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**TWISTOR-LIKE TYPE II SUPERSTRING
WITH THE HETEROTIC $(2,0)$
AND $(4,0)$ WORLD SHEET SUPERSYMMETRY**

V. Chikalov
Institute of Applied Physics, 244007 Sumy, Ukraine

and

A. Pashnev*
International Centre for Theoretical Physics, Trieste, Italy
and
Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
Dubna, Head Post Office, P.O.Box 79, 101 000 Moscow, Russian Federation.†

ABSTRACT

The twistor-like formulation of type II Green-Schwarz superstring with $N = (2, 0)$ and $(4, 0)$ heterotic worldsheet supersymmetry is constructed. Both of the Virasoro constraints are solved with the help of twistors in dimensions $D = 4$ and $D = 6$. As in the case of $N = (1, 0)$ worldsheet supersymmetry, considered earlier, any worldsheet gauge fields enter the actions. On the component level the resulting Lagrangians are classically equivalent to the Green - Schwarz one.

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*E-mail: pashnev@thsun1.jinr.dubna.su

†Permanent address.

1 Introduction

The construction during the last years of the double supersymmetric (twistor-like) models for the superparticle, superstring and supermembranes [2] - [20] made it possible to understand the geometrical nature of the κ - symmetry. The interpretation of the κ - symmetry as transformations belonging to the diffeomorphism group of the corresponding superworldline or superworldsheet has been worked in details for the superparticle [2] - [10] in all ($D = 3, 4, 6, 10$) dimensions. These results have been further extended to the case of the heterotic superstring [6] - [17] moving in the gravitational and Yang - Mills backgrounds in the same dimensions and to the 11 - dimensional supermembrane with $N = 8$ world volume supersymmetry.

The commuting spinor variables, which naturally arise in twistor - like models as superpartners of the target - space Grassmann spinor coordinates θ , play the central role in enforcing of the mass-shell condition for the superparticle and one of the Virasoro constraints for the heterotic superstring. To have both of the Virasoro constraints in the superstring one needs to include an additional set of the commuting spinors. As was shown in [18] such commuting spinors are actually the superpartners of the second Grassmann spinor coordinate θ^2 of the type II superstring and both of Virasoro constraints are on the same footing. In this paper the superworldsheet of the type II superstring was taken to be the heterotic one with $(1, 0)$ supersymmetry. It means that only one of κ - transformations has geometrical nature.

The $(1, 1)$ supersymmetric worldsheet for the type II superstring was considered in detail in [19] and thus the total description of the κ - symmetry of the nonheterotic superstring in $D = 3$ was obtained. However, the attempts to use the same formalism to the $D = 4, 6, 10$ nonheterotic superstring are unsuccessful, because some of the Lagrange multipliers become dynamical.

In the present paper we obtain the Lagrangians for the type II superstring in the flat target superspace with the heterotic $(2, 0)$ and $(4, 0)$ worldsheet supersymmetry. It follows, that half of the κ - transformations in $D = 4$ and $D = 6$ are described geometrically. The resulting component Lagrangians are classically equivalent to the Green - Schwarz [1] Lagrangian with $n = 2$ target - space supersymmetry.

2 $D = 4, N = (2, 0)$

In what follows we use the definitions of the paper [16]. The superworldsheet of the superstring is parametrized by two bosonic ξ^-, ξ^+ and two fermionic $\eta, \bar{\eta}$ coordinates. In the superspace with coordinates $\xi^-, \eta, \bar{\eta}$ act two covariant derivatives

$$D = \partial/\partial\eta + i\bar{\eta}\partial/\partial\xi^{(-)}, \quad \bar{D} = -\partial/\partial\bar{\eta} - i\eta\partial/\partial\xi^{(-)}.$$

Additional bosonic coordinate ξ^+ plays the role of parameter - it means we consider the heterotic $N = (2, 0)$ worldsheet supersymmetry. We introduce left $X_L^\mu, \Theta^{A\alpha}$ and right $X_R^\mu = \overline{X_L^\mu}, \bar{\Theta}^{A\dot{\alpha}} = \overline{\Theta^{A\alpha}}$, chiral superfields, which satisfy, respectively

$$\bar{D}X_L^\mu = \bar{D}\Theta^{A\alpha} = 0, \quad DX_R^\mu = D\bar{\Theta}^{A\dot{\alpha}} = 0, \quad (2.1)$$

The index A corresponds to type II superstring and takes values $A = 1, 2$.

In terms of these chiral superfields and additional real superfield P_μ (Lagrange multiplier), we propose the following action for the $D = 4$, $N = (2, 0)$ type II superstring

$$\begin{aligned}
S = & \frac{1}{2} \int d^2\xi d\eta d\bar{\eta} \left\{ P_\mu \left(\frac{i}{2} (X_L^\mu - X_R^\mu) + \Theta^A \sigma_\mu \bar{\Theta}^A \right) \right. \\
& + X_L^\mu (\partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1 - \partial_+ \Theta^2 \sigma_\mu \bar{\Theta}^2) + X_R^\mu (\Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1 - \Theta^2 \sigma_\mu \partial_+ \bar{\Theta}^2) \\
& - \frac{i}{2} \Theta^1 \sigma^\mu \bar{\Theta}^1 (\partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1 - \Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1) \\
& + \frac{i}{2} \Theta^2 \sigma^\mu \bar{\Theta}^2 (\partial_+ \Theta^2 \sigma_\mu \bar{\Theta}^2 - \Theta^2 \sigma_\mu \partial_+ \bar{\Theta}^2) \\
& - i \Theta^1 \sigma^\mu \bar{\Theta}^1 (\partial_+ \Theta^2 \sigma_\mu \bar{\Theta}^2 - \Theta^2 \sigma_\mu \partial_+ \bar{\Theta}^2) \\
& \left. + i \Theta^2 \sigma^\mu \bar{\Theta}^2 (\partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1 - \Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1) \right\}
\end{aligned} \tag{2.2}$$

The reduction to the case of heterotic superstring $\Theta^2 = 0$ leads to the action of Delduc, Ivanov and Sokatchev [16] with slightly modified superfield P_μ :

$$P_\mu^{DIS} = P_\mu + 2i(\partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1 - \Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1). \tag{2.3}$$

The superfield P_μ is more convenient, because it is inert under the transformations of the $n = 2$ rigid target - space supersymmetry. All the other superfield transform in a standard way:

$$\delta \Theta^A = \epsilon^A, \quad \delta X_L^\mu = 2i\theta \sigma^\mu \bar{\epsilon}. \tag{2.4}$$

The simplest way to establish the correspondence of the action (2.2) with the Green-Schwarz superstring action [1] is to integrate over one of the Grassmann coordinates of the superworldsheet and compare the result with the $N = (1, 0)$ action [18] for the type II superstring. We represent to this end complex coordinate η , integration measure and covariant derivatives D and $\bar{D}$ through the real ones

$$\begin{aligned}
\eta &= \frac{1}{\sqrt{2}}(\eta_1 - i\eta_2), \quad \bar{D} = D_1 - iD_2, \\
D_i &= \frac{\partial}{\partial \eta_i} + i\eta_i \partial_-, \quad d\eta d\bar{\eta} = -2id\eta_1 d\eta_2.
\end{aligned} \tag{2.5}$$

Instead of integration over Grassmann coordinate η_2 we will find the covariant derivative of the Lagrangian $D_2 \mathcal{L} |_{\eta_2=0}$. Let us denote

$$\begin{aligned}
P_\mu |_{\eta_2=0} &= p_\mu, \quad \frac{i}{2} D_2 P_\mu |_{\eta_2=0} = \Pi_\mu, \\
(X_L^\mu + X_R^\mu) |_{\eta_2=0} &= 2X^\mu, \quad (X_L^\mu - X_R^\mu) |_{\eta_2=0} = 2iY^\mu.
\end{aligned} \tag{2.6}$$

As a consequence of the chirality conditions (2.1) the following relations take place:

$$\begin{aligned}
D_2 X_L^\mu &= -iD_1 X_L^\mu, \quad D_2 \Theta^A = -iD_1 \Theta^A, \\
D_2 X_R^\mu &= +iD_1 X_R^\mu, \quad D_2 \bar{\Theta}^A = +iD_1 \bar{\Theta}^A.
\end{aligned} \tag{2.7}$$

After slightly lengthy but straightforward calculations with the help of relations (2.7) the following form of the action (2.2) arises

$$S = \int d^2\xi d\eta_1 \left\{ (-ip_\mu + \Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1 - \partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1 \right.$$

$$\begin{aligned}
& -\Theta^2 \sigma_\mu \partial_+ \bar{\Theta}^2 + \partial_+ \Theta^2 \sigma_\mu \bar{\Theta}^2) \Omega_0 \\
& + (D_1 \Theta^1 \sigma_\mu \bar{\Theta}^1 + \Theta^1 \sigma_\mu D_1 \bar{\Theta}^1 - D_1 \Theta^2 \sigma_\mu \bar{\Theta}^2 - \Theta^2 \sigma_\mu D_1 \bar{\Theta}^1) \Omega_1 \\
& - i(D_1 \Theta^1 \sigma_\mu \bar{\Theta}^1 + \Theta^1 \sigma_\mu D_1 \bar{\Theta}^1)(\Theta^2 \sigma^\mu \partial_+ \bar{\Theta}^2 - \partial_+ \Theta^2 \sigma^\mu \bar{\Theta}^2) \\
& + i(\Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1 - \partial_+ \Theta^1 \sigma_\mu \bar{\Theta}^1)(D_1 \Theta^2 \sigma^\mu \bar{\Theta}^2 + \Theta^2 \sigma^\mu D_1 \bar{\Theta}^2) \\
& + 2\tilde{\Pi}_\mu (Y^\mu + \Theta^A \sigma^\mu \bar{\Theta}^A) + \text{total derivative},
\end{aligned} \tag{2.8}$$

where

$$\Omega_0 = D_1 X - i D_1 \Theta^A \sigma^\mu \bar{\Theta}^A - i \Theta^A \sigma^\mu D_1 \bar{\Theta}^A, \tag{2.9}$$

$$\begin{aligned}
\Omega_1 = & \partial_+ X^\mu - i \partial_+ \Theta^1 \sigma^\mu \bar{\Theta}^1 + i \Theta^1 \sigma^\mu \partial_+ \bar{\Theta}^1 \\
& - i \partial_+ \Theta^2 \sigma^\mu \bar{\Theta}^2 + i \Theta^2 \sigma^\mu \partial_+ \bar{\Theta}^2,
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\tilde{\Pi}_\mu = & \Pi_\mu + i D_1 \Theta^1 \sigma_\mu \partial_+ \bar{\Theta}^1 - i \partial_+ \Theta^1 \sigma_\mu D_1 \bar{\Theta}^1 \\
& - i D_1 \Theta^2 \sigma_\mu \partial_+ \bar{\Theta}^2 + i \partial_+ \Theta^2 \sigma_\mu D_1 \bar{\Theta}^2.
\end{aligned} \tag{2.11}$$

This is exactly the action of the paper [18] for the case of $D = 4$ with additional auxiliary fields $\tilde{\Pi}_\mu$ and Y^μ , which can be excluded due to its equations of motion.

3 $D = 6, N = (4, 0)$

Our conventions in the case of $N = (4, 0)$ heterotic worldsheet supersymmetry once again coincide with those of Ref.[16]. The $D = 6, n = 2$ target superspace coordinates are $X^{\alpha\beta} = -X^{\beta\alpha}$ and $\Theta^{iA\alpha}$, which satisfy the pseudo Majorana reality condition $\bar{\Theta}^{iA\alpha} = \epsilon_{ij} C_\alpha^{\dot{\alpha}} \Theta^{iA\dot{\alpha}}$. Here α, β and i, j are correspondingly $SU(4)$ and $SU(2)$ spinor indices and $A = 1, 2$ as before.

The worldsheet superspace coordinates are $\xi^{(+)}, \xi^{(-)}, \eta^i, \bar{\eta}_i$ together with the harmonic coordinates $u_i^\pm$. All target space coordinates are analytic harmonic superfields:

$$D^+ X^{\alpha\beta} = \bar{D}^+ X^{\alpha\beta} = 0, D^+ \Theta^{+A\alpha} = \bar{D}^+ \Theta^{+A\alpha} = 0 \tag{3.1}$$

where

$$D^\pm = u_i^\pm D^i, \bar{D}^\pm = u_i^\pm \bar{D}^i, \{D_i, \bar{D}^j\} = i \delta_i^j \partial_{(-)}. \tag{3.2}$$

We propose the following action for $D = 6, N = (4, 0)$ type II superstring

$$\begin{aligned}
S = & \int d^2 \xi [du] d^2 \eta^+ \left\{ P_{\alpha\beta} (D^{++} X^{\alpha\beta} - i \Theta^{+A\alpha} \Theta^{+A\beta}) + Q_\alpha^{-A} D^{++} \Theta^{+A\alpha} \right. \\
& \left. + \partial_{(+)} X^{\alpha\beta} (\Theta^{+1\gamma} \Theta^{+1\delta} - \Theta^{+2\gamma} \Theta^{+2\delta}) \epsilon_{\alpha\beta\gamma\delta} \right\},
\end{aligned} \tag{3.3}$$

which almost trivially generalizes the action for the $D = 6, N = (4, 0)$ heterotic superstring [16]. Note that this action, opposite to the actions for $N = (1, 0)$ and $N = (2, 0)$ heterotic worldsheets, does not contain terms quadratic in Θ 's. The Lagrange multipliers $P_{\alpha\beta}$ and Q_α^{-A} are analytic harmonic superfields and $D^{++} = u^{+i} \partial / \partial u^{-i} + i \eta^+ \bar{\eta}^+ \partial_{(-)}$ is the harmonic covariant derivative in the analytic basis.

The action (3.3) is invariant under $n = 2$ global supersymmetry transformations in target superspace ($\epsilon^{\pm A\alpha} = u_i^\pm \epsilon^{iA\alpha}$):

$$\delta X^{\alpha\beta} = i(\epsilon^{-A\alpha} \Theta^{+A\beta} - \epsilon^{-A\beta} \Theta^{+A\alpha}), \delta \Theta^{+A\alpha} = \epsilon^{+A\alpha}, \tag{3.4}$$

$$\delta P_{\alpha\beta} = -2 \epsilon_{\alpha\beta\gamma\delta} (\epsilon^{-1\gamma} \partial_{(+)} \Theta^{+1\delta} - \epsilon^{-2\gamma} \partial_{(+)} \Theta^{+2\delta}), \tag{3.5}$$

$$\delta Q_\alpha^{-A} = 2(-1)^A \epsilon_{\alpha\beta\gamma\delta} \partial_{(+)} X^{\beta\gamma} \epsilon^{-A\delta} + 2i P_{\alpha\beta} \epsilon^{-A\beta}. \tag{3.6}$$

The simplest way to see that the action (3.3) describes Green - Schwarz superstring is to integrate over the Grassmann and harmonic coordinates. From the equations of motion for Lagrange multipliers Q_α^{-A}

$$D^{++}\Theta^{+A\alpha} = 0 \quad (3.7)$$

it follows

$$\Theta^{+A\alpha} = \theta^{\alpha i} u_i^+ + \eta^+ \lambda^\alpha + \bar{\eta}^+ \bar{\lambda}^\alpha - i\eta^+ \bar{\eta}^+ \partial_{(-)} \theta^{\alpha i} u_i^-. \quad (3.8)$$

In terms of the following component fields

$$x^{\alpha\beta} = \int [du] X^{\alpha\beta} |_{\eta=0} \quad (3.9)$$

$$\theta^{i\alpha} = \int [du] u^{-i} \Theta^+ |_{\eta=0} \quad (3.10)$$

$$\lambda^\alpha = \int [du] D^- \Theta^+ |_{\eta=0}, \quad (3.11)$$

$$\bar{\lambda}^\alpha = \int [du] \bar{D}^- \Theta^+ |_{\eta=0} \quad (3.12)$$

$$p^{\alpha\beta} = \int [du] P^{\alpha\beta} |_{\eta=0} \quad (3.13)$$

after the integration the action (3.3) takes form

$$\begin{aligned} S = \int d^2\tau \{ & p_{\alpha\beta} (\pi_{(-)} + 2\bar{\lambda}^1 \lambda^1 + 2\bar{\lambda}^2 \lambda^2)^{\alpha\beta} + 2\pi_{(+)}^{\alpha\beta} (\bar{\lambda}^1 \lambda^1 - \bar{\lambda}^2 \lambda^2)^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \\ & + i\partial_{(+)} x^{\alpha\beta} (\partial_{(-)} \theta^{1i} \theta_i^1 - \partial_{(-)} \theta^{2i} \theta_i^2)^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \\ & - i\partial_{(-)} x^{\alpha\beta} (\partial_{(+)} \theta^{1i} \theta_i^1 - \partial_{(+)} \theta^{2i} \theta_i^2)^{\gamma\delta} \epsilon_{\alpha\beta\gamma\delta} \\ & - (\partial_{(-)} \theta^{1i\alpha} \theta_i^{1\beta} \partial_{(+)} \theta^{2j\gamma} \theta_j^{2\delta} - \partial_{(+)} \theta^{1i\alpha} \theta_i^{1\beta} \partial_{(-)} \theta^{2j\gamma} \theta_j^{2\delta}) \epsilon_{\alpha\beta\gamma\delta} \}, \end{aligned} \quad (3.14)$$

where

$$\pi_{(\pm)}^{\alpha\beta} = \partial_{(\pm)} x^{\alpha\beta} + \frac{1}{2} i \partial_{(\pm)} \theta^{i\alpha} \theta_i^\beta - \frac{1}{2} i \partial_{(\pm)} \theta^{i\beta} \theta_i^\alpha \quad (3.15)$$

In this form the action simply reproduces the action written in [18] for the $D = 6, n = 2$ target superspace.

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