condition of radiative electroweak symmetry breaking implies strong correlations between the supersymmetric parameter $\mu_0$ and the soft supersymmetry breaking term $M_{1/2}$ and $\delta = B_0 - (6r/7)A_0$, where $r$ is the ratio at the electroweak scale of the top quark Yukawa coupling to its infrared fixed point value. These correlations allow a precise determination of the bottom mass corrections, which become significantly large for the large values of $\tan \beta$ consistent with the unification of the three Yukawa couplings of the third generation. This, in turn, implies that the top quark mass predictions are quite different from those ones obtained if the bottom mass corrections were neglected. In section 2 we present a general discussion of the model and our choice of low energy parameters. In section 3 we discuss the radiative electroweak symmetry breaking conditions and its implication for the low energy parameters of the Higgs potential. We present an approximate analytical expression for the one loop renormalization group running of the mass parameters of the Higgs and supersymmetric particles, for the case in which the top and bottom Yukawa couplings unify at $M_{GUT}$. In section 4 we present a detailed numerical analysis of the implications of the radiative $SU(2)_L \times U(1)_Y$ breaking for the supersymmetric mass parameter $\mu$ and the supersymmetry breaking parameters at the grand unification scale, and we compare it with the approximate analytical solution. In section 5 we analyse the one loop corrections to the bottom and tau masses and their implications for the top quark mass predictions. In section 6 we analyse the spectrum, together with the constraints coming from the bounds on the $b \to s\gamma$ decay rate. We reserve section 7 for the conclusions.
2 Gauge and Yukawa Coupling Unification Predictions

We begin with a short discussion of the predictions for the top quark mass following from the unification of the gauge and Yukawa couplings (before imposing the requirement of radiative electroweak breaking), recalling and slightly extending some of the results presented in Refs. [3] - [12]. The gauge coupling unification condition gives predictions for the weak mixing angle $\sin^2 \theta_W(M_Z)$ as a function of the strong gauge coupling $\alpha_3(M_Z)$. The unification condition implies (at the two loop level) the following numerical correlation [12]

$$\sin^2 \theta_W(M_Z) = 0.2324 - 0.25(\alpha_3(M_Z) - 0.123) \pm 0.0025$$  (1)

where the central value corresponds to an effective supersymmetric threshold scale $T_{SU SY} = M_Z$ and the error $\pm 0.0025$ is the estimated uncertainty in the prediction arising from possible supersymmetric threshold corrections (corresponding to vary the effective supersymmetric threshold scale $T_{SU SY}$ from 15 GeV to 1 TeV), threshold corrections at the unification scale as well as from higher dimensional operators. On the other hand, $\sin^2 \theta_W(M_Z)$ is given by the electroweak parameters $G_F$, $M_Z$, $\alpha_{em}$ as a function of the physical top quark mass $M_t$ (at the one loop level) by the formula [3]:

$$\sin^2 \theta_W(M_Z) = 0.2324 - 10^{-7}GeV^{-2} \left( M_t^2 - (138GeV)^2 \right) \pm 0.003$$  (2)

Therefore, the predictions from gauge coupling unification agree with experimental data provided

$$M_t^2 = (138GeV)^2 + 0.25 \times 10^{7}GeV^2 (\alpha_3(M_Z) - 0.123 \pm 0.01)$$  (3)

The above $M_t - \alpha_3(M_Z)$ correlation defines a band whose lower bound is shown in Fig. 1 (the upper bound is above 0.13 for $M_t > 110$ GeV). We observe that the top quark mass $M_t > 110 (155) GeV$ implies $\alpha_3(M_Z) > 0.11 (0.115)$.

Another issue is that of unification of the bottom and tau Yukawa couplings. In this work the unification of Yukawa couplings is always studied numerically at the two-loop level. However, for a qualitative discussion we refer to the one-loop renormalization group equation for the ratio of the bottom to tau Yukawa couplings $r = h_b/h_\tau$, which, in the limit of vanishing electroweak gauge couplings, reads

$$\frac{dr}{dt} = \frac{r}{8\pi} \left( \frac{16\alpha_3}{3} - 3Y_b - Y_t + 3Y_\tau \right)$$  (4)

where $t = \ln(M_Z/Q)^2$ and $Y_i = h_i^2/(4\pi)$. Starting from values of the ratio $r$ above one at the scale $M_b$, as required by experimentally allowed values of the bottom mass
$M_b = (4.9 \pm 0.3)$ GeV [25] and the tau mass, $M_\tau = 1.78$ GeV, $r$ is strongly renormalized and in the limit of negligible Yukawa couplings for values of $\alpha_3$ within the experimentally determined range it becomes lower than one at scales far below the grand unification scale $M_{GUT}$. Hence, in order to get $r(M_{GUT}) = 1$, for a given value of $\alpha_3$, the Yukawa couplings in Eq. (4) should be adjusted to compensate the strong gauge coupling effect. For low and moderate values of $\tan \beta$, $(Y_\tau, Y_\tau \ll Y_t)$, it is the top quark Yukawa coupling which is fixed as a function of $\alpha_3$, by the bottom–tau Yukawa unification requirement. As we discussed in the introduction, this perturbative unification requires values that are within 10% of the top quark mass infrared quasi fixed point value.

Here we are primary concerned with the large $\tan \beta$ solution. Then, the bottom and the top quark Yukawa couplings are of the same order of magnitude and both are important to get $r(M_{GUT}) \approx 1$. The unification of the three Yukawa couplings takes place not only for a particular value of $Y_t$ but also of $\tan \beta$, for given values of $M_t$, $M_\tau$ and $\alpha_3(M_Z)$, implying a fixed $M_t$. An important remark is in order here. The bottom mass which is directly relevant for the top mass prediction following from the Yukawa coupling unification is the tree level running mass $m_b(M_t)$. As we discussed in the previous section, in the large $\tan \beta$ case it may receive large loop corrections from sparticle exchange loops, at least in some range of parameters of the model. The physical (pole mass) $M_b$ is obtained from the running mass $m_b(M_b)$ (which is related to $m_b(M_Z)$ by the Standard Model RG equations) by inclusion of QCD corrections, which are universal for the Standard Model and its supersymmetric version. At the two loop level, they are given by [24]

$$m_b(M_b) = \frac{M_b}{1 + \frac{4\alpha_3(M_b)}{3\pi} + K_b \left( \frac{\alpha_3(M_b)}{\pi} \right)^2},$$

where $K_b = 12.4$. The loop corrections to the running bottom mass at $M_Z$ induced through sparticle exchange loops are an important issue for models with radiative breaking of the electroweak symmetry. In order to distinguish them from QCD corrections, we introduce the pole bottom mass $\check{M}_b$, which is obtained from the unification condition in the case in which the supersymmetric one loop corrections to $m_b(M_Z)$ are ignored. Due to the fact that the supersymmetric corrections could be quite sizeable, for the allowed solutions of the model, the mass $\check{M}_b$ may be significantly different from the physical mass $M_b$.

The predictions for $M_t$ and $\tan \beta$, following from the unification of the three Yukawa couplings, are shown in Fig. 1 for several values of the mass $\check{M}_b$ as a function of $\alpha_3(M_Z)$. The supersymmetric particle masses were set at the scale $M_Z$, while the unification scale
was defined as the scale at which the electroweak gauge couplings unify. Fig. 1 shows also the region in the $\alpha_3(M_Z) - M_t$ plane consistent with the unification of gauge couplings, after considering the experimental dependence of $\sin^2 \theta_W(M_Z)$ on the top quark mass and threshold corrections at the supersymmetric and grand unification scale, Eq.(3).

From Fig. 1 we draw the following conclusions: In case that the supersymmetric loop contributions to the bottom mass were negligible, $\tilde{M}_b = M_b$, and taking into account the experimentally acceptable values for the physical bottom mass, $M_b \approx 4.9 \pm 0.3$ GeV [25], the unification of the gauge and Yukawa couplings drives the top quark mass towards large values (Note the fact that for $Y_b = Y_t$ the IR fixed point solution is lower than for $Y_b \ll Y_t$) [8]. Although the predictions for $M_t$ are no longer so strongly constrained to be close to its infrared quasi fixed point values as for the low and moderate values of tan $\beta$ (as explained above, strong renormalization effects in the running of $h_b$ are partially cancelled by $h_b$ itself), for the values of $\alpha_3(M_Z)$ consistent with gauge coupling unification the top quark mass is still close to the appropriate infrared fixed point solution. For instance, for a physical bottom quark mass $M_b = 5.2$ GeV, $\alpha_3(M_Z) \approx 0.12$ and tan $\beta = 50$, the top quark mass is predicted to be $M_t \approx 175$ GeV. In general, as it is clear from Fig. 1, if the supersymmetric corrections to the running bottom mass were small, the top quark mass would acquire values $M_t > 165$ GeV within this scheme [17]. In Fig. 1 we also plot the predictions obtained for values of $\tilde{M}_b$ larger than the experimental upper bound for the bottom mass, $M_b < 5.2$ GeV, which will become of interest while studying the supersymmetric corrections to the bottom mass. Indeed, the values of the top quark mass in the range, say (140–160) GeV are compatible with unification of couplings provided $\tilde{M}_b > M_b$ and sizeable supersymmetric loop corrections to the bottom mass are induced. As we shall show below, this is the case in the minimal supergravity model with minimal $SO(10)$ Yukawa unification. It is relevant to contrast this situation with what happens for low and moderate values of tan $\beta$, for which the consistency of a moderately heavy top quark, $M_t < 160$ GeV, with bottom–tau Yukawa unification requires the ratio of vacuum expectation values to be very close to one, tan $\beta \approx 1.4$, unless large threshold corrections to both gauge and Yukawa couplings are present at the grand unification scale [7], [9],[12].

3 Higgs Potential Parameters

In order to analyze the radiative electroweak symmetry breaking condition, one should concentrate on the Higgs potential of the theory. In the Minimal Supersymmetric Standard Model, and after the inclusion of the leading–logarithmic radiative corrections, it
may be written as [11], [26]-[28]

\[ V_{\text{eff}} = m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2 - m_3^2 (H_1^T i \tau_2 H_2 + h.c.) + \frac{\lambda_1}{2} (H_1^+ H_1)^2 + \frac{\lambda_2}{2} (H_2^+ H_2)^2 + \lambda_3 (H_1^+ H_1) (H_2^+ H_2) + \lambda_4 \left| H_1^T i \tau_2 H_2^T \right|^2 \]

(6)

where the quartic couplings may be obtained by the corresponding renormalization group equations and the fact that, at scales at which the theory is supersymmetric the running quartic couplings \( \lambda_j \), with \( j = 1 - 4 \), must satisfy the following conditions Refs. [26]-[29]:

\[ \lambda_1 = \lambda_2 = \frac{g_1^2 + g_2^2}{4}, \quad \lambda_3 = \frac{g_3^2 - g_1^2}{4}, \quad \lambda_4 = -\frac{g_2^2}{2}. \]

(7)

The masses \( m_i^2 \), with \( i = 1 - 3 \) are also running mass parameters, whose renormalization group equations may be found in the literature [23]-[31]. As we explained in section 1, in the numerical analysis we considered the two loop renormalization group evolution of gauge and Yukawa couplings, while the supersymmetric and Higgs mass parameters, as well as the low energy Higgs quartic couplings are evolved at the one loop level with the leading supersymmetric threshold corrections included. The minimization conditions read

\[ \sin(2\beta) = \frac{2m_2^2}{m_A^2}, \]

(8)

\[ \tan^2 \beta = \frac{m_1^2 + \lambda_2 v^2 + (\lambda_1 - \lambda_2) v_1^2}{m_2^2 + \lambda_2 v^2}, \]

(9)

where \( \tan \beta = v_2/v_1 \), \( v_i \) is the vacuum expectation value of the Higgs fields \( H_i \), \( v^2 = v_1^2 + v_2^2 \), and \( m_A \) is the CP-odd Higgs mass,

\[ m_A^2 = m_1^2 + m_2^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4) v^2 \]

(10)

and we define the mass parameter \( m_3^2 \) to be positive.

Apart from the mass parameters \( m_i^2 \), appearing in the effective potential, the evolution of the supersymmetric mass parameter \( \mu \) appearing in the superpotential \( f \),

\[ f = h_t c_t Q^T U H_2^+ + h_u c_u Q^T D H_1^+ + h_r c_r L^T E H_1^+ + \mu c_{ij} H_1^T H_2^+ \]

(11)

(where \( Q^T = (T \ B) \) is the top-bottom left handed doublet superfield and \( U, D \) and \( L \) are \( SU(2)_L \) singlet superfields) is relevant for the analysis of the radiative electroweak symmetry breaking conditions. The bilinear mass term proportional to \( m_3^2 \) appearing in the Higgs potential may be rewritten as a soft supersymmetry breaking parameter \( B \) multiplied by the Higgs bilinear term appearing in the superpotential, that is \( m_3^2 = B \mu \).
Analogously, the scalar potential may contain a scalar trilinear breaking term proportional to the $h_f -$ Yukawa dependent part of the superpotential, with a trilinear coupling $A_f$.

In order to get an understanding of the numerical results, we will present approximate analytical formulae, for the relations required by the electroweak symmetry breaking conditions, in which the radiative corrections to the quartic couplings are ignored. There are several features of the Higgs potential which are characteristic for large $\tan \beta$ values. They can be easily discussed in a qualitative way on the basis of the supersymmetric tree level potential. Eq.(9) simplifies to

$$\tan^2 \beta = \frac{m_1^2 + M_Z^2/2}{m_2^2 + M_Z^2/2},$$  \hspace{1cm} (12)

so, for large $\tan \beta$ (already, say, $\tan \beta > 30$), either

$$m_2^2 \approx -\frac{M_Z^2}{2},$$  \hspace{1cm} (13)

if $m_1^2, m_2^2$ are of the order of the $Z^0$ boson mass squared, or

$$m_1^2 \approx \tan^2 \beta m_2^2$$  \hspace{1cm} (14)

when $m_1^2, m_2^2 \gg M_Z^2$. In general, the smaller is the cancellation in the denominator of Eq.(12), the larger is the hierarchy between $m_1^2$ and $m_2^2$. The second relation, Eq.(14), is, however, unnatural when $Y_t \approx Y_b$. Indeed, if all supersymmetric particle masses are below a few TeV, Eq.(13) holds, within a good approximation (Although the inclusion of radiative corrections modifies the low energy convergence of the $m_2^2$ parameter, the relation $|m_2^2| \approx \frac{1}{2} M_Z^2$ is preserved, what is sufficient for the understanding of the properties discussed below). Eq.(13), combined with the condition $M_A^2 \simeq m_1^2 + m_2^2 > 0$, gives a useful constraint:

$$m_1^2 - m_2^2 > M_Z^2,$$  \hspace{1cm} (15)

Another very important property is

$$m_3^2 \simeq \frac{M_A^2}{\tan \beta},$$  \hspace{1cm} (16)

or, equivalently, $m_1^2 \gg m_3^2$. Since in the $Y_t \simeq Y_b$ case a large hierarchy between $m_1^2$ and $m_2^2$ is highly unnatural, the above condition, Eq.(16), implies also $|m_3^2| \gg m_3^2$ ($M_Z^2 \gg m_3^2$).

Thus, in order to study the implication of the electroweak symmetry breaking condition, one can effectively replace Eq.(16) with the condition $m_3^2 \simeq 0$. 

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To go further with the analysis, it is very useful to obtain approximate analytical solutions for the one loop renormalization group evolution of the mass parameters, whose validity may be proven by comparing them with our numerical solutions. We will assume universality of the soft supersymmetry breaking parameters, that is to say a common scalar mass $m_0$ and a common gaugino mass $M_{1/2}$, as well as the boundary conditions for the parameters $A_i$ ($A_t$ and $A_r$), $B$ and $\mu$, at the grand unification scale to be given by $A_0$, $B_0$ and $\mu_0$, respectively. In the region of large values of $\tan \beta$, for which the bottom Yukawa coupling is of the order of the top Yukawa coupling, an approximate analytical solution for the one loop evolution of the mass parameters may be obtained. For this, we identify the bottom and top Yukawa couplings and neglect the tau Yukawa coupling effects. Furthermore, all supersymmetric threshold corrections are ignored at this level. The solution for $Y = Y_t \simeq Y_b$ reads

$$Y_i(t) = \frac{4\pi Y_i(0) E(t)}{4\pi + 7Y_i(0) F(t)}$$

where $E$ and $F$ are functions of the gauge couplings,

$$E = (1 + \beta_3 t)^{16\beta_3/3} (1 + \beta_2 t)^{3\beta_2/2} (1 + \beta_1 t)^{13\beta_1/5}, \quad F = \int_0^t E(t') dt'$$

with $\beta_i = \alpha_i(0)b_i/4\pi$, $b_i$ the beta function coefficient of the gauge coupling $\alpha_i$, $t = 2 \log(M_{GU/T}/Q)$ and we identify the right bottom and the right top hypercharges. As we said, the fixed point solution is obtained for values of the top quark Yukawa coupling which become large at the grand unification scale, that is, approximately,

$$Y_f(t) = \frac{4\pi E(t)}{7F(t)}$$

As had been anticipated in Ref. [8], the fixed point solution for the $Y_t \simeq Y_b$ case differs in a factor $6/7$ from the corresponding solution in the low $\tan \beta$ case, for which $Y_t \gg Y_b$. From here, by inspecting the renormalization group equation for the mass parameters, we obtain the approximate analytical solutions

$$m^2_{H_1} \simeq m^2_{H_2} = m_0^2 + 0.5M_{1/2}^2 - \frac{3}{7} \Delta m^2$$

$$m^2_U \simeq m^2_D = m_0^2 + 6.7M_{1/2}^2 - \frac{2}{7} \Delta m^2$$

$$m^2_Q \simeq m_0^2 + 7.2M_{1/2}^2 - \frac{2}{7} \Delta m^2$$

$$m^2_{U} \simeq m^2_{D} = m_0^2 + 6.7M_{1/2}^2 - \frac{2}{7} \Delta m^2$$

$$m^2_{Q} \simeq m_0^2 + 7.2M_{1/2}^2 - \frac{2}{7} \Delta m^2$$
where \( m_i^2 = \mu^2 + m_{H_i}^2 \), with \( i = 1, 2 \), \( m_Q \), \( m_D \), \( m_U \) are the squark doublet, right bottom squark and right stop quark mass parameters respectively and

\[
\Delta m^2 \quad \simeq \quad \frac{3\alpha_0^2}{\tan \beta} - 4.6 A_{0/2} \frac{Y}{Y_{f}} \left( 1 - \frac{Y}{Y_{f}} \right) \\
+ \quad A_{0}^{2} \frac{Y}{Y_{f}} \left( 1 - \frac{Y}{Y_{f}} \right) + \frac{M_{1/2}^2}{14 \left( \frac{Y}{Y_{f}} \right)^2}.
\]

Here we have concentrated on the above mass parameters, because they are the only relevant ones for the study of the properties of the radiative electroweak symmetry breaking solutions in the approach of ref. [21]. We will discuss the properties of the mass spectrum in more detail in section 6. Moreover, the supersymmetric mass parameter renormalization group evolution gives,

\[
\mu^2 = 2 \mu_0^2 \left( 1 - \frac{Y}{Y_{f}} \right)^{6/7},
\]

while the running of the soft supersymmetry breaking bilinear and trilinear coupling read,

\[
A_t = A_0 \left( 1 - \frac{Y}{Y_{f}} \right) - M_2 \left( 4.2 - 2.1 \frac{Y}{Y_{f}} \right),
\]

\[
B \simeq \delta(Y) + M_{1/2} \left( 2 \frac{Y}{Y_{f}} - 0.6 \right),
\]

with

\[
\delta(Y) = B_0 - \frac{6 Y}{7 Y_{f}} A_0.
\]

The coefficients characterizing the dependence of the mass parameters on the universal gaugino mass \( M_{1/2} \) are functions of the exact value of the gauge couplings. In the above, we have taken the values of the coefficients that are obtained for \( \alpha_3(M_Z) \simeq 0.12 \).

The approximate solutions, Eqs. (20–25), become weakly dependent on the parameter \( A_0 \), the dependence being weaker for top quark Yukawa couplings closer to the fixed point value. The strongest dependence on the parameter \( A_0 \) comes through the parameter \( \delta(Y) \) introduced above. Similar properties are obtained in the low tan \( \beta \) regime [22], although the explicit form of the parameter \( \delta \) is different in this case. From Eq.(24), it follows that the coefficient relating \( \mu \) to \( \mu_0 \) tends to zero as \( Y \rightarrow Y_{f} \). The coefficients scales faster to zero than in the low \( \tan \beta \) case.
4 Radiative Breaking of $SU(2)_L \times U(1)_Y$

In the following we present a complete numerical analysis of the constraints coming from the requirement of a proper radiative electroweak symmetry breaking in the large $\tan \beta$ regime. As described in the Introduction, we use the bottom–up approach of ref. [21]. For a fixed value of the top quark mass $M_t$ we search for all solutions to radiative breaking, which give a chosen value of $\tan \beta$, by scanning over the CP odd Higgs mass and the low energy stop mass parameters. The latter are very convenient as the input parameters as they fix the leading supersymmetric threshold corrections to the Higgs potential. While studying the model from low energies we have chosen for definiteness an upper bound of 2 TeV on the scanned parameters. For a somewhat larger upper bound, larger values of the soft supersymmetry breaking parameters are allowed, but the general features of the solutions are preserved. It is natural to expect that the supersymmetric parameters are at most of order of a few TeV, if supersymmetry is to solve the hierarchy problem of the Standard Model. In Figs. 2 - 5 we present the results which show interesting correlations among the soft supersymmetry breaking parameters.

As discussed in section 3, the one loop corrections to the effective Higgs potential, necessary to perform a proper analysis of the radiative electroweak symmetry breakdown, were included in the numerical analysis. The gauge and Yukawa couplings were evolved with their two loop renormalization group equations between $M_Z$ and $M_{GUT}$. In their evolution, we have treated all supersymmetric particle masses as being equal to $M_Z$. Although this procedure introduces small uncertainties on the predicted values of $a_3(M_Z)$ and $M_t$ (which will be considered in our analysis), it keeps all the essential features of the radiative electroweak symmetry with unification of bottom and top Yukawa couplings, makes possible the comparison of our results with the ones of Fig. 1 and allows an easy analytical interpretation of the numerical results. In addition, the small uncertainties on $a_3(M_Z)$ and $M_t$ may be treated by analytical methods [8], [16].

Analogously to the low $\tan \beta$ scenario [22], it is possible to derive approximate analytical relations, which are useful in the understanding of the numerical results. Indeed, considering the conditions for a proper radiative electroweak symmetry breaking, Eq.(13), the approximate solutions for the mass parameters, Eqs. (20) - (23), and ignoring radiative corrections to the quartic couplings the following analitycal expression is obtained,

$$\mu^2 = m_0^2 \left( \frac{9 Y}{7 Y_f} - 1 \right) - M_t^{1/2} \left[ 0.5 - \frac{6 Y}{Y_f} + \frac{18}{7} \left( \frac{Y}{Y_f} \right)^2 \right]$$
In the analytical presentation we will always keep the expressions as a function of the low energy parameter $\mu$. The reason is that in the one loop approximation $\mu$ and $\mu_0$ are linearly related, Eq.(24), and $\mu$ becomes a more appropriate parameter for the description of the solution properties, particularly for large values of the top quark mass where $\mu_0$ strongly depends on the degree of proximity to the fixed point value. The $\mu_0$ dependence may be always recovered by using Eq.(24).

In the above we have taken the expression of $m^2_1$ obtained in the analytical approximation in which $m^2_1 \simeq m^2_2$. In the explicit numerical solution to the mass parameters, however, we obtain

$$m^2_1 - m^2_2 = \alpha M^2_{1/2} + \beta m^2_0$$

where for $Y/Y_f \simeq 1$ ($M_t \simeq 190$ GeV), $\alpha \simeq 0.2$, and $\beta \simeq -0.2$, while for $Y/Y_f \simeq 0.6$ ($M_t \simeq 150$ GeV), $\alpha \simeq 0.1$ and $\beta \simeq -0.1$. Hence, the coefficient $\alpha$ is small and positive, and $\beta$ is negative and small in magnitude. The order of magnitude of the coefficients $\alpha$ and $\beta$ can be easily inferred from the renormalization group equations. Indeed, it is easy to show that under the condition of unification of the three third generation Yukawa couplings $\alpha$ comes mainly from the difference in the running of bottom and top Yukawa couplings, together with the different hypercharges of the right top and bottom quarks, which induce a different gaugino dependence of the stop and sbottom parameters. The negative values of $\beta$ are mainly due to the $\tau$ lepton Yukawa effects. We see that, due to the restriction $m^2_1 - m^2_2 > M^2_Z$, Eq.(15), values of $m^2_0 > M^2_{1/2}$ make the radiative breaking of the electroweak symmetry impossible, in the approximation which neglects supersymmetric threshold corrections to the Higgs potential. In the numerical analysis, which includes those corrections, the only solutions are still obtained for $M^2_{1/2}$ of the order of, or larger than $m^2_0$, as seen in Fig. 2.a and Fig. 2.b. It is also important to remark that, due to the smallness of the parameters $\alpha$ and $\beta$, the dependence of the mass parameters $m^2_1$ and $m^2_2$ on the gaugino mass $M_{1/2}$ is well described by Eq.(28) (which was obtained in the approximation $m^2_1 = m^2_2$), while the dependence on the mass parameters $A_0$, $m_0$, remains weak for $M_{1/2} > m_0$. In general, the corrections to the approximate solutions given in Eqs.(19 - 22) and Eq.(28) are small, and, hence they provide useful information for the analysis of the electroweak symmetry breaking condition.

The values of the top quark mass, $M_t = 190$ GeV and $M_t = 150$ GeV, and of the ratio of the Higgs vacuum expectation values, $\tan \beta = 55$ and $\tan \beta = 38$, used above are such that unification of gauge and Yukawa couplings is achieved for $M_b \simeq 5.4$ GeV, $\alpha_3(M_Z) \simeq 0.129$.
and $\bar{M}_6 \approx 5.85$ GeV, $\alpha_3(M_Z) \approx 0.124$, respectively. Considering a Yukawa coupling solution sufficiently close to the infrared fixed point, the values of $M_{1/2}^2 \geq m_0^2$ as required by the radiative breaking conditions, and $M_{1/2}^2 > M_Z^2$ (as follows from Eqs.(15, 29), we obtain from Eq.(28) that,

$$\mu^2 \approx 3M_{1/2}^2,$$

i.e. there is a strong linear correlation between $\mu$ and $M_{1/2}$. If, instead, we consider the case $Y/Y_t = 0.6$, (corresponding to $M_t \approx 150$ GeV) as a representative one of what happens when we depart from the fixed point value we obtain

$$\mu^2 = -0.23m_0^2 + 2.2M_{1/2}^2 - 0.47A_0M_{1/2} + 0.1A_0^2 - M_Z^2/2.$$  

(31)

There is a stronger dependence on the supersymmetry breaking parameter $A_0$. However, due to the relation $M_{1/2}^2 \geq m_0^2$, the bounds on $A_0$ and $B_0$ coming from the stability condition and the requirement of the absence of a colour breaking minima [30], and the smallness of the coefficients associated with the $A_0$ dependence, one gets that the correlation between $\mu$ and $M_{1/2}$ is conserved over most of the parameter space,

$$\mu^2 \approx DM_{1/2}^2,$$

(32)

where $D \approx 2$. The predictions coming from the above analysis, based on the approximate relations Eqs.(30–32), must be compared with the results of the numerical analysis, in which the running of gauge and Yukawa couplings have been considered at the two loop level, and all one loop threshold corrections to the quartic couplings and masses have been included. The resulting correlations between $\mu$ and $M_{1/2}$ are depicted in Fig. 3.a and Fig. 3.b, which are in good agreement with the analytical results, although the coefficient $D$ in Fig. 3b is somewhat smaller than the analytical prediction, Eq.(32).

The information above may be used to get a further understanding of the properties of Fig. 2. The lower bound on $M_{1/2}$, for instance, follows from the condition $m_{1/2}^2 - m_2^2 > M_Z^2$, which yields

$$M_{1/2} > \frac{M_Z}{\sqrt{\alpha}},$$

(33)

where, as we said above, $\alpha \approx 0.2$ for $M_t \approx 190$ GeV and $\tan \beta = 55$, while $\alpha \approx 0.1$ for $M_t \approx 150$ GeV and $\tan \beta = 38$. From Fig. 2 we observe that, although the lower limit on $M_{1/2}$ for $M_t = 150$ GeV is well described by Eq.(33), the one for $M_t = 190$ GeV is somewhat higher than the predicted one from Eq.(33). This difference is a reflection of
the size of the one loop radiative corrections to the quartic couplings, which grow with the fourth power of the top quark mass and were ignored for the obtention of Eq. (33).

As we explained above, the condition \( m_1^2 - m_2^2 > M_Z^2 \) also excludes the points with \( m_0 \geq M_{1/2} \). Furthermore, low values of \( m_0 \), although consistent with the condition of radiative breaking induce large mixings in the stau sectors which yield stau masses lower than the neutralino ones. The fact that low values of \( m_0 \) leads to a stau lighter than the neutralinos was already noticed in Ref. [20]. In the Figures we impose the condition of a neutral supersymmetric particle to be the lightest one as an additional experimental constraint. Under these conditions, the lightest supersymmetric particle is always a bino, with mass \( M_B \approx 0.4 M_{1/2} \). In order to get a quantitative understanding of the lower limit on \( m_0 \), we recall that, ignoring small tau Yukawa coupling effects, the left and right slepton mass parameters are given by [31],

\[
m_L^2 \simeq 0.5 M_{1/2}^2 + m_0^2, \quad m_R^2 \simeq 0.15 M_{1/2}^2 + m_0^2, \tag{34}
\]

while the mixing term for large \( \tan \beta \) is dominated by the \( \mu \) parameter

\[
m_{LR}^2 \simeq - h_\tau \mu v_2. \tag{35}
\]

Using the fact that, at energies of the order of \( M_Z \), \( h_\beta/h_\tau \approx 1.7 \), and the bottom - top unification condition, the condition \( m_\tau > M_B \) approximately yields,

\[
m_0^2 \geq -0.15 M_{1/2}^2 + \sqrt{(0.15 M_{1/2}^2)^2 + \mu^2 m_t^2/3}. \tag{36}
\]

Recalling Eqs.(30) and (32), and using Eq.(36), one can get an understanding of the \( m_0 \) region, indicated as experimentally excluded in the Figs. 2.a and 2.b (see also Fig. 10).

Close to the infrared quasi fixed point solution the condition \( m_3^2 \simeq 0 \) yields \( B \approx 0 \) (from Eqs. (30),(32) \( \mu^2 > M_{1/2}^2 > M_Z^2 \)), i.e.

\[
\delta \equiv B_0 - \frac{6 A_0}{7} \approx -1.4 M_{1/2} \tag{37}
\]

In the numerical analysis, we studied the correlations between \( A_0/M_{1/2} \) and \( B_0/M_{1/2} \), and compared it with the results coming from Eq.(37). The results are depicted in Fig 4.a. The numerical results confirm in a good degree the analytical expectations. Analogously, for \( Y/Y_f \approx 0.6 \), we obtain

\[
\frac{B_0}{M_{1/2}} - \frac{0.5 A_0}{M_{1/2}} = -0.6. \tag{38}
\]

The correlation, resulting in this case from the numerical analysis is depicted in Fig 4.b, being in good agreement with Eq.(38), too.
The strong correlation between the parameter $\delta$ and $M_{1/2}$, together with the $\mu - M_{1/2}$ correlation, Eqs. (30) - (32), implies also a strong correlation between $\mu$ and $\delta$. The numerical correlation is presented in Figs. 5.a and 5.b, for which we chose to plot the GUT scale parameter $\mu_0$ instead of the renormalized parameter $\mu$. From Figs. 3 and 5 we can hence obtain also information about the relation between $\mu$ and $\mu_0$, which agrees well with the analytical prediction, Eq. (24).

Observe that the condition $m_3^2 \approx 0$ is a property of the radiative breaking solutions with large values of $\tan \beta$ and a not too heavy supersymmetric spectrum, and in this sense is independent of the condition of unification of top and bottom quark Yukawa couplings. Since very low values of $\mu$ ($\mu \approx 0$) are not consistent with the condition of radiative breaking of the electroweak symmetry, equations analogous to Eqs. (37) and (38) will be obtained even if we relax the bottom-top Yukawa unification condition. We exemplify this by taking two solutions with $Y_t(0)/Y_\ell(0) \approx 2$ and large values of $\tan \beta$ and studying the numerical solutions. The resulting correlations are depicted in Fig. 4.c and 4.d.

When the condition of unification of bottom and top Yukawa couplings is relaxed, however, large values of $M_{1/2}$ are not longer needed to get the necessary hierarchy between $m_1^2$ and $m_2^2$. As the bottom and tau Yukawa couplings decrease compared with the top one, the coefficients $\alpha$ and $\beta$, Eq.(29), increase, $\beta$ becoming positive for $Y_t(0)/Y_\ell(0) > 1.6$. Hence, for $Y_t(0)/Y_\ell(0) > 1.6$, acceptable radiative breaking solutions may be also obtained by taking large values of $m_0^2 \gg M_{1/2}^2$. For these solutions the strong correlation between $\mu$, and $M_{1/2}$ is lost, together with the hierarchical relation between $M_{1/2}$ and $m_0$. These results are depicted in Figs. 2.c and 2.d. 3.c and 3.d, and 5.c and 5.d, where $Y_t(0)/Y_\ell(0) \approx 2$.

In summary, in general the condition of radiative breaking of the electroweak symmetry with large values of $\tan \beta$ implies a strong correlation between the parameters $M_{1/2}$ and $\delta$. This correlation is a reflection of, in principle, a strong degree of fine tuning, implied by the condition $m_3^2 \approx 0$. However, it is tempting to speculate that this correlation has some fundamental origin, what would imply the necessity of redefining the naive fine tuning criteria.

If the top quark-bottom quark Yukawa coupling unification is required, the parameters $M_{1/2}$ and $\delta$ are also strongly related with the supersymmetric mass parameter $\mu$. These properties do not strongly depend on the proximity to the infrared fixed point solution, although the exact value of the coefficient relating the different parameters and the strength of the correlation does depend on the top quark Yukawa coupling value.
Radiative breaking of the electroweak symmetry is driven by the gaugino mass $M_{1/2}$ and $M_{1/2}^2 > m_0^2 > M_Z^2$. For a large enough departure from the exact top quark–bottom quark Yukawa unification ($Y_t(0)/Y_b(0) > 1.6$) solutions with radiative breaking driven by $m_0^2$ are also possible, for which both the correlation between $\mu$ and $M_{1/2}$ and the hierarchical relation between $M_{1/2}$ and $m_0$ are destroyed.

5 Radiative Corrections to $M_b$ and $M_t$ and the Predictions for the Top Quark Mass

Fig.1 summarizes the predictions for the top quark mass as a function of $\alpha_3(M_Z)$ for given values of $\tilde{M}_t$, which follow from unification of the three Yukawa couplings. As explained in Section 2, the pole mass $\tilde{M}_b$ is obtained from the unification condition in the case in which the supersymmetric one-loop corrections to the bottom mass are ignored (i.e. it includes only QCD corrections). In this section we calculate the supersymmetric one-loop corrections to the bottom mass in the model with radiative breaking. For large values of tan $\beta$, they are not only large but, due to the strong correlations between the soft supersymmetry breaking parameters present in the large tan $\beta$ solutions with $Y_t \approx Y_b$, for fixed tan $\beta$ they are almost constant in the whole parameter space allowed by radiative breaking. Thus, in the first approximation, for fixed tan $\beta$ and $M_t$, $M_b = \tilde{M}_b + \text{const}$. If this value of $M_b$ is in the range of the experimentally acceptable values for the physical bottom mass, $M_b \approx 4.9 \pm 0.3$ GeV, then the corresponding values of $M_t$ and tan $\beta$ are the predictions for the top quark mass and tan $\beta$, consistent with radiative breaking. Of course, all uncertainties taken into account, the actual prediction is a band of values for $M_t$ and tan $\beta$.

There is a higher order ambiguity due to the choice of the scale at which the supersymmetric one-loop corrections are calculated. A natural choice is between the electroweak and supersymmetric ($M_{3/2}$) scales and we choose to work with $m_3(M_Z)$. The corrected running bottom quark mass $m_b$ reads [17],[18]

$$m_b = h_b v_1 (1 + \Delta(m_b)).$$

(39)

$\Delta(m_b)$ receives contributions coming from bottom squark–gluino loops and top squark–chargino loops, and is given by,

$$\Delta(m_b) = \frac{2\alpha_3}{3\pi} M_{3/2} \mu \tan \beta I(m_{i,1}^2, m_{i,2}^2, M_{3/2}^2)$$

$$+ \frac{Y_t}{4\pi} A_t \mu \tan \beta I(m_{i,1}^2, m_{i,2}^2, \mu^2),$$

(40)
where the integral function \( I(a, b, c) \) is given by

\[
I(a, b, c) = \frac{ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a - b)(b - c)(a - c)},
\] (41)

with \( M_{\tilde{g}} \) and \( m_{\tilde{e}_i} (m_{\tilde{t}_i}) \) are the gluino and sbottom (stop) eigenstate masses respectively. The integral function may be parametrized as \( I(a, b, c) = K_1 / a_{\text{max}} \), where \( a_{\text{max}} \) is the maximum of the three squared masses appearing in the functional integral and the coefficient \( K_1 \approx 0.5 - 0.9 \) if there is no large hierarchy between the three different masses. Observe that the minimum value of \( K_1 = 0.5 \) is only obtained when the three masses are equal. As we will discuss below, for the typical values of the mass parameters appearing in the radiative electroweak symmetry breaking solutions, \( K_1 \approx 0.6 \) gives a good approximation to the integral.

The tau mass corrections are, instead, dominated by the bino exchange contribution, which is negligible for the bottom quark case. Indeed,

\[
m_\tau = h_\tau v_1 (1 + \Delta(m_\tau)),
\] (42)

with

\[
\Delta(m_\tau) = \frac{\alpha_1}{4\pi} M_\tilde{B} \mu \tan \beta \left( M_{\xi_{1,2}}^2, M_{\tilde{B}}^2 \right)
\] (43)

Observe that although the effect is expected to be small due to the presence of the weak gauge coupling, it is partially enhanced by the fact that the particles appearing in the loop are lighter than in the bottom case. We will discuss it in more detail below.

Due to the approximate dependence of \( A_t \) on \( A_0 \) and \( M_{1/2} \), Eq.(25), close to the fixed point there is a strong correlation between \( A_t \) and the gluino mass. Indeed, for \( Y/Y_f \approx 1 \),

\[
A_t \simeq -\frac{2M_{\tilde{g}}}{3}.
\] (44)

For values of \( Y/Y_f \approx 0.6 \), \( A_t \) is shifted towards larger values in most of the parameter space,

\[
A_t \simeq -M_{\tilde{g}}.
\] (45)

These correlations are observed in the numerical analysis. The relations above, Eqs.(44), (45), are only violated for large values of \( A_0 \), close to the upper bound on this quantity (For the numerical bounds on \( A_0 \) see Fig. 4). Due to the minus signs in Eqs.(44) and (45), there is an effective cancellation between both bottom mass correction contributions. Interestingly enough, due to the fact that \( A_t \) is larger when the Yukawa coupling \( Y \) is
smaller, the cancellation between both bottom mass correction contributions for $Y/Y_f \simeq 0.6$ is similar to the one appearing for $Y/Y_f \simeq 1$.

The bottom mass corrections become very relevant for large values of $\tan \beta \geq 30$. In order to reduce the bottom mass corrections, while fulfilling the requirement $m_{\tilde{g}}^2 \simeq 0$, the authors of Ref.[17] imposed a Peccei Quinn symmetry $\mu \to 0$, which is explicitly broken, its breakdown being characterized by the (assumed) small parameter $\epsilon_{PQ} = \frac{\mu}{m_{\tilde{g}}}$. They also required the presence of an approximate continuous $R$ symmetry, present in the limit $B \to 0$, $M_{\tilde{g}} \to 0$, $A \to 0$, which breaking is characterized by the (assumed) small parameter

$$\epsilon_R = \frac{B}{m_{\tilde{g}}} \simeq \frac{A}{m_{\tilde{g}}} \simeq \frac{M_{\tilde{g}}}{m_{\tilde{g}}}$$

which would protect both $\tan \beta$ and the bottom mass corrections.

However, the electroweak symmetry radiative breaking solutions with universal soft supersymmetry parameters at the grand unification scale and exact top quark–bottom quark Yukawa coupling unification is inconsistent with the approximate preservation of these symmetries. Indeed, as we have discussed in the last section, the only solutions satisfying these requirements are obtained for $M_{1/2}^2$ of the order of, or larger than $m_{\tilde{g}}^2$. Under these conditions, the squark mass is of the order of the gluino mass and not much larger than it, as required by $\epsilon_R$. In addition, as explained above, the mass parameter $A_t$ is of the order of the gluino mass and, hence, the bottom mass corrections are not suppressed in the minimal supergravity model. Indeed, due to the strong correlation between $\mu$ and $M_{1/2}$, the $\mu$ parameter is strongly correlated with the gluino mass, and an approximate expression for the integrals $I(a, b, c)$ may be obtained. Using these correlations, we obtain that the integrals are well approximated by setting $K_1 \simeq 0.6$,

$$\Delta(m_{\tilde{b}}) \simeq 1.2 \tan \beta \frac{\mu}{M_{\tilde{g}}} \left( \frac{\alpha_3}{3\pi} + \left( \frac{3}{2} \right) \frac{Y_f A_t}{8\pi M_{\tilde{g}}} \right)$$

where the factor $3/2$ is to account for the fact of having written a factor $M_{\tilde{g}}^2$ in the denominator, instead of the appropriate factor $m_{\tilde{g}}^2$ (The correlation between the gluino and squark masses will be discussed in the next section). The above expression gives a good approximation to the bottom mass corrections in most of the parameter space consistent with radiative breaking of the electroweak symmetry and bottom–top Yukawa coupling unification. Observe that, due to the strong correlations between $A_t$ and $M_{\tilde{g}}$, and using the fact that the fixed point value of the top and bottom quark Yukawa coupling is approximately given by $Y_f \simeq 16\alpha_3/21$, there is an effective cancellation between the two contributions, which reduces the gluino contribution by about a 30%. In the more precise
numerical result this cancellation is of the order of 25\%. Taking this into account, and the fact that \( M_\beta \simeq \alpha_3 M_{1/2}/\alpha_G \), with \( \alpha_G \) the unifying value of the gauge couplings, the relative bottom mass corrections are given by

\[
\Delta(m_b) \simeq 0.0045 \tan \beta \frac{\mu}{M_{1/2}}.
\]

(48)

From Fig. 1 we observe that, in general, independent of the bottom mass value, the condition of unification of the three Yukawa couplings is such that the larger is \( \tan \beta \), the closer is the top quark Yukawa coupling to its fixed point value. Values of the top quark Yukawa coupling close to its fixed point are obtained for \( \tan \beta \approx 60 \), while \( \frac{Y_t}{Y_f} = 0.6 \) is obtained for \( \tan \beta \approx 40 \). Once Eq.(48) is combined with the numerical results shown in Fig. 3, we obtain a relative bottom mass correction of the order of 45\% for values of \( \tan \beta \) and the top quark mass consistent with the fixed point, while for \( \frac{Y_t}{Y_f} = 0.6 \), the relative bottom mass correction is of the order of 20\%.

An analogous procedure can be applied for the estimation of the relative tau mass corrections. In this case, the heaviest stau mass is of the order of 2.5 times the bino mass, while the lightest stau mass is of the order or somewhat larger than the bino mass, depending on the relative value of \( m_0 \) with respect to \( M_{1/2} \). Under these conditions, over most of the allowed parameter space the loop integral may be approximated by a factor \( K_I \) of the order of 0.85. One can check that, under these conditions the tau mass corrections are not larger than 6\% of the tau mass. Moreover, a relatively large tau mass correction is always associated with a large left - right stau mass mixing, for which the lightest stau becomes the lightest supersymmetric particle. Once the condition of a neutralino being the lightest supersymmetric particle is imposed, the relative tau mass corrections are bounded to be lower than 4\% over most of the allowed parameter space. Since, in addition, a relative tau mass variation affect less the unification condition than a relative bottom mass correction, the tau mass correction effects on the top quark mass predictions are small.

For chosen values of \( M_t \) and \( \tan \beta \) we are now able to calculate the physical bottom mass by running down the corrected \( m_b(M_Z) \) with the standard Model RGE to the scale \( M_b \) and applying the appropriate QCD corrections, Eq.(5). The results are shown in Fig. 6, for the same representative values of the top quark mass and \( \tan \beta \) chosen in the previous figures (which, as we discussed in section 4, for the cases a) and b) correspond to \( M_b \simeq 5.4 \) GeV and \( M_b \simeq 5.85 \) GeV, respectively). The two branches of \( M_b \) correspond to two signs of \( \mu \), the lowest values of \( M_b \) corresponding to negative values of \( \mu \times M_{1/2} \) (or equivalently, positive values of \( \mu \times A_t \)). From Fig. 6 and recalling the results presented in Fig.1, we see
that large corrections to the bottom mass, of the order of 30\% may be used to reconcile $Y_t = Y_b = Y$ with $M_t \leq 160$ GeV. Moreover, it is easy to see that, due to the size of the characteristic corrections, for $\alpha_3(M_Z) > 0.11$, there are no solutions with $\hat{M}_t < M_t$ and a top quark Yukawa coupling within the range of validity of perturbation theory.

The above results may be used to set an upper bound on the top quark mass as a function of the strong gauge coupling value. This upper bound will correspond to the maximum values of $\tan \beta$ and $Y_t$ consistent with a physical bottom mass $M_b \simeq 4.6$ GeV. Larger top quark mass values will correspond to lower values of $\hat{M}_t$ and larger values of $|\Delta m_b|$, implying a physical bottom quark mass outside the present experimental bounds on this quantity, $M_t < 4.6$ GeV. The upper bound may be hence estimated as follows: For a given value of $\alpha_3(M_Z)$, $M_t$ and $\tan \beta$, the relative bottom mass corrections may be computed by using the supersymmetric mass parameters allowed by the condition of radiative breaking of the electroweak symmetry. In addition, $M_t$ may be obtained from the correlations between $M_t$, $\hat{M}_t$ and $\tan \beta$ depicted in Fig. 1. We perform a scanning over the values of $M_t$ and $\tan \beta$, looking for the maximum value consistent with a physical bottom mass $M_b \geq 4.6$ GeV. This value of $M_t$ gives an estimate of the upper bound on this quantity for this given value of $\alpha_3(M_Z)$. The uncertainties associated with this procedure will be discussed below.

For example, for $\alpha_3(M_Z) = 0.12$, the upper bound on the top quark mass is approximately given by $M_t \leq 150$ GeV while the upper bound on the ratio of vacuum expectation values is given by $\tan \beta \leq 39$. These bounds are associated with a bottom mass $\hat{M}_b \simeq 5.6$ GeV. From Eqs. (40), (48) it follows that the approximate bottom mass corrections under these conditions (corresponding to the lowest value of $\mu/M_{1/2} \simeq 1$) are of the order of 18\%. Hence, as required for the solution associated to the upper bound on the top quark mass, the physical bottom mass will be approximately equal to the lower experimental bound on this quantity $M_b \simeq 4.6$ GeV. Analogously, for $\alpha_3(M_Z) = 0.13$ (0.11) the upper bounds read $M_t \leq 170$ (130) GeV and $\tan \beta \leq 43$ (34), for which the bottom mass $\hat{M}_b \simeq 6$ (5.3) GeV. The lowest bottom mass corrections are of the order of 23\% (13 \%) (corresponding to a ratio $\mu/M_{1/2} \simeq 1.2$ (0.85)) and hence the physical bottom mass is approximately equal to 4.6 GeV.

It is important to discuss the uncertainties on the estimate of the top quark mass upper bounds presented above. For the obtention of Fig. 1, the supersymmetric spectrum has been taken to be degenerate at a mass $M_Z$. However, the squark and gluino spectrum arising from the bottom - top Yukawa unification condition is heavy and hence, the top
quark mass could be modified by the supersymmetric particle threshold effects. These effects have been estimated in Ref. [17]. For the characteristic spectrum obtained in these solutions, the resulting top quark mass uncertainties are of the order of 5 - 10 GeV. In addition, in the above we have ignored the possible effects of tau mass corrections. The tau mass corrections are correlated in sign with the bottom mass corrections and their effects on the top quark mass predictions may be hence estimated by a lowering of the relative bottom mass corrections in an amount of the order of the relative tau mass corrections. A modification of the order of 3% of the relative bottom mass corrections gives variations of the top quark mass prediction of the order of 5 - 10 GeV, too. Finally, there is the already discussed $\alpha_3$ - scale uncertainty in the evaluation of the bottom mass corrections, which can also modify the top quark mass predictions in a few percent.

From the above discussion, it follows that the estimate for the upper bound on the top quark mass quoted above may be away from the real bound in 10 - 20 GeV. However, it is important to remark that even after the inclusion of these uncertainties the allowed top quark mass values become much lower than the values obtained for the case in which a negligible bottom mass correction is assumed [16],[8],[17]. Indeed, even after the uncertainties are included, the upper bounds on the physical top quark mass obtained above are of the order of the lower bounds for the same quantity for the case in which the bottom mass corrections are negligible.

A lower bound on the top quark mass is also obtained. However, the lower bounds on the top quark mass is given by $M_t \geq 120$ GeV for $\alpha_3(M_Z) = 0.13$, while for values of $\alpha_3(M_Z) < 0.13$ the lower bound is below the present experimental limit on the top quark mass. In general, for $\alpha_3(M_Z) < 0.13$, large values of the top quark mass $M_t \geq 180$ GeV will be only possible for the case in which we relax the condition of unification of the three Yukawa couplings. For instance, a top quark mass $M_t \simeq 190$ GeV may be achieved for $\tan \beta = 50$, for $Y_t/Y_b \simeq 2$, $\alpha_3(M_Z) \simeq 0.125$ and $\tilde M_b \simeq 5.2$ GeV. As we explained above, since $m_0$ under these conditions may be much larger than $M_{1/2}$, the approximate symmetries required in Ref. [17], Eq.(46), become now possible, and hence the bottom mass corrections can be small, $\tilde M_b \simeq \tilde m_0$.

6 Supersymmetric Particle Spectrum

The properties of the sparticle spectrum are to a large extent determined by the correlation of the mass parameter $\mu$ and the soft supersymmetry breaking parameters and their
large values necessary to fulfill the condition of radiative breaking of the electroweak symmetry (see section 4). For instance, since large values of the parameters $\mu$ and $M_{1/2}$ are required, there will be little mixing in the chargino and neutralino sectors. The lightest (heaviest) chargino is given by a wino (charged Higgsino) with mass equal to $M_2 \simeq 0.8M_{1/2} (|\mu|)$. The lightest neutralino will be given by a bino of high degree of purity and mass $M_B \simeq 0.4M_{1/2}$. These issues have been already discussed in Refs. [19] - [21], and survive in our more precise numerical correlation. We will hence concentrate on the predictions which depend stronger on the precise values of the top quark mass and hence are more sensitive to the change on the top quark mass predictions induced by the bottom mass corrections studied in the previous section. In addition, we will present an analysis of the constraint on the soft supersymmetry breaking parameters coming from the present experimental bounds on the \( b \to s\gamma \) decay rate.

In Fig. 7 we give the behaviour of the CP odd Higgs mass as a function of the lightest chargino mass. As we remarked above, due to the large values of $M_{1/2}$ and $\mu$ appearing in this scheme, the lightest chargino is almost a pure wino, with mass $m_{\chi^+} \simeq 0.8M_{1/2}$, while the heaviest chargino is a Higgsino with mass equal to $|\mu|$. The CP odd Higgs mass squared is given by $m_A^2 = \alpha M_{1/2}^2 + \beta m_0^2 + \text{const.}$, where the constant term is negative. Since $\alpha$ is positive and $\beta$ is negative, we get an upper bound on $m_A^2$,

$$m_A^2 < \alpha M_{1/2}^2$$

which is visible in the figures. Observe that the sensitivity under top quark mass variations of this bound comes through the dependence of the parameter $\alpha$ on $M_t$. The largest values of $m_A$ are obtained for low values of $m_0$, which, could lead to a stau to be the lightest supersymmetric particle (see section 4 and Ref. [20]). From Figs. 7.a and 7.b, we see that for the allowed parameter space consistent with bottom - top Yukawa unification the CP odd Higgs becomes light. Very low values of $m_A$ are, however, excluded by experimental limits. Moreover, the CP odd Higgs mass becomes

$$m_A \leq m_Q \sqrt{\frac{\alpha}{5}}$$

where the factor 5 comes from the strong correlation between $m_Q$ and $M_{1/2}$ (see below). For squark mass parameters $m_Q < 2 \text{ TeV}$, as set in our study and $\alpha \simeq 0.1$ as obtained for $M_t \simeq 150 \text{ GeV}$ and $\tan \beta \simeq 38$, the upper limit on the CP odd Higgs mass, $m_A^u \simeq 250$ GeV. Similar bounds on $m_A$ were obtained in Ref. [20]. However, the upper bounds on $M_{1/2}$ in that work come from an estimate of the constraints on the soft supersymmetry breaking parameters coming from the relic density bound $\Omega h^2 < 1$. Observe that in
Ref. [20], larger values of the top quark mass were used, corresponding to the predictions without bottom mass corrections.

As it is shown in Figs. 7.c and 7.d, once the condition of unification of top and bottom Yukawa couplings is relaxed, the upper bound on the CP odd Higgs mass becomes weaker. The behaviour of the CP odd Higgs mass simply reflects a change in sign of the parameter $\beta$, which becomes positive for these values of the top quark mass and $\tan \beta$.

In Fig. 8 we show the numerical solutions for the lightest stop mass as a function of the gluino mass. We observe a very strong correlation between these two quantities in Figs. 8.a and 8.b, for which unification of top and bottom Yukawa couplings holds. This is easily understood from the behaviour of the squark mass parameters, Eqs. (21) and (22). Indeed, we see that the $A_0$ and $m_0$ dependence of the mass parameters $m_Q^2$ and $m_U^2$ is weak, becoming weaker for top quark mass values close to the infrared fixed point solution. In addition, values of $m_0 \geq M_{1/2}$ are forbidden by the radiative electroweak symmetry breaking condition. Hence, $m_U^2 \simeq m_Q^2 \simeq 5M_{1/2}^2$ for both values of $M_t$. The mixing is dominated by the $A_t$ term. Hence, the lightest stop mass is given by

$$m_{\tilde{t}}^2 \simeq 5M_{1/2}^2 + m_t^2 - A_t m_t.$$

Recalling Eq. (25), and the fact that $M_{1/2} \geq 300$ GeV, we get that in both cases $m_t \simeq KM_{1/2}$ with $K \simeq 2.1 - 2.3$. This implies that $m_t \simeq 0.75M_\beta$, which qualitatively describes the results shown in Figs. 8.a and 8.b.

In Figs. 8.c and 8.d we shows what happens when we depart from the condition of exact unification. Under these conditions large values of $m_0$ are allowed and hence the strong correlation between the gluino mass and the lightest stop quark mass is lost.

In Fig. 9 we plot the charged and lightest CP even Higgs mass spectrum. After radiative corrections, the charged Higgs mass becomes of the order of the CP odd Higgs mass and is hence tightly bounded from above when exact unification of bottom and top Yukawa couplings is required. Due to the moderate values of the top quark mass necessary to achieve unification and the low values of the CP odd Higgs mass, there is a large region of the allowed parameter space where the CP even Higgs mass becomes lighter than $M_Z$. This tendency, however, changes as the charged Higgs particle mass is above 150 GeV, for which the CP even Higgs mass reaches a maximum value, which varies only slightly for larger values of the charged Higgs mass. This behaviour is a general feature of the large $\tan \beta$ solutions and do not depend on the condition of unification of bottom and top Yukawa couplings. From Fig. 9.b, we observe that for a top quark mass $M_t \simeq 150$
GeV and $\tan\beta \simeq 38$, the upper bound on the CP even Higgs mass, $m_h \leq 110 \text{ GeV}$.

In Fig. 10 we present the lightest stau mass spectrum as a function of the lightest chargino one. In Figs. 10.a and 10.b it is easy to identify the region excluded by the requirement of the lightest stau being heavier than the bino. As we discussed before, since for these cases the mixing in the chargino sector is small, the lightest chargino is almost a pure wino with mass $m_{\tilde{\chi}^+} \simeq 0.8 M_{1/2} \simeq 2 M_B$. Hence, this requirement implies that the stau mass should be larger than approximately a half of the lightest chargino mass. Figs. 10.c and 10.d show what happens when we depart from the condition of exact Yukawa unification, the larger values of $m_{\tilde{\chi}^+}$ being associated with larger values of the soft supersymmetry breaking parameter $m_0$.

### 6.1 Experimental constraints coming from $b \to s \gamma$

In our discussion above, we have not addressed the experimental constraints coming from the bounds on the $b \to s \gamma$ decay rate. These bounds can be very relevant in defining the allowed parameter space, particularly in models with a large hierarchy between the squark and CP odd Higgs masses [32]-[33], as occur for the large $\tan\beta$ scenario when the condition of unification of top and bottom Yukawa couplings is required. In addition, for large values of $\tan\beta$ there is an enhancement of the chargino - exchange contribution, with similar physical origin as the one that enhances the bottom mass corrections for this case. The gluino-exchange contributions are also enhanced, although they are still much lower than the chargino - exchange ones [34].

For large values of $\tan\beta$, hence, the chargino contributions becomes sizeable even for a characteristic squark and Higgsino spectrum $m_Q \simeq m_{\tilde{\chi}} \simeq \mathcal{O}(1 \text{ TeV})$, as appears in the model under study. The sign of these contributions, as happens with the bottom mass corrections, depend on the sign of the product of the parameters $\mu$ and $A_t$, related to the chargino masses and the eigenstate stop mass splitting, respectively.

We have set a calculation of the $b \to s \gamma$ rate, following the procedure suggested in Ref. [33]. The gluino contributions were neglected, and the mixing of the first and second generation up squarks was ignored. We computed the rate numerically, according to the results presented in Refs. [32] - [33]. For the allowed soft supersymmetry breaking parameters required for a radiative breaking of $SU(2)_L \times U(1)_Y$ with exact unification of top and bottom Yukawa couplings, $M_t \simeq 150 \text{ GeV}$ and $\tan\beta \simeq 38$, we find,
a) For positive values of $A_t \times \mu$, the rate is enhanced as compared to the one of the Standard Model with one extra Higgs doublet.

b) For negative values of $A_t \times \mu$, the rate is smaller as the one of the Standard Model with one extra Higgs doublet. The chargino contributions partially cancel the charged Higgs ones and the rate becomes of the order of the Standard Model one for squark and Higgsino masses above a lower bound, which is of the order of 300 GeV.

As we discussed above, if we insist in exact bottom - top Yukawa unification, positive values of $A_t \times \mu$ are needed in order to get the right values for the bottom mass. This means that the lower bounds on the CP odd mass will become stronger than in the case of the Standard Model with one additional Higgs doublet. In order to estimate a bound on the charged Higgs mass and the soft supersymmetry breaking parameters, however, the uncertainties in the computation of the rate $b \to s\gamma$ must be addressed. As discussed in Ref. [35], by far the largest uncertainty in the $b \to s\gamma$ rate is the one coming from the choice of the renormalization scale of the Wilson coefficients. This uncertainty is as large as 20 - 30 % of the computed rate [35]. Taking this uncertainty into account, conservative bounds on the soft supersymmetry breaking parameters may be obtained by demanding [35]

$$BR^{\text{theor}}[B \to Xs\gamma] - 2 \times \epsilon < BR^{\mu,\text{exp}}$$

where $\epsilon$ is the theoretical error bar and $BR^{\mu,\text{exp}} \simeq 5.4 \times 10^{-4}$ is the experimental upper bound on this branching ratio [36].

We have computed the bounds on our model according to Eq.(52) and the uncertainties estimated in Ref. [35]. For $M_t \simeq 150$ GeV, tan $\beta \simeq 38$, $A_t \simeq -M_3$, $\mu \simeq -M_{1/2}$ and $m_{H^\pm}^2 \simeq \alpha M_{1/2}^2 + \beta m_0^2$, as approximately required by exact unification with $M_6 \simeq 4.6$ GeV, we find a lower bound on $M_{1/2}$ which is approximately given by 550 GeV. If the theoretical uncertainties are, instead, considered at the one sigma level, the lower bound on $M_{1/2}$ is approximately 700 GeV. No significant bound on $m_0$ arises for the model under study.

Hence, if the above described estimate of theoretical uncertainties is correct, $b \to s\gamma$ will have relevant implications for the model under study. Indeed, if we insist in a squark spectrum below a few TeV, the squark, chargino and Higgs spectrum will be very well defined as follows from Figs. 7 - 9 and the lower bounds on $M_{1/2}$ given above. A more complete study of the rate $b \to s\gamma$, including higher loop effects to cancel the large uncertainties in the rate computation will be necessary, however, before a definite
statement in this direction can be made.

7 Conclusions

In this work, we have studied the conditions for a proper breakdown of the electroweak symmetry in a minimal supersymmetric model with unification of the three third generation fermion Yukawa couplings and universal soft supersymmetric parameters at the grand unification scale. We have shown that the condition of radiative electroweak symmetry breaking implies strong correlations between the different soft supersymmetry breaking parameters. These correlations have implications in the supersymmetric particle spectrum of the theory, which strongly depends only on the universal soft supersymmetry breaking gaugino mass $M_{1/2}$. Moreover, a minimum value of $M_{1/2} \geq 300$ GeV, is implied by the condition of radiative breaking of $SU(2)_L \times U(1)_Y$, leading to a lower bound on the squark and gluino masses of the order of a few hundred GeV. The lightest supersymmetric particle becomes hence a bino with mass $M_{\tilde{B}} \simeq 0.4 \ M_{1/2}$. In addition, the CP odd Higgs and charged Higgs masses become much lower than the characteristic squark spectrum of the theory. For squark masses lower than a few TeV, the heavy Higgs acquire masses of the order of the weak scale.

We have shown that, due to large bottom mass corrections induced through sparticle exchange loops, the predicted values of the top quark mass and $\tan \beta$ are much lower than the values previously estimated in the literature under the assumption that these corrections were negligible. Indeed, we have shown that, for $\alpha_3(M_Z) \simeq 0.12$, a top quark mass $M_t \geq 170$ GeV is difficult to achieve within this scheme. This tight bound on the top quark mass may only be avoided by a relaxation of either the exact unification of the Yukawa couplings or of the universality of the soft supersymmetry breaking parameters at the grand unification scale.

Finally, we have shown that the large negative bottom mass corrections are also associated with an enhancement of the rate of the decay $b \to s\gamma$ with respect to the one of the standard model with one extra Higgs doublet. A lower bound on the universal gaugino mass may be hence obtained from requiring the total decay rate to be below the experimental upper bound on this quantity $M_{1/2} \geq 550$ GeV. This bound depends, however, on an estimate of the QCD uncertainties associated with the rate computation. Consistency between this bound and the requirement of a not too heavy supersymmetric spectrum makes the model very predictable and, hence, easy to test experimentally.
Acknowledgements. S.P. would like to thank P. Chankowski, I. Hall, J. Louis, H. P. Nilles and S. Raby for useful discussions. M.C. and C.W. would like to thank G. Giudice, R. Hempfling and F. Zwirner for useful discussions. M.O. and S.P. are partially supported by the Polish Committee for Scientific Research. The work of M.C. and C.W. is partially supported by the Worldlab.
FIGURE CAPTIONS

Fig. 1. Top quark mass predictions as a function of the strong gauge coupling, within the framework of exact unification of the three Yukawa couplings of the third generation. The solid lines represent constant values of the bottom mass $M_b$, equal to A) 4.6 GeV, B) 4.9 GeV, C) 5.2 GeV, D) 5.5 GeV and E) 5.8 GeV. The dashed lines represent constant values of $\tan \beta$, equal to a) 40, b) 45, c) 50, d) 55 and e) 60. The region to the right of the dotted curve is that one consistent with the unification of gauge couplings and the experimental correlation between $M_t$ and $\sin^2 \theta_W(M_Z)$. The upper dashed line represents the infrared quasi fixed point values for the top quark mass, for which $Y_t(0) \simeq 1$.

Fig. 2. Values of the universal gaugino mass $M_{1/2}$ and the soft supersymmetry breaking parameter $m_0$ consistent with the condition of radiative electroweak symmetry breaking. The plots are done for two characteristic values of the top quark mass and $\tan \beta$ consistent with exact Yukawa coupling unification: a) $M_t = 190$ GeV, $\tan \beta = 55$ and b) $M_t = 150$ GeV, $\tan \beta = 38$, and two values for which $Y_t(0) \simeq 2Y_b(0)$: c) $M_t = 190$ GeV, $\tan \beta = 50$ and d) $M_t = 150$ GeV, $\tan \beta = 30$. The points allowed by the radiative breaking condition, but excluded experimentally, are represented by dots.

Fig. 3. The same as Fig. 2, but for the supersymmetric mass parameter $\mu$ and the universal gaugino mass $M_{1/2}$.

Fig. 4. The same as Fig. 2, but for the ratios of soft breaking parameters $A_0/M_{1/2}$ and $B_0/M_{1/2}$.

Fig. 5. The same as Fig. 2, but for the supersymmetric mass parameter at the grand unification scale $\mu_0$ and the soft supersymmetry breaking parameter $\delta$.

Fig. 6. Predictions for the physical bottom mass as a function of the gluino mass for the same characteristic values of $M_t$ and $\tan \beta$ as in Fig. 2.

Fig. 7. The same as Fig. 2, but for the CP - odd Higgs and the lightest chargino masses.
Fig. 8. The same as in Fig. 2, but for the lightest stop and the gluino masses.

Fig. 9. The same as Fig. 2, but for the charged and the lightest CP - even Higgs masses.

Fig. 10. The same as Fig. 2 but for the lightest stau and the lightest chargino masses.
References


    M.S. Berger, P. Ohmann and R.J.N. Phillips, U. of Wisconsin prep. MAD/PH/755,
    April 1993.


    (1994) 110.


P.H. Chankowski, Diploma Thesis (1990), University of Warsaw;
A. Seidl, Diploma Thesis (1990), Technical University, Munich;


[34] F. M. Borzumati, DESY preprint, DESY 93-090, August 1993.

    A.J. Buras, M. Misiak, M. Munz and S. Pokorski, Max Planck preprint,
    MPI-Ph/93-77, Nov. 1993