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Gluon bremsstrahlung from massive quarks in high energy
collisions of polarized electrons and positrons

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Abstract

The gluon bremsstrahlung cross section, $e^+e^- \rightarrow q\bar{q}g$, including effects of finite quark and antiquark masses, is calculated for arbitrarily spin polarized electron-positron beams. Mass effects and polarization effects are given and are shown to have sizeable influence on the cross section. It is shown, however, that for the left-right asymmetry, A_{LR} , the mass corrections and radiative corrections vanish at the Z_0 pole. The use of longitudinal polarized electrons in measurements of the forward-backward asymmetry, A_{FB} , may give sizeable enhancements.

1 Introduction

Techniques of obtaining spin polarized high energy electron beams have improved over the last years. Linear polarizations of average value 22.4 % have recently been obtained at the SLAC Linear Collider [1] and at the HERA electron storage ring transverse electron polarizations up to nearly 60 % have been obtained [2]. Calculations of gluon bremsstrahlung from massless quarks in high energy electron positron annihilation for arbitrary electron and positron polarizations [3] show that beam polarizations affect cross sections and asymmetries in distinct ways. It has been proposed [4] that flavour separation may be obtained by means of transverse

electron and positron beam polarizations. Further it has been shown that gluon linear and circular polarizations are influenced by electron-positron beam polarizations [4].

In the present paper we take into account the finite mass of the quark and antiquark. The first calculation of gluon bremsstrahlung from massive quarks for unpolarized beams, was made by Grunberg et al [6] (photon exchange only) and by Jersák et al [7] who included Z_0 exchange.

2 The gluon bremsstrahlung cross section

The cross section for the process

$$e^+ + e^- \rightarrow \gamma, Z_0 \rightarrow q + \bar{q} + (g)$$

where a quark q , an antiquark $\bar{q}$ and a gluon g are created in the collision of an electron e^- and a positron e^+ , with a photon or a Z_0 -boson in the intermediate state, is given by

$$\frac{d^5\sigma}{d\Omega d\chi dx d\bar{x}} = \frac{1}{64} \frac{1}{(2\pi)^5} \sum_{color, s_q, s_{\bar{q}}, e} |M^f|^2. \quad (2.1)$$

Here the matrix element for flavour f is

$$M^f = -\frac{ie^2 g_s T_a}{s} \left(-Q_f L_\gamma^\mu H_{\mu\gamma}^f + f(s) L_Z^\mu H_{\mu Z}^f \right), \quad (2.2)$$

with the electron and quark charges $-e$ and eQ_f respectively and g_s the strong coupling constant. T_a is the colour matrix normalized such that

$$\sum_{a,b} Tr(T_a T_b) = 4.$$

The leptonic currents for γ -exchange, L_γ^μ , and for Z_0 -exchange, L_Z^μ , are given by

$$\begin{aligned} L_\gamma^\mu &= \bar{v}(p_+, s_+) \gamma^\mu u(p_-, s_-), \\ L_Z^\mu &= \bar{v}(p_+, s_+) \gamma^\mu (v - a\gamma_5) u(p_-, s_-), \end{aligned} \quad (2.3)$$

for specified momenta p_+ and p_- and polarizations s_+ and s_- .

The hadronic matrix elements including emission of a gluon with polarization e_μ are similarly given by

$$\begin{aligned} H_{\mu\gamma}^f &= \bar{u}_f(q, s_q) \left[\not{\epsilon} \frac{\not{q} + \not{s} + m_f}{2qg} \gamma_\mu - \gamma_\mu \frac{\not{q} + \not{s} + m_f}{2\bar{q}g} \not{\epsilon} \right] v_f(\bar{q}, s_{\bar{q}}), \\ H_{\mu Z}^f &= \bar{u}_f(q, s_q) \left[\not{\epsilon} \frac{\not{q} + \not{s} + m_f}{2qg} \gamma_\mu (v_f - a_f \gamma_5) \right. \\ &\quad \left. - \gamma_\mu (v_f - a_f \gamma_5) \frac{\not{q} + \not{s} + m_f}{2\bar{q}g} \not{\epsilon} \right] v_f(\bar{q}, s_{\bar{q}}), \end{aligned} \quad (2.4)$$

for specified quark, q, s_q , and antiquark, $\bar{q}, s_{\bar{q}}$, momenta and polarizations respectively.

In Eq. (2.2) m_f is the mass of the quark of flavour f , and $f(s)$ is proportional to the ratio of Z_0 and the photon propagators

$$f(s) = \frac{1}{4 \sin^2 2\theta_w} \frac{s}{s - M_z^2 + iM_z \Gamma_z^{tot}}.$$

where θ_w is the weak mixing angle, M_z is the mass and Γ_z^{tot} the total width of the Z_0 . The standard model coupling constants are

$$\begin{aligned} \text{for } e^- & \quad v = -1 + 4 \sin^2 \theta_W & a = -1 \\ \text{for } u, c, t & \quad v_f = 1 - \frac{8}{3} \sin^2 \theta_W & a_f = -1 & Q_f = \frac{2}{3} \\ \text{for } d, s, b & \quad v_f = -1 + \frac{4}{3} \sin^2 \theta_W & a_f = -1 & Q_f = -\frac{1}{3} \end{aligned}$$

In Eq. (2.1) we sum over quark, antiquark and gluon polarizations, while the electron and positron polarizations are specified by the invariants

$$S_\pm^\mu S_{\pm,\mu} = -\mathbf{P}_\pm^2 = -(\mathbf{P}_\pm^\parallel^2 + \mathbf{P}_\pm^\perp^2),$$

where $\mathbf{P}_\pm^\parallel$ and $\mathbf{P}_\pm^\perp$ are the positron/electron longitudinal and transverse polarizations in the rest system of the particles, respectively. In the laboratory system the polarization four vectors are

$$S_\pm = (S_0, \mathbf{S})_\pm = \left(P_\pm^\parallel \frac{|\mathbf{P}_\pm|}{m}, \mathbf{P}_\pm^\perp + \frac{E_\pm}{m} \mathbf{P}_\pm^\parallel \right), \quad (2.5)$$

satisfying the invariant relation

$$S_\pm p_\pm = 0.$$

For a pure spin state $S_\mu S^\mu = -\mathbf{P}_\pm^2 = -1$. Partial polarized states, $\mathbf{P}_\pm^2 < 1$, are described by the density matrices

$$\rho(p_\pm, S_\pm) = \frac{1}{2} (1 + \gamma_5 \not{s}_\pm) (\not{p}_\pm \mp m_e),$$

with $S_{\pm}$ given by Eq. (2.5). For high electron-positron energies ρ simplifies

$$\rho(p_{\pm}, S_{\pm}) \simeq \frac{1}{2} \left[1 + \gamma_5 (\not{S}_{\pm} \mp P_{\pm}^{\parallel}) \right] \not{p}_{\pm}. \quad (2.6)$$

The cross section Eq. (2.1) is obtained from the matrix element, Eq. (2.2),

$$\begin{aligned} \frac{d^5\sigma}{d\Omega d\chi dx d\bar{x}} &= \frac{\alpha^2}{(2\pi)^2} \frac{\alpha_s}{s} \\ &\times \sum_f \left\{ L_{\gamma\gamma}^{\mu\nu} H_{\gamma\gamma\mu\nu}^f + 2\text{Re}f(s) L_{\gamma Z}^{\mu\nu} H_{\gamma Z\mu\nu}^f + |f(s)|^2 L_{ZZ}^{\mu\nu} H_{ZZ\mu\nu}^f \right\}, \end{aligned} \quad (2.7)$$

where χ is the azimuthal angle of $\mathbf{p}_-$ in the coordinate system with the z-axis along $\mathbf{q}$, and the leptonic tensors are given by

$$\begin{aligned} L_{\gamma\gamma}^{\mu\nu} &= 4L_{\gamma}^{\mu} L_{\gamma}^{\nu *} = 4Tr \gamma^{\mu} \rho(p_-, s_-) \gamma^{\nu} \rho(p_+, s_+) \\ &= \Xi L_1^{\mu\nu} + \xi L_2^{\mu\nu} - L_3^{\mu\nu}, \\ L_{\gamma Z}^{\mu\nu} &= 4L_{\gamma}^{\mu} L_Z^{\nu *} \\ &= -(v\Xi - a\xi) L_1^{\mu\nu} - (v\xi - a\Xi) L_2^{\mu\nu} + v L_3^{\mu\nu} + a L_4^{\mu\nu}, \\ L_{ZZ}^{\mu\nu} &= 4L_Z^{\mu} L_Z^{\nu *} \\ &= \left[(v^2 + a^2)\Xi - 2va\xi \right] L_1^{\mu\nu} + \left[(v^2 + a^2)\xi - 2va\Xi \right] L_2^{\mu\nu} \\ &\quad - (v^2 - a^2) L_3^{\mu\nu}, \end{aligned} \quad (2.8)$$

with

$$\begin{aligned} L_1^{\mu\nu} &= 4 (p_+^{\mu} p_-^{\nu} + p_-^{\mu} p_+^{\nu} - g^{\mu\nu} p_+ p_-), \\ L_2^{\mu\nu} &= -4i\varepsilon^{\mu\nu\alpha\beta} p_+^{\alpha} p_-^{\beta}, \\ L_3^{\mu\nu} &= 4(p_+ p_-) \left(P_+^{\perp\mu} P_-^{\perp\nu} + P_-^{\perp\mu} P_+^{\perp\nu} \right) + (\mathbf{P}_+^{\perp} \mathbf{P}_-^{\perp}) L_1^{\mu\nu}, \\ L_4^{\mu\nu} &= 4i\varepsilon_{\alpha\beta\gamma\delta} \left[P_+^{\perp\alpha} p_+^{\beta} g^{\gamma\mu} (P_-^{\perp\delta} p_-^{\nu} - P_-^{\perp\nu} p_-^{\delta}) \right. \\ &\quad \left. - (p_+, P_+^{\perp} \leftrightarrow p_-, P_-^{\perp}) \right]. \end{aligned} \quad (2.9)$$

Here $\Xi = 1 - P_+^{\parallel} P_-^{\parallel}$, $\xi = P_-^{\parallel} - P_+^{\parallel}$ with the four vectors $P_{\pm}^{\perp} = (0, \mathbf{P}_{\pm}^{\perp})$.

The hadronic tensors are similarly given by

$$\begin{aligned}
H_{\gamma\gamma\mu\nu}^f &= \sum_{\text{colours}, S_q, S_{\bar{q}}, e} H_{\mu\gamma}^f H_{\nu\gamma}^{f*} = 8sQ_f^2 H_{V\mu\nu}^f, \\
H_{\gamma Z\mu\nu}^f &= \sum H_{\mu\gamma}^f H_{\nu Z}^{f*} = 8sQ_f \left[v_f H_{V\mu\nu}^f - a_f H_{A\mu\nu}^f \right], \\
H_{ZZ\mu\nu}^f &= \sum H_{\mu Z}^f H_{\nu Z}^{f*} = 8s \left[(v_f^2 + a_f^2) H_{V\mu\nu}^f - 2a_f v_f H_{A\mu\nu}^f \right. \\
&\quad \left. + 2a_f^2 m_f^2 H_{V\mu\nu}^{Zf} \right],
\end{aligned} \tag{2. 11}$$

where

$$\begin{aligned}
H_{V\mu\nu}^f &= \frac{4}{(qg)(\bar{q}g)} \left[\left(Qq - m_f^2 \frac{Qg}{\bar{q}g} \right) (Q_\mu q_\nu + Q_\nu q_\mu - g_{\mu\nu}(Qq)) \right. \\
&\quad \left. - \left(Q^2 - 2m_f^2 \frac{Qg}{\bar{q}g} \right) q_\mu \bar{q}_\nu + m_f^2 ((Qg)g_{\mu\nu} - g_\mu g_\nu) + (q \leftrightarrow \bar{q}) \right], \\
H_{A\mu\nu}^f &= \frac{-4i}{(qg)(\bar{q}g)} \varepsilon_{\mu\nu\alpha\beta} \left[\left(Qq - m_f^2 \frac{Qg}{\bar{q}g} \right) q^\alpha Q^\beta - (q \leftrightarrow \bar{q}) \right], \\
H_{V\mu\nu}^{Zf} &= \frac{4}{(qg)(\bar{q}g)} \left[\left(q\bar{q} - m_f^2 \frac{qg}{\bar{q}g} \right) g_{\mu\nu} + g_\mu g_\nu + (q \leftrightarrow \bar{q}) \right].
\end{aligned} \tag{2. 12}$$

The cross section may then be written in the form

$$\begin{aligned}
\frac{d^5\sigma}{d\Omega d\chi dx d\bar{x}} &= \frac{\alpha^2}{(2\pi)^2} \frac{\alpha_s}{s} \frac{1}{(1-x)(1-\bar{x})} \\
&\times \sum_f \left\{ h_f^{(1)}(s, P_-^\parallel P_+^\parallel) X_0 + h_f^{(2)}(s, P_-^\parallel P_+^\parallel) Y_0 + h_f^{(5)}(s, P_-^\parallel P_+^\parallel) Z_0 \right. \\
&\quad \left. + h_f^{(3)}(s) X_s + h_f^{(6)}(s) Y_s + h_f^{(4)}(s) Z_s \right\},
\end{aligned} \tag{2. 13}$$

where the coupling functions, depending on energy, flavour and linear polarization

are given by

$$\begin{aligned}
h_f^{(1)}(s, P_-^{\parallel} P_+^{\parallel}) &= Q_f^2 \Xi - 2Q_f \text{Ref}(s)(v\Xi - a\xi)v_f \\
&\quad + |f(s)|^2 \left[(v^2 + a^2)\Xi - 2va\xi \right] (v_f^2 + a_f^2), \\
h_f^{(2)}(s, P_-^{\parallel} P_+^{\parallel}) &= -2Q_f \text{Ref}(s)(a\Xi - v\xi)a_f \\
&\quad - 2|f(s)|^2 \left[(v^2 + a^2)\xi - 2va\Xi \right] v_f a_f, \\
h_f^{(3)}(s) &= Q_f^2 - 2Q_f \text{Ref}(s)vv_f + |f(s)|^2(v^2 - a^2)(v_f^2 + a_f^2), \quad (2. 14) \\
h_f^{(4)}(s) &= 2Q_f \text{Im}f(s)av_f, \\
h_f^{(5)}(s, P_-^{\parallel} P_+^{\parallel}) &= 2|f(s)|^2 \left[(v^2 + a^2)\Xi - 2va\xi \right] a_f^2, \\
h_f^{(6)}(s) &= -2|f(s)|^2(v^2 - a^2)a_f^2.
\end{aligned}$$

It is convenient to define $h_f^{(1)+} = h_f^{(1)}$, $h_f^{(1)-} = h_f^{(1)} - h_f^{(5)}$:

$$\begin{aligned}
h_f^{(1)\pm}(s, P_-^{\parallel} P_+^{\parallel}) &= Q_f^2 \Xi - 2Q_f \text{Ref}(s)(v\Xi - a\xi)v_f \\
&\quad + |f(s)|^2 \left[(v^2 + a^2)\Xi - 2va\xi \right] (v_f^2 \pm a_f^2),
\end{aligned}$$

and in the same way $h_f^{(3)+} = h_f^{(3)}$, $h_f^{(3)-} = h_f^{(3)} - h_f^{(6)}$:

$$h_f^{(3)\pm}(s) = Q_f^2 - 2Q_f \text{Ref}(s)vv_f + |f(s)|^2(v^2 - a^2)(v_f^2 \pm a_f^2). \quad (2. 15)$$

Here $h_f^{(1)}(s)$ - $h_f^{(4)}(s)$ are the same function as in reference [3]. The X -, Y - and Z -functions depending on angle and particle energies and momenta and on transverse

electron and positron polarizations are obtained as

$$\begin{aligned}
X_0 &= \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x}\right) \left[x^2 (1 + \beta_x^2 \cos^2 \theta) + \bar{m}_f^2 \right] \\
&\quad + \frac{\bar{m}_f^2}{4} \left[x_g^2 (1 + \cos^2 \theta_g) - 8x_g \right] + (x \leftrightarrow \bar{x}, \theta \leftrightarrow \bar{\theta}), \\
Y_0 &= 2 \left\{ \left(x - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) x \beta_x \cos \theta - (x \leftrightarrow \bar{x}, \theta \leftrightarrow \bar{\theta}) \right\}, \\
Z_0 &= -\frac{\bar{m}_f^2}{4} \left\{ 4 \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) - x_g^2 (1 - \cos^2 \theta_g) - 4x_g \right\}, \quad (2.16) \\
X_s &= P_-^\perp P_+^\perp \left\{ \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) x^2 \beta_x^2 \sin^2 \theta \cos(2\phi - \phi_+ - \phi_-) \right. \\
&\quad \left. + \frac{\bar{m}_f^2}{4} x_g^2 \sin^2 \theta_g \cos(2\phi_g - \phi_+ - \phi_-) + (x \leftrightarrow \bar{x}, \theta \leftrightarrow \bar{\theta}, \phi \leftrightarrow \bar{\phi}) \right\}, \\
Y_s &= P_-^\perp P_+^\perp \frac{\bar{m}_f^2}{4} x_g^2 \sin^2 \theta_g \cos(2\phi_g - \phi_+ - \phi_-), \\
Z_s &= -P_-^\perp P_+^\perp \left\{ \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) x^2 \beta_x^2 \sin^2 \theta \sin(2\phi - \phi_+ - \phi_-) \right. \\
&\quad \left. + \frac{\bar{m}_f^2}{4} x_g^2 \sin^2 \theta_g \sin(2\phi_g - \phi_+ - \phi_-) + (x \leftrightarrow \bar{x}, \theta \leftrightarrow \bar{\theta}, \phi \leftrightarrow \bar{\phi}) \right\}.
\end{aligned}$$

Here β_x and $\beta_{\bar{x}}$ are the quark and antiquark velocities, respectively, with $x\beta_x$, $\bar{x}\beta_{\bar{x}}$ the scaled momenta and $\bar{m}_f = m_f/E$ the scaled quark mass of flavour f . The polar angle θ_g is the angle between $\mathbf{p}_-$ and the gluon momentum $\mathbf{g}$; from the $\mathbf{q}$, $\bar{\mathbf{q}}$, $\mathbf{g}$ -triangle follows

$$x_g \cos \theta_g = -x\beta_x \cos \theta - \bar{x}\beta_{\bar{x}} \cos \bar{\theta}.$$

The azimuthal angles are related to $\mathbf{p}_-$ as polar axis, defined in a right handed sense; the $\mathbf{q}$, $\bar{\mathbf{q}}$, $\mathbf{g}$ -triangle gives

$$\begin{aligned}
x_g \sin \theta_g \cos \phi_g &= -x\beta_x \sin \theta \cos \phi - \bar{x}\beta_{\bar{x}} \sin \bar{\theta} \cos \bar{\phi}, \\
x_g \sin \theta_g \sin \phi_g &= -x\beta_x \sin \theta \sin \phi - \bar{x}\beta_{\bar{x}} \sin \bar{\theta} \sin \bar{\phi}. \quad (2.17)
\end{aligned}$$

The transverse polarizations are in the same way described in a plane perpendicular to $\mathbf{p}_-$

$$\mathbf{P}_\pm = P_\pm (\cos \phi_\pm, \sin \phi_\pm, 0).$$

The cross section differential in angles and energies for initially arbitrarily spin polarized electrons and positrons may then be written in the form

$$\begin{aligned}
\frac{d^5\sigma}{d\Omega d\chi dx d\bar{x}} &= \frac{\alpha^2}{(2\pi)^2} \frac{\alpha_s}{s} \frac{1}{(1-x)(1-\bar{x})} \\
&\times \sum_f \left\{ \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) \left[h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) x^2 (1 + \beta_x^2 \cos^2 \theta) + h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \bar{m}_f^2 \right. \right. \\
&\quad \left. \left. + P_-^\perp P_+^\perp x^2 \beta_x^2 \sin^2 \theta \left(h_f^{(3)+}(s) \cos(2\phi - \phi_+ - \phi_-) - h_f^{(4)}(s) \sin(2\phi - \phi_+ - \phi_-) \right) \right] \right. \\
&\quad \left. + \left(2h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) - h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \right) \frac{\bar{m}_f^2}{4} x_g^2 + h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \frac{\bar{m}_f^2}{4} x_g^2 \cos^2 \theta_g \right. \\
&\quad \left. - \left(h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) + h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \right) \bar{m}_f^2 x_g \right. \\
&\quad \left. + P_-^\perp P_+^\perp \frac{\bar{m}_f^2}{4} x_g^2 \sin^2 \theta_g \left[h_f^{(3)-}(s) \cos(2\phi_g - \phi_+ - \phi_-) \right. \right. \\
&\quad \left. \left. - h_f^{(4)}(s) \sin(2\phi_g - \phi_+ - \phi_-) \right] \right. \\
&\quad \left. + (x \leftrightarrow \bar{x}, \beta_x \leftrightarrow \beta_{\bar{x}}, \theta \leftrightarrow \bar{\theta}, \phi \leftrightarrow \bar{\phi}) \right. \\
&\quad \left. + 2h_f^{(2)}(s, P_-^\parallel P_+^\parallel) \left[\left(x - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x} \right) x \beta_x \cos \theta - (x \leftrightarrow \bar{x}, \theta \leftrightarrow \bar{\theta}) \right] \right\}. \quad (2.18)
\end{aligned}$$

This way of presenting the gluon bremsstrahlung cross section shows clearly the relation to the cross section for creating $q\bar{q}$ pairs from annihilation of polarized electrons and positrons, $e^+e^- \rightarrow q\bar{q}$, which is easily obtained as

$$\begin{aligned}
\frac{d^2\sigma}{d\Omega} &= \frac{3}{4} \frac{\alpha^2}{s} \beta \\
&\times \sum_f \left\{ h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) (1 + \beta^2 \cos^2 \theta) + h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \bar{m}_f^2 \right. \\
&\quad \left. + P_-^\perp P_+^\perp \beta^2 \sin^2 \theta \left[h_f^{(3)}(s) \cos(2\phi - \phi_+ - \phi_-) - h_f^{(4)}(s) \sin(2\phi - \phi_+ - \phi_-) \right] \right. \\
&\quad \left. + 2h_f^{(2)}(s, P_-^\parallel P_+^\parallel) \beta \cos \theta \right\}, \quad (2.19)
\end{aligned}$$

where β is the velocity of the quark or the antiquark. It should be noted that β_x , $\beta_{\bar{x}}$ and β in Eqs. (2.18) and (2.19) always appear in the momentum components $x\beta_x \cos \theta$, $x\beta_x \sin \theta$ as compared to the case of massless quarks of reference [3], where the β 's are all unity.

The cross section as a function of the angle and energy of the quark and the energy of the antiquark is obtained from Eq. (2.18) by integrating over the azimuth angle χ of $\mathbf{p}_-$ with $\mathbf{q}$ the polar axis. The equations expressing the antiquark emission angles $\bar{\theta}$ and $\bar{\phi}$ in terms of the quark angles θ , ϕ and χ and of the angle between the quark and antiquark momenta ϑ are

$$\begin{aligned}\sin \bar{\theta} \cos \bar{\phi} &= (-\cos \theta \cos \chi \sin \vartheta + \sin \theta \cos \vartheta) \cos \phi - \sin \vartheta \sin \chi \sin \phi, \\ \sin \bar{\theta} \sin \bar{\phi} &= (-\cos \theta \cos \chi \sin \vartheta + \sin \theta \cos \vartheta) \sin \phi + \sin \vartheta \sin \chi \cos \phi, \\ \cos \bar{\theta} &= \cos \vartheta \cos \theta + \sin \vartheta \sin \theta \cos \chi,\end{aligned}\quad (2. 20)$$

where ϑ is given by

$$\begin{aligned}x^2 \beta_x^2 \bar{x}^2 \beta_{\bar{x}}^2 \sin^2 \vartheta &= 4(1-x)(1-\bar{x})(1-x_g) - \bar{m}_f^2 x_g^2, \\ x \beta_x \bar{x} \beta_{\bar{x}} \cos \vartheta &= x \bar{x} + 2(1-x-\bar{x}) + \bar{m}_f^2.\end{aligned}\quad (2. 21)$$

The cross section differential in the angles θ and ϕ and in the energies x and $\bar{x}$ is found to be given by

$$\begin{aligned}\frac{d^4 \sigma}{d\Omega dx d\bar{x}} &= \frac{\alpha^2 \alpha_s}{2\pi s} \frac{1}{(1-x)(1-\bar{x})} \\ &\times \sum_f \left\{ \mathcal{F}_1(x, \bar{x}) \left(h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) (1 + \cos^2 \theta) \right. \right. \\ &\quad \left. \left. + P_-^\perp P_+^\perp \sin^2 \theta \left[h_f^{(3)+}(s) \cos(2\phi - \phi_+ - \phi_-) - h_f^{(4)}(s) \sin(2\phi - \phi_+ - \phi_-) \right] \right) \right. \\ &\quad \left. + \frac{\bar{m}_f^2}{2} \mathcal{F}_2(x, \bar{x}) \left(h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \cos^2 \theta \right. \right. \\ &\quad \left. \left. + P_-^\perp P_+^\perp \sin^2 \theta \left[h_f^{(3)-}(s) \cos(2\phi - \phi_+ - \phi_-) \right. \right. \right. \\ &\quad \left. \left. \left. - h_f^{(4)}(s) \sin(2\phi - \phi_+ - \phi_-) \right] \right) \right. \\ &\quad \left. + 2h_f^{(2)}(s, P_-^\parallel P_+^\parallel) \mathcal{F}_3(x, \bar{x}) \cos \theta + h_f^{(1)+}(s, P_-^\parallel P_+^\parallel) \mathcal{F}_4(x, \bar{x}) \right. \\ &\quad \left. + \frac{\bar{m}_f^2}{4} h_f^{(1)-}(s, P_-^\parallel P_+^\parallel) \mathcal{F}_5(x, \bar{x}) \right\},\end{aligned}\quad (2. 22)$$

where the quark and antiquark energies are contained in the functions:

$$\begin{aligned}
& \mathcal{F}_1(x, \bar{x}) \\
&= \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x}\right) x^2 \beta_x^2 + \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-\bar{x}}\right) \bar{x}^2 \beta_{\bar{x}}^2 \left(1 - \frac{3}{2} \sin^2 \vartheta\right), \\
& \mathcal{F}_2(x, \bar{x}) \\
&= x^2 \beta_x^2 + \bar{x}^2 \beta_{\bar{x}}^2 \left(1 - \frac{3}{2} \sin^2 \vartheta\right) + 2x\beta_x \bar{x}\beta_{\bar{x}} \cos \vartheta = x_g^2 - \frac{3}{2} \bar{x}^2 \beta_{\bar{x}}^2 \sin^2 \vartheta, \\
& \mathcal{F}_3(x, \bar{x}) \\
&= \left(x - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-x}\right) x\beta_x - \left(\bar{x} - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-\bar{x}}\right) \bar{x}\beta_{\bar{x}} \cos \vartheta, \tag{2. 23} \\
& \mathcal{F}_4(x, \bar{x}) \\
&= 2 \left(1 - \frac{\bar{m}_f^2}{2} \frac{x_g}{1-\bar{x}}\right) \bar{x}^2 \beta_{\bar{x}}^2 \sin^2 \vartheta + \bar{m}_f^2 \left[2 + x_g^2 \left(1 - \frac{\bar{m}_f^2}{2(1-x)(1-\bar{x})}\right) \right], \\
& \mathcal{F}_5(x, \bar{x}) \\
&= \bar{x}^2 \beta_{\bar{x}}^2 \sin^2 \vartheta - 2(x_g^2 + 4x_g - 4) - 2\bar{m}_f^2 \frac{x_g^2}{(1-x)(1-\bar{x})},
\end{aligned}$$

with ϑ given by Eq. (2.21).

It should be noted that polarization and quark mass effects are contained in different functions: $h_f^{(1)\pm}(s, P_-^\parallel P_+^\parallel)$ and $h_f^{(2)}(s, P_-^\parallel P_+^\parallel)$ depend on s and the longitudinal polarizations only, while $\mathcal{F}_1(x, \bar{x}) - \mathcal{F}_5(x, \bar{x})$ depend on $x, \bar{x}$ only and contain the quark mass $\bar{m}_f$.

3 Longitudinally polarized electron beam

Experimentally the most interesting case is at present one polarized beam. For longitudinally polarized electrons to a degree $P_-^\parallel$ and unpolarized positrons the

cross section, Eq. (2.22) simplifies

$$\begin{aligned}
\frac{d^4\sigma}{d\Omega d\mathbf{x} d\bar{\mathbf{x}}} &= \frac{\alpha^2 \alpha_s}{2\pi s} \frac{1}{(1-x)(1-\bar{x})} \\
&\times \sum_f \left\{ h_f^{(1)+}(s, P_-^\parallel) \mathcal{F}_1(x, \bar{x}) (1 + \cos^2 \theta) + h_f^{(1)-}(s, P_-^\parallel) \frac{\bar{m}_f^2}{2} \mathcal{F}_2(x, \bar{x}) \cos^2 \theta \right. \\
&\quad + 2h_f^{(2)}(s, P_-^\parallel) \mathcal{F}_3(x, \bar{x}) \cos \theta + h_f^{(1)+}(s, P_-^\parallel) \mathcal{F}_4(x, \bar{x}) \\
&\quad \left. + h_f^{(1)-}(s, P_-^\parallel) \frac{\bar{m}_f^2}{4} \mathcal{F}_5(x, \bar{x}) \right\}, \tag{3. 1}
\end{aligned}$$

with $\Xi = 1$ and $\xi = P_-^\parallel$:

$$\begin{aligned}
h_f^{(1)\pm}(s, P_-^\parallel) &= Q_f^2 - 2Q_f \text{Re} f(s)(v - aP_-^\parallel)v_f \\
&\quad + |f(s)|^2 \left[(v^2 + a^2) - 2vaP_-^\parallel \right] \left(v_f^2 \pm a_f^2 \right), \\
h_f^{(2)}(s, P_-^\parallel) &= -2Q_f \text{Re} f(s)(a - vP_-^\parallel)a_f \\
&\quad - 2|f(s)|^2 \left[(v^2 + a^2)P_-^\parallel - 2va \right] v_f a_f, \\
h_f^{(5)}(s, P_-^\parallel) &= 2|f(s)|^2 \left[(v^2 + a^2) - 2vaP_-^\parallel \right] a_f^2, \tag{3. 2}
\end{aligned}$$

A measure of the importance of the polarization may be obtained from the quantities

$$R_f^{(i)}(E)P_-^\parallel = \frac{h_f^{(i)}(s, P_-^\parallel) - h_f^{(i)}(s, -P_-^\parallel)}{h_f^{(1)}(s, P_-^\parallel) + h_f^{(1)}(s, -P_-^\parallel)}, \quad i = 1, 2, 5. \tag{3. 3}$$

We show in Fig. 1 $R_f^{(i)}(E)$ for a range of energies. We use $\sin^2 \theta_W = 0.23$ and $M_Z = 91.2 \text{ GeV}$, $\Gamma_Z = 2.48 \text{ GeV}$.

In order to indicate the effects of the quark mass we give in Fig. 2 the functions $\mathcal{F}_1(x, \bar{x})$ and $\mathcal{F}_3(x, \bar{x})$ for $x = \bar{x}$, i.e. $x_g = 2(1-x)$, $(1+m_f^2)/2 \leq x \leq 1$, for $\bar{m}_f = m_f/E = 0.0, 0.05$ and 0.5 .

4 Transversely polarized electrons and positrons

For electron and positron antiparallel transverse polarizations, $\phi_+ = \phi_- + \pi$, the cross section Eq. (2.22) becomes

$$\begin{aligned}
\frac{d^4\sigma}{d\cos\theta d\phi_- dx d\bar{x}} &= \frac{\alpha^2 \alpha_s}{2\pi s} \frac{1}{(1-x)(1-\bar{x})} \\
&\times \sum_f \left\{ \mathcal{F}_1(x, \bar{x}) \left(h_f^{(1)+}(s)(1 + \cos^2\theta) \right. \right. \\
&\quad \left. \left. + P_-^\perp P_+^\perp \sin^2\theta \left[h_f^{(3)}(s) \cos 2(\phi - \phi_-) - h_f^{(4)}(s) \sin 2(\phi - \phi_-) \right] \right) \right. \\
&\quad \left. + \frac{\bar{m}_f^2}{2} \mathcal{F}_2(x, \bar{x}) \left(h_f^{(1)-}(s) \cos^2\theta + P_-^\perp P_+^\perp \sin^2\theta \left[h_f^{(3)-}(s) \cos 2(\phi - \phi_-) \right. \right. \right. \\
&\quad \left. \left. \left. - h_f^{(4)}(s) \sin 2(\phi - \phi_-) \right] \right) \right. \\
&\quad \left. + 2h_f^{(2)}(s) \mathcal{F}_3(x, \bar{x}) \cos\theta + h_f^{(1)+}(s) \mathcal{F}_4(x, \bar{x}) + \frac{\bar{m}_f^2}{4} h_f^{(1)-}(s) \mathcal{F}_5(x, \bar{x}) \right\}, \quad (4.1)
\end{aligned}$$

where $h_f^{(n)}(s) = h_f^{(n)}(s, 0, 0)$ ($\Xi = 1, \xi = 0$) for transverse polarizations, and where the spin azimuthal angle $\phi_- - \phi$ is measured from the $\mathbf{p}_- - \mathbf{q}$ plane.

5 The cross section at the Z_0 -pole with longitudinally polarized electrons

We consider here specifically the cross section at the Z_0 pole. From Eq. (2.19) we obtain for the two jet cross section, $e^+e^- \rightarrow q\bar{q}$, for electron polarization $P_-^\parallel$ and unpolarized positrons $P_+ = 0$

$$\begin{aligned}
\frac{d^2\sigma}{d\Omega} &= \frac{3\alpha^2}{4s} |f(s)|^2 \\
&\times \left\{ (v^2 + a^2 - 2vaP_-^\parallel) \sum_f \left[(1 + \beta^2 \cos^2\theta)(v_f^2 + a_f^2) + \bar{m}_f^2(v_f^2 - a_f^2) \right] \right. \\
&\quad \left. - 4 \left[(v^2 + a^2)P_-^\parallel - 2va \right] \beta \cos\theta \sum_f v_f a_f \right\}. \quad (5.1)
\end{aligned}$$

Correspondingly, for three jets, $e^+e^- \rightarrow q\bar{q}g$, from Eq. (3.1)

$$\begin{aligned} \frac{d^4\sigma}{d\Omega dx d\bar{x}} &= \frac{\alpha^2 \alpha_s}{2\pi s} \frac{|f(s)|^2}{(1-x)(1-\bar{x})} \\ &\times \left\{ (v^2 + a^2 - 2vaP_-^\parallel) \sum_f \left[(1 + \cos^2 \theta) (v_f^2 + a_f^2) \mathcal{F}_1(x, \bar{x}) \right. \right. \\ &\quad \left. \left. + \bar{m}_f^2 \cos^2 \theta (v_f^2 - a_f^2) \mathcal{F}_2(x, \bar{x}) + (v_f^2 + a_f^2) \mathcal{F}_4(x, \bar{x}) + \frac{\bar{m}_f^2}{4} (v_f^2 - a_f^2) \mathcal{F}_5(x, \bar{x}) \right] \right. \\ &\quad \left. - 2 \left[(v^2 + a^2) P_-^\parallel - 2va \right] \beta \cos \theta \cdot v_f a_f \mathcal{F}_3(x, \bar{x}) \right\}, \end{aligned} \quad (5.2)$$

6 Asymmetries with longitudinally polarized electrons at the Z_0 -pole

We discuss here the left-right, A_{LR} , and the forward-backward, A_{FB} , asymmetries with longitudinally polarized electrons.

From Eqs. (5.1) and (5.2) follows that at the Z_0 pole the left-right asymmetry, A_{LR} , for two- and three jets are equal:

$$\begin{aligned} A_{LR} &= \frac{1}{P_-^\parallel} \frac{\sigma(-P_-^\parallel) - \sigma(P_-^\parallel)}{\sigma(-P_-^\parallel) + \sigma(P_-^\parallel)} \quad \text{2-jets, 3-jets} \\ &= \frac{2va}{a^2 + v^2} = \frac{2(1 - 4 \sin^2 \theta_W)}{1 + (1 - 4 \sin^2 \theta_W)^2}, \end{aligned} \quad (6.1)$$

where $\sigma(\pm P_-^\parallel)$ is the total cross section for two or three jets. This implies that A_{LR} is without radiative and quark mass correction to first order in α_s . The independence of the radiative corrections is a consequence of the fact that the decay of the Z_0 -particle is independent of the polarization of the created Z_0 , the left-right asymmetry depends on the initial state only. The independence of radiative corrections is therefore true to all orders in α_s . Note also that the asymmetry for differential three-jets cross sections is given by the same result

$$\frac{1}{P_-^\parallel} \frac{\frac{d^2\sigma(-P_-^\parallel)}{dx d\bar{x}} - \frac{d^2\sigma(P_-^\parallel)}{dx d\bar{x}}}{\frac{d^2\sigma(-P_-^\parallel)}{dx d\bar{x}} + \frac{d^2\sigma(P_-^\parallel)}{dx d\bar{x}}} = \frac{2va}{a^2 + v^2}. \quad (6.2)$$

For the forward-backward asymmetry, A_{FB} , an enhancement of the cross section may be obtained with the use of polarized electrons. From Eqs. (5.1) and (5.2) one

finds that for A_{FB} at the Z_0 -pole

$$A_{FB} = \frac{\sigma_F(P_-^{\parallel}) - \sigma_B(P_-^{\parallel})}{\sigma_F(P_-^{\parallel}) + \sigma_B(P_-^{\parallel})} = \frac{1}{A_{LR}} \frac{A_{LR} - P_-^{\parallel}}{1 - P_-^{\parallel} A_{LR}} \cdot A_{FB}(P_-^{\parallel} = 0), \quad (6.3)$$

where as given in Eq. (6.1)

$$A_{LR} = \frac{2va}{a^2 + v^2},$$

and $A_{FB}(P_-^{\parallel} = 0)$ is the usual forward-backward asymmetry for unpolarized electrons.

Equation (6.3) shows that considerable enhancement of A_{FB} may be obtained with polarized electrons with polarization $|P_-^{\parallel}| \gg A_{LR}$. Note that Eq. (6.3) also is correct for pure leptonic forward-backward asymmetries, in which case $A_{FB}(P_-^{\parallel} = 0)$ is particularly small. The use of polarized electrons may therefore in this case be particularly useful.

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Figure caption

Fig. 1.

The polarization asymmetries $R_f^{(i)}(E)$, Eq. (3.3), as functions of the total e^+e^- energy. The solid curves: $Q_f = 2/3$ -quarks, u, c, t . Dashed curves: $Q_f = -1/3$ -quarks, d, s, b .

Fig. 2.

The form factors $\mathcal{F}_1(x, \bar{x})$ and $\mathcal{F}_3(x, \bar{x})$ with $x = \bar{x}$ for $\bar{m}_f = m_f/E = 0.0$ (solid curve) $\bar{m}_f = 0.05$ (dashed curve) and $\bar{m}_f = 0.5$ (dot-dashed curve).

Fig.1

Fig.2