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THE NEUTRON/PROTON RATIO OF FAST NUCLEON EMISSION IN ANTIPROTON ANNIHILATION ON NUCLEI¹

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Abstract

The measurement of fast protons and neutrons emitted after antiproton annihilation at rest on ^{238}U and ^{63}Cu reveals a large neutron/proton ratio R . Its value for ^{63}Cu is larger than expected from the conventional model of intranuclear pion rescattering. A value of R essentially constant over the whole range of mass number, from ^{12}C up to ^{238}U , is also announced by the experimentalists. It is shown that, on the contrary, the conventional scheme predicts a regular increase of R with mass number. Alternative explanations of the effect are looked for. Within the usual scheme, an excess of negative pions, leading to more emitted neutrons, is not compatible with final pion multiplicities. The influence of meson resonances on the neutron/proton ratio is considered, as well as the possible occurrence of annihilations on two nucleons. They cannot warrant an increase of R at low mass targets sufficient to explain a constant value over the whole range of target masses.

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1. Introduction

Experimental results have been obtained in the last few years concerning fast nucleons emitted in antiproton annihilation on nuclei — especially at rest [1-3]. They appear intriguing by their large neutron/proton ratio. We try here to understand this feature in the light of simple model assumptions.

To start with, we adopt the conventional picture of the annihilation mechanism at rest, where the antiproton annihilates on a single nucleon in the outer nuclear fringe, emitting about five pions. A fraction of the pions cross the nucleus, ejecting nucleons (and a small amount of light nuclides) and being possibly absorbed. The intranuclear cascade (INC) model [4] embodies this picture and has been shown in the last decade to reproduce several aspects of the experiments to a fair level of accuracy. In the present study, we borrow some properties of INC without dwelling into the complete space-time structure of the cascade. The later phases of nuclear deexcitation are not considered here.

2. The experimental results for neutron and proton yield

In their recent study of nucleon emission from $\bar{p}$ annihilation on ^{63}Cu and ^{238}U , Polster et al. [3] have identified in the neutron spectrum a high temperature component with a Maxwell shape, of temperature $T = (49.6 \pm 4.8) \text{ MeV}$ for ^{63}Cu and $T = (46.5 \pm 3) \text{ MeV}$ for ^{238}U . We assume that these particles originate in the fast stage of the pion cascade: INC calculations [4] have obtained for nucleons directly knocked out of the nucleus during the hot phase of the cascade a Maxwell-like spectrum, with a "temperature" of the same order as the ones cited above, in spite of the fact that one cannot speak of a totally thermalized source. For the protons, the authors of ref. [3] have made a fit imposing the same temperature as for the neutrons, replacing E in the Maxwell form by $(E - B_c)$, B_c being the height of the Coulomb barrier. The mean multiplicities of neutrons M_n and protons M_p obtained by integration of the so-determined distributions are shown in Table 1. To account for the neutron/proton asymmetry of the target, the neutron/proton ratio is normalized by division by the N/Z value of the target. One would then naively expect the normalized ratio :

$$R = \frac{M_n^* Z}{M_p^* N} \quad (2.1)$$

to be close to 1. The values obtained for R are given in the third line of the Table. They deviate quite significantly from 1. Similar values for the multiplicities of energetic nucleons have been found in other measurements for uranium target : 1.34 ± 0.37 for protons [1], 4.49 ± 0.75 for neutrons [2] i.e. a normalized neutron/proton slightly larger but compatible with the result of Ref. [3]: $R = 2.11 \pm 0.39$. (However, the temperatures are then larger and different for protons (≈ 76 MeV) and for neutrons (≈ 99 MeV)).

Meanwhile the number of protons and neutrons effectively observed are not those which have been involved in intranuclear scattering by pions. To single out the properties of the intrinsic pion cascade inside the nucleus, the multiplicities should be corrected for nuclear effects. For the protons, a correction has to be made for the presence of the Coulomb barrier. The phase space factor appropriate for the protons being $\sqrt{E - B_c}$ instead of $\sqrt{E}$ for the neutrons, we scale up the proton multiplicity obtained in the fit of the measured spectrum by $\exp\left(\frac{B_c}{T}\right)$, with $B_c = 6.78$ MeV for ^{63}Cu and $B_c = 14.52$ MeV for ^{238}U . To take into account the production of fast fragments [5], supposedly initiated by fast nucleons, we assume that the fragments are formed by the coalescence of scattered protons and neutrons and also take account of the production of ^3He and ^3H fragments, assumed to result from pick-up by deuterons. Referring to data by Hofman et al. [5], we parametrize the number of fragment per proton as $5.2 \cdot 10^{-2} A^{1/3}$, and add the corresponding number to the proton and neutron multiplicities. The result of the two modifications is given in Table 1, where M_n^* and M_p^* are our estimates for the mean multiplicities of neutrons and protons recoiling inside the nucleus from the cascading pions.

We end up with estimated mean multiplicities of nucleons $M^* = M_n^* + M_p^*$: 3.82 ± 0.32 for Cu and 5.51 ± 0.41 for U. The number of nucleons which would have been emitted in the absence of the effects considered above is M^* , while $M = M_n + M_p^*$ is the number actually emitted. The values of M^* are consistent with the predictions of the version [4] of INC, where charges were not included; however, the new

normalized neutron/proton ratio

$$R^* = \frac{M_n^* Z}{M_p^* N} \quad (2.2)$$

still deviates from 1, quite significantly for Cu. Now, the primordial pion system emitted in the antiproton-nucleon annihilation bears, on average, an excess of negative charge due to annihilation on neutrons and it is tempting to look there for an explanation of the large neutron/proton ratio. We therefore investigate the charge properties in the pion cascade to find out whether the asymmetry of the original pion set can explain the effect.

3. The charge structure of the pion cascade

Since the pions have their energies in the region where the pion-nucleon interaction is dominated by $\Delta(1232)$, we retain the isobar picture, where pion scattering results from :

$$\pi N \rightarrow \Delta, \Delta \rightarrow \pi N, \quad (3.1)$$

to determine the charge branching ratios.

In the Appendix, we follow the variation of the pion charge and the number and charge of the knocked nucleons resulting from the scattering of a pion of definite charge through a medium with a definite (N, Z) neutron-proton composition. We use the results obtained there in the following way for a first comparison with the data. We assume that the annihilation of the antiproton with a nucleon yields, on the average, exactly five pions (for simplicity) with the following distribution of charges : for antiproton-proton annihilation $n_+ = n_0 = n_-$, and for antiproton-neutron annihilation $n_+ = n_0 - \frac{1}{2} = n_- - 1$. Each pion crossing the nucleus is assumed to scatter independently. We thus consider a weighted average of pion charges corresponding to annihilation on a (N, Z) target, with equal probability for annihilation on proton and neutron, and we follow the scattering of such a "beam" propagating through the target medium.

The resulting value of R^* , as a function of the total number of scatterings, is given in Figure 1, where the results are shown for the (N, Z)

values corresponding to ^{238}U , ^{63}Cu and a symmetric nucleus. Note that the number of scattered nucleons refers to one pion, and is not equal to M^* obtained after $\bar{p}$ annihilation (see below); however, for the limited range of multiplicity of physical interest, the neutron/proton ratio can be considered linear in the nucleon number. In ^{238}U , the normalizing N/Z factor entails a low initial value for R^* , which increases steadily as scatterings occur in a neutron-rich medium, whilst for a symmetric medium the impact of the initial charge asymmetry of the pions is progressively damped out. The differences in R^* , however, are not large. More comments on the values of R^* can be found at the end of the Appendix.

Since the number of interacting pions is ≈ 2 for a large nucleus [4], the mean number of knocked nucleons per interacting pion, referring to the multiplicities obtained above, is $\approx 2 - 3$, depending on the value of A . It appears, therefore, that the ratio R^* is not expected to turn greater than ≈ 1.1 for Cu and U . This is at variance with the value we deduced from the experiment in the case of Cu . Furthermore, from unpublished data, the value measured for the ratio R —the ratio of the neutrons to protons effectively emitted— turns out essentially constant over the whole range of A values, from $A = 12$ to $A = 238$ [6]. This is surprising as the corrections introduced above for the multiplicities (Coulomb barrier, production of fragments) become less important as A decreases, reducing the difference between R and R^* ; the result just quoted thus implies that the corresponding value of R^* increases when A decreases, and reaches values definitely larger than expected from Fig. 1.

The presence of the neutron excess observed in the experiment does not fit with the assumption of a simple pion cascade. To vindicate this conclusion, we have devised a simulation concentrating on the charge structure of the cascade, which furthermore includes the possibility of pion absorption.

4. A simple cascade calculation with pion absorption

The model includes a simulation for pion trajectories through the nucleus, registering the scattering of nucleons. Random directions are generated from the edge of a spherical homogeneous nucleus of specified Z and $A = Z + N$ values, with radius $1.15 A^{1/3}$. For pion emission, we use the multiplicities given by a statistical model [7] in good agreement with the data of $\bar{p}p$ and $\bar{p}n$ annihilation and only slightly different from the result of the simple assumption made in Section 3. Scattering inside nuclear matter,

averaged over the momentum of the pions, is characterized by a scattering length λ (in the $I=3/2$ state of the pion-nucleon system). Straight line propagation of the pion is assumed, scattering being supposed to occur along it according to the mean free path. The charge of the target nucleon is chosen according to the Z and N values, which are redefined after each interaction. The charges of the nucleon and pion emerging from scattering are, as in Sect. 3, obtained from the Clebsch-Gordan coefficients implied by (3.1). The possibility of pion absorption which, in the isobar picture, corresponds to the occurrence of

is included through the parameter β , the probability that Δ "recombines" with a nucleon in the $I=1$ state before it has a chance to decay.

We have adjusted the two parameters λ and β on the results for ^{238}U (see Table 2) to reproduce the mean multiplicity of scattered nucleons — protons plus neutrons — and the measured multiplicities of pions [8]. The numbers of nucleons scattered in the simulated trajectories are modified for nuclear effects (Coulomb barrier, fragment formation, see Sect. 2), leading to $M > M^*$. We give in Table 2 the results of the simulation concerning the nucleon mean multiplicities for ^{238}U , ^{63}Cu and ^{12}C . Compared to the evaluation made in section 2, the simulation uses a distribution of pion trajectory lengths and accounts for pion absorption. The change in the neutron/proton ratio is quite small (compare Table 2 and Fig. 1).

The values of the scattering length λ retained to fit M or M^* , respectively 1.6 and 1.8 fm, correspond to isospin-averaged values of 2.4 and 2.7 fm in the case of a symmetric pion set, scattering on a symmetric nucleus. The order of magnitude compares with 2.1 fm used by Besch et al. [9] for annihilation at low $\bar{p}$ momentum. The absorption probability β refers to the $I=1$ state; therefore the retained values 0.6 or 0.7 correspond in a symmetric system to a probability $3\beta/8$, equal to 0.22 or 0.26, for a Δ to be absorbed before decaying.

The simulation confirms a property anticipated in section 2, namely that the neutron-proton ratio depends only weakly on the nucleon

multiplicity. As the comparison of the set of results reproducing either M or M^* shows, the change of 12 % in λ entails a variation around 10% in the number of nucleons but it hardly changes the value of the neutron/proton ratio, in agreement with the slopes in Figure 1 (for ^{238}U , the slope of R^* is ≈ 0.02).

With the value of β retained for ^{238}U , the absorption probability is predicted too large for ^{12}C . The trend of the effect can be understood as follows. The absorption probability is expected to increase with the length of the pion trajectory, since a slower pion, and hence a slower Δ , is more likely to lead to absorption. For ^{12}C (where the mean number of scatterings per interacting trajectory is only 1.8 against 2.7 in ^{238}U), the value 0.4 instead of 0.6 for β gives the measured value of $M(\pi)$.

In any case, the value predicted for R is practically independent of β . We have calculated the value of R for several nuclei over the whole mass range from ^{12}C to ^{238}U . The actual values of R are slightly scattered around the smooth curve in Fig. 2. There is a definite slope, in contradiction with the result for Cu [3] and with the general property of R being essentially constant over the whole mass range [6]. Since the measured neutron/proton ratio cannot be understood within the scheme adopted above, we look for modified assumptions which could explain a different distribution of the charge among the nucleons.

5. Other possible schemes

In spite of everything, the dominance of Δ isobar in pion-nucleon scattering appears as the most efficient mechanism to translate an excess of negative charge in the primordial pion system as well as a neutron excess of the target into a larger fraction of neutrons among the knocked out nucleons, albeit unsufficiently. It is easy to show that πN scattering in isospin $1/2$ state leads to a value of R^* lower than 1 (0.91 for ^{238}U after one scattering length). We have neglected intranuclear rescattering of nucleons, but its occurrence tends to reduce any deviation from 1 for R^* and therefore does not help.

One possibility for a larger value of the neutron/proton ratio is a different distribution of charges in the initial pion set, which could result from a deviation from the N/Z ratio for $\bar{p}n$ / $\bar{p}p$ annihilations. It has been

proposed by Bugg et al [10] that the annihilation probability is smaller for $\bar{p}n$ than for $\bar{p}p$. To reproduce the difference between π^- and π^+ in ^{12}C , they assumed $\gamma = T(\bar{p}n)/T(\bar{p}p) \approx 0.63$. This works in the wrong direction for the neutron/proton ratio : while it seems to give a better π^-/π^+ ratio in the final pions, at least for ^{238}U , it decreases somewhat R (Table 2).

Let us also mention that the signature of a neutron halo, an effect which is in a sense in the direction opposite to what has just been considered, has been observed in $\bar{p}$ annihilation on ^{232}Th [11], for annihilation events where the mass number is reduced only by 1. The large observed ratio of $^{231}\text{Th} / ^{231}\text{Ac}$ residues corresponds to a large predominance of annihilations on neutrons. However, this class of annihilations is a filter for very peripheral events, which are not expected to represent more than $\approx 10\%$ of all annihilations [4] and do not yield scattered nucleons. Anyway, in order to modify the value of R significantly, we have to force a large global asymmetry in the occurrence of (not too peripheral) $\bar{p}n$ and $\bar{p}p$ annihilations. We can simulate an extreme effect : Table 2 gives the results for the extreme, unrealistic case of annihilations occurring only on neutrons ($\gamma \rightarrow \infty$) ; the increase in the neutron/proton ratio is not spectacular and R has even a greater slope than in the normal case. Besides, it leads to a π^-/π^+ ratio much larger than what is observed. Incidentally, the sum of charged pions is essentially independent of detailed assumptions whereas the π^-/π^+ ratio is directly affected.

Another possibility to obtain a different neutron/proton ratio is the role of meson resonances. The ω meson which lives sufficiently to propagate well inside the nucleus has a substantial rate ($\approx 0.2 \omega$ per annihilation). Elastic scattering of the ω meson gives $R^* = Z/N$; another possible interaction is $\omega N \rightarrow \pi N$, via N^* formation and decay, giving a ratio

$$R^* = Z (N + 2Z) / [N (2N + Z)] \quad (5.1)$$

which is lower than 1 for $Z/N < 1$. A similar argument applies to the less abundant η . We fail to find an explanation for a large value of the neutron/proton ratio.

A more drastic possibility implies resonances of isospin 1, with a shorter lifetime. It is now generally accepted that nucleon-antinucleon annihilation at rest and at low momenta is dominated by two-body modes, i.e. the emission of two —often wide— mesons which decay, directly or in steps, into long-lived mesons, principally pions [12,13]. The use of a pion cascade to study the interaction of the annihilation products with the nucleus is justified by the fact that, due to a mean decay length ($\leq 16m$) likely shorter than their mean free path, these objects almost always decay before interacting (except, probably, for ω and η). Let us, however, consider for a moment the possibility that a short interaction length in nuclear matter leads to scattering before decay into pions has the chance to occur. The scheme

forbids the positive mesons for $\bar{p}n$ annihilation, assuming equal probability for the three charges in $\bar{p}p$ annihilation, we have the charge distribution $M^+ : M^0 : M^- = 2 : 5 : 5$ for a symmetric nucleus. This is a bias in favor of negative charge larger than in direct emission of five pions. For intranuclear scattering, we keep on with the spirit of a cascade and look at processes of the type:

where B is an intermediate baryon state and M' is identical to M or not. The isospins of M and M' can be 0 or 1, and the isospin of B, $1/2$ or $3/2$. To get an idea of the maximum value possibly reached by the neutron/proton ratio, we make extreme assumptions able to grant a large ratio, by taking $I(M)=I(M')=1$, $I(B)=3/2$. We assume that the first two scattering lengths are covered in the form of a resonance, the rest of the scatterings being made by daughter pions. The maximum enhancement of R^* is obtained for a small nucleus. We consider the case of ^{12}C , with a total emission of $M^* = 2.71$ nucleons (see Table 2). Over the two scattering lengths, 1.33 nucleon is emitted, split up into 0.535 proton and 0.798 neutron, as follows from

application of the appropriate formulas for $b_{p,n}$ in the Appendix. Pions then scatter ($2.71-1.33$) = 1.38 nucleon, with the ratio 1.17 (see Fig.1), thus 0.636 proton and 0.744 neutron, (assuming for pions a charge distribution similar to the primordial one in the conventional case, which, if anything, is a bias in favor of neutrons). The final outcome is 1.17 proton and 1.54 neutron, with a ratio $R^*=1.32$ which does not represent a very large increase, it does not quite reach the value for a pion cascade with an initial pion set corresponding to $\gamma \rightarrow \infty$. Remember that we have considered particularly favorable conditions, probably too drastic to be realized. The conclusion is that large values of R^* cannot be explained in the framework of resonance interaction.

The last possibility which we investigate is the occurrence of a significant fraction of annihilations on two nucleons (the so-called $B=1$ annihilations, against $B=0$ for conventional ones). We use the statistical model to generate the products of the reaction of annihilation. The model for $B=0$ [7], used above, is extended to the $B=1$ case by means of the rule retained in Ref. [14] for the dimensional parameter. The mean multiplicities of nucleons and pions are given in Table 3.

Referring again to the case of ^{12}C , we have the contribution of the primordial nucleon, 0.454 proton and 0.546 neutron, to which we add 1.71 nucleon resulting from the scattering of a pion system distributed in charge as in Table 3. It gives 0.790 proton and 0.920 neutron, so that we end up with a ratio $R^* = 1.466/1.244 = 1.18$, a result lower than in other conditions which we have examined. Let us point out that a more direct signature of the presence of $B=1$ annihilations should be searched in a high energy tail of the nucleon spectrum or, possibly, in an enhancement of strangeness in the final products. From the consideration of the shape of the high-momentum tail of the proton spectrum, a fraction of $27 \pm 2\%$ ($15 \pm 2\%$) of all events in carbon (uranium) has been ascribed to $B > 0$ annihilations [1]. These figures are pretty large but do not help to solve the question of the neutron/proton ratio.

6. Conclusion

We have examined the production of fast nucleons emitted from annihilation of antiprotons at rest on nuclei. The data for U and Cu [1] show

a neutron/proton ratio, normalized for the neutron excess of the target, significantly larger than 1. The experimental result for Cu is not in agreement with charge properties in the frame of conventional intranuclear scattering of pions. The fact that the neutron/proton ratio maintains a large value down to ^{12}C [4], contrarily to the obvious expectation, has motivated an investigation.

In spite of various alternative assumptions about the process of interaction, we have not been able to spot any possibility to account for the observed behavior. Of all the possibilities which we have considered, the conventional picture of pions scattering through the nucleus, emitting nucleons in a scattering scheme involving production and decay of the Δ state is the more apt to generate a value of the neutron/proton ratio greater than 1, however unsufficiently. The possibility of an initial set of pions strongly biased in favor of negative ones is not compatible with the observation of final pions. The predicted neutron/proton ratio is robust, in the sense that, within a definite scheme, it depends only marginally on the total number of nucleons, for reasonable variations of this number. We have briefly examined the possibility of other mechanisms (meson resonances, annihilation on two nucleons) only to find that they do not seem able to warrant a significant increase in the neutron/proton ratio.

To sum up, the isobar picture (3.1) predicts the largest neutron/proton ratio for the fast nucleons emitted from $\bar{p}$ annihilation on nucleons. It is compatible with the ^{238}U results within the measurement errors but the results for ^{63}Cu and for lighter nuclei deviate from its predictions. The observed values of the neutron/proton ratio cannot be understood in any simple picture we could think of. More accurate data are needed. If the numbers are confirmed, they are likely to require a new explanation, beyond a conventional scheme.

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Table 1. Nucleon multiplicities and ratios measured in experiment [1] and deduced after correction for nuclear effects

	^{63}Cu	^{238}U
Measured neutron multiplicity M_p [3, 6]	2.20 ± 0.27	3.17 ± 0.31
Measured proton multiplicity M_p [3, 6]	0.99 ± 0.08	1.04 ± 0.12
Normalized ratio $R = \frac{M_n}{M_p} \frac{Z}{N}$	1.90 ± 0.28	1.92 ± 0.29
Measured nucleon multiplicity M	3.19 ± 0.28	4.21 ± 0.33
Proton multiplicity corrected for Coulomb	1.14 ± 0.09	1.42 ± 0.16
Fragment multiplicity (estimated)	0.24 ± 0.03	0.46 ± 0.05
Modified neutron multiplicity M_n^*	2.44 ± 0.30	3.63 ± 0.35
Modified proton multiplicity M_p^*	1.38 ± 0.11	1.88 ± 0.21
Normalized ratio $R^* = \frac{M_n^*}{M_p^*} \frac{Z}{N}$	1.51 ± 0.22	1.22 ± 0.18
Measured nucleon multiplicity M^*	3.82 ± 0.32	5.51 ± 0.41

Table 2. Predictions of the intranuclear pion cascade with Δ isobar (with several sets of parameters, see text) for nucleon multiplicities and ratio and for pion multiplicities, compared to measurements when available.

	λ	β	γ	M_p	M_n	M	R	
Exp.				1.04 ± 0.12	3.17 ± 0.31	4.21 ± 0.33	1.92 ± 0.29	[3]
238 U	1.60	0.60	1	1.13	3.11	4.23	1.74	
	1.80	0.70	1	1.02	2.82	3.84	1.75	
	1.60	0.60	0.63	1.14	3.07	4.21	1.70	
	1.80	0.70	0.63	1.03	2.77	3.80	1.69	
	1.60	0.60	∞	1.05	3.35	4.40	2.01	
Exp.				0.99 ± 0.08	2.20 ± 0.27	3.19 ± 0.28	1.90 ± 0.28	[3]
63 Cu	1.60	0.60	1	1.31	2.09	3.40	1.37	
	1.80	0.70	1	1.18	1.89	3.07	1.37	
	1.60	0.60	0.63	1.33	2.03	3.36	1.30	
	1.80	0.70	0.63	1.20	1.86	3.05	1.32	
	1.60	0.60	∞	1.20	2.28	3.48	1.62	
Exp.				0.72 ± 0.05				[1]
12 C	1.60	0.60	1	1.07	1.33	2.40	1.24	
	1.80	0.70	1	0.97	1.21	2.18	1.24	
	1.60	0.40	1	1.06	1.31	2.37	1.23	
	1.60	0.40	0.63	1.07	1.28	2.35	1.19	
	1.80	0.45	0.63	0.97	1.16	2.13	1.20	
	1.60	0.40	∞	0.97	1.42	2.39	1.46	

continued next page

Table 2 (continued)

	M_p^*	M_n^*	M^*	R^*	$M(\pi^+)$	$M(\pi^0)$	$M(\pi^-)$	$M(\pi)$	
Exp.	1.88 ± 0.21	3.63 ± 0.35	5.51 ± 0.41	1.22 ± 0.18	0.99 ± 0.05	1.36 ± 0.09	1.48 ± 0.08	3.84 ± 0.13	[8]
238 U	2.27	3.84	6.11	1.07	0.96	1.36	1.53	3.85	
	2.05	3.48	5.53	1.07	0.95	1.35	1.52	3.82	
	2.29	3.81	6.11	1.05	1.00	1.37	1.49	3.85	
	2.08	3.44	5.52	1.04	0.99	1.36	1.48	3.83	
	2.12	4.04	6.16	1.20	0.84	1.34	1.67	3.85	
Exp.	1.36 ± 0.11	2.44 ± 0.30	3.82 ± 0.32	1.51 ± 0.22					
63 Cu	1.90	2.48	4.38	1.12	1.09	1.48	1.55	4.12	
	1.71	2.24	3.95	1.12	1.09	1.47	1.54	4.10	
	1.93	2.43	4.36	1.07	1.14	1.48	1.50	4.12	
	1.74	2.22	3.96	1.09	1.12	1.48	1.50	4.10	
	1.74	2.64	4.38	1.29	0.92	1.45	1.75	4.12	
Exp.					1.25 ± 0.06	1.73 ± 0.11	1.59 ± 0.08	4.57 ± 0.16	[8]
12 C	1.27	1.48	2.75	1.17	1.19	1.58	1.64	4.42	
	1.15	1.35	2.50	1.17	1.18	1.59	1.62	4.39	
	1.26	1.46	2.71	1.16	1.25	1.63	1.68	4.56	
	1.27	1.43	2.70	1.12	1.30	1.64	1.63	4.56	
	1.14	1.30	2.44	1.13	1.29	1.65	1.63	4.57	
	1.16	1.55	2.71	1.35	1.04	1.62	1.91	4.57	

Table 3. Mean particle multiplicities in $B = 1$ annihilation

Initial state	Emitted particle				
	p	n	π^+	π^0	π^-
$\bar{p} p p$	0.549	0.451	1.654	1.654	1.203
$\bar{p} p n$	0.451	0.549	1.203	1.654	1.654
$\bar{p} n n$	0.365	0.635	0.7993	1.5295	2.1644
$Z = N$ average	0.454	0.546	1.215	1.623	1.669

Appendix : Algebra of charge in consecutive pion scatterings in the isobar model

We consider a succession of scattering processes, assumed to occur according to (3.1), of a pion propagating through a target with a fixed $z = Z/A$ value ; pion absorption is not included here. We are interested in the transformation of the pion charge and in the charge and number of the nucleons which recoil in the scatterings.

We denote by

$$v_i = (a_+(i), a_0(i), a_-(i)) \quad (A.1)$$

the vector describing the mean abundances (summing up to 1) in the three pion charges after the i th scattering length. The asymptotic pion abundances are given by

$$v = A v \quad (A.2)$$

where A reflects the isospin structure in (3.1)

$$A = \frac{1}{9} \begin{pmatrix} 7+2z & 2z & 0 \\ 2(1-z) & 7 & 2z \\ 0 & 2(1-z) & (9-2z) \end{pmatrix}. \quad (A.3)$$

We find

$$a_+(\infty) = \frac{z^2}{1-z+z^2} \quad (A.4a)$$

$$a_0(\infty) = \frac{z(1-z)}{1-z+z^2} \quad (A.4b)$$

$$a_-(\infty) = \frac{(1-z)^2}{1-z+z^2}, \quad (A.4c)$$

leading to the interesting relation

$$a_+(\infty)a_-(\infty) = [a_0(\infty)]^2 \quad (A.5)$$

independently of the z -value of the target. Note that the often used ratio

$$\rho = \frac{2a_0}{a_+ + a_-} \quad (A.6)$$

takes the form

$$\rho(\infty) = \frac{2z(1-z)}{1-2z+z^2} \quad (A.7)$$

which gives exactly 1 only for $z = 1/2$.

From the knowledge of v_{i-1} , the production of nucleons in the i th step is given by

$$w_i = (b_p(i), b_n(i)) \quad (A.8)$$

obtained from the application of the 3×2 matrix B

$$B = \frac{1}{9} \begin{pmatrix} 2+7z & 2(1+z) & z \\ (1-z) & 2(2-z) & (9-7z) \end{pmatrix} \quad (A.9)$$

$$w_i = B v_{i-1}. \quad (A.10)$$

The problem of computing the pion charge abundances and the proton and neutron recoils collected after n scattering lengths can be solved by the search of the eigenvalue and eigenvectors of the matrix

$$C = \begin{pmatrix} A & O_1 \\ B & O_2 \end{pmatrix} \quad (A.11)$$

operating on vectors of type $V \equiv (v, w)$. In (A.11), the O_1 and O_2 matrices are respectively 2×3 and 2×2 matrices identically equal to zero. The pion charge abundances are given by the first three components of

$$V_n = C^n V_0 \quad (A.12)$$

while the nucleon numbers are given by the last two components of

$$X_n = \sum_{i=1}^n A^i V_0 = \sum_{i=1}^n V_i. \quad (A.13)$$

The results are given below.

Putting the following set of definitions

$$\lambda_{1,2} = \frac{7 \pm 2\sqrt{z(z-1)}}{9} \quad (A.14a)$$

$$\alpha, \beta = \frac{\lambda_{1,2}}{1-\lambda_{1,2}} (1-\lambda_{1,2})^n \quad (A.14b)$$

$$Q_n^\pm = \lambda_1^n \pm \lambda_2^n \quad (A.14c)$$

$$D = 2(1-z+z^2), \quad (A.14d)$$

we have, for $a_+(0) = 1, a_0(0) = a_-(0) = 0$:

$$a_+(n) = \frac{\left[2z^2 + (1-z)\sqrt{z(1-z)} Q_n^- + (1-z) Q_n^+ \right]}{D} \quad (A.15a)$$

$$a_0(n) = \frac{\left[2z(1-z) + (1-z)^2 \sqrt{\frac{(1-z)}{z}} Q_n^- - (1-z)z Q_n^+ \right]}{D} \quad (A.15b)$$

$$a_-(n) = \frac{\left[2(1-z)^2 - (1-z) \sqrt{\frac{(1-z)}{z}} Q_n^- - (1-z)^2 Q_n^+ \right]}{D}. \quad (A.15c)$$

Putting, furthermore

$$E = 2(4z^2 - 4z + 49) \quad (A.16a)$$

$$\gamma = \frac{(\alpha - \beta)\sqrt{z(z-1)}}{E} \quad (A.16b)$$

$$\delta = \frac{(\alpha + \beta)}{E}, \quad (A.16c)$$

we have

$$b_p(n) =$$

$$\frac{2 \left[\frac{D}{3} z(1+2z^2) - \gamma z - 1(2z^3 + 27z^2 + 3z + 14) - 5\delta(z-1)(2z^3 - 3z^2 + 6z + 2) \right]}{D} \quad (A.17a)$$

$$b_n(n) =$$

$$\frac{2}{3} \frac{\left\{ \frac{n}{z} (-2z^3 + 6z^2 - 7z + 3) - 5\gamma \frac{z-1}{z} (2z^3 - 3z^2 + 6z - 7) - \delta(z-1) (2z^3 - 33z^2 + 63z - 46) \right\}}{D} \quad (A.17b)$$

Since $\lambda_1, \lambda_2 < 1$, only the first terms in the expressions (A.15), (A.17) remain asymptotically; (the limit cases $z = 0$ and $z = 1$, of little physical interest, should be excluded). For the pion abundances, the result has already been obtained above. The rate of approach towards the asymptotic values is quite slow, as seen from the fairly simple expressions obtained in the particular case $z = 1/2$

$$a_{\pm}(n) = \frac{1}{3} \left\{ 1 \pm \frac{1}{2} \left[3 \left(\frac{8}{9} \right)^n + \left(\frac{6}{9} \right)^n \right] \right\} \quad (A.18a)$$

$$a_0(n) = \frac{1}{3} \left[1 - \left(\frac{6}{9} \right)^n \right] \quad (A.18b)$$

$$b_{p,n}(n) = \frac{n \pm \frac{5}{2}}{3} \left[1 - \left(\frac{8}{9} \right)^n \right]. \quad (A.18c)$$

The particle numbers in the case $a_-(0) = 1, a_+(0) = a_0(0) = 0$ are immediately obtained from the (A.15, A.17) by the simultaneous substitutions

$$a_+ \leftrightarrow a_-, \quad b_p \leftrightarrow b_n, \quad z \leftrightarrow 1-z.$$

In the case $a_0(0) = 1, a_+(0) = a_-(0) = 0$, we have

$$a_+(n) = \frac{\left[2z^2 + \sqrt{z(1-z)} (1-z) Q_n^- - z^2 Q_n^+ \right]}{D} \quad (A.19a)$$

$$a_0(n) = \frac{\left[2z(1-z) - \sqrt{z(1-z)} Q_n^- + (2z^2 - 2z + 1) Q_n^+ \right]}{D} \quad (A.19b)$$

$$a_-(n) = \frac{\left[2z(1-z)^2 + z\sqrt{z(1-z)} Q_n^- - (1-z)^2 Q_n^+ \right]}{D} \quad (A.19c)$$

and

$$b_p(n) = \frac{2}{3} \frac{\left\{ \frac{n}{z} (1 + 2z^2) + \gamma (8z^3 - 42z^2 + 27z - 4) - \delta(2z-1) (6z^3 + 8z^2 + 7z + 14) \right\}}{D} \quad (A.20a)$$

$$b_n(n) =$$

$$\frac{2}{3} \frac{\left\{ \frac{n}{z} (-2z^3 + 6z^2 - 7z + 3) - \gamma(2z-1) (4z^2 + 11z - 11) - \delta(2z-1) (6z^3 + 26z^2 + 41z - 35) \right\}}{D} \quad (A.20b)$$

The results are symmetric for the simultaneous interchanges :
 $a_+ \leftrightarrow a_- \quad b_p \leftrightarrow b_n \quad z \leftrightarrow (1-z)$, as it must.

For $z = 1/2$, they take the particularly simple form :

$$a_+(n) = \frac{1}{3} \left[1 - \left(\frac{6}{9} \right)^n \right] \quad (A.21a)$$

$$a_0(n) = \frac{1}{3} \left[1 + 2 \left(\frac{6}{9} \right)^n \right] \quad (A.21b)$$

$$b_{p,n}(n) = \frac{n}{3}. \quad (A.21c)$$

The asymptotic values of $R^* = \frac{b_n}{b_p} \cdot \frac{z}{1-z}$ are independent of the pion charge. They are equal to 1.35, 1.11 and 1.00 for the values of z corresponding respectively to ^{238}U , ^{63}Cu and a symmetric nucleus (compare to Fig. 1) : the approach towards the asymptotic values is quite slow at the extremities of the range of physical z values.

As for the behaviour at low nucleon number, let us consider the situation after one scattering length (this corresponds to $2/3$ for the nucleon number in the case of $z = 1/2$, and a slightly larger value for $z < 1/2$). Straightforward algebra involving the Clebsch-Gordan coefficients and the initial, averaged, pion charge distribution as assumed in Sect. 3 gives for the (non normalized) neutron/proton ratio

$$\frac{b_n}{b_p} = \frac{8z - 71z + 9z^2}{17 + 44z + 9z^2}. \quad (A.22)$$

R^* can be written, in terms of the relative neutron excess $E = \frac{N-Z}{A} = 1 - 2z$,

to first order :

$$R^* = \frac{195}{135} \left(\frac{R_L}{a_1} \right)^2$$

The values of the $\frac{R_L}{a_1}$ ratio are $\frac{195}{135}$ and of $R^* = \frac{195}{135} \left(\frac{R_L}{a_1} \right)^2$

Nucleotide

N = $\frac{195}{135}$ Z
630 $\frac{195}{135}$ U
238 $\frac{195}{135}$ 8U

Z S
0.5 2.0
0.460 0.460
0.387 0.387

to first order :

$$R^* \approx \frac{195}{135} (1 - 0.72 E) . \quad (A.23)$$

The values of this ratio and of $R^* = \frac{b_n z}{b_p (1 - z)}$ are given below.

Nuclide	z	n/p	R*
N = Z	0.5	1.182	1.182
⁶³ Cu	0.460	1.308	1.116
²³⁸ U	0.387	1.581	0.996

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Figure captions

Fig. 1. Normalized ratio of mean neutron/proton multiplicities in the scattering of one pion (averaged over the charges in $\bar{p}N$ annihilation, see text), as a function of the number of scattered nucleons, in targets with different neutron-proton content.

Fig. 2. Curve R : Normalized neutron/proton ratio in fast nucleon emission resulting from $\bar{p}$ annihilation, predicted from pion rescattering in the isobar model. The two data points are the results for ^{63}Cu and ^{238}U .

Curve R^* : Normalized neutron/proton ratio for intranuclear scattering, before correction for Coulomb barrier and fragment production.

Fig. 1

Neutron/proton ratios