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ABSTRACT

We study the constraints on the density profile of the total mass in a cluster of galaxies that can be obtained when a radial arc produced by gravitational lensing is observed. We apply this to the cluster MS2137-23, which contains a radial arc, a tangential arc, and three other arclets. As shown by Mellier et al. , the positions and elongations of all these five images are well reproduced as arising from two different background galaxies, if the mass distribution in the cluster is elliptical, is exactly centered on the brightest cluster galaxy, and has the same ellipticity and position angle as this galaxy. The galaxy is therefore identified as the center of the cluster potential well.

We show that the ratio θ_r/θ_i of the distance from the radial arc to the center of the cluster, to the distance from the other image of the same source to the center, gives a powerful constraint on the form of the dark matter density profile. Models that are favored have well-defined core radii of $\sim 30h^{-1}$ kpc, with the slope changing rapidly from almost flat to roughly isothermal around this radius. Density profiles that remain steeper than r^{-1} all the way to the center are probably ruled out, since they predict θ_r/θ_i to be smaller than observed. We also show that θ_r/θ_i is significantly affected by the mass in stars in the central galaxy. The stellar mass tends to move the radial arc closer to the center, since its profile is very steep. This implies an upper limit to the mass-to-light ratio of the stars, similar to the dynamical measurements of the mass-to-light ratio of elliptical galaxies. The constraints on the profile will be improved once the position of the radial arc is measured more accurately, and the arc redshifts are determined.

All the models for the density profile that are consistent with the observed position of the radial arc predict a velocity dispersion for the cluster galaxies of $\sim 1200 \text{ km s}^{-1}$, and a central velocity dispersion for the stars of $\sim 350 \text{ km s}^{-1}$, consistent with the observed velocity dispersions in other cD galaxies. The models also predict that the velocity dispersion of the cD halo should rise rapidly with radius, and that the two sources producing the five arcs should be at similar redshifts. This can be tested by future observations.

Subject headings: gravitational lenses - galaxies: clustering - galaxies: cD - cosmology

1. INTRODUCTION

Since gravitational lensing in clusters of galaxies was discovered (Soucail et al. 1987; Lynds & Petrosian 1989), one of the applications that has most frequently been considered is to constrain the mass distribution in the clusters. The long arcs have given values of the mass-to-light ratio at the radius of the arcs, which are typically $\sim 200h M_{\odot}/L_{\odot}$ (see Fort 1992, Blandford & Narayan 1992 for reviews), and are the best evidence we have of the presence of dark matter, independent of any dynamical arguments. They have also given upper limits to any core radius within which the density is roughly constant (Grossman & Narayan 1988, 1989). In addition, they have indicated the presence of substructure on small scales (Pelló et al. 1991, 1992; Kassiola, Kovner & Blandford 1992; Kneib et al. 1993; Miralda 1993).

Another interesting feature of the mass distribution in clusters of galaxies that has been clarified by the long arcs, although not often pointed out in the literature, is that the mass is generally concentrated in halos around the brightest cluster galaxies, and in this sense follows the light distribution. In all the cases where there is a dominant cD galaxy (e.g., A963, Lavery & Henry 1988; MS2137-23, Fort et al. 1992; A2218, Pelló et al. 1988, 1992), or a central clump of galaxies (such as in Cl0024+16, Kassiola, Kovner & Fort 1993) the arcs are centered on it. In A370, where there are two similarly bright galaxies close to the center, the observed arcs cannot be understood as produced by an elliptical potential, but they agree with a double potential where two massive halos are centered on the two galaxies (Kneib et al. 1993). A similar conclusion is reached for Cl0302 (Mathez et al. 1992). The only possible exception is A2390, where a straight arc seems to indicate the presence of a double clump, but only one bright galaxy is observed (Pelló et al. 1991); however, this case may be due to a peculiarity of the source (Kassiola et al. 1992; Smail et al. 1993).

Together with the fact that cD galaxies are centered on the peaks of the cluster X-ray emission in cases where this emission is regular (Jones et al. 1979), this is the best evidence we have that cD galaxies are not similar to other galaxies orbiting in a cluster, but they are instead the bottom of the cluster potential well itself. In this case, the cD galaxy and the cluster are part of the same object, with a single total mass density profile $\rho(r)$, and a stellar density profile $\rho_*(r)$, which includes the cD halo and any intracluster light (see Dressler 1979). What is usually called a “cD galaxy” should be nothing but the stars which are in orbit in the potential well of the cluster, in the same way as the galaxies, although the stars are much more concentrated to the center. This higher concentration of the stars to the center is similar to the situation in an isolated galaxy, where the dark matter is also much more extended than the stars. In fact, in any theory of hierarchical gravitational collapse, there should be a continuity of systems between what we call galaxies and clusters. The main structural difference between a field elliptical galaxy and a cluster is that, in the elliptical galaxy, most of the stars are in the stationary equilibrium system orbiting in the main halo, whereas in a cluster most of the stellar mass is in satellite galaxies.

If that was not the case, and the cD galaxy was orbiting around some other cluster center, then the cD should typically appear several hundred kpc away from the center of the lens (a distance typically larger than the distances of the arcs from the center); in the cases mentioned above, cD galaxies appear coincident with the centers of the dark matter halos producing the lensing, with accuracies better than a few arc seconds.

The fact that cD galaxies often have large peculiar velocities relative to the average of the cluster

galaxies (their velocity dispersion is about $1/3$ of that of the clusters; Malumuth 1992; Zabludoff et al. 1993) has been used as an argument against their identification as cluster centers; however, clusters are continuously merging, and their density peaks do not need to coincide with their centers of mass. Substructure will cause the density peaks to move, in response to the gravitational forces of the infalling material. The cluster A370 is a clear example of a merging cluster, where two cD galaxies moving at a relative velocity of 1000 km s^{-1} are associated with massive dark matter halos, as evidenced from gravitational lensing (Kneib et al. 1993). A recent high-resolution X-ray image from ROSAT reveals a double-peaked X-ray emission, with the two peaks coincident with the two cD galaxies (Mellier and Böhringer 1993, priv. communication).

The determination of the form of the density profiles of dark matter halos in the universe (in particular, those of clusters) would be of great interest as a fundamental test for the origin and formation of these halos. For example, in theories of hierarchical gravitational collapse by collisionless dark matter, the profiles of the halos can be predicted as higher resolution simulations can be done (Quinn, Salmon, & Zurek 1986; Katz 1991; Dubinski & Carlberg 1991). In this paper, we are interested in the constraints that can be obtained on the form of the density profiles in clusters from the long arcs. Since the long arcs appear at distances from the cluster center of $30 - 100 h^{-1} \text{ kpc}$, they may constrain the form of the density profile in this range of radius. At larger radius, the weak lensing distortion may be used for the same purpose (Tyson, Valdes & Wenk 1990; Kochanek 1990; Miralda 1991; Kaiser & Squires 1992).

In an elliptical potential, the radius of curvature of an arc compared to the distance to the center is sensitive to the steepness of the profile within the arc. In the case of A963, the radius of curvature indicates a profile shallower than isothermal (Miralda 1993), i.e., where the density increases more slowly than r^{-2} toward the center, but in general the interpretation of the arc curvatures is complicated by the presence of substructure, and by possible effects from the surface brightness distribution in the sources. Hammer (1991) has argued that the profiles are steeper than isothermal, on the basis of the thinness of the observed arcs. For profiles steeper than isothermal, the width is demagnified compared to the source diameter, whereas it is magnified for shallower profiles. However, this conclusion depends on the assumed radius for the sources, and on the details of how an arc width is observationally determined; it can also be affected by the ellipticity of the sources, and by clumpiness in their surface brightness (Miralda 1993).

A much more clear-cut case, proving that the mass profile is shallower than isothermal within the arcs, has been found in the cluster MS2137-23 (Fort et al. 1992; Mellier et al. 1993), where a radial arc appears about $5''$ from the center of the cD galaxy. A radial arc is an image of a background galaxy elongated in the radial direction, which appears closer to the center of the lens than the usual tangential arcs (in this case, the word “arc” is rather inappropriate because the radially elongated images are almost perfectly straight, i.e., the direction of the shear is constant along them; however, historically all elongated images of background galaxies lensed by clusters have been designated by the term “arc”). Mellier et al. (1993) found a value for the core radius, in a particular model of the density profile. Here, we shall investigate in more detail the constraints imposed on the form of the density profile by the observed radial arc. This will be discussed in § 2 and § 3.

From a dynamical point of view, it is also clear that the profiles of the lensing clusters must be shallower than isothermal at the radius of the long arcs. Otherwise, it would be impossible to understand the typical velocity dispersions of cD galaxies of $\sim 300 \text{ km s}^{-1}$, when the splitting angles

of such lenses are in the range $20'' - 60''$. A singular isothermal sphere with $\sigma = 300 \text{ km s}^{-1}$ would produce splitting angles of only $\sim 5''$. We shall show in § 4 that this is not at all contradictory, provided that the dark matter profile is sufficiently shallow, and that the velocity dispersion of the stars increases with radius, reaching typical values of $700 - 1000 \text{ km s}^{-1}$ at the radius of the tangentially elongated arcs. In fact, in any consistent dynamical model of the cluster, a rise of the stellar velocity dispersion must take place at some radius, in order to match the velocity dispersion of the galaxies. Such a rise of the stellar velocity dispersion was observed in A2029 by Dressler (1979); surprisingly, very few additional observations of this phenomenon have been done since that time (Carter et al. 1981).

Our results and the future prospects for determining the form of the dark matter halos in clusters will be discussed in § 5.

2. RADIAL ARCS AND THE MEASUREMENT OF THE MASS PROFILE

In this section, we show how the observation of radial arcs can put significant constraints on the form of the mass profile of the lensing system. For this, we need to see the conditions for producing radial arcs, and to calculate the radius where they will appear.

Let $\rho_0(r) = \rho_D(r) + \rho_s(r)$ be the density profile of the system comprising the cluster of galaxies and the central cD galaxy, where $\rho_s(r)$ is the density associated with stars in the galaxy, and $\rho_D(r)$ is the density of mass in all other forms. We will generally refer to $\rho_D(r)$ as the density of the dark component, since it is mostly dark matter, although a small fraction of it is the mass in the X-ray emitting gas and in the galaxies other than the central one. The corresponding surface densities are $\Sigma_0(r) = \Sigma_D(r) + \Sigma_s(r)$. We assume that $\Sigma_s(r)$ is proportional to the observed light profile. If the mass distribution is elliptical, then the surface density can be written as $\Sigma(r, \phi) = \Sigma_0[r(1 - \epsilon \cos 2\phi)^{1/2}]$, where ϵ is the ellipticity, and ϕ is the angle with respect to the major axis. We only consider the case where ϵ is the same for the stellar and dark components, and is independent of radius. Keeping only terms to first order in ϵ , this can be approximated as

$$\Sigma(r, \phi) = \Sigma_0(r) - \frac{\epsilon}{2} r \frac{d\Sigma_0}{dr} \cos 2\phi. \quad (1)$$

This approximation is equivalent to consider only the quadrupole term produced by the elliptical mass distribution, and is valid for small ellipticities.

When a source of light is behind the cluster, its image will be distorted by gravitational lensing. The action of gravitational lensing can be thought of as a mapping from the image coordinates x_I to the source coordinates x_S , given by the lens equation $x_S = x_I - \alpha$. Here, α is the deflection angle. The image plane contains the images that are actually observed, while the source plane consists of the image that would have been observed if the lens was absent. The amplification matrix is defined as $A_{ij}^{-1} = \partial x_{S,i} / \partial x_{I,j} = \delta_{ij} - \partial \alpha_i / \partial x_{I,j}$, and measures the distortion of the images at every point.

In general, the amplification matrix is of the form

$$A^{-1} = \begin{pmatrix} 1 - \kappa - \lambda & -\mu \\ -\mu & 1 - \kappa + \lambda \end{pmatrix}, \quad (2)$$

where $\kappa = \Sigma/\Sigma_{crit}$, λ and μ are the two components of the shear, $\Sigma_{crit} \equiv (c^2/4\pi G)(D_s/D_{ls}/D_l)$ is the critical surface density, and D_l , D_s and D_{ls} are the angular diameter distances to the cluster acting as a lens, to the source producing the radial arc, and from the lens to the source, respectively (see, e.g., Schneider, Ehlers & Falco 1992). For an elliptical lens, it is useful to write the amplification matrix at every point in a frame where the x-axis is in the tangential direction with respect to the center of the lens, and the y-axis is in the radial direction. When the lens is circular, the amplification matrix is diagonal in this frame (i.e., $\mu = 0$). Tangential arcs will appear when the eigenvalue $1 - \kappa - \lambda = 0$, since that implies infinite magnification along the tangential direction (the magnification always remains finite for a source of finite size), while radial arcs will be formed when $1 - \kappa + \lambda = 0$. When a quadrupole term is introduced in the surface density [eq. (1)], the convergence and the components of the shear in this frame can be expressed as

$$\kappa(\theta, \phi) = \kappa_0(\theta) - \frac{\epsilon}{2} \theta \kappa'_0(\theta) \cos 2\phi, \quad (3a)$$

$$\lambda(\theta, \phi) = \lambda_0(\theta) + \frac{\epsilon}{2} \lambda_1(\theta) \cos 2\phi, \quad (3b)$$

$$\mu(\theta, \phi) = \frac{\epsilon}{2} \mu_1(\theta) \sin 2\phi, \quad (3c)$$

where $\theta = r/D_l$ is the angular distance to the center of the lens, and $\kappa'_0 = d\kappa_0/d\theta$. The quantities κ_0 and λ_0 are the convergence and the shear for the circularly symmetric lens: $\kappa_0 = \Sigma_0/\Sigma_{crit}$, and $\lambda_0 = \bar{\kappa}_0 - \kappa_0$ (Schneider et al. 1992), where $\bar{\kappa}_0$ is the average value of κ_0 within radius θ ,

$$\bar{\kappa}_0 = \frac{2}{\theta^2} \int_0^\theta d\theta' \theta' \kappa_0(\theta'). \quad (4)$$

The quantities λ_1 and μ_1 give the perturbation to the shear due to the ellipticity, and are given by

$$\lambda_1(\theta) = \frac{6\psi_1}{\theta^2} - 2\kappa_0 + \theta \frac{d\kappa_0}{d\theta}, \quad (5a)$$

$$\mu_1(\theta) = \frac{6\psi_1}{\theta^2} - 4\kappa_0, \quad (5b)$$

$$\psi_1(\theta) = \frac{2}{\theta^2} \int_0^\theta d\theta' \theta'^3 \kappa_0(\theta'). \quad (5c)$$

(see Miralda 1993; eqs. [5] are obtained from eqs. [10a] and [10b] of that paper).

The equations for the critical lines are obtained by requiring that the determinant of the inverse magnification matrix in equation (2) vanishes. If we neglect terms of order ϵ^2 , these are simply the equation $\kappa + \lambda = 1$ for the tangential critical line, and $\kappa - \lambda = 1$ for the radial critical line. However, if a tangential and a radial arc are observed in a cluster, they will generally be produced by two different sources at two different redshifts. The critical surface densities of the two sources are proportional to the distance ratios, D_{ls}/D_s . Let D_t be the distance ratio of the source producing the tangential arc, and D_r the distance ratio of the source producing the radial arc. Then, if the convergence and the shear are defined using the critical surface density for the radial arc, the equation for the tangential critical line at the redshift of the tangential arc is $\kappa + \lambda = D_r/D_t$. We then have:

$$\bar{\kappa}_0(\theta_t) + \left(\frac{3\psi_1(\theta_t)}{\theta_t^2} - \kappa_0(\theta_t) \right) \epsilon \cos 2\phi_t \equiv F_t(\theta_t, \phi_t) = D_r/D_t, \quad (6)$$

and

$$2\kappa_0(\theta_r) - \bar{\kappa}_0(\theta_r) - \left(\frac{3\psi_1(\theta_r)}{\theta_r^2} + \theta_r \kappa'_0(\theta_r) - \kappa_0(\theta_r) \right) \epsilon \cos 2\phi_r \equiv F_r(\theta_r, \phi_r) = 1 , \quad (7)$$

where θ_t , ϕ_t , θ_r and ϕ_r are the observed distances and angles of the tangential and radial arcs, and we have defined the functions F_t and F_r for later use.

Any source producing a radial arc must also have another image on the other side of the lens. The radius at which this other image is observed, θ_i , can be compared with θ_r to constrain the form of the profile $\kappa_0(\theta)$. Neglecting terms of order ϵ^2 , the radius θ_i is determined by the condition

$$\frac{\alpha_\theta(\theta_r, \phi_r) + \alpha_\theta(\theta_i, \phi_i)}{\theta_r + \theta_i} \equiv F_i(\theta_r, \phi_r, \theta_i, \phi_i) = 1 , \quad (8)$$

where α_θ is the radial component of the deflection angle (notice that the effect of the azimuthal component of the deflection angle on the value of θ_i is second order in ϵ). For the surface density of equation (3), α_θ is

$$\alpha_\theta = \theta \bar{\kappa}_0 + \theta (\psi_1/\theta^2 - \kappa_0) \epsilon \cos 2\phi . \quad (9)$$

We now separate the surface density profile into the dark component and the stellar component. We define

$$f_s \equiv \bar{\Sigma}_s(\theta_i)/\Sigma_{crit} . \quad (10)$$

where Σ_{crit} is, as before, the critical surface density at the redshift of the *radial* arc. Put into words, f_s is a measure of the mass-to-light ratio of the cD envelope, in units of the value required to make the projected mass within θ_t equal to the critical value (for $\epsilon = 0$ and $D_r = D_t$, f_s is the ratio of stellar mass to total mass within θ_t). The functions F_{Dt} , F_{Dr} and F_{Di} for the dark component can be defined in terms of the surface density as in equations (6), (7), (8) and (5), but using the dark profile $\kappa_D = \Sigma_D/\Sigma_{crit}$ instead of the total profile κ_0 , and the same with F_{st} , F_{sr} and F_{si} for the stellar profile. We also define the quantities $R_{st} \equiv F_{st}/f_s$, and the same for R_{sr} and R_{si} . These quantities are independent of Σ_{crit} , and therefore can be calculated from the observed light profile of the central cluster galaxy, independently of any normalization. Since $F_t = F_{Dt} + F_{st}$ (and the same for F_r and F_i), equations (6), (7) and (8) can be written as

$$F_{Dt} = \frac{D_r}{D_t} - f_s R_{st} , \quad (11a)$$

$$F_{Dr} = 1 - f_s R_{sr} , \quad (11b)$$

$$F_{Di} = 1 - f_s R_{si} . \quad (11c)$$

Of these three conditions on the dark profile, one is related to the normalization. If only a radial arc is observed, then the ratio of equations (11b) and (11c) gives one condition on the *form* of the profile. If a tangential arc is also present, this gives another constraint on the form of the profile, although it depends on D_t/D_r , and so the two redshifts must be known to make the constraint useful (notice that, if the tangential arc was absent, f_s could be defined for any fiducial radius θ_t , although eq. [11a] will of course not be applicable). These constraints apply directly to the form of the *dark* profile, with all the uncertainty due to the presence of the stellar component being parameterized in the constant f_s , since the quantities R_s can be determined from the light profile.

For spherical potentials, the conditions on the form of the profile are simplified. Dividing (11b) by (11a), and (11b) by (11c), we obtain

$$\frac{F_{Dr}}{F_{Dt}} = \frac{2\Sigma_D(\theta_r) - \bar{\Sigma}_D(\theta_r)}{\bar{\Sigma}_D(\theta_t)} = \frac{1 - f_s R_{sr}}{D_r/D_t - f_s R_{st}} \equiv C_t . \quad (12a)$$

$$\frac{F_{Dr}}{F_{Di}} = \frac{[2\Sigma_D(\theta_r) - \bar{\Sigma}_D(\theta_r)] (\theta_r + \theta_i)}{\theta_i \bar{\Sigma}_D(\theta_i) + \theta_r \bar{\Sigma}_D(\theta_r)} = \frac{1 - f_s R_{sr}}{1 - f_s R_{si}} \equiv C_i . \quad (12b)$$

Notice that, since equations (12) involve only ratios of surface densities, we may use Σ_D or κ_D .

To illustrate the use of these equations in a simple example, we consider pure power-law profiles, where $\Sigma_D(\theta) \propto \theta^{-\eta}$. Equation (12a) then gives

$$\theta_r/\theta_t = \left[\frac{(1-\eta)}{C_t} \right]^{1/\eta} . \quad (13)$$

The case $\eta = 1$ corresponds to the singular isothermal sphere, which has no radial caustic. For any value $\eta < 1$, and for the case $C_t = 1$, radial arcs are formed at successively larger radius as η decreases, reaching the maximum value $\theta_r/\theta_t = e^{-1}$ as η approaches zero. Only if $C_t < 1$, it is possible to have $\theta_r/\theta_t > e^{-1}$.

It is interesting to notice here that the stellar halos of cD galaxies are generally steeper than isothermal, except near the core, which is much smaller than the distances of any observed arcs to the center. This implies that the quantity $2\Sigma_s(\theta_r) - \bar{\Sigma}_s(\theta_r)$ will be negative, and so will R_{sr} . On the other hand, R_{st} is always positive, and so from equation (12b) we see that the presence of the stellar component can only have the effect of *increasing* C_i . Thus, $C_i < 1$ is only possible when the radial arc is at higher redshift than the tangential arc.

From equation (12b), we obtain, for the power-law profile,

$$C_i \left(\frac{\theta_i}{\theta_r} \right)^{1/\eta} - (1-\eta) \frac{\theta_i}{\theta_r} + 1 - \frac{1-\eta}{C_i} = 0 . \quad (14)$$

In this case, C_i does not depend on any redshifts, and is equal to 1 when $f_s = 0$. For the same reasons outlined above, C_i increases with f_s . For $C_i = 1$, the above equation gives a value of θ_r/θ_i that increases as the profile becomes softer, starting at zero when $\eta = 1$, and reaching the maximum value 0.278 as η approaches zero. In the presence of a contribution from the stellar profile, $C_i > 1$ and θ_r/θ_i is lower.

We now consider profiles of the form

$$\Sigma_D(\theta) = \Sigma_c \left[1 - \left(\frac{\theta}{\theta_c} \right)^\eta \right] . \quad (15)$$

This equation is only intended to parameterize the form of the profile close to the center, and can only be correct for $\theta \ll \theta_c$, since for $\theta > \theta_c$ the surface density would be negative. For $0 < \eta < 1$, the density $\rho_D(\theta)$ diverges at low radius as $\theta^{\eta-1}$. For $1 < \eta < 2$, the density is continuous at the center, but its derivative still diverges. Finally, for $\eta = 2$, equation (15) is a Taylor expansion of any profile which is smooth at the origin. Substituting into equations (12a,b), and considering only the case $C_t = 1$, $C_i = 1$, we obtain:

$$\theta_r/\theta_t = (1+\eta)^{-1/\eta} , \quad (16)$$

and

$$\left(\frac{\theta_i}{\theta_r}\right)^{1+\eta} - (\eta+1) \frac{\theta_i}{\theta_r} - \eta = 0. \quad (17)$$

These are actually the same equations as (13), (14), although with a different sign of η due to the different definition (for C_t and C_i not equal to 1 the equations are different and depend on θ_c). As η increases, both ratios θ_r/θ_t and θ_r/θ_i increase, reaching the values $1/\sqrt{3}$ and $1/2$, respectively, at $\eta = 2$.

We notice that all the profiles of the form (15) have a slope that tends to be flat at the origin, yet the ratios θ_r/θ_t and θ_r/θ_i increase with η . These ratios do not only depend on the slope of the profile, but also on the "curvature", or the rate at which the slope is varying; the ratios are larger when the curvature is high. The limiting case is when η tends to infinity in equation (15), where the profile is very flat and declines suddenly to zero close to θ_c . Both ratios would be close to unity in such a profile. However, realistic profiles for the surface density cannot have such high curvatures, and as we shall see in the next Section, the ratios obtained for the case $\eta = 2$ are the maximum that can be found for any realistic profiles.

3. RESULTS

A radial arc, which is made of two merged images of opposite parity, is always accompanied by another image of the same source. In this Section, we calculate the ratio of the distances to the center of the lens of a radial arc and its other image (which we write as θ_r/θ_i). We also calculate the ratio of the distance of a radial arc, to the distance of a tangential arc produced by a different source (θ_r/θ_t). We shall do this for several different models of the dark profile $\Sigma_D(\theta)$, for the case of spherical lenses, and for elliptical ones applied to the case of MS2137-23.

In Figure 1, we show an example of a model reproducing the observed arcs in the cluster MS2137-23 (Fort et al. 1992). The model is very similar to the ones in Mellier et al. (1993). The cross at the origin indicates the location of the central cD galaxy. The solid triangles A01 and A02 indicate the brightest two spots in the tangential arc, which has a total length of about $14''$ (see Figure 2 in Mellier et al. 1993; we follow their nomenclature for the various observed images). The triangle A1 is the center of the radial arc, and A2, A4 and A5 indicate the positions of three more observed arclets, taken from Table 1 of Fort et al. 1992.

The striking fact that was emphasized by Mellier et al. (1993) is that a simple elliptical model, which has the same center, ellipticity and orientation as the central cD galaxy, reproduces all of the observed images quite remarkably, as coming from two sources. In Figure 1, the two thick circles indicate the position and sizes of two circular sources assumed in the model, as would be observed if the lens was absent. One of them is on top of the astroid caustic (dotted line), and therefore produces the tangential arc made of two images on the tangential critical line (shown as a dashed line). This source also produces the images A2 and A4. The other source, on top of the radial caustic, produces the radial arc A1 and the arclet A5. For this model we have used the surface density of equation (1), with the profile $\Sigma_0(\theta) \propto (\theta^2 + \theta_c^2)^{-1/2}$, an ellipticity $\epsilon = 0.22$, and a position angle of -32° , consistent with those of the cD galaxy (Fort et al. 1992). The only parameters that are left are the core radius θ_c (equal to $4''.3$ in the model), and the mass normalization, which determine the size of

the two critical lines. The model also assumes that the two sources are at the same redshift, as in Mellier et al.

The small differences in the positions of the arclets A2, A4, A5, and of the orientation and shape of the arc A01-A02, between the models and the observations, may arise from a variety of reasons : a small amount of substructure in the mass distribution associated with the galaxies (see the models of Mellier et al. 1993; there is a cluster galaxy on the eastern end of the arc A0, and another close to A5), differences in the functional form of the density profile, variations of the ellipticity and twisting of the major axis with radius, the ellipticity and irregularity in the surface brightness of the sources, and a difference in the redshift of the sources. Clearly, the more parameters one adds to the model, the more accurately can the observations be reproduced. However, it is clear that a simple elliptical potential provides a very good approximation.

Our main goal in this paper is not to construct a specific model which can reproduce with the greatest accuracy all the details of the observed images, but rather to explore a large variety of forms for the surface density profile, to see which are allowed and which are not by the observations. As we have seen in § 2, the distances to the center of the tangential arc A01-A02, the radial arc A1, and the other image of the radial arc A5, give us useful conditions on the form of the profile. On the other hand, the positions of the images A2 and A4 do not give much additional information on this form, since they are part of a “broken” Einstein ring together with the long arc A01-A02. The positions along this ring where A2 and A4 appear are quite sensitive to the ellipticity and its variation with radius, or the way in which the spherical symmetry of the lens is broken, and they are less sensitive to the form of the radial profile. Similarly, the position angle of A5 relative to the radial arc A1 depends mostly on the ellipticity; it is their relative distance that is most sensitive to the profile.

In MS2137-23, the distance of A5 to the center is $\theta_i = 22''.5$ (Fort et al. 1992). For the distance of the tangential arc, we shall use the average distance of the segments A01 and A02, which is $\theta_t = 15''.6$. The distance of the radial arc is unfortunately more uncertain, since it is not exactly clear how much it extends into the cD galaxy (the distance given in Table 1 of Fort et al. 1992 is $6''.2$, but their text mentions a distance of $5''$). From Figure 2 of Fort et al. 1992, which shows a B-R image where the cD galaxy is approximately subtracted, we measure a distance for the center of A1 of $\theta_r = 5''.4$, which we shall adopt here. This value, however, could have an error as large as $\sim 10\%$. A more recent image of the cluster also suggests that the distance of the radial arc to the center is between $5''.1$ and $6''.0$ (Fort 1993, priv. communication). Future high-resolution observations may reveal details in the source surface brightness, and allow one to identify the exact position of the radial critical line.

We now proceed to calculate the ratios θ_r/θ_i and θ_r/θ_t for various models of the density profile, using equations (11), with the definitions in (6), (7), and (8).

3.1.. Models with a Flat Core

Our first model (Model 1) for the dark density profile is

$$\text{Model 1 : } \rho_D(\theta) = \frac{\rho_0}{1 + \theta^2/\theta_c^2} . \quad (18)$$

This gives a surface density $\Sigma_D \propto (\theta^2 + \theta_c^2)^{-1/2}$ (the same as that for the model in Figure 1). We start with the case of spherical lenses ($\epsilon = 0$). In Figure 2a, the two ratios θ_r/θ_t and θ_r/θ_i are plotted as a function of θ_r/θ_c . The solid lines give the solutions for the case $C_i = 1$, $C_t = 1$ in equations (12). In the limit where θ_c is very large, the surface density profile can be expanded as $\Sigma_D(\theta) = A + B\theta^2$, and θ_r/θ_i tends asymptotically to 0.5, and θ_r/θ_t goes to $3^{-1/2}$, as was found in the last section. When θ_c is very small, the profile approaches that of a singular isothermal sphere, and the two ratios tend to zero. The dashed horizontal lines indicate the values $\theta_r/\theta_i = 0.240$, and $\theta_r/\theta_t = 0.346$, observed for the lensed images in MS2137-23. Thus, to reproduce the observed θ_r/θ_i , one needs a core radius $\theta_c = \theta_r/1.82 = 3''.0$.

To investigate the effects from the mass in the stellar component of the cD galaxy, we need to calculate the constants R_s in equations (11), as defined in § 2. For the cluster MS2137-23, we have calculated these constants using the observed light profile in the R band in Fort et al. (1992), which is shown in Figure 7 as the filled squares. We have obtained fits to this surface brightness profile, given by the curves in Figure 7, which will be described in detail in § 4. Here, we shall use the fit shown as the solid line (which we shall call Model S1 in § 4). This gives the following constants for the spherical case: $R_{st} = 1$, $R_{sr} = -2.740$, and $R_{si} = 1.482$. If $f_s = 0.02$ (0.04), then from equations (12) we have $C_i = 1.087$ (1.180), and $C_t = 1.076$ (1.156) (assuming $D_r = D_t$, i.e. the two sources are at the same redshift). The ratios θ_r/θ_i and θ_r/θ_t for these two values of f_s in the stellar component are plotted as the two dotted lines below each one of the solid lines. As expected from the discussion in § 2, the two ratios decrease as the contribution from the cD galaxy is increased. Basically, the profile becomes steeper, and therefore the radial arc appears closer to the center.

The interesting feature of the dotted lines is that, contrary to the solid lines, they reach a maximum value. If this maximum value is less than the observed ratio, the model is ruled out. Thus, for any given model for the form of the profile, *the observed ratio θ_r/θ_i gives an upper limit to the mass in the stellar component*. In MS2137-23, the observation $\theta_r/\theta_i = 0.24$ implies that $f_s < 0.07$, when Model 1 is used for the form of Σ_D . To relate f_s to a stellar mass-to-light ratio (see eq. [10]), we calculate the critical mass for lensing at the radius of the tangential arc: $M_{crit} = (c^2/4G)\theta_t^2 D_l (D_s/D_{ls}) = 1.7 \times 10^{13} h^{-1} (D_s/D_{ls}) M_\odot$ (assuming $\Omega = 1$). The apparent magnitude $R = 17.0$ of the light within θ_t (Fort et al. 1992) implies a luminosity $L_B = 9 \times 10^{10} h^{-2} L_\odot$ (the observed R band corresponds to the rest-frame blue band at the cluster redshift $z_s = 0.313$). The stellar mass-to-light ratio is therefore $M_s/L_B = 190 h (D_{ls}/D_s) f_s M_\odot/L_\odot$. The redshift of the radial arc is unfortunately not known; for $z_r = 1$, we have $D_{ls}/D_s = 0.565$, and $M_s/L_B = 340 h f_s M_\odot/L_\odot$. If $f_s < 0.07$, an upper limit of $24 h$ is obtained for the mass-to-light ratio of the stellar component, about twice higher than the dynamical measurements in elliptical galaxies (van der Marel 1991). The core radius for this maximum value of f_s is $\sim 7''$; we see that the derived core radius is highly sensitive to f_s . The observed θ_r/θ_t is in this case not useful to constrain f_s , since the redshifts are not known, and the value of C_t would be decreased if the tangential arc was at lower redshift than the radial arc.

Figure 2b shows the same curves for an elliptical model, with ellipticity $\epsilon = 0.22$. In equations (6), (7) and (8) we have used the values $\epsilon \cos 2\phi_t = -0.02$, $\epsilon \cos 2\phi_r = -0.11$, and $\epsilon \cos 2\phi_i = -0.20$, derived from the position angles ϕ with respect to the major axis (assumed to be oriented along -32° , as in Fig. 1) for the center of the arc A0, the radial arc A1, and the arclet A5, respectively (the positions are taken from Table 1 of Fort et al. 1992). The main effect of the ellipticity in this

case is to increase the ratio θ_r/θ_t , owing to the fact that the tangential critical line is elongated in the same way as the lens, whereas the radial critical line is elongated in the perpendicular direction (see the example in Fig. 1). Since both the radial and tangential arcs are closer to the minor axis of the lens, θ_t decreases and θ_r increases with the ellipticity. On the other hand, the deflection angle is also increased with the ellipticity for objects close to the minor axis, and this causes θ_i to increase. In fact, the ratio θ_r/θ_i happens to remain roughly constant with ellipticity, as is seen by comparing Figures 2a and 2b. For this reason, the upper limit on f_s is roughly the same as in the spherical model, and is insensitive to the value of the ellipticity adopted. The core radius derived from θ_r/θ_i when the ellipticity is included is $3''.1$ when $f_s = 0$, but can increase to much higher values when $f_s > 0$. Therefore, only the *lower limit* $\theta_c > 3''.1$ can be derived. This lower limit is increased to $3''.6$ if we consider that $f_s = 0.02$ gives the minimum reasonable mass-to-light ratio for the stellar component, about $7 h M_\odot / L_\odot$ (although this mass-to-light ratio depends on the arc redshift).

The dotted curves are analogous to those in Figure 2a. The constants R_s change very slightly due to the ellipticity: we find $R_{s,t} = 0.994$, $R_{s,r} = -2.753$, $R_{s,i} = 1.486$ (a fixed f_s therefore corresponds to slightly different values of C_t and C_i , through equations 12). We can use the dotted curves for θ_r/θ_i to constrain the difference in redshifts of the two sources. If we assume that the two sources are at the same redshift, the observed θ_r/θ_i is then almost consistent with the same core radius derived from θ_r/θ_i . For example, if $f_s = 0.02$, then Figure 2b indicates that the core radius $\theta_c = \theta_r/1.48 = 3''.6$ reproduces $\theta_r/\theta_i = 0.24$, and it predicts $\theta_r/\theta_t = 0.36$, almost equal to the observed value. In order to agree exactly with the value $\theta_r/\theta_t = 0.346$, we need just increase f_s from 0.02 to 0.03. The value of f_s is of course unique, but we see from equation (12a) that this increase in f_s is equivalent to assuming $D_r/D_t = 0.965$. This corresponds to a small difference in redshift: if the radial arc is at $z_r = 1$, then the tangential arc should be at $z_t = 0.93$. Thus, we see that if Model 1 is correct, the two redshifts should be rather close and, if anything, the tangential arc should be at lower redshift than the radial arc.

3.2.. Models with Central Cusps

Let us now consider other models for the form of the profile $\Sigma_D(\theta)$. Our Model 2 is:

$$\text{Model 2 : } \rho_D(\theta) = \frac{\rho_0}{(1 + \theta/\theta_c)^2} . \quad (19)$$

At large radius, the density is proportional to θ^{-2} , just as in Model 1. However, at the center, the density has a discontinuous derivative, even though it is finite. The slope of the corresponding surface density profile, $\Gamma \equiv -d \log \Sigma_D / d \log \theta$, changes from 0 at the origin to 1 at large radius, just as in Model 1, but the curvature $K \equiv -d^2 \log \Sigma_D / d(\log \theta)^2$ is smaller for Model 2.

The results for the ratios θ_r/θ_i and θ_r/θ_t are shown in Figure 3a for a spherical lens, and in Figure 3b for an elliptical lens, with the same ellipticity and position angles for the various images that were used in Model 1. The curves for this model are generally lower; the dotted curves, which are for the same values of f_s as in Figures 2(a,b), reach lower maximum values, so the upper limit on f_s is more stringent for Model 2: $f_s \lesssim 0.04$, corresponding to a mass-to-light ratio $14 h M_\odot / L_\odot$ for a radial arc redshift $z_r = 1$. The difference in redshifts for the radial and tangential arc required to reproduce both ratios are similar than in Model 1.

In Figures (4a,4b,5a,5b,6a,6b), the same curves are plotted for Models 3, 4 and 5 of the dark matter density profile:

$$\text{Model 3 : } \rho_D(\theta) = \frac{\rho_0 \theta_c}{\theta [1 + (\theta/\theta_c)^2]^{3/4}} . \quad (20)$$

$$\text{Model 4 : } \rho_D(\theta) = \frac{\rho_0 \theta_c}{\theta (1 + \theta/\theta_c)^{3/2}} . \quad (21)$$

$$\text{Model 5 : } \rho_D(\theta) = \frac{\rho_0 \theta_c}{\theta (1 + \theta/\theta_c)} . \quad (22)$$

For these models, the density diverges as θ^{-1} toward the origin, and falls as $\theta^{-2.5}$ at large radius in Models 3 and 4, and as θ^{-2} for Model 5. Since the profile is less soft in the center, the ratios θ_r/θ_i and θ_r/θ_s are lower for these models, as seen in the figures. In fact, in Figure 4b, we see that for Model 3 the upper limit for f_s is only about 0.02, giving a mass-to-light ratio of $\sim 7 h M_\odot / L_\odot$, and a core radius $\theta_c \simeq 20''$. In this case, the difference in redshifts between the two sources should be even smaller than for Models 1 and 2.

Models 4 and 5 have lower curvature than Model 3, so the ratios of distances are decreased even further. The upper limits to f_s are in this case so low (~ 0.01), that the models are practically ruled out, unless the distance θ_r is found to be significantly lower than our adopted value $\theta_r = 5''.4$. The difference in the outer slope between Models 4 and 5 has almost no effect on the ratios of distances.

3.3.. Other Models

We have also tried a de Vaucouleurs model for the surface density profile of the dark matter. The maximum values of θ_r/θ_i are obtained when the de Vaucouleurs radius is as large as 10^3 times θ_r , in order to make the profile sufficiently shallow. The profile cannot then apply up to the de Vaucouleurs radius, since this would be larger than the typical turnaround radius of a rich cluster. Even when the de Vaucouleurs radius is so large, the maximum of θ_r/θ_i is only 0.2 for $f_s = 0.02$, so a de Vaucouleurs profile is ruled out for the dark matter halo. The reason is that the curvature of the de Vaucouleurs profile is too small.

Finally, we have considered the profile $\Sigma_D \propto (2 + \theta^2/\theta_c^2)/(1 + \theta^2/\theta_c^2)^{3/2}$. This is the profile that was used in the models of Mellier et al. (1993), as can be obtained from the laplacian of their potential. This model is similar to our Model 1, since it has a flat core and a high curvature. The core radius obtained from $\theta_r/\theta_i = 0.24$ is $\theta_c = 5''$, for $f_s = 0$. This is significantly different than the core radius of $8''$ obtained by Mellier et al. (1993). The reason for the discrepancy is that the model of Mellier et al. does not reproduce correctly the distance of the radial arc from the center, it predicts it to be further than observed (see their Fig. 4c). This distance, as we have argued in this paper, is the main constraint on the form of the profile. If f_s is increased to 0.04, then the core radius required increases to $6''.5$ for this model.

4. VELOCITY DISPERSION PROFILES OF THE STARS AND THE GALAXIES

In addition to the lensing constraints, the total density profile of the cluster $\rho(r)$ must also be consistent with the velocity dispersion profiles of the galaxies and the stars, under the assumption

of virial equilibrium. In this section we shall only consider spherical models for the cluster. For any population of objects with density $\nu(r)$, the equation of dynamical equilibrium for the radial velocity dispersion $\sigma_r(r)$ in a spherically symmetric potential is (eqs. 4.54, 4.60 of Binney & Tremaine 1987)

$$\frac{1}{\nu} \frac{d(\nu \sigma_r^2)}{dr} + 2\beta \frac{\sigma_r^2}{r} = -\frac{GM(r)}{r}, \quad (23)$$

where $M(r)$ is the mass contained within radius r for the density profile $\rho(r)$, and $\beta(r)$ is the anisotropy of the velocity dispersion ($\beta = 1 - \sigma_\theta^2/\sigma_r^2$, where σ_θ is the tangential velocity dispersion). If the velocity dispersion is isotropic, $\beta = 0$, and if all the orbits are radial, $\beta = 1$. The line-of-sight velocity dispersion $\sigma_{los}(R)$, where R is the projected radius, is given by

$$I(R)\sigma_{los}^2(R) = 2 \int_R^\infty \left(1 - \beta \frac{R^2}{r^2}\right) \frac{\nu \sigma_r^2 r dr}{\sqrt{r^2 - R^2}}, \quad (24)$$

$$I(R) = 2 \int_R^\infty \frac{\nu r dr}{\sqrt{r^2 - R^2}}. \quad (25)$$

Here, we shall calculate the velocity dispersion of the stellar component and the galaxies expected in the cluster MS2137-23, using the various models for the density profile of § 3, with the values of the core radius that can account for the observed ratio θ_r/θ_i . To calculate this velocity dispersion using the above equations, we need also the stellar density profile in MS2137-23, which was observed in the R band by Fort et al. 1992. We shall use two models for this profile with the following form:

$$\rho_s(r) = \frac{\rho_{s0} (1 + r^2/r_h^2)^{2-\gamma/2}}{(1 + r^2/r_c^2)^2}. \quad (26)$$

Our two models have the following parameters:

$$\text{Model S1 : } r_c = 1''.5, r_h = 10''.5, \gamma = 2$$

$$\text{Model S2 : } r_c = 1''.5, r_h = 7''.5, \gamma = 2.5$$

Both models have a flat core in the center, fall as r^{-4} over a range between r_c and r_h , and have an extended envelope falling as $r^{-\gamma}$ for $r > r_h$. Figure 7 shows the projected surface densities of these two models, compared with the observed profile in Fort et al. (1992; black squares). As we see, both models are a good fit (except near the center, where the observed profile is smoothed by seeing), even though the asymptotic slope at large radius is different. We consider the two models in order to see how sensitive the result for the velocity dispersion is to the slope of the outer stellar profile.

We will also calculate the velocity dispersion profile expected for the cluster galaxies. For the density profile of the galaxies, we simply take the form $\rho_g(r) = \rho_{g0}/(r^2 + r_c^2)^\gamma$, with $r_c = 30''$ and $\gamma = 2.7$. Profiles of cluster galaxies are uncertain and difficult to measure (see Bahcall 1977; Dressler 1978; West, Dekel & Oemler 1987), but the galaxy velocity dispersion we shall derive is not very sensitive to this profile. The only purpose in calculating the velocity dispersion of the galaxies is to show that our models for the mass profile are consistent with a value of $\sim 1000 - 1500 \text{ km s}^{-1}$, typical of the clusters where lensing has been observed.

Our models for the velocity dispersion will be normalized by requiring that the critical radius for a source at a redshift of 1 is equal to $15''.6$ (the observed radius of the tangential arc), assuming

again spherical symmetry [i.e. we use equation (6) with $\epsilon = 0$]. As we have seen in § 2 and 3, the ellipticity has almost no effect on this radius since the tangential arc lies very close to 45° from the major axis. For other values of the redshift of the tangential arc, the velocity dispersions scale as the square root of the distance ratio D_{ls}/D_s .

Figure 8 gives the line-of-sight velocity dispersion profile, for our Model 2 for the total density profile. The thick lines are for the stellar component, and the thin ones for the galaxies. The solid curve assumes Model S2 for the stellar profile, isotropic velocity dispersion ($\beta = 0$), $f_s = 0.02$ for the fraction of mass in the stellar component, and a core radius $\theta_c = 3''.45$ for the dark profile (as is implied for this case from Fig. 3b). The other lines are the result of varying each one of the parameters at a time: the dashed line is for $\beta = 0.5$, independent of radius (i.e. more radial orbits for the stars), the dotted line is for Model S1, and the dash-dot line shows the effect of increasing the fraction of mass in the stars to $f_s = 0.04$, and the core radius of the dark profile to $\theta_c = 8''.3$, using the lensing constraint of Figure 3b.

As we can see, these changes produce small variations in the velocity dispersion profiles. All models predict a rapid rise in the stellar velocity dispersion profile between $3''$ and $20''$. This is a consequence of the large splitting angle of $\sim 30''$ of the lens, which demands that the velocity dispersion rises to a value close to 1000 km s^{-1} at the radius of the tangential arc, $\theta_t \simeq 15''$, and the presence of the radial arc, which demands a shallow profile for the total mass, while the stellar profile is much steeper. The line-of-sight velocity dispersion predicted for the stars at small radius is consistent with the velocity dispersions of typical cD galaxies (REF). We note that the flattening in σ_{los} for the stars at small radius in Figure 8 is an artifact due to the flat core assumed for the stellar profile in Models S1 and S2. The true profile is probably more cuspy, which would imply a faster decline of σ_{los} toward the center for $r \lesssim 3''$. Better predictions for σ_{los} at small radius can be made when higher resolution observations of the cD galaxy become available.

In Figure 9, we fix $f_s = 0.02$, $\beta = 0$, and we take Model S2, and consider various models for the total mass profile, as indicated in the figure. We choose the following values of the core radius θ_c which reproduce the ratio θ_r/θ_t for $f_s = 0.02$ (see Figs. 2b, 4b, 6b): $\theta_c = 3''.65$ for Model 1, $\theta_c = 20''$ for Model 3, and $\theta_c = 30''$ for Model 5. Again, there is not much difference in the velocity dispersion profiles between different models; the constraints obtained from lensing are sufficiently restrictive to allow a larger variation. The small differences that are present in Figure 9 (and also between the solid and dash-dot curves in Fig. 8) indicate that the stellar velocity dispersion rises more slowly when the dark matter profile is shallower at the center. The velocity dispersion of the galaxies is much larger at the center, ($\sim 1200 \text{ km s}^{-1}$), owing to the large core radius of $30''$, or $100h^{-1} \text{ kpc}$, that we assume. The variations at large radius depend on the outer slope of the mass profile, and are obviously much less constrained by the observed arcs; only the “weak lensing” can be useful at large radius.

5. DISCUSSION AND CONCLUSIONS

In this paper, we have argued that observations of gravitational lensing in clusters of galaxies provide very strong evidence that the dark matter halo of a cD galaxy should be identified with that of the cluster as a whole. The cD galaxy, and any intracluster light, is identified as a continuous

stellar component orbiting in the potential well of the cluster. We have shown that radial arcs provide particularly powerful constraints on the form of the density profile of the lensing objects (rather than the normalization, or mass-to-light ratio at a given radius). This has been applied to MS2137-23, the only case of a radial arc known so far, which is also probably the clearest case where the cD galaxy is exactly at the center of the lens and has the same ellipticity as the lens (Mellier et al. 1993). Basically, the distance from the radial arc to the center, compared to the distance of the other image of the same source, gives constraints on the slope and the curvature of the total density profile, defined as the first and second logarithmic derivative. Models which agree best with the radial arc in MS2137-23 are those which have shallow slopes and large curvature (i.e., the slope increases rapidly with radius in the region of the observed arcs, between 15 and $100h^{-1}$ kpc). Models where the density profile of the dark halo increases as r^{-1} all the way to the center, as is suggested by the numerical simulations of gravitational collapse of halos in Hernquist (1990) and Dubinski & Carlberg (1991), are practically ruled out (our Models 4 and 5), although they can still be consistent if they are highly curved (our Model 3). Models with flat cores (1 and 2) are more favored. More accurate determinations of the distance from the radial arc to the center, and observations of the arc redshifts, are needed to strengthen these conclusions.

We have also seen that the contribution to the mass of the stellar component in the cluster cannot be neglected when modeling radial arcs, due to their proximity to the center. As shown in § 3, the lensing constraints on the form of the density profile give an upper limit to the mass-to-light ratio of the stellar component. This upper limit is consistent with the dynamically determined values in elliptical galaxies for our Models 1 and 2, but is unrealistically low for the other models (except, possibly, for Model 3).

The constraints imposed by lensing on the mass distribution allow us to predict the velocity dispersion as a function of radius of any population of objects in the cluster with a given density profile. We have shown that, for the stars, one expects a central velocity dispersion of $300 - 400 \text{ km s}^{-1}$, while for the galaxies it should be $1000 - 1400 \text{ km s}^{-1}$. This is consistent with observations in other clusters and cD galaxies, and it shows that the difference in velocity dispersions of the two populations arises from their different density profiles: the stars are concentrated in a small region in the center, where the dark matter profile is very flat, whereas the galaxies are further out. However, at large radius the stellar velocity dispersion must increase to that of the galaxies. Thus, a fundamental test of the model should be provided by velocity dispersion profiles of cD halos.

Observations of the stellar velocity dispersion have not been made in MS2137-23. The best observation of the velocity dispersion profile in a cD galaxy was done in A2029 by Dressler (1979). He found that the velocity dispersion in this cluster increases from a central value of 370 km s^{-1} to about 430 km s^{-1} at $10h^{-1}$ kpc, and to about 500 km s^{-1} at $50h^{-1}$ kpc. At the redshift of MS2137-23 ($z_c = 0.313$), these distances would correspond, respectively, to $3''$ and $14''$. From Figures 8 and 9, we see that all our models predict a much faster rise.

A possible reason might be that different clusters have different profiles, and the dark matter in A2029 happens to be less centrally concentrated than in MS2137-23. However, we believe that the rise in the stellar velocity dispersion of A2029 may have been underestimated by Dressler (1979), who fitted the absorption line profiles observed at different radii to gaussians. If the velocity dispersion rises with radius as steeply as in the models of Figures 8 and 9, it is clear that the distribution of velocities will be very far from gaussian at any point. The stars coming up from the center will

be mostly at low velocities, while stars coming down from the outer halo will be moving rapidly. This is true even when the velocity dispersion tensor is isotropic. If the stars on the outer halo are preferentially on radial orbits, the non-gaussianity of the projected velocity distribution should be further increased, producing profiles with narrow cusps and broad tails (van der Marel & Franx 1993). When fitted with gaussian profiles, the velocity dispersion will generally be underestimated.

The information given by gravitational lensing on the structure of the density peak in a cluster of galaxies may have some consequences for the theory of formation of cD galaxies. The origin of cD galaxies has been usually ascribed to “galactic cannibalism”, or the merging of many galaxies with a giant elliptical at the center of the cluster (Gallagher & Ostriker 1972; Ostriker and Hausman 1977; Merritt 1985). However, this process should probably be viewed as a progression of the phenomenon of cluster mergers, or mergers of collisionless matter in halos. When two clusters merge, the low density parts of the outer halos at large radius will tidally disrupt, merge, and reach a new virial equilibrium as soon as the merger starts. On the other hand, the denser parts in the two cores will not be affected for several orbits, but they will be disrupted only when they come sufficiently close. As long as the orbital time within an individual core is shorter than the orbital time of the two cores around each other, the structure of every core is preserved due to adiabatic invariance. Since the orbital times depend only on the density, tidal disruption at a given radius generally occurs when the density of an individual core is similar to the density of the halo in which it is merging. The final merger of the two central galaxies is then simply the last stage of the cluster merger.

It remains to be shown if the process of hierarchical collapse, starting from galactic halos with a distribution of stars as observed in galaxies, can preserve a population of low velocity dispersion stars (which constitute the cD galaxy) at the bottom of the clusters that are formed via successive mergers. If that was not the case, and it was found that the stellar distributions in clusters formed by hierarchical merging should be much more extended and have higher velocity dispersions than the observed cD galaxies, then one would have to invoke dissipation and star formation from the intracluster medium as the process of formation of cD galaxies (Fabian, Nulsen & Canizares 1990).

The exact form of the density profile of dark matter halos should be a powerful test for the presently favored theories of hierarchical gravitational collapse. Of all the known objects in the universe associated with dark matter halos, the cluster MS2137-23 is probably the one where the density profile is most highly constrained, thanks to the observation of a radial arc. Calculations of the probability to observe radial arcs indicate that, for a cluster similar to MS2137-23, the frequency of radial arcs should be $\sim 10\%$ that of tangential arcs of the same length and surface brightness (see Fig. 4 in Miralda 1993). This suggests that many more radial arcs should be found, since the number of known tangential arcs longer than the radial arc in MS2137-23 is much larger than 10, but they may have been missed because they lie close to the central galaxies. Similar constraints on the density profile could then be found in other clusters. Several future observations can improve At the same time, the determination of the density profile in MS2137-23. Higher resolution images of the radial arc will show the exact location of the radial critical line, since the radial arc is made of two images of opposite parity. Deeper observations may reveal other multiply imaged sources of lower surface brightness, as well as large numbers of weakly distorted images at larger radius, which will constrain the density profile at larger radius using the weak gravitational lensing effect. This will also show if there is any substructure in the mass distribution which might affect the modeling of the density profile in this paper, based on the assumption of an elliptical lens. The determination of the

two redshifts of the sources will provide a test for our models, which predict that the two redshifts ought to be very similar, and will add new constraints. Observations of the velocity dispersion of the cluster galaxies and the stars will offer another test of the model predictions.

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REFERENCES

- Blandford, R. D., & Narayan, R. 1986, ApJ, 310, 568
——— 1992, ARA&A, 30, 311
- Carter, D., Efstathiou, G., Ellis, R. S., Inglis, I., & Godwin, J. 1981, MNRAS, 195, 15p
- Dressler, A. 1979, ApJ, 231, 659
- Dubinski, J. & Carlberg, R. G. 1991, ApJ, 378, 496
- Faber, S. M. & Gallagher, J. S. 1979, ARA&A, 17, 135
- Fort, B. 1992, in *Gravitational Lenses*, ed. R. Kayser, T. Schramm & L. Nieser (Springer-Verlag), p. 267
- Fort, B., Le Fèvre, O., Hammer, F. & Cailloux, M. 1992, ApJ, 399, L125
- Fort, B., Prieur, J. L., Mathez, G., Mellier, Y., & Soucail, G. 1988, A&A, 200, L17
- Gallagher, J. S. & Ostriker, J. P. 1972, AJ, 77, 288
- Grossman, S. A., & Narayan, R. 1988, ApJ, 324, L37
——— 1989, ApJ, 344, 637
- Hammer, F. 1991, ApJ, 383, 66
- Jones, C., Mandel, E., Schwarz, J., Forman, W., Murray, S. S., & Harnden, F. R. 1979, ApJ, 234, L21
- Kaiser, N. & Squires, G. 1992, ApJ, 404, 441
- Katz, N. 1991, ApJ, 368, 325
- Kassiola, A., Kovner, I., & Blandford, R. D. 1992, ApJ, 396, 10
- Kneib, J. P., Mellier, Y., Fort, B. & Mathez, G. 1993, A&A, 273, 367
- Kochanek, C. S. 1990, MNRAS, 247, 135
- Lynds, R., & Petrosian, V. 1989, ApJ, 336, 1
- Malumuth, E. M. 1992, ApJ, 386, 420
- Mellier, Y., Fort, B. & Kneib, J.-P. 1993, ApJ, 407, 33
- Merritt, D. 1985, ApJ, 289, 18
- Miralda-E., J. 1991, ApJ, 370, 1
- Miralda-E., J. 1993, ApJ, 403, 497
- Ostriker, J. P., & Hausman, M. A. 1977, ApJ, 217, L125
- Ostriker, J. P., & Tremaine, S. D. 1975, ApJ, 202, L113
- Pelló, R., LeBorgne, J. F., Sanahuja, B., Mathez, G. & Fort, B. 1992, A&A, 266, 6
- Pelló, R., Le Borgne, J. F., Soucail, G., Mellier, Y., & Sanahuja, B. 1991, ApJ, 366, 405
- Pelló, R., Soucail, G., Sanahuja, B., Mathez, G., and Ojero, E. 1988, A&A, 190, L11
- Quinn, P. J., Salmon, J. K. & Zurek, W. H. 1986, Nature, 322, 329
- Schneider, P. 1984, A&A, 140, 119
- Schneider, P., Ehlers, J. & Falco, E. E. 1992, *Gravitational Lenses* (Berlin: Springer-Verlag)
- Smail, I., Ellis, R. S., Aragón-Salamanca, A., Soucail, G., Mellier, Y. & Giraud, E. 1993, MNRAS, 263, 628
- Soucail, G., Fort, B., Mellier, Y., & Picat, J. P. 1987, A&A, 172, L14
- Soucail, G., Mellier, Y., Fort, B., Mathez, G., & Cailloux, M. 1988, A&A, 191, L19
- Turner, E. L., Ostriker, J. P., & Gott, J. R. 1984, ApJ, 284, 1
- Tyson, J. A. 1988, AJ, 96, 1
- Tyson, J. A., Valdes, F., & Wenk, R. 1990, ApJ, 349, L1

- van der Marel, R. 1991, MNRAS, 253, 710
van der Marel, R. P., & Franx, M. 1992, ApJ, 407, 525
West, M. J., Dekel, A., Oemler, A. 1987, ApJ, 316, 1
Zabludoff, A. I., Geller, M. J., Huchra, J .P. & Vogeley, M. S. 1993, AJ, 106, 1273

FIGURE CAPTIONS

Fig. 1: Example of a model for the arcs in MS2137-23. The cross indicates the center of the lens, identified with the center of the galaxy. The two sources are the thick circles, and are assumed to be at the same redshift. The dotted lines are caustics, and the dashed lines are critical lines. The solid triangles are the observed positions of the images, from Table 1 of Fort et al. (1992). The source on the astroid caustic generates the arc A01-A02, the arclets A2 and A4, and a small image in the center which is not observed (it is very faint, and behind the bright central galaxy). The other source generates the radial arc A1 and the arclet A5. The model has the same ellipticity and position angle as the central galaxy, and has the density profile of our Model 1, with $\theta_c = 4''.3$.

Fig. 2: The upper solid line shows the ratio of the distance of the radial arc to the center to the distance of the tangential arc to the center, θ_r/θ_t , assuming the two redshifts are the same, as a function of θ_r/θ_c , where θ_c is the core radius, for Model 1 of the density profile. The upper dashed line is the observed value for this ratio in MS2137-23. The lower solid line gives the ratio θ_r/θ_i , where θ_i is the distance to the center of the other image of the radial arc (A5 in MS2137-23), and the lower dashed line gives the observed value. The two dotted lines below each solid line give the values of the two ratios when a second mass component, with a density profile proportional to the observed stellar profile in the R band, is added to the dark matter profile of Model 1, such that the fraction f_* of projected mass in the stellar profile within the radius θ_i is 0.02 and 0.04. Figure 2a is for a spherical lens, and Figure 2b for an elliptical lens with ellipticity $\epsilon = 0.22$, and using the position angles with respect to the major axis of each image observed in MS2137-23. Since the dotted lines for θ_r/θ_i reach a maximum value, an upper limit of ~ 0.06 is obtained for f_* . The fact that the ratio θ_r/θ_i is consistent with roughly the same core radius as θ_r/θ_c implies that the two sources are at similar redshift.

Fig. 3: Same as Fig. 2, but for Model 2 of the density profile. The upper limit to the mass of the stellar component is only 0.04.

Fig. 4: Same as Fig. 2, for Model 3 of the density profile. The upper limit for the mass in the stellar component is only 0.02, or

Fig. 5: Same as Fig. 2, for Model 4 of the density profile. The mass in the stellar component is now required to be so low that the model is ruled out, unless the distance of the radial arc to the center is smaller than the value assumed here of $5''.4$.

Fig. 6: Same as Fig. 2, for Model 5 of the density profile. This model is ruled out for the same reason as Model 4.

Fig. 7: Surface brightness profile of the cD galaxy in MS2137-23. The black squares are the observations in Fort et al. (1992) in the R band, the solid line is our Model S1 and the dotted line our Model S2. The two models differ in the slope at large radius.

Fig. 8: Velocity dispersion profile in MS2137-23 for the stars (thick lines) and the galaxies (thin lines). The galaxies are assumed to have a density profile with a flat core of $30''$, which causes their velocity dispersion to be large at the center, whereas the stars are much more concentrated to the center than the mass. We assume Model 2 for the density profile of the dark matter. The model called “standard” (solid line) assumes a fraction of mass in the stars within the radius of the tangential arc (equal to $15''.6$) $f_s = 0.02$, and a core radius $\theta_c = 3''.45$ for the dark matter in eq. (19); it also assumes isotropic velocity dispersion ($\beta = 0$) and Model S2 for the stellar profile (with outer slope $\Sigma_s \propto \theta^{-1.5}$). The other curves are the result of varying one of these parameters: the dashed line assumes a predominance of radial orbits in the stellar component, with $\beta = 0.5$; the dotted line assumes Model S1 for the stellar profile; and the dash-dot line is for $f_s = 0.04$, with the core radius of the dark matter increased to $\theta_c = 8''.3$, to keep satisfy the lensing constraints of Fig. 3b.

Fig. 9: Same as Fig. 8, for different models of the dark density profile. All the curves assume isotropic velocity dispersion, $f_s = 0.02$, and Model S2 for the stellar profile.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 5a

Fig. 5b

Fig. 7

Fig. 8

Fig. 9