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Effects of the Back Reaction in the Brans-Dicke-Dilaton Cosmology

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Abstract

Effects of the back reaction are studied in the Brans-Dicke-Dilaton cosmology in the 4-dimensional spacetime. Especially, the extended inflationary model is examined in the presence of the back reaction. As quantized matter, conformally-invariant quantum fields are considered. In the conformally flat spacetime, the expectation value of the stress-tensor is exactly obtained. Cosmological solutions are examined in the radiation-dominant and non-vanishing cosmological constant universes. Most important contributions come from the trace-anomaly part of the stress tensor. In the most of the parameter region and boundary conditions, solutions behave quite differently from the classical solutions even for late epoch of the big bang. Only by fine tuning of the boundary conditions, solutions, whose behaviours are close to the classical ones, are obtained. The sign of the fourth-derivative term of the scale factor in the stress-tensor is very important, and it determines whether there exist stable solutions or not.

1 Introduction

Effects of the back reaction have been a long-standing problem in the quantum field theory in the curved spacetime. In the final stage of the black hole evaporation and the early stage of the big bang cosmology, the back reaction must give significant effects. Since the work by Callan, Giddings, Harvey and Strominger on black hole evaporation in the 2-dimensional dilaton gravity[1], there has been renewed interest in this problem.

In the present paper, we shall study the effects of the back reaction in the Brans-Dicke-Dilation (BDD) cosmology in the 4-dimensional spacetime. As quantized matters, we shall consider conformally-invariant quantum fields. We shall also restrict ourselves to the conformally flat spacetime. In this case, we can exactly obtain the expectation value of the stress-energy tensor[2]. Therefore in a sense, study in the present paper can be viewed as a 4-dimensional extension of the dilaton cosmology in 2 dimensions[3]. However, as the structure of the stress-energy tensor is very different in 4 and 2 dimensions, it is expected that the results must be very different in these two cases.

Though the couplings of the BDD field to the gravity and other fields are very restricted by the present observations, it might have played an important role in the early stage of the big bang. In fact in the framework of the BDD gravity, the extended inflationary cosmology was proposed[4]. Contrary to the old inflationary model, it predicts a power-law inflation and gives a natural solution to the "graceful exit" problem. Though a few modifications are necessary, the basic idea of the soft inflationary scenario[5] is contained in the extended inflationary model. Therefore, we shall examine the extended inflationary model in the presence of the back reactions, and see if desirable behaviours of classical solutions remain unchanged.

In Sec.II, we shall explain the BDD gravity and give the field equations. Classical solutions for the radiation dominant universe and also for the universe with cosmological constant are given. The renormalized stress-energy tensor and the trace anomaly

are explained. In Sec.III, qualitative study of the field equations is given. In the Einstein gravity, the stability of the Friedmann universe in the presence of the back reactions has been studied in various ways[6, 7, 8]. Some of those arguments are also useful for the present BDD cosmological model. Approximate solutions in the BDD gravity for the radiation dominant universe are given. In Sec.IV, results of the numerical study are given. We investigate solutions with various boundary conditions. Only by the fine tuning of the boundary conditions, solutions which behave closely to the classical solutions are obtained. The sign of the fourth-derivative term of the scale factor in the stress-energy tensor is very important, and it determines whether there exist stable solutions or not. This result agrees with the qualitative discussion in Sec.III. In Sec.V, the stability of the Minkowski spacetime in the presence of the quantum back reaction will be studied. It will be shown that the sign of the fourth-derivative term determines the stability of the Minkowski spacetime. Section VI is devoted for conclusion.

2 The BDD theory and the field equations

Action of the BDD theory[9] is given by

$$S = \int d^4x \sqrt{-g} \left[-\Phi R + \omega \left(\frac{\partial_\mu \Phi \partial^\mu \Phi}{\Phi} \right) \right] + (\text{matter part}), \quad (2.1)$$

where Φ is the BDD field and ω is a dimensionless constant which parametrizes the theory. At present, the value of ω is restricted as $\omega > 500$ by time-delay experiments. As matters, we shall consider conformally-invariant quantized fields in the framework of the semiclassical approximation. Contribution from the cosmological constant is also considered. From the action (2.1), field equations are obtained in the semiclassical approximation as,

$$-\nabla_\mu \nabla_\nu \Phi + g_{\mu\nu} \square \Phi + \omega \left\{ \frac{g_{\mu\nu} (\partial\Phi)^2}{2\Phi} - \frac{(\partial_\mu \Phi \partial_\nu \Phi)}{\Phi} \right\}$$

$$+\Phi(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R) = 8\pi(T_{\mu\nu}^{class} + <T_{\mu\nu}>_{ren}). \quad (2.2)$$

$$2\omega\Box\Phi - \omega\frac{(\partial\Phi)^2}{\Phi} + R\Phi = 0, \quad (2.3)$$

where the convention of Birrell and Davies[2] is used, and $T_{\mu\nu}^{class}$ is the classical part of the stress-energy tensor coming from particles in thermal equilibrium, the cosmological constant, etc, and $<T_{\mu\nu}>_{ren}$ is the renormalized stress-energy tensor in the curved spacetime[10]. As we are considering conformally-invariant matters in the conformally flat spacetime, the above quantities have well-defined expressions. Explicit form of the renormalized stress-energy tensor is the following

$$\begin{aligned} <T_{\mu\nu}>_{ren} &= -\frac{\alpha^{(1)}}{6}H_{\mu\nu} + \beta^{(3)}H_{\mu\nu}, \\ {}^{(1)}H_{\mu\nu} &= 2R_{\mu\nu} - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}, \\ {}^{(3)}H_{\mu\nu} &= \frac{1}{12}g_{\mu\nu}R^2 - R^{\rho\sigma}R_{\rho\mu\sigma\nu} \\ &= R_{\mu}^{\rho}R_{\rho\nu} - \frac{2}{3}RR_{\mu\nu} - \frac{1}{2}R_{\rho\sigma}R^{\rho\sigma}g_{\mu\nu} + \frac{1}{4}R^2g_{\mu\nu}, \end{aligned} \quad (2.4)$$

where the parameters α and β have the following values for spin= 0, 1/2 (two-component) and 1 particles,

spin	$(2880\pi^2)\alpha$	$(2880\pi^2)\beta$
0	1	1
$\frac{1}{2}$	3	$\frac{11}{2}$
1	-18 or 12	62.

(2.5)

In the rest of the discussion, we shall restrict ourselves to the homogeneous, isotropic and spatially flat spacetime. Then the spacetime is described by the $k=0$ Robertson-Walker metric,

$$ds^2 = dt^2 - a^2 \sum_{k=1}^3 (dx_k)^2, \quad (2.6)$$

where the scale factor a is a function of t , $a = a(t)$.

The BDD field Φ has the dimension (energy)². It is convenient to use the following parameterization,

$$\Phi = G^{-1}e^{-2\phi}, \quad a = e^h, \quad (2.7)$$

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where the fields ϕ and h are dimensionless functions of t . There is an ambiguity in the above parameterization of Φ . We always take $\phi(t_0) = 0$, where t_0 is the unit time at which the boundary condition of the field equations is given in the later calculation. We use the "Planck mass" $M_P^2 \equiv \frac{3}{8\pi G}$ as unit for the energy density ρ and the pressure P in the stress-energy tensor,

$$\rho = T_{00}^{class} = M_P^4 \hat{\rho}, \quad P = -T_i^{i, class} = M_P^4 \hat{P} \quad (i = 1, 2, 3). \quad (2.8)$$

We also use unit $M_P^{-1}T$ for time t

$$t = (M_P^{-1}T)\tau, \quad (2.9)$$

where T is a numerical constant and we take $T = 10^4$ in the later calculation.

Field equations of ϕ and h are obtained from eqs.(2.2) and (2.3). At the classical level, i.e., by ignoring the contribution from $<T_{\mu\nu}>_{ren}$, they are given as[11].

$$\phi'' - 2(\phi')^2 + 3h'\phi' + \frac{T^2 e^{2\phi}}{2(3+2\omega)}(\hat{\rho} - 3\hat{P}) = 0, \quad (2.10)$$

$$(h')^2 = T^2 e^{2\phi} \hat{\rho} + \frac{2\omega}{3}(\phi')^2 + 2h'\phi', \quad (2.11)$$

where $'$ denotes the derivative with respect τ . Conservation of the stress-energy tensor also gives the following equation,

$$\hat{\rho}' = -3h'(\hat{\rho} + \hat{P}). \quad (2.12)$$

To solve the above classical field equations, we need equation of state, i.e., the relation between $\hat{\rho}$ and $\hat{P}$, and the boundary conditions, e.g., $h(\tau_0)$, $\phi'(\tau_0)$ and $\hat{\rho}(\tau_0)$. In the radiation dominant universe, the equation of state is $\hat{P} = \frac{1}{3}\hat{\rho}$. In this case, eqs.(2.10),(2.11) and (2.12) are easily solved and the solution is

$$\begin{aligned} \frac{\Phi}{G} &= e^{-2\phi} = 1, \\ a &= e^h = \tau^{1/2}, \\ \hat{\rho} &= \{4T^2\tau\}^{-1}. \end{aligned} \quad (2.13)$$

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The above solution coincides with that in the Einstein gravity. Similarly for the universe with non-vanishing cosmological constant $\dot{\rho} = -\dot{P} = \dot{\rho}_0$ (=constant), the solution is

$$\begin{aligned}\frac{\Phi}{G} &= e^{-2\phi} = \left[1 + \frac{T\sqrt{\tilde{\rho}_0}}{\Omega}(\tau - \tau_0)\right]^2, \\ a = e^{\phi} &= \left[1 + \frac{T\sqrt{\tilde{\rho}_0}}{\Omega}(\tau - \tau_0)\right]^{\omega + \frac{1}{2}},\end{aligned}\quad (2.14)$$

where $\Omega^2 = \frac{(3+2\omega)(5+6\omega)}{12}$. This solution gives the extended inflationary scenario in the BDD gravity, and the scale factor increases in the power-law for large τ , in contrast with the exponential increase in the old inflation in the Einstein gravity. We shall examine if the above desirable behaviours of the classical solutions remain unchanged in the presence of the quantum back reaction.

Including the effect of the quantum back reaction, the field equations are given as,

$$\phi'' - (\phi')^2 + 3h'\phi' + \frac{3}{2\omega}(h'' + 2(h')^2) = 0, \quad (2.15)$$

$$(h')^2 = T^2 e^{2\phi} \dot{\rho} + \frac{2\omega}{3}(\phi')^2 + 2h'\phi' + e^{2\phi}(m(2h'h''' - (h')^2 + 6(h')^2 h'') + l(h')^4), \quad (2.16)$$

where the parameters m and l are

$$\begin{aligned}m &= \frac{3}{T^2} \sum_{particles} \alpha, \\ l &= \frac{3}{T^2} \sum_{particles} \beta,\end{aligned}\quad (2.17)$$

and the above summations are taken over particles which are effectively massless at that period of the universe. As the boundary condition for the numerical calculation, we shall input the values of $h'(\tau_0)$, $h''(\tau_0)$ and $\phi'(\tau_0)$.

We shall solve the above field equations by numerical calculation in later sections. Before presenting results of the calculation, we shall discuss the qualitative behaviour of solutions in the following section.

3 Qualitative study of the field equations

The results of the numerical calculation of the field equations (2.15) and (2.16) will be given in Sec.IV. In this section, we examine the qualitative behaviours of the solutions. In the Einstein gravity, the effects of the back reaction on the cosmological solutions have been studied by various methods. In Ref.[7], it was shown that the field equations can be mapped to a potential problem with a (anti-)dissipation. We find that this argument is useful also for the qualitative study of the cosmological solutions in the BDD gravity. Therefore, let us start with reviewing the case of the Einstein gravity.

The Einstein gravity corresponds to the $\omega \rightarrow +\infty$ limit of the BDD gravity. In this limit, $\phi \rightarrow 0$, and the field equation is

$$(h')^2 = T^2 \dot{\rho} + m(2h'h''' - (h'')^2 + 6(h')^2 h'') + l(h')^4. \quad (3.18)$$

From eqs.(2.5) and (2.17), we assume $l > 0$. Eq.(3.18) can be mapped to the equation of motion of a (anti-)dissipative system with a potential. For the radiation-dominant universe $\dot{\rho} = c/a^4$, eq.(3.18) is rewritten as,

$$x \equiv \left(\epsilon \frac{a'}{a}\right)^{1/2}, \quad \epsilon \equiv \text{sgn}(a'), \quad (3.19)$$

$$\begin{aligned}\frac{d}{d\tau} \left(\frac{1}{2} (x')^2 + V_R \right) &= -3\epsilon x'^2 (x')^2 + \frac{\epsilon c}{2m} \frac{1}{a^4}, \\ V_R &= \frac{1}{8m} \left[-\frac{c}{a^4} \frac{T^2}{x^2} - x^2 + \frac{l}{3} x^6 \right].\end{aligned}\quad (3.20)$$

Similarly for the universe with non-vanishing cosmological constant.

$$\begin{aligned}\frac{d}{d\tau} \left(\frac{1}{2} (x')^2 + V_{CC} \right) &= -3\epsilon (x')^2 (x')^2, \\ V_{CC} &= \frac{1}{8m} \left[-\frac{\dot{\rho}_0}{3} \frac{T^2}{x^2} - x^2 + \frac{l}{3} x^6 \right].\end{aligned}\quad (3.21)$$

We assume that the energy density of (classical part of) the matters is sufficiently low compared with the energy density given by the Planck mass, i.e., $a(\tau_0)^4 > 4lT^2 \dot{\rho}(\tau_0) =$

$4/T^2 \frac{c}{a(\tau_0)^3}$ for the radiation-dominant universe, and $1 > 4/T^2 \tilde{\rho}_0$ for the universe with the cosmological constant. Then the potentials V_R and V_{CC} have the form in Fig.1. and there are two extrema A and B. For V_R , the position of A moves as $x_A \sim (a(\tau))^{-4}$. It is easily seen that the extremum A essentially corresponds to the classical solutions[7]. In fact, if we assume that the system stays always at A (by ignoring the movement of A), the scale factor a is given by

$$a(\tau) \sim (\sqrt{cT\tau})^{1/2}$$

for the expanding radiation-dominant universe, and

$$a(\tau) \sim e^{\sqrt{\frac{2n}{3}}T\tau}$$

for the universe with the cosmological constant $\tilde{\rho}_0$. It is obvious from the form of the potential V_R and V_{CC} that these solutions, which correspond to the extremum A, are unstable for $m > 0$ and stable $m < 0$. Actually for the radiation-dominant universe, an asymptotic solution for large τ is obtained for $m < 0$ as[12]

$$h' \simeq \frac{2}{3\tau} \left[1 + \sin\left(\frac{\tau - \tau_0}{n}\right) + \frac{2n}{3\tau} \cos\left(\frac{\tau - \tau_0}{n}\right) \sin\left(\frac{\tau - \tau_0}{n}\right) + \dots \right], \quad (3.22)$$

where $n = \sqrt{-m}$ and we have assumed $n \ll 1$. On the other hand, the extremum B corresponds to solutions which expand or contract very rapidly for the Planck time scale, and they are not physically acceptable cosmological solutions.

We examined solutions to eq.(3.18) by the numerical method, and found that the result supports the above qualitative argument. For $m > 0$, with any boundary condition, the solution is unstable and expands or contracts very rapidly for the Planck time scale. On the other hand for $m < 0$, solutions are stable. However, with the most of the boundary conditions, they oscillate with the period of the Planck time scale. Therefore, the asymptotic solution (3.22) exhibits general features of solutions for $m < 0$.

The above qualitative discussion in terms of the potential problem can be applied to the case of the BDD cosmology with small modification. It is not so difficult to

show that eqs.(2.15) and (2.16) can be rewritten in terms of $x = \left(\epsilon \frac{\phi'}{a}\right)^{1/2}$ as follows.

$$\begin{aligned} \phi'' - (\phi')^2 + 2\epsilon x^2 \phi' + \frac{2}{\omega}(\epsilon x x' + x^4) &= 0, \\ \frac{d}{d\tau} \left(\frac{1}{2} (x')^2 + V(x) \right) &= -3\epsilon x^2 (x')^2 + \frac{3}{8m} \epsilon (1 + \kappa) T^2 \tilde{\rho} \\ &\quad + \frac{\epsilon^{-2\phi}}{4m} \left\{ \phi' x^2 + \frac{2\omega}{3x^3} (\phi')^2 x' + \frac{2\epsilon}{x} \phi' x' \right\}, \quad (3.23) \\ V(x) &\equiv \frac{1}{8m} \left[-\frac{T^2 \tilde{\rho}}{x^2} - \epsilon^{-2\phi} x^2 + \frac{l}{3} x^6 \right], \end{aligned}$$

where $\kappa = \frac{1}{3}$ and $\tilde{\rho} = \frac{\epsilon}{a^4}$ for the radiation-dominant universe, and $\kappa = -1$ and $\tilde{\rho} = \tilde{\rho}_0$ for the universe with the cosmological constant $\tilde{\rho}_0$. Therefore, the form of the potential is the same with V_R and V_{CC} . As $\epsilon^{-2\phi}$ increases for the expanding universe, the extremum A moves to the y -axis as V_R in the Einstein gravity. As in the Einstein gravity, we can expect that stable solutions exist for $m < 0$. It is obvious that most of the terms on the RHS of eq.(3.23) work as dissipation to the potential system for the case of $m < 0$ and the expanding universe. Field equation of ϕ does not contain higher-derivative terms, and therefore stable behaviour of ϕ is expected for stable $x = x(\tau)$. Actually for sufficiently large ω , the asymptotic solution for the radiation-dominant universe (3.22) gives

$$\phi' \simeq -\frac{3}{2\omega} h' + \text{const.} \quad (3.24)$$

It is not so difficult to verify that (3.22) and (3.24) are asymptotic solution to (2.15) and (2.16) for large τ

4 Numerical calculations

In this section, we shall give the solutions to eqs.(2.15) and (2.16) obtained by the numerical method. Solutions exhibit different behaviours for $m < 0$, $m = 0$ and $m > 0$, as expected from the qualitative discussion in Sec.III. We shall consider these three cases separately.

4.1 $m < 0$

We shall show the results for $2880\pi^2 \sum \alpha = -18$ and $2880\pi^2 \sum \beta = 62$. However, qualitative behaviours of the solutions are insensitive to the values of α and β . We found that the effect of the back reaction is very important even for small α and β .

A. Radiation-dominant universe; $P = \frac{1}{2}\rho$

As the boundary condition, we input the values of $\phi'(\tau_0)$, $h'(\tau_0)$ and $h''(\tau_0)$, and fix the energy density as $\bar{\rho}|_{\tau=\tau_0} = \frac{1}{4\tau_0^2}$. We investigated the numerical solutions with various boundary conditions. We summarize the result in the following.

(A.1) The classical boundary condition; $\phi' = 0$, $h' = \frac{1}{2}$, $h'' = -\frac{1}{2}$; Fig. 2.A.1

From the solutions to the classical field equation, we can fix the boundary condition. We call this boundary condition the classical boundary condition. We obtained the solution by the numerical method with this boundary condition, and we found that the solution is very close to the classical solution (2.13). There are no oscillating terms in both $a(\tau)$ and $\phi(\tau)$.

(A.2) $\phi' = 0$, $h' = \frac{1}{2}$, $h'' = -\frac{T}{2}$; Fig. 2.A.2

This boundary condition corresponds to $|R| \sim 10^{\frac{M_P^2}{T^2}} |h''| \sim \frac{10}{T} M_P^2$ at $\tau = \tau_0$. The solution is close to the classical one (2.13). However, both $a(\tau)$ and $\phi(\tau)$ oscillate with the period $\frac{T}{M_P} (2\pi\sqrt{-m}) \sim \frac{1}{4M_P}$. We also verified $< h'(\tau)\tau > \sim \frac{1}{2}$, which means $a(\tau) \sim t^{1/2}$ instead of $t^{2/3}$ in the asymptotic solution (3.22).

(A.3) $\phi' = 0$, $h' = \frac{1}{2}$, $h'' = \frac{2}{3\sqrt{-m}}$; Fig. 2.A.3

This boundary condition corresponds to $|R| \sim \frac{10M_P^2}{T^2\sqrt{-m}}$ at $\tau = \tau_0$. The scale factor $a(\tau)$ expands more rapidly than the classical solution. Both $a(\tau)$ and $\phi(\tau)$ have oscillating terms with the period $\sim \frac{1}{4M_P}$. We observed $< h'(\tau)\tau > \sim 0.84$.

(A.4) $\phi' = -\frac{1}{\omega}$, $h' = \frac{2}{3}$, $h'' = \frac{2}{3\sqrt{-m}}$; Fig. 2.A.4

This boundary condition corresponds to the asymptotic solution (3.22), and $|R| \sim \frac{10M_P^2}{T^2\sqrt{-m}}$ at $\tau = \tau_0$. Both $a(\tau)$ and $\phi(\tau)$ oscillate with the period $\sim \frac{T}{M_P} (2\pi\sqrt{-m})$. We also observed $< h'(\tau)\tau > \sim 0.73$. However, the asymptotic solution (3.22) predicts $< h'(\tau)\tau > \sim \frac{2}{3} = 0.67$ Therefore, there remains small but finite effect which is not

taken into account by the adiabatic approximation.

(A.5) $\phi' = 0$, $h' = \frac{1}{2}$, $h'' = -\frac{T^2}{2}$; Fig. 2.A.5

At $\tau = \tau_0$, this boundary condition gives $|R| \sim 10M_P^2$. The behaviour of the solution is very different from the classical solution. Both $a(\tau)$ and $\phi(\tau)$ oscillate, and because of the effect of dissipation, the amplitudes of $a(\tau)$ and $\phi(\tau)$ decrease fairly rapidly.

B. Universe with non-vanishing cosmological constant; $P = -\rho$

We fix the value of the cosmological constant as $\bar{\rho} = \frac{1}{T^4}$. However, qualitative behaviours of the solutions are insensitive to the value of the cosmological constant as long as it is much smaller than M_P^4 . We obtained solutions with various boundary conditions.

(B.1) The classical boundary condition; $\phi' = -\frac{1}{\Omega}$, $h' = \frac{\omega+\frac{1}{2}}{\Omega}$, $h'' = -\frac{\omega+\frac{1}{2}}{\Omega^2}$, $\Omega^2 = \frac{(3+2\omega)(5+6\omega)}{12}$; Fig. 3.B.1

With this boundary condition, the solution is very close to the classical solution (2.14), and there are no oscillating terms in both $a(\tau)$ and $\phi(\tau)$. We observed

$$h' \left[1 + \frac{1}{\Omega} (\tau - 1) \right] \sim \frac{\omega + \frac{1}{2}}{\Omega}, \phi' \left[1 + \frac{1}{\Omega} (\tau - 1) \right] \sim -\frac{1}{\Omega}.$$

(B.2) $\phi' = -\frac{1}{\Omega}$, $h' = \frac{\omega+\frac{1}{2}}{\Omega}$, $h'' = -\frac{\omega+\frac{1}{2}}{\Omega^2} T$; Fig. 3.B.2

This boundary condition corresponds $|R| \sim \frac{10\omega+\frac{1}{2}}{T} M_P^2$. Expansion rate is close to the classical one, and we observed $< h' [1 + \frac{1}{\Omega} (\tau - 1)] > \sim \frac{\omega+\frac{1}{2}}{\Omega}$. However both $a(\tau)$ and $\phi(\tau)$ oscillate with the period $\sim \frac{T}{M_P} 2\pi\sqrt{-m}$.

(B.3) $\phi' = -\frac{1}{\Omega}$, $h' = \frac{\omega+\frac{1}{2}}{\Omega}$, $h'' = -\frac{\omega+\frac{1}{2}}{\Omega^2} 10^2 \cdot T^2$; Fig. 3.B.3

This boundary condition corresponds to $|R| \sim \frac{10^3}{T^2} \cdot \frac{\omega+\frac{1}{2}}{\Omega^2} M_P^2$. Expansion rate of this solution is different from that of the classical solution, and we observed

$$< h' \left[1 + \frac{1}{\Omega} (\tau - 1) \right] > \sim 1.2 \cdot \frac{\omega + \frac{1}{2}}{\Omega}.$$

The periods of the oscillation of $a(\tau)$ and $\phi(\tau)$ are both $\sim \frac{T}{M_P} 2\pi\sqrt{-m}$.

(B.4) $\phi' = -\frac{1}{\Omega}$, $h' = \frac{\omega+\frac{1}{2}}{\Omega}$, $h'' = -\frac{\omega+\frac{1}{2}}{\Omega^2} T^2$; Fig. 3.B.4

This boundary condition corresponds to $|R| \sim \frac{\omega+\frac{1}{2}}{\Omega^2} M_P^2$. The solution $a(\tau)$ and $\phi(\tau)$

behaves quite differently from the classical solution. Strong effect of dissipation is observed.

In conclusion, for $m < 0$ there are stable solutions to the semi-classical field equations for most of the boundary conditions. However, fine tuning of the boundary condition is required to obtain the solutions which are close to the classical ones for sufficiently large t .

4.2 $m = 0$

In the case $m = 0$, there are no higher-derivative terms in the semi-classical field equations. We numerically studied the solutions for the radiation-dominant universe and the universe with the cosmological constant. For both cases, we took the classical boundary conditions; $h' = \frac{1}{2}, \dot{\rho}|_{r=r_0} = \frac{1}{4T^2}$ for the radiation-dominant universe, and $h' = \frac{\omega+\frac{1}{2}}{\alpha}, \dot{\rho} = \frac{1}{T^2}$ for the universe with the cosmological constant. As shown in Fig. 4, the solutions are very close to the classical ones in both cases.

4.3 $m > 0$

We show the results for $2880\pi^2\alpha = 18$ and $2880\pi^2\beta = 62$. The coefficients of the trace anomaly are small, but the back reaction gives significant effect for the cosmological solutions. We numerically obtained solutions for both radiation-dominant and non-vanishing cosmological constant universes. As shown in Fig. 5, the solutions exhibit singular behaviour within the time $t < \frac{1}{M_P}$, even for the classical boundary conditions. This result agrees with the qualitative discussion on the field equations given in Sec. 3.

5 Stability of the Minkowski spacetime

In the previous sections, we showed that the sign of the higher-derivative term in the stress-energy tensor is important for the stability problem of the cosmological solutions. For $m < 0$, with appropriate boundary conditions, solutions which are

close to the classical ones are obtained. This result is very important for the soft-inflationary scenario which is based on the power-law expanding solution for the universe with non-vanishing cosmological constant. However for $m > 0$, for any boundary condition, there are no regular solutions which have physical interpretation.

Stability of the Minkowski spacetime has been also discussed in the presence of the quantum back reaction in the Einstein gravity[13, 7]. It is important to see if the above regular or singular behaviour of the cosmological solutions is related with the stability or instability of the Minkowski spacetime in the BDD gravity. We shall consider this problem in this section.

To this end, we shall follow the discussion by Horowitz and Wald[13] We parameterize the fields $g_{\mu\nu}$ and Φ as

$$g_{\mu\nu} = (1+w)\eta_{\mu\nu}, \quad \Phi = G^{-1}(1-2\varphi), \quad (5.25)$$

and to the first order of w and φ , the field equations (2.15) and (2.16), by setting $\rho = P = 0$, are

$$\begin{aligned} (\partial_\mu \partial_\nu - \eta_{\mu\nu} \Box_M)(w - \varphi - 8\pi G \frac{T^2}{3} m \Box_M w) &= 0, \\ \Box_M(2\omega\varphi - 3w) &= 0, \end{aligned} \quad (5.26)$$

where $\Box_M = \eta^{\mu\nu} \partial_\mu \partial_\nu$. It is not so difficult to show that the general solution to eq.(5.26) is given as follows,

$$\begin{aligned} w &= r^\alpha x_\alpha + r + \frac{1}{2\omega+3} g(x) + f(x), \\ \varphi &= q^\alpha x_\alpha + q + \left(1 - \frac{3}{2\omega(2\omega+3)}\right) g(x) - \frac{3}{2\omega} f(x), \end{aligned} \quad (5.27)$$

where r^α, r, q^α and q are constants, and functions $g(x)$ and $f(x)$ satisfy the following equations,

$$\begin{aligned} \left[\Box_M - \frac{2\omega+3}{2\omega K}\right]f(x) &= 0, \quad K = \frac{8\pi T^2}{3} m, \\ \Box_M g(x) &= 0. \end{aligned} \quad (5.28)$$

In the solution (5.27), the term $r^\alpha x_\alpha + r$ can be eliminated by a gauge transformation. For $m > 0$, the mass squared of $f(r)$, $-\frac{2\omega+2}{2\omega K}$, is negative, and because of that the Minkowski spacetime is unstable. On the other hand for $m < 0$, $f(r)$ is massive and it describes stable fluctuation. However, in contrast with the Einstein gravity, there is always massless fluctuation which is described by $g(r)$. Its appearance is naturally expected because there is no potential term for the BDD field Φ . To see whether $g(x)$ causes instability of the Minkowski spacetime, we have to examine higher-order terms. Also the linear term in the solution φ , $q^\alpha x_\alpha + q$, has physical meaning, and the Minkowski spacetime is generally unstable to the "linear-dilaton" spacetime.

We have checked the above observation by the numerical method. The results are shown in Fig 6.

6 Conclusion

In the present paper, we studied effects of the back reaction in the BDD cosmology. Main motivation is to see if desirable behaviours of the classical solutions for the extended-inflationary scenario remain unchanged in the presence of the back reaction. According to the sign of the parameter in the stress-energy tensor, there appears stability or instability in the cosmological solutions. However, as local counter-terms of the form $\int d^4x \sqrt{-g} R^2$ change the value of this parameter, it is not clear if this result gives any restriction for the contents of the elementary particles.

The stability of the Minkowski spacetime was also discussed in the presence of the back reaction. For $m > 0$, there is an unstable fluctuation and the Minkowski spacetime is quite unstable to singular spacetime. For $m < 0$, there are massless and massive fluctuations. Numerical calculation shows that in most of cases the Minkowski spacetime is unstable to the regular "linear-dilaton" spacetime.

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- [11] The eq.(2.10) is obtained from the trace part of eq.(2.2) plus eq.(2.3). It is convenient to use this form of equation for numerical calculation.
- [12] The asymptotic solution (3.22) was obtained by the adiabatic approximation which assumes that $a(\tau)$ and $E = \frac{1}{2}(\dot{x}')^2 + \dot{V}_R$ are constant during one oscillation.
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Figure Captions

Fig.1: The form of the potentials V_R and V_{CC} . There are two extrema A and B.

Fig.2.A.1 ~ 2.A.5: Numerical solutions of the scale factor $a(\tau)$ and the BDD field $\Phi(\tau)$ for the radiation-dominant universe. The boundary conditions are explained in the text. We set $\omega = \frac{1}{5}$ in the numerical calculation. However, qualitative behaviours of solutions are insensitive to the value of ω .

Fig.3.B.1 ~ 3.B.4: Numerical solutions for the universe with cosmological constant. The boundary conditions are explained in the text. We set $\omega = \frac{1}{5}$ in the numerical calculation. However, qualitative behaviours are insensitive to the value of ω .

Fig.4: Numerical solutions for $m = 0$ cases. 4.a is the results for the radiation-dominant universe and 4.b is the results for the universe with cosmological constant.

Fig.5: Numerical solutions for $m > 0$ cases. 5.a is the results for the radiation-dominant universe and 5.b is the results for the universe with cosmological constant.

Fig.6: Stability and instability of the Minkowski spacetime.

6.a: For $m > 0$, the Minkowski spacetime is unstable to singular spacetime. The result is shown for the boundary condition $\phi' = 0, h' = \frac{1}{T^2}, h'' = 0$. On the other hand for $m < 0$, the Minkowski spacetime is unstable to the regular "linear-dilaton" spacetime. The boundary conditions are following,

$$6.b: \phi' = 0, h' = \frac{1}{T^2}, h'' = \frac{1}{T^3},$$

$$6.c: \phi' = \frac{1}{T^2}, h' = \frac{1}{T^2}, h'' = 0.$$

Fig.1

Fig. 2.A.1

Fig. 2.A.2

Fig. 2. A. 2

Fig. 2. A. 3

Fig. 2. A. 3

Fig. 2. A. 4

Fig. 2. A. 4

Fig. 2. A. 5

Fig. 2. A. 5

$$h' \left[1 + \frac{1}{\Omega} (\tau - 1) \right] \Omega / \left(\omega + \frac{1}{2} \right)$$

Fig. 3. B. 1

Fig. 3. B. 2

Fig. 3. B. 2

$$h'[1 + \frac{1}{\Omega}(\tau - 1)]\Omega/(\omega + \frac{1}{2})$$

Fig. 3. B. 3

Fig. 3. B. 3

Fig. 3. B. 4

Fig. 3. B. 4

radiation-dominant

Fig. 4 a

cosmological constant

Fig. 4 b

radiation-dominant

cosmological constant

$e^{-2\phi}$

Fig. 5 a

$e^{-2\phi}$

Fig. 5 b

Fig. 6 a

Fig. 6 b

Fig. 6 c