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IAEA-ICTP 93-404
SW 9408
IC/93/404

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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OF TESTING MACROREALISM
VERSUS QUANTUM MECHANICS

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MIRAMARE - TRIESTE,

December 1993

Summary. In order to evaluate its relevance, we reconsider critically the recent proposal by Leggett and Garg to test macrorealism against quantum mechanics by resorting to experiments involving noninvasive measurement processes on a SQUID. Our conclusion is that, in spite of the fact that the proposed experiment would neither constitute a test of macrorealism nor a test of macrocontextuality, a simplified form of it represents a (presumably) feasible experiment permitting a direct test of macroscopic quantum coherence. We also analyze the proposal from the point of view of the recent attempts to build up model theories allowing to take, within a purely quantum framework, a macrorealistic position about natural phenomena i.e. the so called dynamical reduction models and we stress that the proposed experiment has no relevance for the dynamical reduction program, as developed so far. However consideration of the SQUID system allows one to test other possible dynamical mechanisms leading to the objectification of macroproperties which could, in principle, be operative. We also briefly sketch experimental procedures to be followed to get all relevant information concerning macrocoherence.

1. Introduction.

Recently in a series of interesting papers⁽¹⁻⁶⁾ some of the best known and most debated problematic aspects of quantum theory, in particular those related to the measurement process and, more generally, to the description of macroscopic systems, have been reconsidered from a new point of view. More specifically, the basic question addressed by the above papers is whether the assumption that quantum predictions about macroscopic systems are correct is compatible with a macrorealistic position about natural phenomena. The novelty of the investigations under discussion stems from the fact that they analyze possible experiments aimed to answer the above question.

The issue is obviously of great relevance and its reconsideration is quite appropriate if one takes into account that all the debate about the

possibility of attributing objective properties to individual physical systems which arose in connection with the EPR-argument, Bell's inequality and the beautiful experiments by A. Aspect and collaborators⁽⁷⁾ is more concerned with microscopic systems and with the locality issue than with macrorealism.

Leggett and Garg⁽¹⁾ tackle directly the macrorealistic issue: they propose experimentally feasible tests of quantum mechanics versus macrorealism, more specifically versus a wide class of theories satisfying quite general assumptions which embody (in the authors' opinion) requests which a macrorealist cannot give up.

In this paper we want to perform a critical analysis of the proposal by identifying to which extent the assumptions on which it is based are really necessary for macrorealism; by this we mean "possessed values macrorealism" i.e. that macroscopic systems almost always have definite macroproperties. Some of the arguments we are going to present have already been raised in various papers⁽⁸⁻¹⁰⁾. Our treatment will be more general and will focus on many conceptually relevant features of the problem. In particular, besides presenting new arguments and more detailed proofs that, as appropriately stated in refs. (8-10), the experiment is not a test of macrorealism, we will discuss its implications for the problem of contextuality and show that the experiment cannot be regarded as a test of contextuality at the macroscopic level. In spite of the above remarks we stress that the proposal is relevant since it could give experimental evidence of quantum coherence effects. Finally, we will discuss the significance of experiments of the kind suggested in ref. (1) from the point of view of those model theories whose aim it is to consent, within a fully quantum context, a macrorealistic position about natural phenomena, i.e., the dynamical reduction models. We conclude the paper by discussing possible relevant experimental programs in this area.

2. The Proposal by Leggett and Garg.

As previously stated the authors identify two general conditions which should be satisfied by any theory in order to be compatible with macrorealism:

1. - If a macroscopic system under certain conditions is, whenever observed, found to be in one of two or more macroscopically

distinguishable states, then we can assign to it at nearly all times, i.e. even when it is not observed, the property of actually being in a particular one of these states. This assumption will be referred to as the hypothesis of macrorealism and will be denoted by MR.

2. - For such a system it is, in principle, possible to observe which of the various possible macrostates it is in, without affecting its subsequent behaviour, at least as regards motion between macrostates. This assumption will be called the hypothesis of Noninvasive Measurability at the Macroscopic Level and will be denoted by NIM.

The basic idea is to test the whole class of macrorealistic theories against quantum mechanics by considering specific physical systems such that, for them, one can put into evidence quantum interference effects between appropriate states which are macroscopically distinguishable and for which there is the possibility of performing (practically) noninvasive measurements aimed to identify such states. The best candidates for this purpose are devices involving Josephson junctions.

More specifically, one takes into account an rf-SQUID, i.e. a superconducting ring with a Josephson junction in an external magnetic field. Under appropriate conditions the dynamical behaviour of the magnetic flux through the ring can be described, to an extremely high degree of accuracy, as a quantum particle in a symmetric double well potential (see fig. 1). The flux performs coherent oscillations between the two wells, with an appropriate frequency Δ . The flux state corresponding to one of the two wells is associated to a "macroscopic current" of about 50 nAmpere circulating clockwise in the ring, the one corresponding to the other well is associated to the same current circulating anticlockwise. The current involves at least⁽⁵⁾ 10^{12} electrons. The two corresponding quantum states, which we will denote as $|+\rangle$ and $|-\rangle$, respectively, can then be considered as macroscopically distinguishable. The hamiltonian of the system induces oscillations between the two states, so that linear superpositions of them occur in the SQUID. Moreover, as discussed by various authors and, in particular, in the exhaustive papers by Tesche⁽¹¹⁾, by coupling the rf-SQUID to hysteretic dc-SQUID switches and to dissipative dc-SQUID magnetometers one can perform a (practically) noninvasive quantum measurement in the sense we will make clear below.

From the point of view of the conceptual aspects of the proposal we can limit our considerations to a very schematic description of the physical processes taking place in the SQUID. Let us consider the linear manifold spanned by the two states $|+\rangle$ and $|-\rangle$ which are identified with the two eigenstates of the 2×2 matrix σ_z . The time evolution is then generated by a very simple hamiltonian:

$$H = \frac{1}{2} \hbar \Delta \sigma_y \quad (2.1)$$

Following the authors of ref.(1), we consider a time dependent observable $Q(t)$ taking the value $+1$ or -1 according to whether, in a measurement at time t , the system is found in the state $|+\rangle$ or $|-\rangle$, respectively, and we define the two-times correlation functions $\langle Q(t_i)Q(t_j) \rangle$ as the probability that both measurements at t_i and t_j give the same result minus the probability that they give opposite results. Leggett and Garg prove that, if assumptions 1 and 2 above, i.e. MR and NIM, are made, then one can derive for the correlations a Bell-type inequality:

$$\langle Q(t_1)Q(t_2) \rangle + \langle Q(t_1)Q(t_3) \rangle - \langle Q(t_2)Q(t_3) \rangle \leq 2 \quad (2.2)$$

According to standard quantum mechanics (obviously in performing the calculation one has to take into account that the first measurement induces wave packet reduction) one has:

$$\langle Q(t_i)Q(t_j) \rangle = \cos \Delta(t_j - t_i) \quad (2.3)$$

For an appropriate choice of the times t_1, t_2, t_3 , and t_4 , Eq.(2.2) can be violated by these quantum values for the correlation functions*.

We stress that in order to derive the inequality (2.2) one needs both previous assumptions, i.e., MR and NIM. Furthermore, from the formal point of view, the NIM assumption plays exactly the same role as the locality assumption in the usual derivation of Bell's inequality. Thus, what the proposal allows one to test is the conjunction of the above two

assumptions, just as an EPR correlation experiment is a test of the conjunction of the requirements of realism and locality. However, according to the authors of ref.(1), and as repeatedly stressed by Leggett himself^(4,6), the conceptual status of NIM with respect to MR is completely different from the one of locality with respect to realism. In particular, while locality is logically independent from realism, in the authors' opinion NIM is a natural consequence of MR and to reject it would amount to plainly reject MR*.

As it will be clear from what follows, we disagree on this point. Actually, as we will show below the assumption of NIM is (even though quite natural) not only logically independent but also by no means a corollary of MR.

3. Non Invasive Measurement Processes.

Before coming to expound our point of view let us be more precise about the idea of a noninvasive measurement. Following the authors of ref.(1) we consider, as a typical non-invasive measurement, an *ideal negative result measurement*, i.e., one obtained by allowing the system to interact for a finite time with another system in a "ready state" $|A_0\rangle$, the interaction hamiltonian being such so as to yield the evolution:

$$|+\rangle |A_0\rangle \Rightarrow |+\rangle |A_0\rangle \quad (3.1a)$$

$$|-\rangle |A_0\rangle \Rightarrow |-\rangle |A_0\rangle \quad (3.1b)$$

The dots at the right hand side of (3.1b) express the fact that one does not commit himself about what the final state (provided it consents the identification of the triggering state) is, the measured system could even be destroyed in the process. We stress that, at this stage, the very idea of noninvasivity is formulated within a (standard) Hilbert space description of the process.

* For later purposes it is useful to quote directly from ref.(6): *It should be emphasized that while NIM is not strictly logically equivalent to MR, it is a very natural corollary of it. Indeed, it is not entirely clear what meaning one could give to the hypothesis that the system is at all times in a definite macrostate, if one had simultaneously to admit that it is not possible, at least in principle, to verify this in a way which did not drastically disturb its subsequent motion.*

If eqs.(3.1) hold, we will say that the measurement is noninvasive for the state $|+\rangle$ and we obviously assume that a similar measurement which is noninvasive for the state $|-\rangle$ is possible. From Eq. (3.1a) the authors argue that, when the system is in the state $|+\rangle$ it does not interact with the apparatus and, consequently, it is not disturbed by the measurement. On the basis of this, making use of the assumption of MR, i.e. that each individual system has at all times the property corresponding to a definite value of $Q(t)$, they conclude that when in the measurement the outcome $Q(t)=+1$ is obtained, the measurement itself cannot have any influence on the future evolution of the system; things simply go on as if no measurement would have been performed^{*}. By considering both types of noninvasive measurements and subjecting the systems which have survived the noninvasive measurements to a second (not necessarily noninvasive) measurement, one can build the two times correlation functions and one easily proves that they must satisfy (2.2).

4. A Critical Investigation of the Above Argument.

Our first remark is that the very formulation of the assumptions of MR and of NIM is not fully unambiguous. In fact it is obvious that the authors are considering hypothetical theories which somehow differ from standard quantum mechanics, either in the interpretation or for their physical implications. This being the case it is possible for observable R to possess a definite value even when the system's quantum state is not an eigenstate of R . Therefore, for SQUIDS, we must carefully distinguish between (i) the SQUID occupying an eigenstate of Q and (ii) the SQUID possessing a definite value for Q . Obviously, what one wants for being allowed to take a macrorealistic position is that macroscopic systems almost always have definite macroproperties. However, when one is going beyond standard formalism, as well known, one can get such a result in various and conceptually very different ways. The distinction mentioned above between being in a state or possessing a property is strictly connected with well analyzed possible attitudes with respect to the standard formalism, i.e. claiming its incompleteness or its being wrong.

^{*} Once more we consider it appropriate to quote directly from ref. (6): *It is clear that in cases where the counter did not register, it is very implausible to hold (within a macrorealistic theory) that the subsequent motion of the system is affected by our having conducted the measurement, since there was in this case no physical interaction; indeed, if we had to admit the existence of an effect under these conditions, it is by no means clear that we would mean by asserting that immediately prior to the measurement the system was definitely in the state corresponding to $Q=+1$.*

The first attitude corresponds to contemplating the possibility that, even though quantum statistical predictions are correct, the theory could be enriched, e.g. by introducing appropriate "hidden" parameters in such a way that choosing the parameters would precisely define the "actual" properties of each individual member of the statistical ensemble. The second one corresponds to accepting a modification of the theory, in particular of its evolution equation, in such a way that, at least at the desired macroscopic level, definite macroscopic properties spontaneously emerge, or, said differently, dynamical objectification of such properties occurs. In what follows, we will investigate the proposal of ref. (1) with reference to both the above mentioned alternatives.

The second remark is that "translating" some feature as NIM which can even look absolutely natural and holding at a certain level (more precisely within a specific theory) to a new level, or, as stated above, to a not completely defined theoretical scheme is an extremely delicate step which can very easily lead one to put forward more stringent requirements than those which are logically necessary. In particular, we stress that in doing so it is extremely important to keep requests of realism clearly distinct from other requests which we will loosely denote as requests of *classicality*.

As a first example to elucidate this point let us consider a very elementary microscopic process (it will become clear soon that the restriction to microsystems does not limit the general validity of the argument).

- Classical Physics. - We have an ensemble of particles which can go from point P to point Q by following two different paths (see Fig.2). Along path 2 there is a particle detector which absorbs the particles hitting it.

- The quantum analogue of noninvasivity. - For those particles which are in a quantum state localized along path 1 (let us call it $|+\rangle$ by analogy with the SQUID case), the measurement is noninvasive and the subsequent dynamics is actually^{*} not affected by the detector along path 2.

^{*} Strictly speaking, since no wave function can have compact support, a state localized along path 1 will nevertheless exhibit a "tail" overlapping path 2. As a consequence all above statements are only approximate, but, as is well known, for sufficiently well localized states and for a sufficient separation of the two paths the approximations which are involved are extremely accurate, certainly much more than those which have been used in dealing with the SQUID dynamics. In particular, the above fact does not invalidate in any way the argument we are developing here.

Let us now consider, within an hypothetical theory agreeing with quantum mechanics but which is realistic about trajectories, particles which are, quantum mechanically, in a superposition of the states corresponding to the two paths. One immediately realizes that the presence of the detector cannot be noninvasive even for those particles which have the property of "going along path 1". We do not want to be misunderstood. We are not raising the same objection as A. Peres⁽¹²⁾ against the argument of ref. (1), i.e. that the detector always has an invasive effect, since it reduces the statevector. To this objection the authors have appropriately answered by stressing that, claiming that the system is at all times in one of the two macrostates under consideration one is already beyond conventional quantum mechanics and therefore one cannot use standard quantum rules (such as applying the reduction postulate to the superposition) to analyze the situation. We perfectly agree on this but we want to emphasize that the violation of NIM for an individual which is, quantum-mechanically, in a superposition, does not at all forbid a realistic position about the process under consideration (see also the remarks of refs.(8-10)).

The simplest way to see this derives from considering a case quite similar to the one of Fig. 2, within the context of the Pilot Wave Theory. For simplicity, let us suppose that along one of the two paths there is a phase shifter so that at point Q there is destructive interference, when no detector is present. According to the Pilot Wave Theory each individual particle actually possesses the property of being at all times *either along path 1 or along path 2*; consequently, such a theory allows one to take a fully realistic position about the path followed by each particle. It has to be stressed that all systems which are in states such that, within quantum mechanics, it is legitimate to assert that they reach Q following path 1, also follow path 1 according to the Pilot Wave Theory and, furthermore, for them the measurement is absolutely noninvasive. But even for particles following (according to the Pilot Wave picture) path 1, if they are associated with states corresponding to quantum superpositions, the measurement turns out to be invasive. In fact, if one wants to account for the measurement process within the considered theory, one has to write the equations describing the coupling between the particle with the detector. For simplicity, let us suppose that the measurement interaction triggers a macroscopic pointer of the absorber, which is displaced whenever the particle follows path 2. Even if one limits his considerations

to the subensemble of the compound system particle plus apparatus for which the position of the pointer does not change (i.e. for which the particle follows path 1), there are particles reaching point Q. Thus, the measurement turns out to be invasive also for the particles following path 1.

If one takes into account this simple example one can easily see that statements as those of ref.(6) (referred to in the footnotes at pag 5 and 6) are by no means stringent and, consequently, that NIM cannot be considered a natural corollary, or an unavoidable element of the assumption of macrorealism.

(Coming back to the subject of our investigation, we point out that, for the process under consideration, one can easily build up deterministic hidden variable theories which agree with quantum mechanics and which imply that each individual SQUID always possesses the property of either having a clockwise or an anticlockwise current flow. For completeness we exhibit a very simple example of this type.

An arbitrary state for the SQUID, $|\Psi\rangle = \alpha|+\rangle + \beta|-\rangle$ uniquely identifies a direction $\mathbf{n}$ such that $\sigma \bullet \mathbf{n} |\Psi\rangle = |\Psi\rangle$, σ_i being the Pauli matrices. The simplified quantum dynamics considered in Section 2 for the SQUID induces a rotation of the vector $\mathbf{n}$ (identified by $|\Psi\rangle$) with a frequency Δ around the y-axis. Following the analogous treatment by J.S. Bell (13) for the spin case, we consider now the following hidden variable model:

-The state of an individual system is completely characterized by a unit vector λ in the three-dimensional real space,

-A quantum ensemble associated to a state $|\Psi\rangle$ corresponding to the direction $\mathbf{n}$ is identified with the inhomogeneous ensemble characterized by a uniform distribution of the hidden variables λ on the hemisphere with axis $\mathbf{n}$.

-The hidden variables dynamics amounts to a rotation of each λ of the ensemble around the y-axis with frequency Δ .

Concerning the attribution to each member of the ensemble of the objective macroproperty pertinent to the observable Q we consider the unit vector $\mathbf{k}$ along the positive z-axis and we stipulate that each member

of an ensemble associated to a (statevector, corresponding to the) direction $\mathbf{p}$ has the property $Q=+1$ or $Q=-1$ according to whether $\lambda \bullet \mathbf{k}' > 0$ or $\lambda \bullet \mathbf{k}' < 0$, respectively. Here $\mathbf{k}'$ is a unit vector obtained by rotating $\mathbf{k}$ towards $\mathbf{p}$ until $1 - (2\theta' / \pi) = \cos\theta'$, where θ' is the angle between $\mathbf{k}'$ and $\mathbf{p}$ and θ is the angle between $\mathbf{k}$ and $\mathbf{p}$. It is easily checked that this model reproduces the quantum probabilities for $Q(t)$.

To complete the picture one has to specify the effect of performing a measurement of Q . Obviously, one has to assume for consistency that the hidden variables of the systems which imply the outcome $+1$ (-1) are somehow reshuffled so that, after the measurement, they turn out to be uniformly distributed on the upper (lower) hemisphere.

It is important to comment on this point. It could seem that introducing a prescription implying a dynamical change of the hidden variables during the measurement we are adopting a dualistic position about natural phenomena, just as it happens if one accepts, within quantum mechanics, the postulate of wave packet reduction. This is not the case. Obviously a complete hidden variable model for the process we are considering should include the ingredients (hidden variables and interaction mechanisms) to describe the physical systems with which the SQUID is interacting. The above prescription is used here just because we confine our analysis (since for our discussion it is not important to be completely general) to the measured SQUID itself. One can explicitly verify that also within hidden variable models which give a complete account of the measurement, the coupling of the hidden variables characterizing the measured system with those of the measuring devices and the ensuing evolution, actually leads to a change in the distribution of the former (see the Appendix for a clarifying example).

Coming back to our model we stress that for an ensemble which at the time in which the measurement takes place corresponds to the eigenstate of Q belonging to the eigenvalue $+1$, by measuring Q one does not alter the subsequent evolution for what concerns the macroproperties we are interested in (compare with the requirement of NIM). However, a selective measurement on an ensemble corresponding to a superposition, implies not only a selection of the systems whose hidden variables ensure that $Q=+1$, but also a uniform redistribution of these variables around $\mathbf{k}$. It is just this reshuffling of the hidden variables which violates NIM rendering the measurement invasive also for those individual systems

which had the property $Q=+1$ before the measurement. Imposing NIM in the sense of ref.(1) would amount to claiming that the measurement simply sorts out of the ensemble those systems which have the property $Q=+1$, without affecting in any way their hidden variables. This request is by no means necessary for realism but it embodies classical demands and would obviously imply a straightforward violation of quantum predictions concerning subsequent measurements of Q . It goes without saying that the model we have just outlined is perfectly realistic about the value of $Q(t)$.

Thus, the conclusion of this first part is that the violation of the inequality (2.2) would not imply the impossibility of a realistic position about the macroproperty that the current is going left or right, but the impossibility of having a theory exhibiting, at the individual level, classical features. The above discussion should have clarified that NIM cannot be considered in any sense a corollary of MR and, consequently, that violation of (2.2) does not constitute a clear cut test disproving macrorealism.

5. Deepening the Analysis.

To really grasp the meaning of the proposal put forward in ref.(1) and to evaluate the opportunity of performing the experimental test suggested in it, it is appropriate to deepen the discussion. We start by recalling the detailed analysis of ref. (8). To begin, the author proves that one does not need to resort to inequality (2.2) to test the conjunction of MR and NIM. The argument goes as follows: quantum mechanics implies the violation of what the author denotes as STAT-NIM, i.e., the noninvasivity of the measurement at the statistical level. With reference to the simplified model of Section 2, comparing, for an appropriate initial state and for an appropriate time t , the probability that $Q(t)=+1$ in the case in which no measurement is performed in the interval $(0,t)$ with the one in which one performs a measurement within the considered interval, one sees that STAT-NIM is violated. Since obviously

$$MR \wedge NIM \supset STAT - NIM$$

a violation of STAT-NIM implies the falsity of $MR \wedge NIM$.

A very elementary way of seeing that quantum mechanics implies the violation of STAT-NIM in the case of negative ideal measurements is

obtained as follows. Let us evaluate the probability $P_+(t_2)$ of getting $Q(t_2) = +1$ for a system prepared at $t=0$ in the state $|+\rangle$, when no measurement whatsoever is performed in the interval $(0, t_2)$. Choosing $t_2 = \pi/\Delta$ we have $P_+(t_2) = \cos^2(\Delta t_2/2) = 0$. On the other hand, if one performs a negative ideal measurement e.g. at time $t_1 = t_2/2$, one has, in general, that the probability $P_+(t_1)$ of getting $Q(t_1) = +1$ is larger or equal to the probability of getting $Q(t_2) = +1$ times the probability of getting $Q(t_2) = +1$ given that $+1$ has been obtained in the first measurement. We thus have $P_+(t_2) \geq \cos^2(\Delta t_1/2) = 1/4$.

Thus, since $P_+(t_2)$ definitely differs from $P_+(t_1)$ one sees that quantum mechanics violates STAT-NIM. As a consequence (if the quantum predictions are correct) to disprove the conjunction of MR and NIM, one can resort to the test just described.

Subsequently, the author of ref. (8) faces the crucial problem of whether a refusal of MR $\wedge$ NIM can be considered equivalent to a plain refusal of MR. Taking into consideration the Pilot Wave Theory (which violates STAT-NIM) he argues, in analogy with what we have done in the previous section, that the above conclusion is illegitimate. However the author, having pointed out how peculiar is the fact that even a particle which can be legitimately claimed to have the property of being along path 1 can be disturbed by the detector in path 2, reaches the conclusion that, if the Leggett and Garg experiment would confirm quantum predictions it would yield "an empirical confirmation of contextuality", a test which "is far from uninteresting".

We must confess that we have not been able to understand the precise meaning which Clifton attaches to the term contextuality in his paper. In spite of this, we consider of some interest to examine whether the Leggett and Garg experiment could be considered as a test of contextuality in the precise technical sense one usually attaches to this expression, i.e., of the impossibility of assigning, for an individual system, definite values to all observables in such a way that they satisfy all the algebraic relations holding among compatible sets of observables^(14,15).

As stressed by Bell, hidden variable theories can circumvent Gleason's theorem, provided one accepts that the observed value of a nonmaximal observable may depend on which other *compatible* observable is simultaneously measured. Obviously, measuring an observable A which is compatible with B cannot (and it does not, both within quantum theory

and within hidden variable models) change the predictions about the outcomes of measurements of B at the ensemble level. However, within hidden variable models reproducing quantum predictions, measuring A *must* in some cases change the outcomes for B at the individual level (this is why if one would have access to the hidden variables one could send faster than light signals). This unavoidable influence at the individual level of measurements of *compatible* observables on each other is the most striking implication of Bell's inequality.

Coming back to our problem, in the Leggett and Garg experiment, should quantum predictions be confirmed, one would have a dependence, as Clifton himself has nicely proved, of statistical predictions about one outcome from the occurrence of the other measurement. But this is not contextuality; this is simply the fact that measurements of $Q(t)$ at different times are equivalent (as one easily sees in Heisenberg picture) to measuring noncompatible observables on the system. It is quite a general feature of quantum mechanics, which has nothing to do with contextuality, that measuring a noncompatible observable changes the statistical predictions about a given observable. As we have shown above, this happens, e.g., in the Pilot Wave theory, which assigns a noncontextual role to positions. Analogously, the hidden variable model for the SQUID system we have presented in the previous section is just built in such a way so as to reproduce quantum predictions for the outcomes of $Q(t)$ which are, within the model, *objective* and *noncontextual* properties of the system.

Thus, in our opinion, experiments involving SQUIDS do not allow to test the contextual nature of macroproperties. Rather, they permit to test the noncommutativity of the observable $Q(t)$ at different times, which is a consequence of the linearity of the evolution generated by an hamiltonian which does not commute with $Q(0)$. This, even though it has nothing to do, in general, with testing quantum mechanics versus macrorealism, is obviously an extremely interesting test since it would yield experimental evidence of quantum coherence effects, a fact that, up to now, has not been possible to test*.

* We stress that such a test would be relevant independently of whether one accepts or rejects the completeness assumption about quantum mechanics. In fact, e.g., even if one adopts a point of view like the one of the Pilot Wave Theory which consents a macrorealistic position, to verify quantum predictions would amount to test that both terms in the superposition contribute in determining the quantum potential governing the process.

In the next section we will analyze the proposal of ref. (1) from the point of view of the other class of model theories which allow one to take a macrorealistic position about natural phenomena, the so called dynamical reduction program.

6. Taking the Dynamical Reduction Point of View.

As it has been repeatedly discussed in recent years⁽¹⁶⁻¹⁸⁾, within a Hilbert space description of physical processes, macrocoherence can be suppressed and, consequently, macrorealism can be regained by accepting that the Schrödinger equation has to be modified by incorporating additional corrective terms that break the unitarity of the evolution and eliminate linear superpositions, though being practically ineffective for microsystems. Various models of dynamical reduction have been presented which imply that the statevector of each individual member of an ensemble of macrosystems is dynamically driven, by the unique dynamics governing all natural processes, within appropriate manifolds corresponding to definite macroproperties. Even though it is extremely important to keep clearly in mind that the considered models induce dynamical reduction at the individual level⁽¹⁸⁾, for the analysis we will perform here we can limit our considerations to the evolution equation for the statistical operator.

Very schematically, the density matrix obeys an evolution equation of the type:

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i}{\hbar} [H, \rho(t)] + T[\rho(t)] \quad (6.1)$$

where $T[\rho(t)]$ transforms states into states.

Among the various possible choices, the model presented in ref. (16) has proved to be very useful to pinpoint the salient features of the dynamical reduction program. Within such a model the corrective term $T[\rho(t)]$ describes a process of spontaneous localization in position for each individual elementary constituent of any physical system within a characteristic length $1/\sqrt{\alpha} \approx 10^{-5}$ cm, occurring with a frequency $\lambda \approx 10^{-16}$ sec⁻¹:

$$T[\rho] = -\lambda \rho + \lambda [\alpha/\pi]^{1/2} \int d^3x \exp[-\frac{\alpha}{2}(q-x)^2] \rho \exp[-\frac{\alpha}{2}(q-x)^2] \quad (6.2)$$

$q = (q_1, q_2, q_3)$ being the position operators of the constituent suffering the localization. The model considered has required important modifications⁽¹⁷⁾ in order to consent the treatment of systems containing identical constituents.

The investigations^(19,20) about the implications of the above mentioned models for macrocoherence in superconducting devices have led to the conclusion that practically there is no appreciable effect concerning the currents circulating in the SQUID. The reason is quite obvious, the suppression of superpositions within the models is related, roughly speaking, to the integral over the whole space of the squared differences of the average (taken over volumes of about 10^{-15} cm³) particle densities associated to the states appearing in the superposition. The particles associated to the macrostates corresponding to the currents circulating clockwise and anticlockwise are diffused over the surface of the superconducting ring, rather than being spatially separated by distances larger than 10^{-5} cm. Consequently, the decoupling mechanism is practically ineffective.

For these reasons we will not deal here with the considered models, but we will adopt the strategy underlying them which is, to our knowledge, the only one allowing to account for the objectification of macroproperties in a consistent way within a strictly quantum framework, i.e., assuming that the statevector gives a complete description of a physical system.

In the case of a SQUID a dynamical reduction process objectifying the properties of being in the macrostates $|+\rangle$ or $|-\rangle$ must contain a reducing term on the corresponding one-dimensional linear manifolds. Let us put:

$$P_+ = |+\rangle\langle+|, \quad P_- = |-\rangle\langle-| \quad (6.3)$$

The modified dynamical equation for the SQUID is then:

$$\frac{\partial \rho(t)}{\partial t} = -\frac{i\hbar}{2} \Delta(\sigma_z \rho(t)) - \lambda \rho(t) + \lambda (P_+ \rho(t) P_+ + P_- \rho(t) P_-) \quad (6.4)$$

Equation (6.4) is easily solved and the evolution is governed by the frequency λ of the reductions and the one $\Delta \approx 10^7$ sec⁻¹ of the oscillations

induced by the hamiltonian between the macrostates $|+>$ and $|->$. Two regimes can occur:

1. The oscillatory case in which the hamiltonian dynamics prevails. This occurs when $\lambda \ll 2\Delta$. The reduction mechanism becomes relevant only asymptotically with respect to the characteristic time of the oscillations of the SQUID. The final state is the equal weights mixture $\frac{1}{2}|P_+> + \frac{1}{2}|P_->$ independently from the initial conditions.

2. The over-damped case in which the frequency of the reductions is so high compared with Δ that the oscillations are inhibited from the very beginning. This occurs when $\lambda \gg 2\Delta$. A state $|\Psi> = a|+> + b|->$ becomes a mixture with weights $|a|^2$ and $|b|^2$ of the states $|+>$ and $|->$ in times of the order of $1/\lambda$, and remains practically unaltered for an extremely long time interval. This is a quasi quantum Zeno effect. Asymptotically the mixture becomes the equal weights one as in the previous case.

We stress that the above analysis shows that in the first case the considered line is not viable for getting definite macroscopic properties for a SQUID. In fact, the oscillating regime violates the MR assumption: linear superpositions of the macroscopically distinguishable states $|+>$ and $|->$ are present at (almost) all times. Therefore, in order to restore MR, λ has to be made large, but then the quantum mechanical predictions about the probabilities of the various outcomes are remarkably violated by the extremely rapid emergence and by the persistence of the mixture.

We will comment on the appropriate way to detect such violations in the next section.

7. Sketching an Experimental Program.

As it should be clear from the the previous analysis, the SQUID, even though it does not allow a conclusive test of macrorealism and/or macrocontextuality, turns out to be quite interesting as a physical system allowing one to test whether macroscopic systems can exhibit a genuinely quantum behaviour. In this section we will focus once more on some crucial points and we will outline, with reference to them, an explicit experimental program aimed at identifying whether and with which precise modalities the physics of the system can turn out to disagree with

quantum predictions. The proposed experiment constitutes an alternative to that of Leggett and Garg. Although our proposal looks simpler, it could be that in real life it is not since it requires a definite initial state.

We start by assuming that we can prepare the system under consideration at the initial time $t=0$ in each of the two macroscopically different states $|+>$ and $|->$. We also assume, for the moment, that we can perform on the system measurements which are (practically) noninvasive in the specific quantum mechanical sense expressed by eqs. (3.1).

We now recall that quantum mechanics (when the approximations which have been made are appropriate) predicts that, for a system prepared in the $|+>$ state and which is left to evolve undisturbed up to time t , the probabilities of finding in a measurement the values $Q(t)=+1$ and $Q(t)=-1$ are $\cos^2(\Delta t/2)$ and $\sin^2(\Delta t/2)$, respectively. In the real case, due to the dissipation arising from the coupling with the environment, as lucidly discussed in the second paper of ref. (11), the oscillations turn out to be damped.

As already stressed we are interested in investigating whether or not the SQUID behaves in a genuinely quantum way. To this purpose we proceed in two steps, requiring two different types of experiments. We have first to check whether the SQUID actually exhibits the oscillations predicted by the theory. This can be done by resorting to experiments in which the system is prepared at time $t=0$ in one of the two states we are interested in and then is left to evolve undisturbed from the preparation time up to the time in which a measurement (it does not matter whether invasive or not, provided it unambiguously allows one to identify the outcome) is performed. One repeats such experiments for various choices of the measurement time and determines the relative frequencies of the outcomes. The above suggested test is relevant, as discussed in the previous section, to test whether dynamical reduction occurs. In fact, as we have seen, for the "interesting" cases of dynamical objectification (i.e.

* Obviously, if one can perform a measurements which is noninvasive for the state $|+>$ ($|->$) one can resort to such a measurement to prepare the system. However we would like to stress that one could devise other procedures to get the preparation, such as, e.g., changing adiabatically the shape of the potential well, by raising one of the two minima with respect to the other so that the particle will go in the well corresponding to the lower one and then going back impulsively to the original shape, a procedure which will not change appreciably its state (such a possibility has been pointed out to us by Profs. Diamanti Palazzi and Comelli).

for the overdamped ones) one expects a remarkable violation of the quantum predictions about the probabilities of the two outcomes.

At this point we can consider two logically possible alternatives: either one discovers that quantum predictions are violated, and then a completely new world opens up for exciting further investigations, or one finds results agreeing with the predictions of quantum mechanics when the damping due to dissipative effects is taken into account.

It has to be noted that this interesting result could be considered as a clear cut proof of macroscopic quantum coherence only if one⁽²⁾ *interprets it strictly in the standard quantum mechanical way*. In fact, if one wants to tackle the issue of macroscopic coherence in a more general way, i.e., with reference to a whole class of theories about the description of macroscopic systems, one could e.g., imagine that all individuals of an ensemble of SQUID systems, all identically prepared in the state $|+\rangle$ ($|-\rangle$) at the initial time, evolve by making sudden jumps between the two considered states (so meeting the MR requirement) in such a way that, when the ensemble evolves undisturbed, the fractions of systems in the two states reproduce the quantum distribution, i.e. the quantum oscillations. It is quite easy to devise models of this type; for a nice and simple example see ref. (9).

When this possibility is taken into account one easily sees that further interesting information can be obtained by resorting to noninvasive measurement processes. They are apt to put into evidence possible nonclassical features of the dynamics. To this purpose let us consider a class of experiments differing from the previous one. In them, one prepares as before the SQUID in one of the two states $|+\rangle$ or $|-\rangle$ at the initial time and then one subjects it to two different measurements at two different times t_1, t_2 ($0 < t_1 < t_2$). We suggest to perform various experiments in the following order.

a. At the time $t_1 = 2\pi/\Delta$ one performs a measurement which is noninvasive for the initial state. Then one performs a second measurement at different times and determines the relative frequencies of the two outcomes. If such a test gives the same predictions as quantum mechanics for the case in which the system is left to evolve undisturbed up to the time of the second measurement (i.e., when no measurement is performed at t_1) we can claim

that we have tested the noninvasivity of the measurement in the strict quantum sense of eqs. (3.1).

b. We follow the same procedure but we choose the noninvasive measurement to take place at the time $t_1 = \pi/2\Delta$ and then we perform, on the subensembles of those systems which have given the result $Q(t_1)=+1$, a second measurement at the time $t_2 = \pi/\Delta$. We recall that, should the first measurement be noninvasive in the sense of the NIM assumption (i.e. it would not change the subsequent evolution of the systems which have been selected by it) we would have at time t_2 no system in the state corresponding to the preparation. In fact we have already checked that quantum predictions in the absence of intermediate measurements are verified, and they imply that no system whatsoever can be found in the initial state at the considered time. This would also be unavoidably true if NIM holds.

Suppose however that this is not the result we find: some particles are found, at time t_2 , in the same state as the initial one. In such a case we would be compelled to accept that the measurement, which we have already tested to be noninvasive for systems which, according to quantum mechanics, would be in the same state as the initial one, turns out to be invasive for systems which, according to quantum mechanics, would be in a linear superposition. This would show that even the less disturbing measurement procedures one can devise turn out to be, in general, unavoidably invasive for the system under consideration. The experimental proof of the unavoidable invasivity obviously constitutes, in turn, an evidence of a nonclassical behaviour of the considered macroscopic system. This is why this second class of experiments gives a richer information than the previous one.

8. Conclusions.

The above analysis leads us to conclude that experiments of the kind proposed in ref. (1) are not a test of macrorealism, and they are not a test of contextuality at the macroscopic level. They can be of some interest from the point of view of the approaches aiming at the objectification of macroproperties within a genuine Hilbert space framework, even though,

* If one takes into account the modifications of the purely hamiltonian evolution due to the coupling with the environment one has to change in an obvious way the above experimental prescriptions.

to experimentally set up this issue one does not need to resort to noninvasive measurements. In spite of the above remarks we stress that this does not diminish the relevance, from a conceptual point of view, of such experiments, as pointed out at the end of the previous section.

Appendix.

We consider a spinless particle in one dimension in the double well potential of Fig.1 and we denote as usual by $|+\rangle$ and $|-\rangle$ the states corresponding to its being localized within the left and right wells, respectively. In the coordinate representations we have:

$$\langle q|+\rangle = f(q); \quad \langle q|-\rangle = f(q-2a) \quad (A.1)$$

where $f(q)$ is a function well localized around $q=a$.

Suppose we are interested in measuring the parity operator $\hat{\Pi}$ of the particle. We couple it to a measuring apparatus by switching on an appropriate interaction for a time interval T . As usual one considers, for simplicity, only the centre of mass motion of the macroscopic pointer of the apparatus, the record of the outcome being given by the final position Q of the pointer. One can easily get this by assuming that the measurement interaction is governed by the hamiltonian

$$H = V\hat{\Pi}\hat{P} \quad (A.2)$$

where V is a c -number constant having the dimensions of a velocity and $\hat{P}$ is the observable which is canonically conjugated to $\hat{Q}$.

Let us denote by $|\pi+\rangle$ and $|\pi-\rangle$ the eigenstates of the operator $\hat{\Pi}$ and by $|Q\rangle$ the improper eigenstate of $\hat{Q}$ belonging to the eigenvalue Q , and let us assume that the ready state of the apparatus is $|Q=0\rangle$. We have:

$$e^{-i\frac{HT}{\hbar}}|\pi+\rangle|Q=0\rangle = |\pi+\rangle|Q=VT\rangle \quad (A.3a)$$

and, analogously

$$e^{-i\frac{HT}{\hbar}}|\pi-\rangle|Q=0\rangle = |\pi-\rangle|Q=-VT\rangle \quad (A.3b)$$

Suppose now we have a particle in the state $|+\rangle$ and we subject it to a parity measurement. We can easily evaluate the evolution by

expressing the state as a linear combination of the two parity eigenstates $\frac{1}{\sqrt{2}}(|+\rangle \pm |-\rangle)$. We then have

$$e^{-i\frac{HT}{\hbar}}|+\rangle|Q=0\rangle = \frac{1}{2}(|+\rangle + |-\rangle)|Q=VT\rangle + \frac{1}{2}(|+\rangle - |-\rangle)|Q=-VT\rangle \quad (A.4)$$

The elementary quantum example we have just discussed, admits obviously a Pilot Wave description. We recall once more that within the Pilot Wave theory the positions of all particles of the universe have at all times precise, objective, and noncontextual values. In our example we have at the initial time an ensemble of particles whose positions are distributed according to $|f(q)|^2$ while all pointers of the ensemble point at $Q=0$. After the interaction the pointers, with equal probability, point either at $Q=VT$ or at $Q=-VT$. According to eq. (A.4), the density distribution of the positions of the particle is, to an extremely high degree of accuracy $(1/2)|f(q)|^2 + (1/2)|f(q-2a)|^2$. The conclusion is that, since in the considered model the positions are the hidden variables, measuring an observable which does not commute with the position operator, requires a remarkable reshuffling of the hidden variables themselves. Actually, roughly speaking, one half of the members of the ensemble have their hidden variable values displaced by the amount $2a$.

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Acknowledgement

This work was supported in part by the Istituto Nazionale di Fisica Nucleare. We are grateful to the referees who have put forward useful remarks on the first version of this paper.

Fig. 1

Fig. 2